Title: Interpretation of Quantum Theory: Lecture 22

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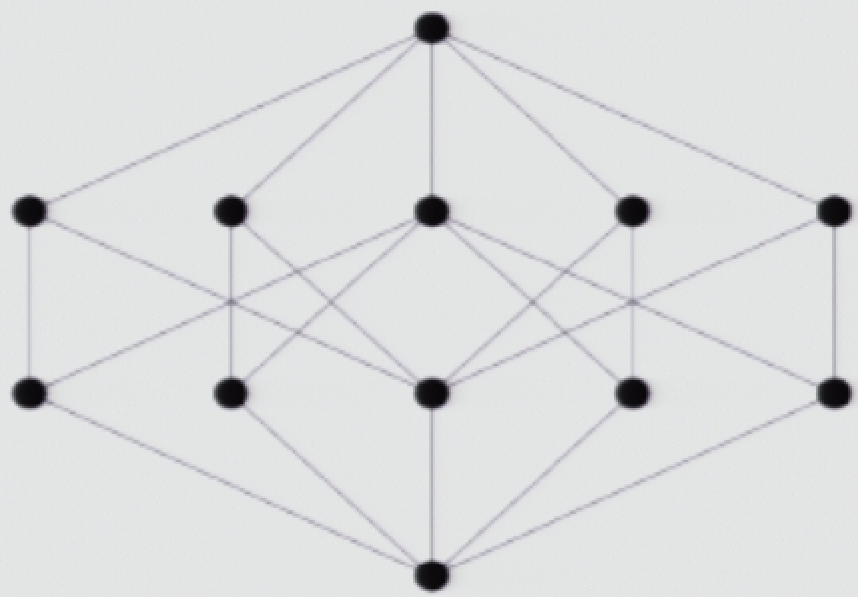
Abstract:

Interpretation of Quantum Mechanics: Current Status and Future Directions

Lecture 22 (March 24 2005)

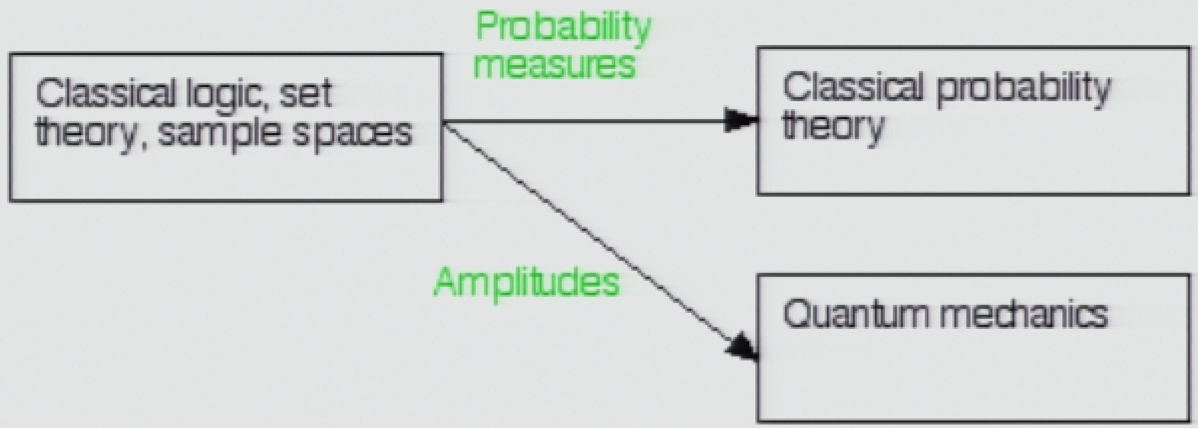
Quantum Logic

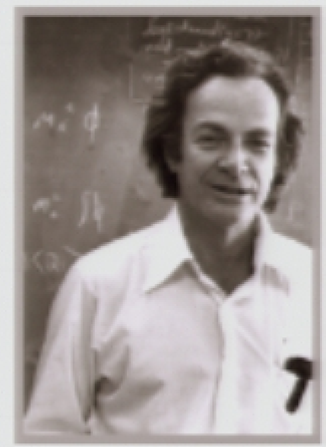


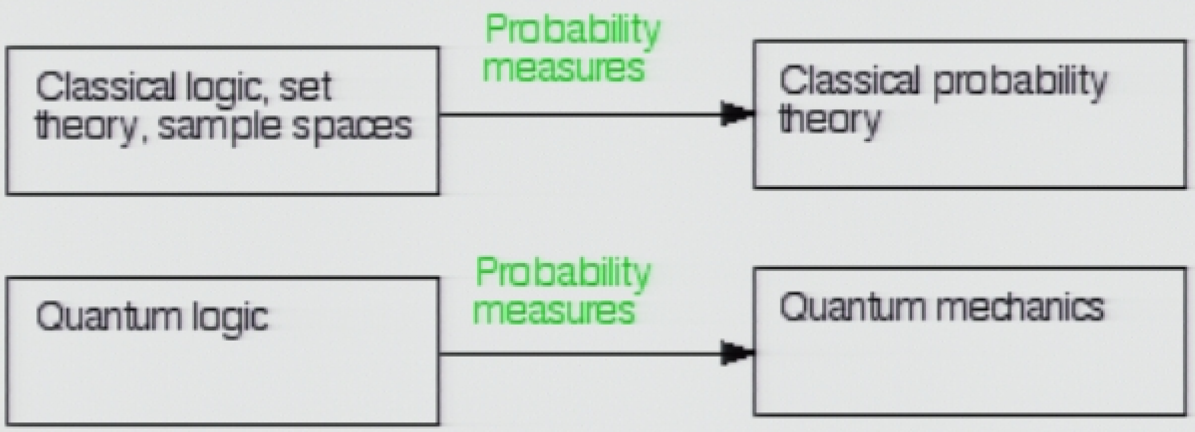
Matthew Leifer

What is quantum logic?

Feynman vs. von Neumann







Outline

1) Classical logic, sets and Boolean lattices

2) Quantum logic, closed subspaces and Hilbert lattices

3) Quantum probability

4) Quantum logic and Hidden Variable Theories

   Partial Boolean algebras and the Bell-Kochen-Specker Theorem

5) The quantum logic interpretation

   Putnam's quantum logical realism - is it really realism?

