Title: Evaluation of effective rigidity of membrane energy dominated universe model

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Abstract: A joint Guelph-Waterloo Gravity Group/Perimeter Institute Seminar -- Observational evidence suggests that the large scale dynamics of the universe is presently dominated by dark energy, meaning a non-luminous cosmological constituent with a negative value of the pressure to density ratio w, which would be unstable if purely fluid, but could be stable if effectively solid with sufficient rigidity. It was suggested by Bucher and Spergel that such a solid constituent might be constituted by an effectively cold (meaning approximately static) distribution of cosmic strings with w=-1/3, or membranes with the observationally favoured value w=-2/3, but it was not established whether the rigidity in such models actually would be sufficient for stabilisation. For cases (exemplified by an approximately O(3) symmetric scalar field model) in which the number of membranes meeting at a junction is even (though not if it is odd) it is easy to obtain an explicit evaluation of the rigidity to density ratio, which is shown to 3/15 in both string and membrane cases, and it is confirmed that this is indeed sufficient for stabilisation.

Membrane lattice model for dark energy in universe.

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Contents

1. Kibble mechanism for defect formation

In radiation era, cosmic temp. Θ determines Hubble rate $H \approx \Theta^2$. For defects, codimension $i = 1$ for membranes, 2 for strings, formed with tension $T \approx \eta^{4-i}$ at symmetry breaking scale η , geometric mean of thermal scale Θ^{-1} and Hubble scale Θ^{-2} gives initial correl. length $\xi \approx \eta^{-3/2}$.

If subsequently frozen into radiation flow with lattice scale $\ell \approx \xi\eta/\Theta$ will give density contrib. $\rho_i \approx T/\ell^i \approx \kappa_i \Theta^i$, corresp. stress-energy contrib. $T_i^{\mu\nu} \approx \rho_i(\gamma_i\, u^\mu u^\nu + w_i\, g^{\mu\nu})$ with $\gamma_i = w_i + 1 = i/3$, and $\kappa_i \approx \eta^{4-i/2}$.

Membrane case with $\gamma_1 = \frac{1}{3}$, $w_1 = -\frac{2}{3}$, has coefficient $\kappa_1 \approx \eta^{7/2}$. Originally proposed by Bucher and Spergel with causal limit estimate $\xi \approx \eta^{-2}$ predicting $\kappa_1 \approx \eta^4$.

2. Multipolytropic equation of state

Contributions to energy density ρ $\propto$ powers of temp. Θ from inflatons, membranes, strings, cold and hot particles :

$\rho = \kappa_0 + \kappa_1\Theta + \kappa_2\Theta^2 + \kappa_3\Theta^3 + \kappa_4\Theta^4$. Define conserved $n = \Theta^3$ ($\propto$ entropy density) and indices $\gamma_i = i/3$:

$\rho = \Sigma_i \kappa_i n^{\gamma_i} \;\Rightarrow\; \mathrm{d}\rho/\mathrm{d}n = \Sigma_i \gamma_i \kappa_i n^{w_i}$ with $w_i = \gamma_i - 1$.

Get pressure $P = n\mathrm{d}\rho/\mathrm{d}n - \rho$, bulk modulus $\beta = n\mathrm{d}P/\mathrm{d}n$ as $P = \Sigma_i w_i \kappa_i n^{\gamma_i} = -\kappa_0 - \frac{2}{3}\kappa_1\Theta - \frac{1}{3}\kappa_2\Theta^2 + \frac{1}{3}\kappa_4\Theta^4$

$\Rightarrow \quad \beta = \Sigma_i \gamma_i w_i \kappa_i n^{\gamma_i} = -\frac{2}{9}(\kappa_1\Theta + \kappa_2\Theta^2) + \frac{4}{9}\kappa_4\Theta^4$.

At low temp. β becomes neg. so fluid unstable, having sound speed c_{s} given by $c_{\mathrm{s}}^2 = \mathrm{d}P/\mathrm{d}\rho = \beta/(\rho + P)$.

