Title: Comments on Spectrum-generating Symmetries for D-branes

Date: Mar 28, 2005 03:15 AM

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Abstract:

Some order in a chaotic landscape ?

C. Bachas
Perimeter I. 03/05

based on work with
M. Gaberdiel, th/0411067

cf also
Hikida, Nozaki, Sugawara
th/0101211

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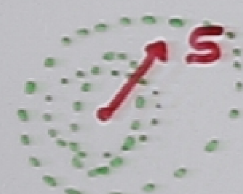
One of early, influential calculations in field theory is that of the Kondo problem:

screening of magnetic impurity by the conduction electrons in a metal



$$H_I = \lambda \vec{S} \cdot \underbrace{\psi_i^{+} \frac{\vec{\sigma}}{2} \psi_i(0)}_{\text{spin density}}$$

λ: 0 $\longrightarrow$ λ^*

asympt. free — IR fixed point

'75 Nozieres; Wilson;
'80 Wiegmann; Andrei

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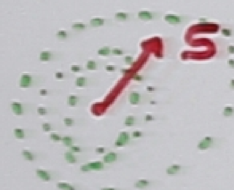

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The problem has been revisited by (B)CFT ists

Cardy '89
Affleck, Ludwig '92

and string theorists, interested in its geometric interpretation.

Alekseev, Schomerus '99
Felder, Fröhlich, Fuchs, Schweigert;
CB, Douglas, Schweigert
Pawelcyk '00

In this latter language, it is a special manifestation of Rob Myers' dielectric effect.

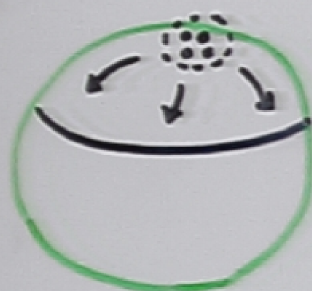
In a nutshell ⤳

$\backslash^{3}$

s-wave electrons:

WZW $\widehat{SU(2)}_k$ model on R^+

$\uparrow$ # of channels participating in process



s D0-branes on S^3 threaded by k units of NS H_3-flux

$\downarrow$

D2-brane with s units of magnetic F_2-flux

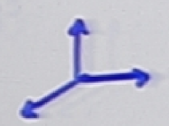
↳ special case of more general situation:

$$s \, |s'\rangle\!\rangle \rightarrow |s'-s-1\rangle\!\rangle \oplus |s'-s+1\rangle\!\rangle \oplus \cdots \oplus |s'+s-1\rangle\!\rangle$$

↲ D2-brane with $F_2 = s'$

The basic mechanism can be understood as follows:

$\mathbb{R}^3$



∴ D0-branes

$$E_{D0} \sim \frac{1}{4}\,\mathrm{tr}\,[\Phi^i,\Phi^j]^2 + $$

superpotential $\quad + \frac{1}{3} H_{ij\beta}\,\mathrm{tr}(\Phi^i\Phi^j\Phi^\beta)$

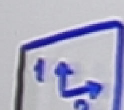
soft term
(can be cancelled
by RR flux)

minimum at $\quad [\Phi^i,\Phi^j] = H^{ij}{}_{\beta}\Phi^\beta$

'fuzzy sphere', breaks $U(N) \longrightarrow U(1)$

↳ A similar story holds for D2-branes

$$E_{D2} \sim \mathrm{tr}\,(H_{123}\Phi^3 + F_{12})^2 + \text{Area}$$



lowered by $F_{12} \sim H_{123}\Phi^3$

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What is probably of greater interest for the audience here is the relevance of this for the study of flux compactifications.

Indeed, $S^3 \times S^3$ compactification was early example of (all but S) moduli stabilisation + graviphoton gauging in het/type II string theory

Antoniadis, CB, Sagnotti '89

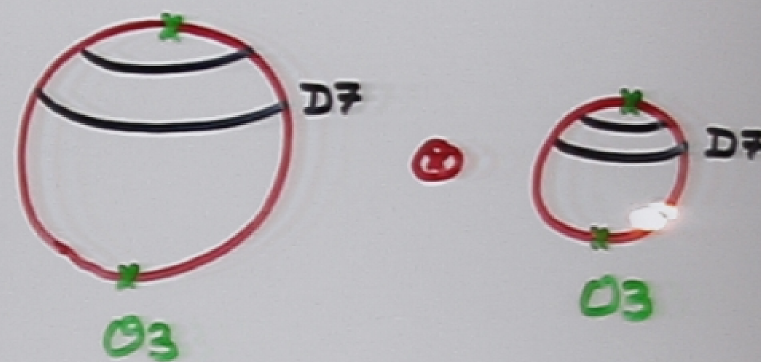
Since $V \sim -\frac{1}{g_s^2}$, the vacuum is of electrovac ($\sim AdS_3 \times \mathbb{R}$) type

Gibbons, Freedman '83?