6) Operational Quantum Logic

   How can two logics coexist? "Firefly in a box" example

   Derivation of Hilbert Space Quantum Mechanics

7) Conclusion

1) Classical Logic

Logic = Syntax + Semantics

1) Classical Logic

Syntax of propositional logic:

Propositions $a, b, c, \ldots, z$

Represent statements like "Waterloo is in Canada", "Frogs are green", "It is raining"

Compound propositions (or sentences) formed using connectives

$\neg$	NOT	negation
$\wedge$	AND	conjunction
$\vee$	OR	disjunction
$\rightarrow$	IF ... THEN ...	implication / conditional
$\leftrightarrow$	IF AND ONLY IF ... THEN ..	equality

Rules for forming sentences

1. If a is a sentence then $\neg a$ is a sentence.
2. If a and b are sentences then $(a \wedge b)$ is a sentence.
3. Intuitive rules for addition and removal of parentheses

1) Classical Logic

Syntax of propositional logic:

Definitions: $a \vee b = \neg(\neg a \wedge \neg b)$

$a \to b = \neg a \vee b$

$a \leftrightarrow b = (a \to b) \wedge (b \to a)$

Example: "If it is raining then Waterloo is in Canada and Frogs are green"

$$a \to (b \wedge c) = \neg a \vee (b \wedge c) = \neg(\neg\neg a \wedge \neg(b \wedge c))$$

1) Classical Logic

Truth Table Semantics of propositional logic:

a	$\neg a$
0	1
1	0

a	b	$a \wedge b$
0	0	0
0	1	0
1	0	0
1	1	1

a	b	$a \vee b$
0	0	0
0	1	1
1	0	1
1	1	1

a	b	c	$\neg a$	$b \wedge c$	$\neg a \vee (b \wedge c)$
0	0	0	1	0	1
0	0	1	1	0	1
0	1	0	1	0	1
0	1	1	1	1	1
1	0	0	0	0	0
1	0	1	0	0	0
1	1	0	0	0	0
1	1	1	0	1	1

1) Classical Logic

Set Theoretical Semantics of propositional logic:

Propositions are associated with the set of objects for which they are true:

"The object is green" $\{\text{frogs, grass, emeralds, leaves, ...}\}$

"It is raining" $\{\text{Monday, Tuesday, Friday}\}$

"Physical system X has a value of quantity Q in the range $y \leq Q \leq z$"

Set of phase-space points

Introduce a *Universal Set*:

$$U = \{\text{cows, cats, frogs, goldfish, grass, diamonds, emeralds, leaves, ...}\}$$

And the *empty set*:

$$\varnothing = \{\ \}$$

1) Classical Logic

Set Theoretical Semantics of propositional logic:

Notation: $[a]$ = the mathematical object associated to proposition a under the semantics we are using.

Negation: $[\neg a] = [a]^c = \{x \mid x \in U, x \notin [a]\}$

Conjunction: $[a \wedge b] = [a] \cap [b] = \{x \mid x \in [a], x \in [b]\}$

Disjunction: $[a \vee b] = [a] \cup [b] = \{x \mid x \in [a] \text{ or } x \in [b]\}$

Example: $U = \{\text{cows, cats, frogs, goldfish, grass, diamonds, emeralds, leaves}\}$

a = "The object is green." $[a] = \{\text{frogs, grass, emeralds, leaves}\}$

b = "The object is an amphibious animal." $[b] = \{\text{frogs, goldfish}\}$

$[\neg a] = \{\text{cows, cats, goldfish, diamonds}\}$ $[a \wedge b] = \{\text{frogs}\}$

$[a \vee b] = \{\text{frogs, goldfish, grass, emeralds, leaves}\}$

1) Classical Logic

Set Theoretical Semantics of propositional logic:

Definitions:

a is a *tautology* iff $[a] = U$ under all possible assignments of sets to elementary propositions.

Example: $a \vee \neg a$

a is a *contradiction* iff $[a] = \varnothing$ under all possible assignments of sets to elementary propositions.

Example: $a \wedge \neg a$

a and b are *equivalent* iff $[a] = [b]$ under all possible assignments of sets to elementary propositions.

Examples: $\neg\neg a = a$ double negation

$\neg(a \wedge b) = \neg a \vee \neg b$

$\neg(a \vee b) = \neg a \wedge \neg b$

de-Moivre's laws

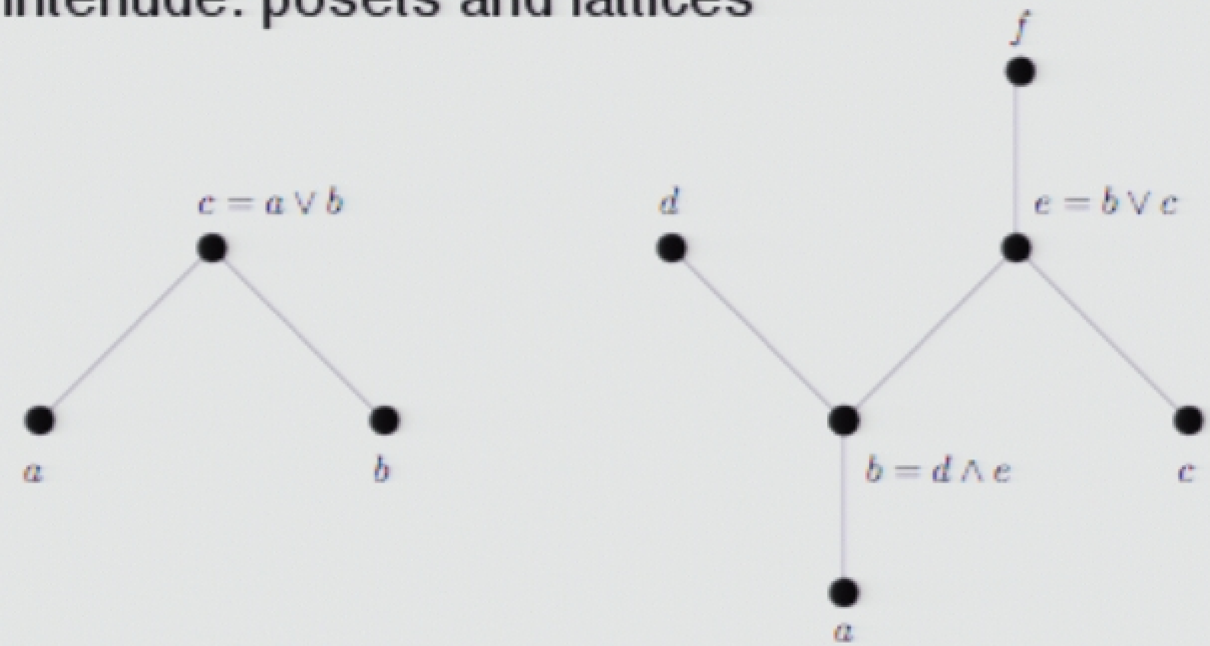
$a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$

$a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$

distributive lavs

Mathematical interlude: posets and lattices

Posets:



A *poset* is a set P with a *partial order relation* $\leq$ satisfying $\forall a,b,c \in P$:

- $a \leq a$
- $a \leq b$ and $b \leq a$ iff $a = b$
- if $a \leq b$ and $b \leq c$ then $a \leq c$

Two elements $a,b \in P$ have a *join* or *least upper bound* if there is an element $a \vee b$ satisfying

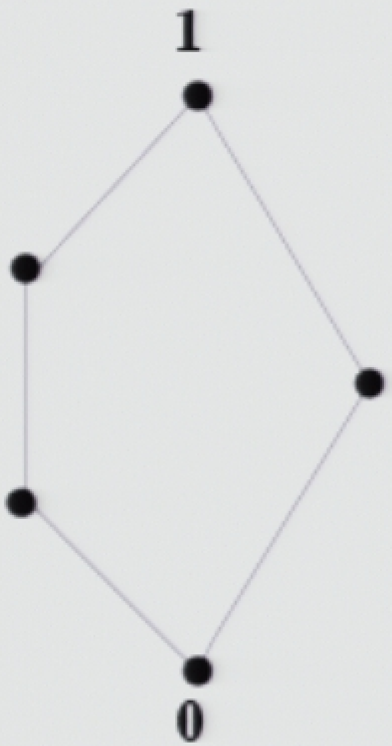
$a \leq a \vee b$ and $b \leq a \vee b$
Any c satisfying $a \leq c$ and $b \leq c$ also satisfies $a \vee b \leq c$.

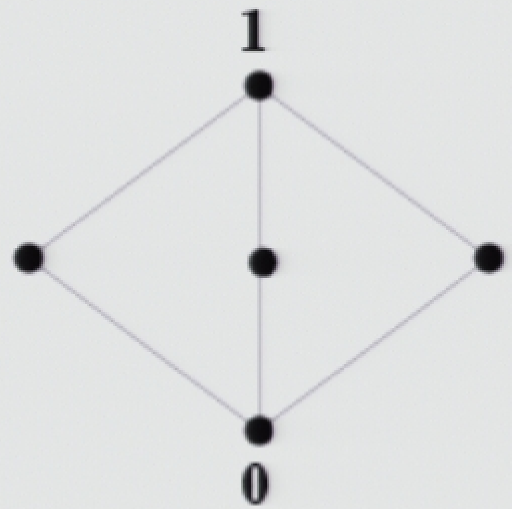
Two elements $a,b \in P$ have a *meet* or *greatest lower bound* if there is an element $a \wedge b$ satisfying

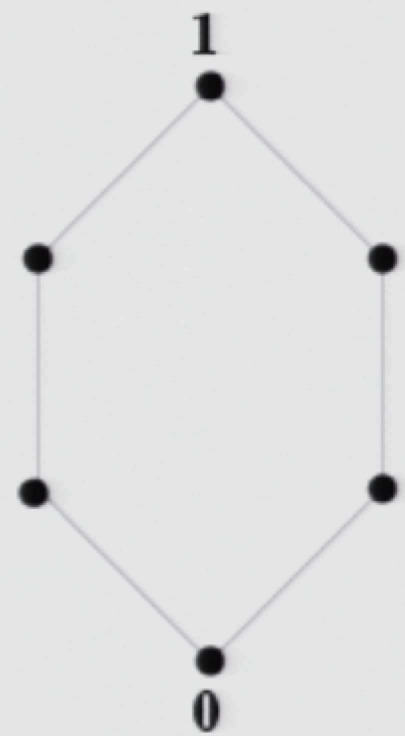
$a \wedge b \leq a$ and $a \wedge b \leq b$
Any c satisfying $c \leq a$ and $c \leq b$ also satisfies $c \leq a \wedge b$.

Mathematical interlude: posets and lattices

Lattices:







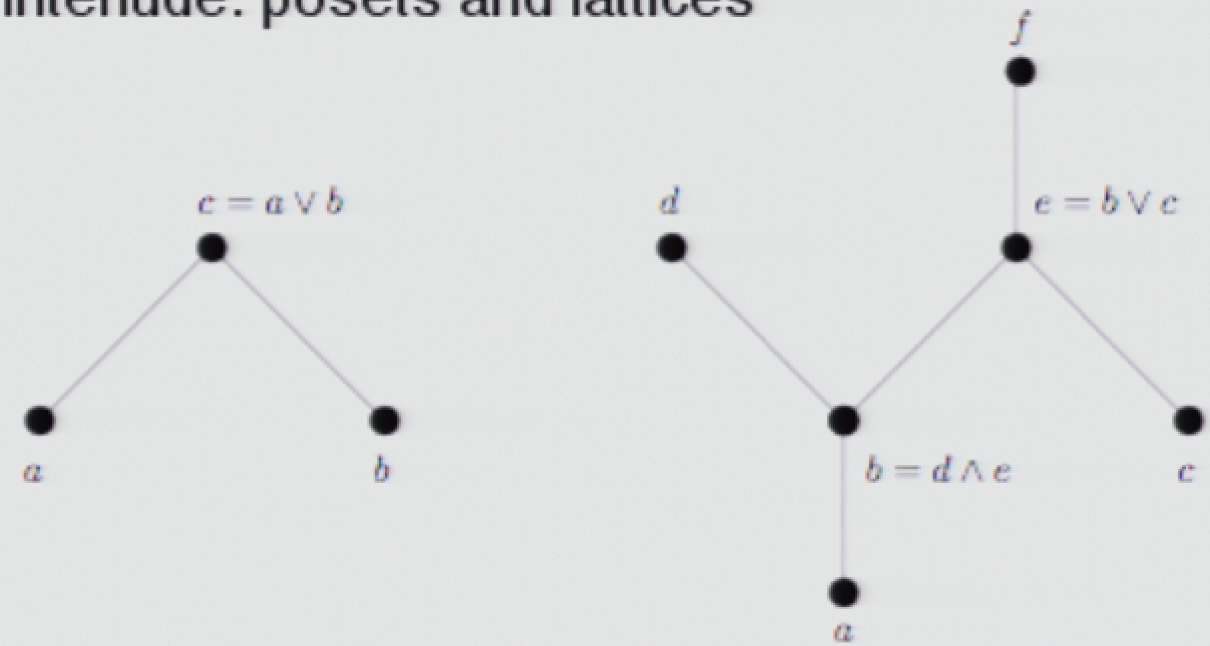
A *lattice* is a poset where every pair of elements has a meet or a join.

We will also require that there is a greatest element $\mathbf{1}$ and a least element $\mathbf{0}$.

Atoms of a lattice are those elements for which $\mathbf{0}$ is the only smaller element.

Mathematical interlude: posets and lattices

Posets:



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Mathematical interlude: posets and lattices