May add D-branes + orientifolds

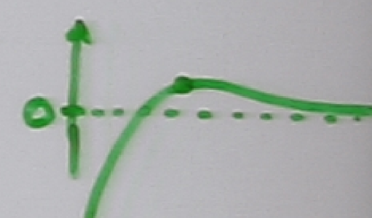
Brunner
'02 Huiszon, Schellekens
cB, Couchoud, Windey

eg.



All moduli fixed, except for string coupling:

$$V \sim -\frac{\#}{g_s^2} + \frac{\#'}{g_s} \cdots$$



unstable de Sitter vacuum

Instantons + higher orders ?

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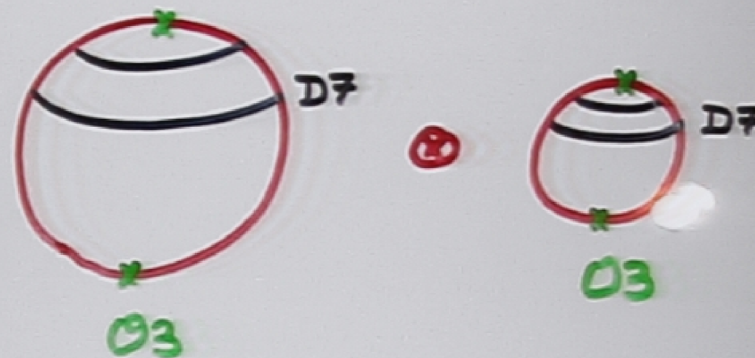

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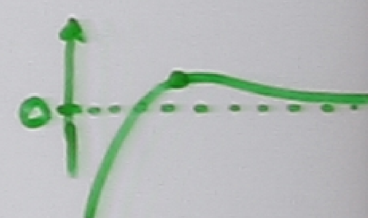


unstable de Sitter vacuum

Instantons + higher orders ?

Alternative (in some ways better) starting point to study non-susy flux vacua, than CY_3 or tori.

Deserves further study, but will not pursue more in this talk.

Here will focus on more theoretical issue: argue that WZW D-branes can be generated by set of symmetries of the field eqns (not necessarily the action) of open string theory.

cf. Ehlers syms of Einstein eqns,
or better: discrete scale inv. of supergravity eqns

eg radius L of $AdS_5 \times S_5$ arbitrary
but discrete

Will now show how to construct these generators explicitly. The method is in some ways intriguing, and could help analyze D-branes in some other less explored contexts.

A D-brane perturbed away from some (classical) equilibrium state is described by the world-sheet evolution operator (in interaction representation):

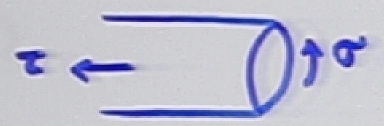
$$T e^{i \int H_I}$$

depends on gauge field & deformation of position

Translates to deformation of
boundary state:

$$|B'\rangle\rangle = \mathrm{tr}\left(P e^{i\oint A^\mu(x)\partial_\sigma X_\mu + Y^\mu(x)\partial_\tau X_\mu}\right) \cdot |B\rangle\rangle$$

matrix val.
gauge & coord. deform.



X^μ: string coords quantized in closed-string channel

Since $A \in (TW)^N$ and $Y \in (\mathcal{N}W)^N$, half of ops in exponent must vanish when evaluated on $|B\rangle\rangle$.

eg for D0 at $x=0$, only

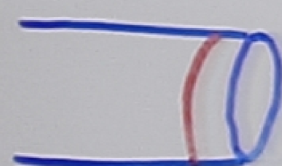
$Y^\mu(0)\,\partial_\tau X_\mu$ really matters.

Let us write $|B'\rangle\rangle = \mathcal{O}\,|B\rangle\rangle$.
Can we define the operator $\mathcal{O}$ independently of $|B\rangle\rangle$?