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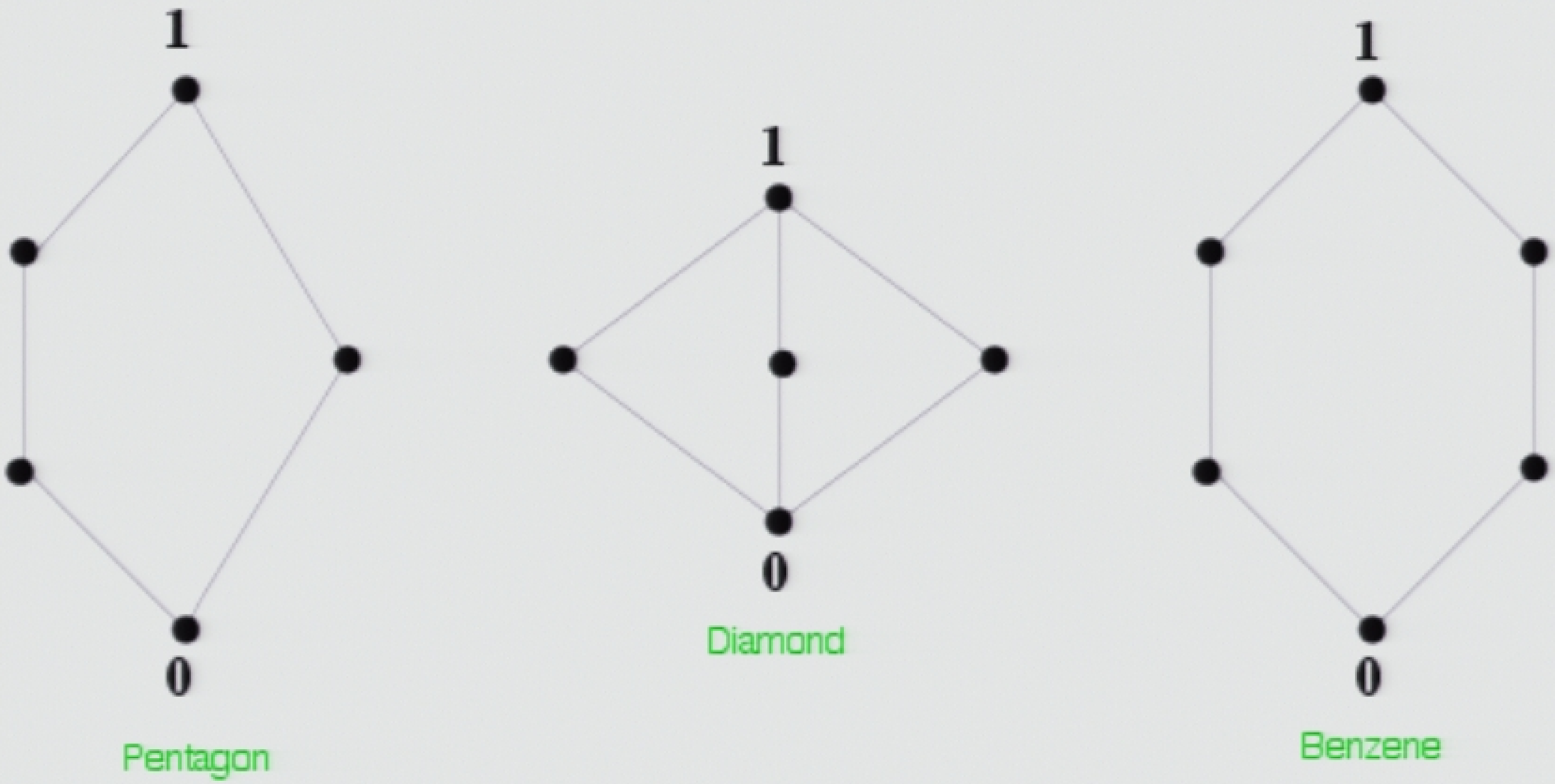

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1) Classical Logic

Logic and Boolean lattices

Consider the logic of propositions that can be formulated from n elementary propositions $a_1, a_2, \ldots, a_n$.

There is a canonical way of associating these with sets of integers:

$$a_1 = \{1\}, a_2 = \{2\}, \ldots, a_n = \{n\}$$

The universal set is $U = \{1,2,\ldots,n\}$

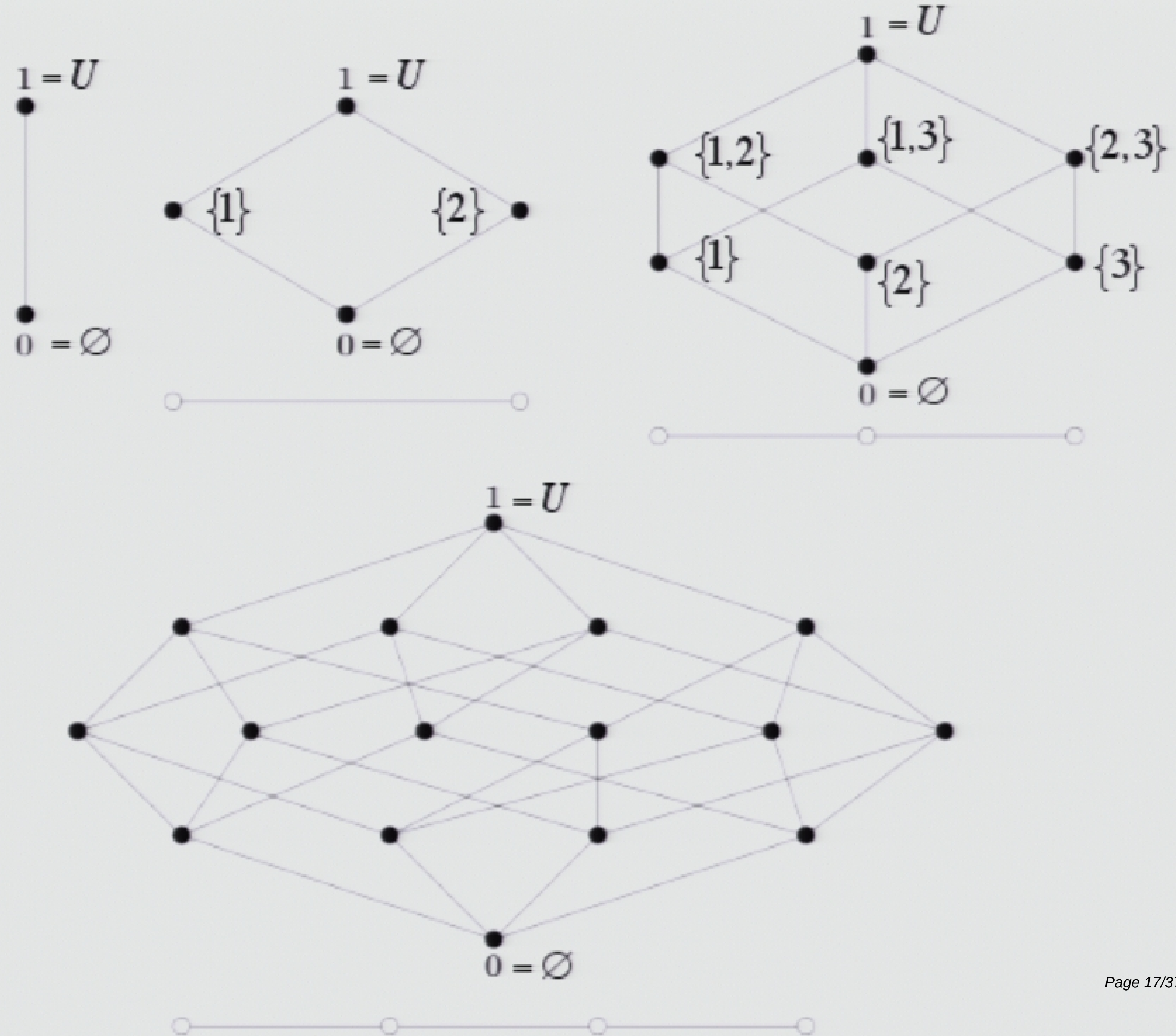
More than one proposition corresponds to the same set, e.g.

$$[\neg a_1] = [a_2 \vee a_3 \vee \ldots \vee a_n] = \{2,3,\ldots,n\}$$

The resulting structure is a *Boolean lattice* with partial order given by subset inclusion.

Greatest and lowest elements $\mathbf{0} = \varnothing, \mathbf{1} = U = \{1,2,\ldots,n\}$

Meet is given by $\cap$ and join is given by $\cup$.


1 = U
0 = ∅
1 = U
{1}
{2}
0 = ∅
1 = U
{1,2}
{1,3}
{2,3}
{1}
{2}
{3}
0 = ∅
1 = U
0 = ∅

2) Quantum Logic

Attaching meaning to heretical propositions

Note that the *syntax* of quantum logic is exactly the same as classical logic, except that we will not define $\rightarrow$ and $\leftrightarrow$. The only difference is in the meaning, i.e. the *semantics*.

"The momentum of the particle is between p and $p + \mathrm{d}p$."

Orthodoxy: Meaningless unless system is in an appropriate eigenstate.

"The position of the particle is between x and $x + \mathrm{d}x$ and he momentum of the particle is between p and $p + \mathrm{d}p$."

Orthodoxy: Always completely and utterly meaningless!

Requirements for a semantics of quantum propositions:

1) Respect the eigenvalue-eigenstate link.
2) Equivalent to classical set-theoretic semantics when propositions are about pairwise commuting observables.

Consider an observable: $A = \sum_j a_j P_j \qquad P_j P_k = \delta_{jk} P_k \quad \sum_j P_j = I$

If the state is an eigenstate $|a_j\rangle$ then $\mathrm{prob}(P_k) = \langle a_j | P_k | a_j \rangle = \delta_{jk}$

2) Quantum Logic

Semantics of Quantum Logic

$[a]$ = the subspace corresponding to proposition a, P_a = the projector onto $[a]$.

Negation: $[\neg a] = [a]^{\perp} = \{ |\psi\rangle \in \mathcal{H} \,|\, \forall |\phi\rangle \in [a] \;\; \langle \phi | \psi \rangle = 0 \}$

$$P_{\neg a} = I - P_a$$

Conjunction: $[a \wedge b] = [a] \cap [b] = \{ |\psi\rangle \in \mathcal{H} \,\|\, \psi\rangle \in [a], |\psi\rangle \in [b] \}$

$$P_{a \wedge b} = \lim_{n \to \infty} (P_a P_b)^n$$

Disjunction:

$$[a \vee b] = [a] \oplus [b] = \{ |\psi\rangle \in \mathcal{H} \,|\, \exists |\phi\rangle \in [a], \exists |\eta\rangle \in [b] \text{ s.t. } |\psi\rangle = \alpha |\phi\rangle + \beta |\eta\rangle \}$$

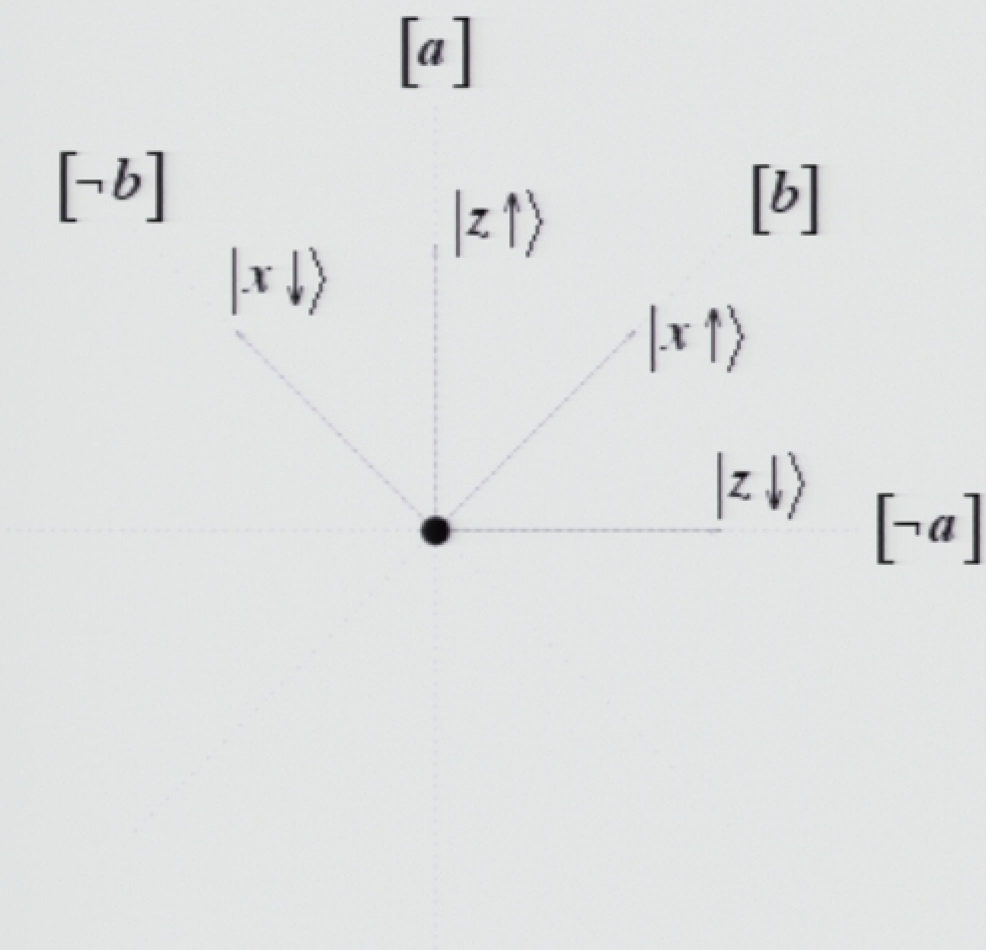
$$P_{a \vee b} = I - \lim_{n \to \infty} ((I - P_a)(I - P_b))^n$$

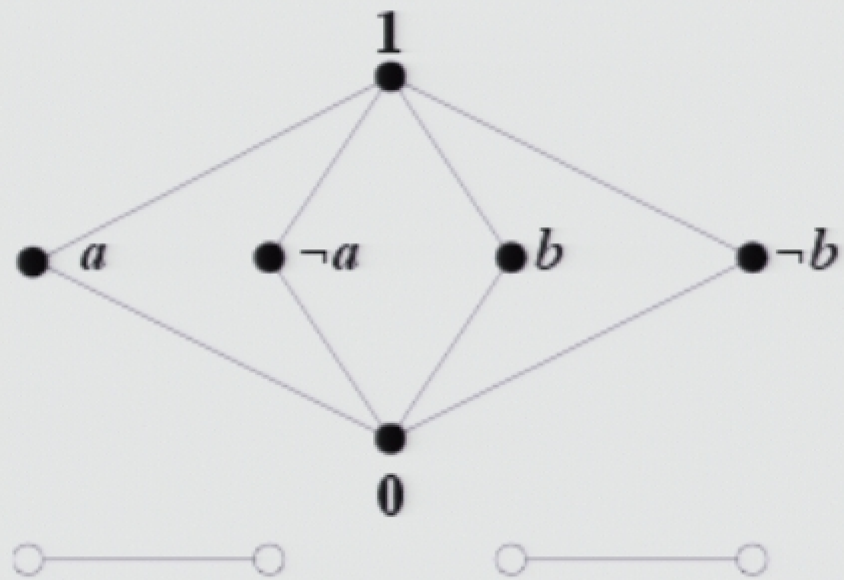
We may define tautologies, contradictions and equivalence in a similar way to the classical case.