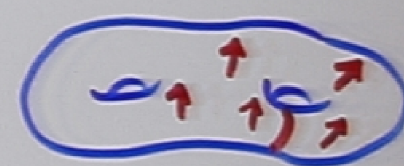
Push forward of Wilson loop to bulk

$\updownarrow$

Push forward of A, Y from
$(TW)^N, (\mathcal{N}W)^N \longrightarrow (T\mathcal{M})^N \oplus (T^*\mathcal{M})^N$



worldsheet



target space

$\mathcal{O}$ defines a **defect line** with general renormalizable couplings

'96 Affleck, Oshikawa ; Petkova, Zuber ;

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The fields A and Y are in general renormalized & flow to IR fixed points where the defect is conformal:

$$[L_n - \bar{L}_{-n}, \mathcal{O}] = 0$$

Pulling back a conformal loop oper. to the bnry generates a D-brane from <u>any</u> other D-brane

$(L_n - \bar{L}_{-n})|B\rangle\rangle = 0 \Rightarrow$

$$(L_n - \bar{L}_{-n})\,\mathcal{O}|B\rangle\rangle = 0$$

provided this action can be defined.

This is $\underbrace{\text{true}}_{\text{for sure}}$ if $\mathcal{O}$ is also chiral $\Rightarrow$

TOPOLOGICAL

Special cases of such operators:
(discrete or continuous) isometries of target manifold:

$$\mathcal{O} = e^{i\lambda J_0}$$

zero mode of chiral U(1) current

But more generally $\mathcal{O}$ reduces the g-factor (tension) of the D-branes, since it results from non-trivial RG flow.

Let's reexamine Kondo problem in this light. Consider

$$\mathcal{O}_r = \mathrm{tr}\, P e^{i\int_0^{2\pi} d\sigma \left(\lambda J^a t^a + \bar{\lambda} \bar{J}^a t^a\right)}$$

G-representation r

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Can determine fixed points by semiclassical argument (analogous to Witten '84):

under $g \longrightarrow u^{-1} g \bar{u}$

$$\begin{cases} J \to u^{-1} J u + ik\, u^{-1} \partial u \\ \bar{J} \to \bar{u}^{-1} \bar{J} \bar{u} + ik\, \bar{u}^{-1} \bar{\partial} \bar{u} \end{cases}$$

$\therefore$ 'Wilson line' is invariant at

$$(\lambda, \bar{\lambda}) = \begin{cases} (1/k, 0) & \text{chiral} \\ (0, 1/k) & \text{antichir.} \\ (1/2k, 1/2k) & \text{diag.} \end{cases}$$

At these special values, $SU(2)_L \times SU(2)_R$ or $SU(2)_{diag}$ symmetry is preserved, so (semiclassically at least)

$$\{ J^a_n + \bar{J}^a_{-n}, \mathcal{O} \} = 0 \Rightarrow \{ L_n - \bar{L}_{-n}, \mathcal{O} \} = 0$$

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Chiral conformal ops ϑ_r^*

measure quantum monodromy :

(generalized string winding & momentum)

$$g_{cl}(\tau,\sigma) = g_+(\tau+\sigma)^{-1}\, g_-(\tau-\sigma)$$

$$g_\pm(x+2\pi) = M\, g_\pm(x)$$

$$\vartheta_r^* = \mathrm{tr}_r\, M = \frac{S_{r\mu}}{S_{0\mu}} \quad \text{in channel } \mu$$

after semiclassical (coadjoint orbit) quantization

Kirillov, Konstant
'89 Fadeev
'90 Alekseev, Shatashvili
⋮

Then simple CFT calculation gives:

$$|B\rangle\rangle = \sum \frac{\psi_{B\mu}}{\sqrt{S_{0\mu}}} |\mu\rangle\rangle_I$$

↳ Ishibashi states

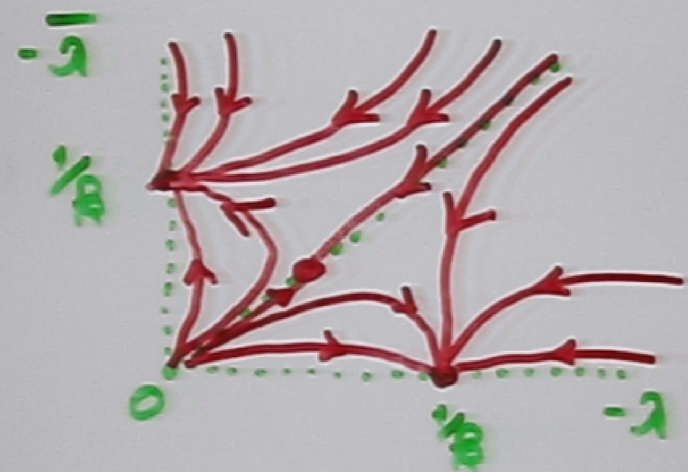
$$\Rightarrow \mathcal{O}_r^* |B\rangle\rangle = \sum_\mu \frac{S_{r\mu}}{S_{0\mu}} \frac{\psi_{B\mu}}{\sqrt{S_{0\mu}}} |\mu\rangle\rangle_I$$