Subspaces of Hilbert space also form a lattice with partial order $[a] \leq [b]$ iff $P_a P_b = P_a$, but not a Boolean one

2) Quantum Logic

A 2D Real Hilbert Space Example



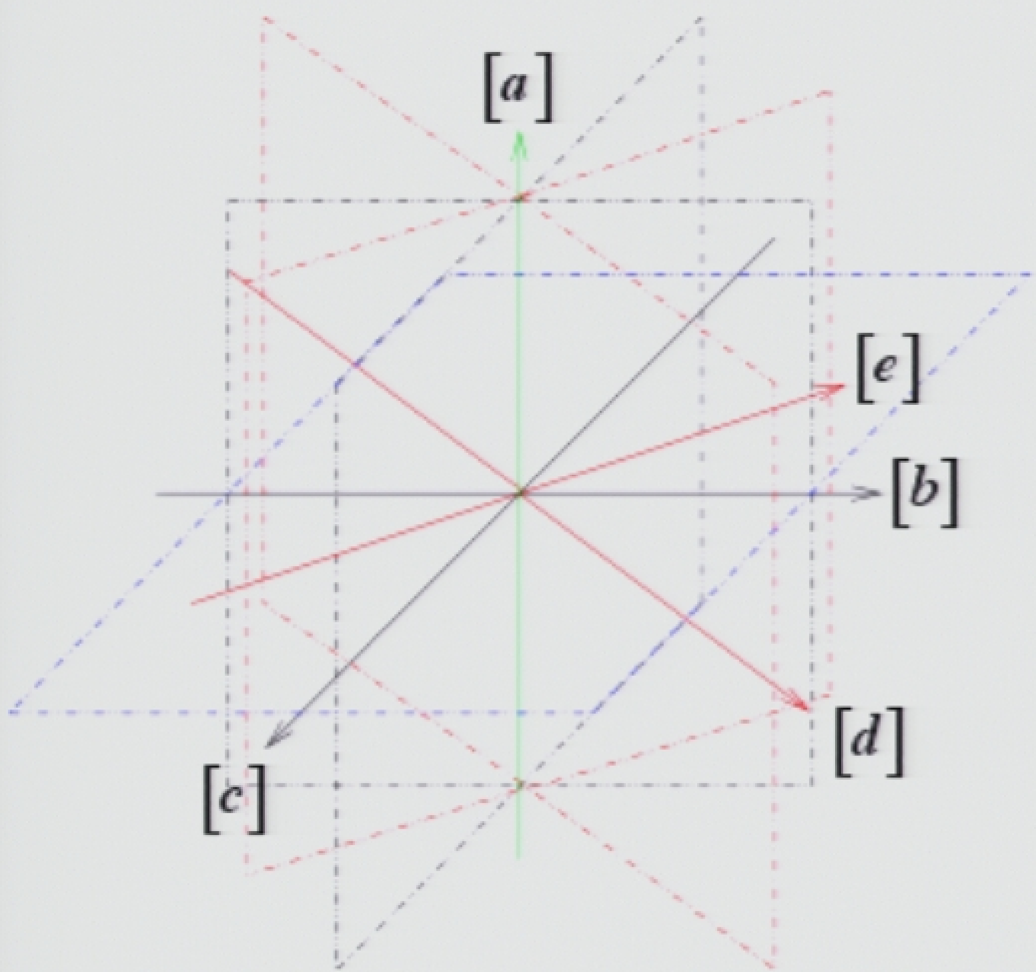


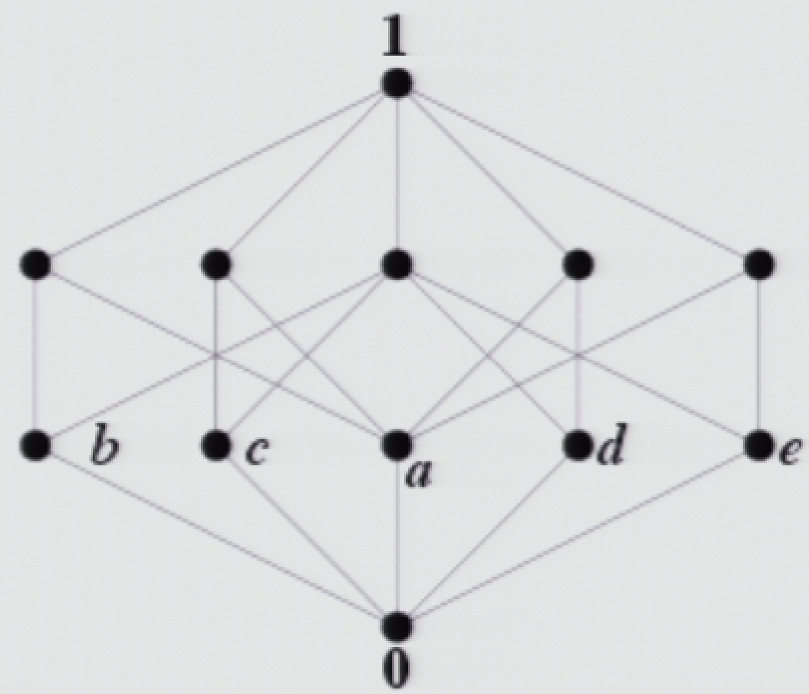
$a =$ "The particle has spin up in the z-direction."

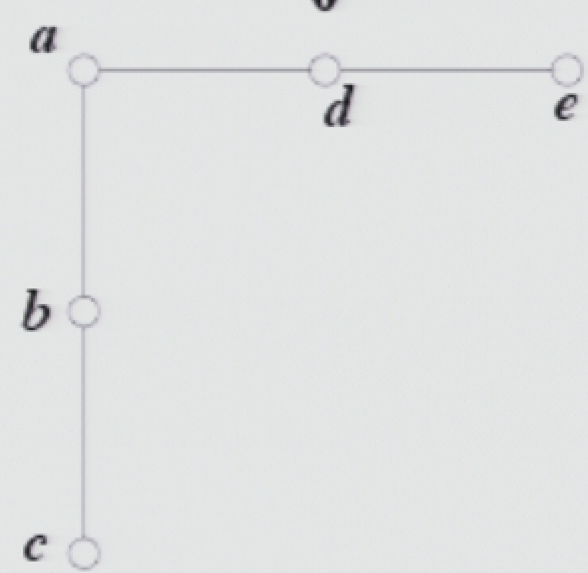
$b =$ "The particle has spin up in the x-direction."

2) Quantum Logic

A 3D Real Hilbert Space Example

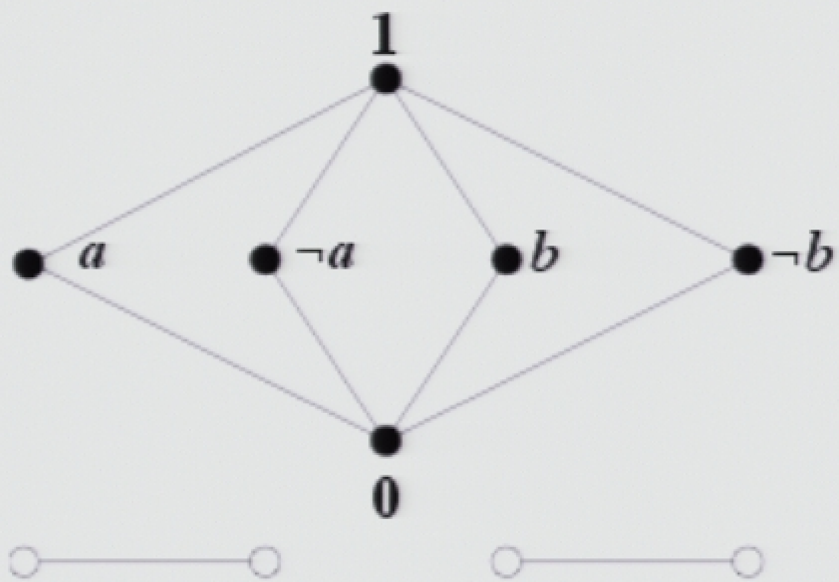






2) Quantum Logic

Quantum Logic is not distributive



$$a \wedge (b \vee \neg b) = a \wedge 1 = a$$

$$(a \wedge b) \vee (a \wedge \neg b) = \mathbf{0} \vee \mathbf{0} = \mathbf{0}$$

Modularity:

If $p \leq q$ then $p \vee (r \wedge q) = (p \vee r) \wedge q$

Orthomodularity:

If $p \leq q$ then $q = p \vee (q \wedge \neg p)$

3) Quantum Probability

Classical Probability measures

In standard probability theory, we define a probability measure on a set of subsets (equivalently on a set of propositions or on a Boolean lattice).

$$\mu : \text{"subsets of } U\text{"} \rightarrow [0,1]$$

Let $\alpha_1, \alpha_2, \ldots, \alpha_n$ be any pairwise disjoint sets, i.e. $\alpha_j \cap \alpha_k = \varnothing$.

Let $\mathrm{A} = \alpha_1 \cup \alpha_2 \cup \ldots \cup \alpha_n$.

Require: $\mu(\mathrm{A}) = \mu(\alpha_1) + \mu(\alpha_2) + \ldots + \mu(\alpha_n)$

$\mu(\varnothing) = 0$ and $\mu(U) = 1$

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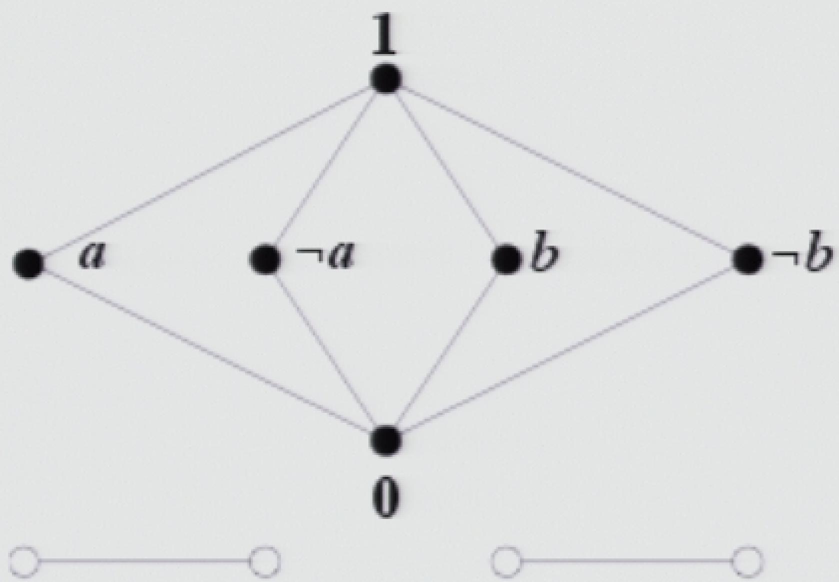

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3) Quantum Probability

Probability measures on Hilbert space PBAs

In quantum probability theory we have similar requirements:

$$\mu\text{: "Projectors on } \mathcal{H}\text{"} \to [0,1]$$

Let $P_1, P_2, \ldots, P_n$ be any projectors onto pairwise orthogonal subspaces, i.e. $P_j P_k = 0$

Let $Q = P_1 + P_2 + \ldots + P_n$

Require: $\mu(Q) = \mu(P_1) + \mu(P_2) + \ldots + \mu(P_n)$

$\mu(0) = 0 \qquad \mu(I) = 1$

Gleason's Theorem:

The only measures on the full PBA of Hilbert spaces of dimension ≥ 3 are of the form

$\mu(P) = \mathrm{Tr}(P\rho)$, where ρ is a positive operator, with $\mathrm{Tr}(\rho) = 1$.

4) Quantum Logic and HVTs

Bell-Kochen-Specker Theorem

Two natural requirements for a Hidden Variable Theory:

1) The complete state of the system determines the value outcome of a measurement of any observable.

2) The outcome assigned to an observable does not depend on which other compatible observables are measured with it (*noncontextuality*).

Bell's version of BKS theorem: Gleason's theorem implies 1) and 2) cannot both be satisfied.

Why? Projectors are observables with e-values 0 and 1, so 1) implies that we must assign 0 or 1 to every projector.

2) and empirical adequacy implies that the assignment is a quantum probability measure.

However, Gleason says all probability measures are $\mathrm{Tr}(\mathrm{P}\,\rho)$ and there is no density operator that assigns probability 0 or 1 to every projector.

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4) Quantum Logic and HVTs

Finite versions of the KS theorem

Mermin-Peres-Kernaghan proof

(1,0,0,0)	(1,0,0,0)	(1,0,0,0)	(1,0,0,0)	(-1,1,1,1)	(-1,1,1,1)	(1,-1,1,1)	(1,1,-1,1)	(0,1,-1,0)	(0,0,1,-1)	(1,0,1,0)
(0,1,0,0)	(0,1,0,0)	(0,0,1,0)	(0,0,0,1)	(1,-1,1,1)	(1,1,-1,1)	(1,1,-1,1)	(1,1,1,-1)	(1,0,0,-1)	(1,-1,0,0)	(0,1,0,1)
(0,0,1,0)	(0,0,1,1)	(0,1,0,1)	(0,1,1,0)	(1,1,-1,1)	(1,0,1,0)	(0,1,1,0)	(0,0,1,1)	(1,1,1,1)	(1,1,1,1)	(1,1,-1,-1)
(0,0,0,1)	(0,0,1,-1)	(0,1,0,-1)	(0,1,-1,0)	(1,1,1,-1)	(0,1,0,-1)	(1,0,0,-1)	(1,-1,0,0)	(1,-1,-1,1)	(1,1,-1,-1)	(1,-1,-1,1)

What is the quantum logical significance of this?

Orthogonality constraints can be formulated as logical statements:

$$A_1 = (a \wedge \neg b \wedge \neg c \wedge \neg d) \vee (\neg a \wedge b \wedge \neg c \wedge \neg d) \vee (\neg a \wedge \neg b \wedge c \wedge \neg d) \vee (\neg a \wedge \neg b \wedge \neg c \wedge d)$$

$$A_2 = (a \wedge \neg b \wedge \neg e \wedge \neg f) \vee (\neg a \wedge b \wedge \neg e \wedge \neg f) \vee (\neg a \wedge \neg b \wedge e \wedge \neg f) \vee (\neg a \wedge \neg b \wedge \neg e \wedge f)$$

$$A_3 = (a \wedge \neg c \wedge \neg g \wedge \neg h) \vee (\neg a \wedge c \wedge \neg g \wedge \neg h) \vee (\neg a \wedge \neg c \wedge g \wedge \neg h) \vee (\neg a \wedge \neg c \wedge \neg g \wedge h)$$

$$\vdots$$

$A_1 \wedge A_2 \wedge \ldots \wedge A_{11}$ is a classical *contradiction* but is *satisfiable* in quantum logic with the MPK assignment.

5) The QL Interpretation

Putnam's main claims

Proposed by Putnam in "Is Logic Emprical?"/"The Logic of Quantum Mechanics" (1969).

This summary is adapted from Gibbins, "Particles and Paradoxes", CUP (1987).

1) Logic is empirical, and open to revision in the light of a new physical theory - just like geometry was seen to be emprical with the advent of General Relativity.