$$= \sum_{\tilde{B}} \mathcal{N}_{rB}{}^{\tilde{B}} |\tilde{B}\rangle\rangle$$

↳ integer fusion coeffs (# of times r appears for open string stretched between B & $\tilde{B}$)

This corresponds precisely to the end result of the Kondo flow discussed in the beginning

With M. Gaberdiel have constructed (regularized, renormalized) this operator explicitly, and checked these semiclassical statements to $o(k^{-4})$. In particular:

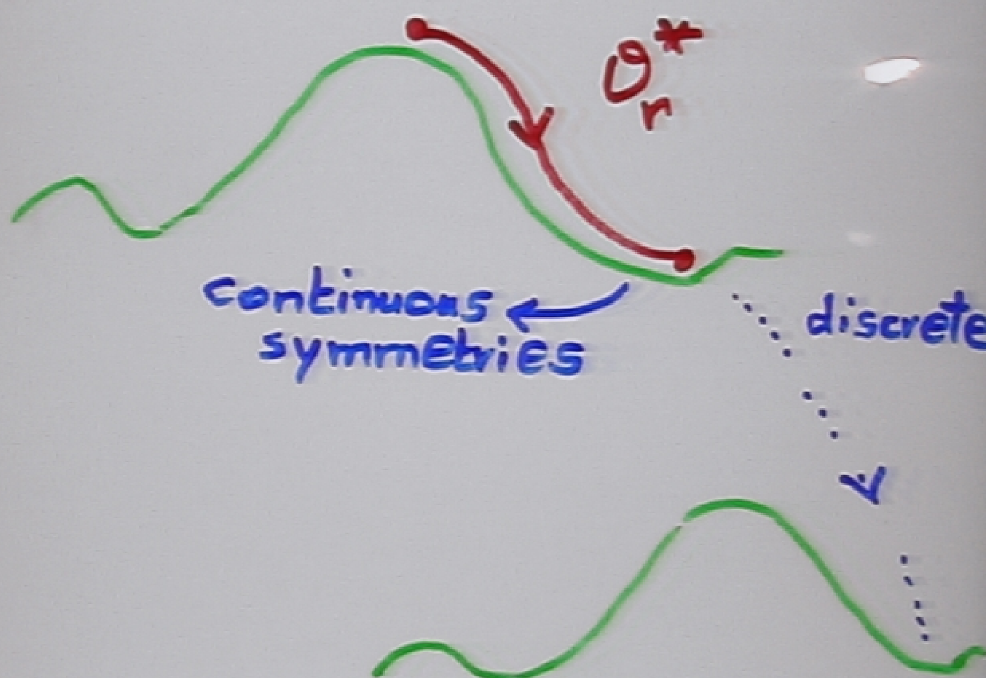


eg $$\frac{d\lambda}{d\log\xi} = \lambda^2 + k\lambda^3 + o(\lambda^4)$$

if $\bar{\lambda} = 0$

Universal, independent of uv D-brane (spin of magnetic impurity, before screening)

So what we learn for this (sub)landscape of D-branes in WZW models:



Universal operators that take us from ridges or mountaintops to valleys !

Is all this very special to WZW models (some sort of 'quantum group' symmetry)?

Perhaps, but some reason for optimism:

* Conformal defects correspond to D-branes in tensor product $\mathcal{M} \otimes \mathcal{M}$.

'02 BDDO

$\mathcal{O} = \mathrm{tr}\, \mathbb{1}$ corresponds to diagonal embedding. More generally, expect interesting solutions for eg $\mathcal{M} = CY_3$.

* Lift to 3D topological F.T.

'03 Fuchs, Runkel, Schweigert

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By connecting ridges with valleys, these could put some order in what looks at present like a chaotic landscape.

Interesting to check whether any of this can extend to Calabi-Yaus.