2) The logic of quantum mechanics is non-Boolean.

3) The peculiarities of quantum mechanics arise from illegitimate uses of classical logic in the description of individual quantum systems. Paradoxes are *resolved* by using quantum logic.

4) Quantum probabilities present no difficulty. They arise in exactly the same way as in classical theories.

5) Quantum logic licenses a *realist* interpretation of quantum mechanics.

6) Ideal measurements *reveal* the values of dynamical variables *posessed* by the system prior to measurement.

7) Although quantum-mechanical states are not classically complete, they do correspond to quantum logically maximally consistent sets of sentences. Indeterminacy arises not because the laws of quantum mechanics are indeterministic but because quantum-mechanical states are not *classically* complete.

8) The meaning of the quantum logical connectives are the same as those of the classical connectives.

5) The QL Interpretation

Quantum logical Realism

Consider an observable $A = \sum_j \alpha_j P_j$, where we include eigenvalue 0 if necessary.

Define $\boldsymbol{a}_j$ = "The value of A is α_j."

Then $P_{a_1 \vee a_2 \vee a_3 \vee \ldots} = P_1 + P_2 + P_3 + \ldots = I \qquad [\boldsymbol{a}_1 \vee \boldsymbol{a}_2 \vee \boldsymbol{a}_3 \vee \ldots] = \mathcal{H}$

$$\boldsymbol{a}_1 \vee \boldsymbol{a}_2 \vee \boldsymbol{a}_3 \vee \ldots = \mathbf{1}$$

But the LHS is just the definition of $\exists j(\boldsymbol{a}_j)$, so we can say "there exists some j s.t. the value of A is α_j."

However, we *cannot* point to any particular value that the observable A actually posseses, unless the system is in an appropriate eigenstate.

Consider the same construction for observables $B, C, \ldots$

$$\exists j(\boldsymbol{a}_j) \wedge \exists k(\boldsymbol{b}_k) \wedge \exists m(\boldsymbol{c}_m) \wedge \ldots = (\boldsymbol{a}_1 \vee \boldsymbol{a}_2 \vee \boldsymbol{a}_3 \vee \ldots) \wedge (\boldsymbol{b}_1 \vee \boldsymbol{b}_2 \vee \boldsymbol{b}_3 \vee \ldots) \wedge (\boldsymbol{c}_1 \vee \boldsymbol{c}_2 \vee \boldsymbol{c}_3 \vee \ldots) \wedge \ldots$$

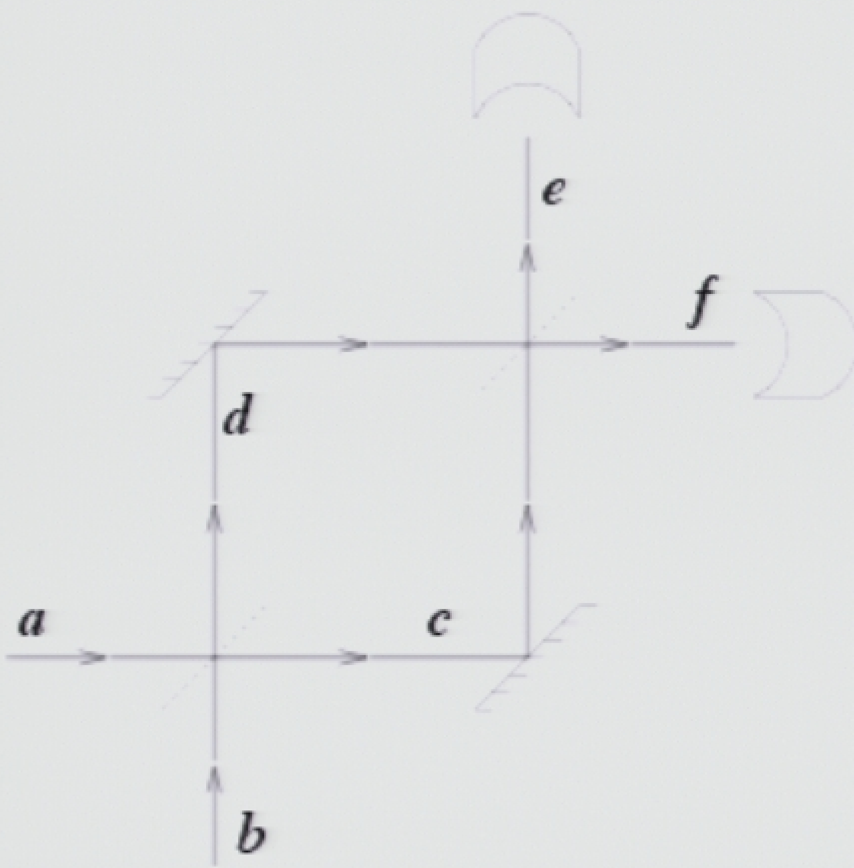
is always true

But..... $\exists j \exists k \exists m \exists \ldots (\boldsymbol{a}_j \wedge \boldsymbol{b}_k \wedge \boldsymbol{c}_m \wedge \ldots) = (\boldsymbol{a}_1 \wedge \boldsymbol{b}_1 \wedge \boldsymbol{c}_1 \wedge \ldots) \vee (\boldsymbol{a}_2 \wedge \boldsymbol{b}_1 \wedge \boldsymbol{c}_1 \wedge \ldots) \vee \ldots$

is always a contradiction.

5) The QL Interpretation

Application: Mach-Zehnder version of double slit experiment



Apologies for extreme abuse of notation:

$$|a\rangle \rightarrow |c\rangle + i|d\rangle \rightarrow |f\rangle$$

$$|f\rangle \sim |c\rangle + i|d\rangle$$

Quantum logically:

$f \wedge (c \vee d)$ is true for the state $|f\rangle$ but...

$$f \wedge c, \quad f \wedge d, \quad (f \wedge c) \vee (f \wedge d)$$

are all contradictions.

5) The QL Interpretation

Standard Objections

1) Classical logic is *necesarily* true.

Response: There is a long tradition of philosophical debate about this, which is not fully resolved. It *can* be plausibly denied.

2) All propositions about experiments can be phrased in terms of classical logic, so the claim that logic is *empirical* is untrue.

Response: QL does not have to be the *unique* way of dealing with propositions, just the most elegant one. In comparison, it is possible to phrase all statements about General Relativity in Euclidean geometry by introducing all sorts of unnatural forces, but that does not mean that GR does not entail that geometry is non-Euclidean.

3) Quantum Logic does not account for the success of classical logic.

Response: Classical logic is valid for propositions about compatible observables. We may be able to make use of decoherence to argue that most propositions we are interested in obey classical logic.

4) Realism essentially *entails* the use of classical logic. Your definition of $\exists$ is incorrect because the very notion of what $\exists$ *means* does not make sense unless it is the classical version of $\exists$.

Response: For a full-blooded realist interpretation I agree with you. However, I never claimed this was a realist interpretation of that type (even if Putnam sometimes did).

5) The *meta-language* you use to talk about quantum logic is classical, so you haven't replaced classical logic after all.

Responses: - Copenhagen has a quantum/classical split as well. QL is an improvement on this because it is explicit about where this split occurs.

- Maybe we can reconstruct all valid reasoning using QL all the way. Admittedly, the

5) Conclusion

Three roads from quantum mechanics