3 Stabilisation by rigidity for a solid

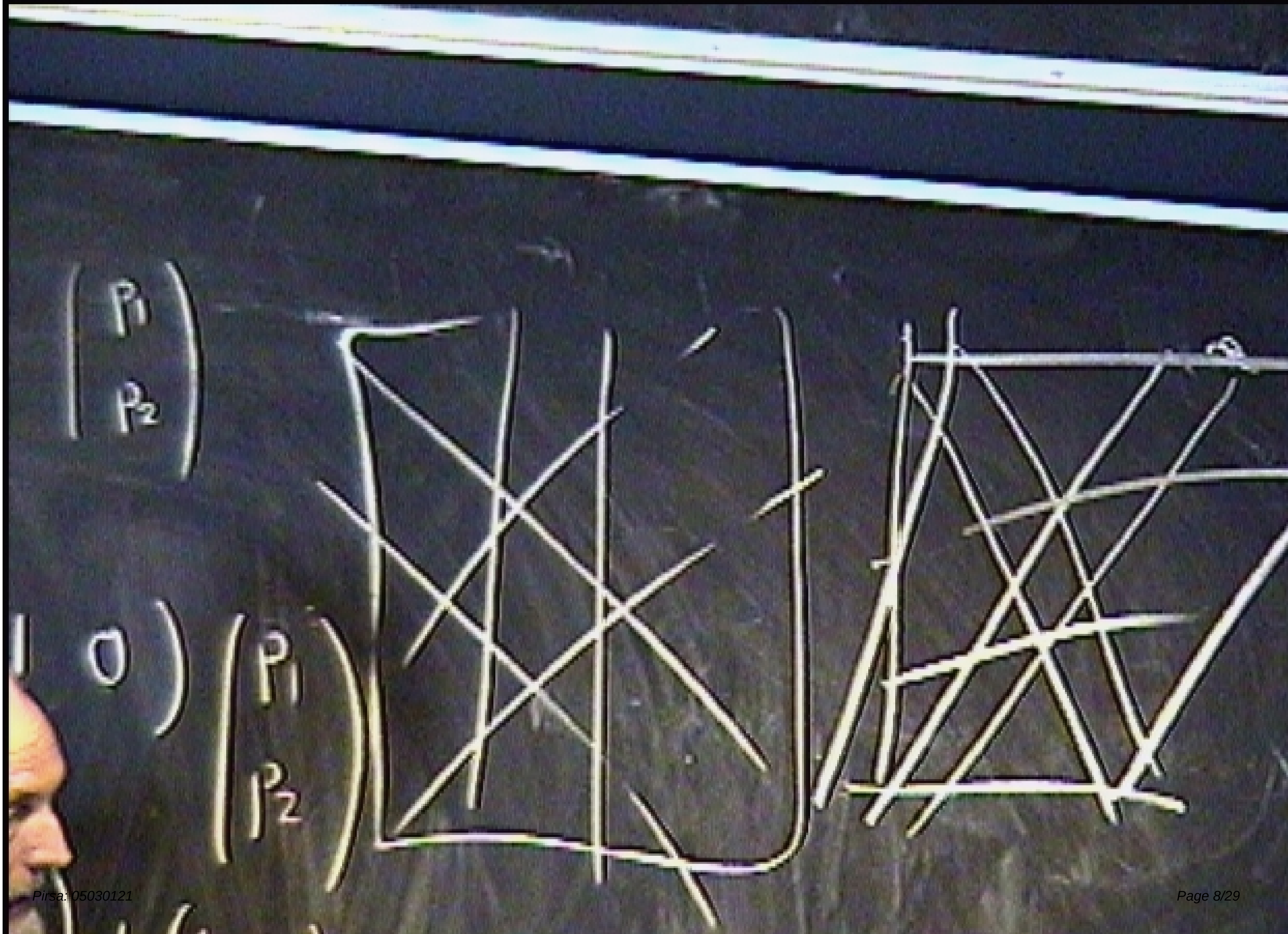
Isotropic solid sustains transverse "shake" modes with speed $c_\perp$ given in terms of shear modulus μ by $c_\perp^2 = \mu/(\rho + P)$. Longitudinal modes have augmented speed $c_\parallel$ given by $c_\parallel^2 = c_s^2 + \frac{4}{3}c_\perp^2 = (\beta + \frac{4}{3}\mu)/(\rho + P)$, so get stability provided $\mu > -\frac{3}{4}\beta$. For strings and membranes we obtained $\beta = -\frac{2}{9}\rho$ so their stability requires $\mu > \frac{1}{6}\rho$, which is hard to satisfy for "odd" type lattice typified by case with Y form junctions. However it does hold for "even" type lattice of straight strings and flat membranes with X form junctions, for which we find $\mu = \frac{4}{15}\rho$.

For the string case this gives $c_\perp^2 = \frac{2}{5}$, $c_\parallel^2 = \frac{1}{5}$, and for the membrane case one gets $c_\perp^2 = \frac{4}{5}$, $c_\parallel^2 = \frac{2}{5}$.

4 Examples of "odd" and "even" systems.

Both kinds obtainable from perturbed O{ N } models with $V = \lambda\,(\Sigma_\alpha \Phi_\alpha^2 - \eta^2)^2 + \mathcal{E}\,\Sigma_\alpha(\Phi_\alpha^2 - \zeta^2)^2$, for N real scalar fields Φ_α, and mass scales η, ζ, where λ and $\mathcal{E}$ are dimensionless with $\mathcal{E} > -\lambda$, $\mathrm{N}\lambda > -\mathcal{E}$.

Exactly O{ N } symmetric if $\mathcal{E} = 0$, $\lambda > 0$, while simplest "even" type example is decoupled limit, $\lambda = 0$, $\mathcal{E} > 0$. Kubotani envisaged "odd" type case, having Y form junctions, for $\mathrm{N} = 3$ with $\mathcal{E} < 0$. With $\mathcal{E} > 0$ (for any N) get potentially stable "even" type case, with X form junctions between walls separating pairs of (the 2^{N} possible) vacuum domains where $\Phi_\alpha^2 = (\lambda\eta^2 + \mathcal{E}\zeta^2)/(\mathrm{N}\lambda + \mathcal{E})$. as shown for $\mathrm{N} = 3$ in Fig. 1.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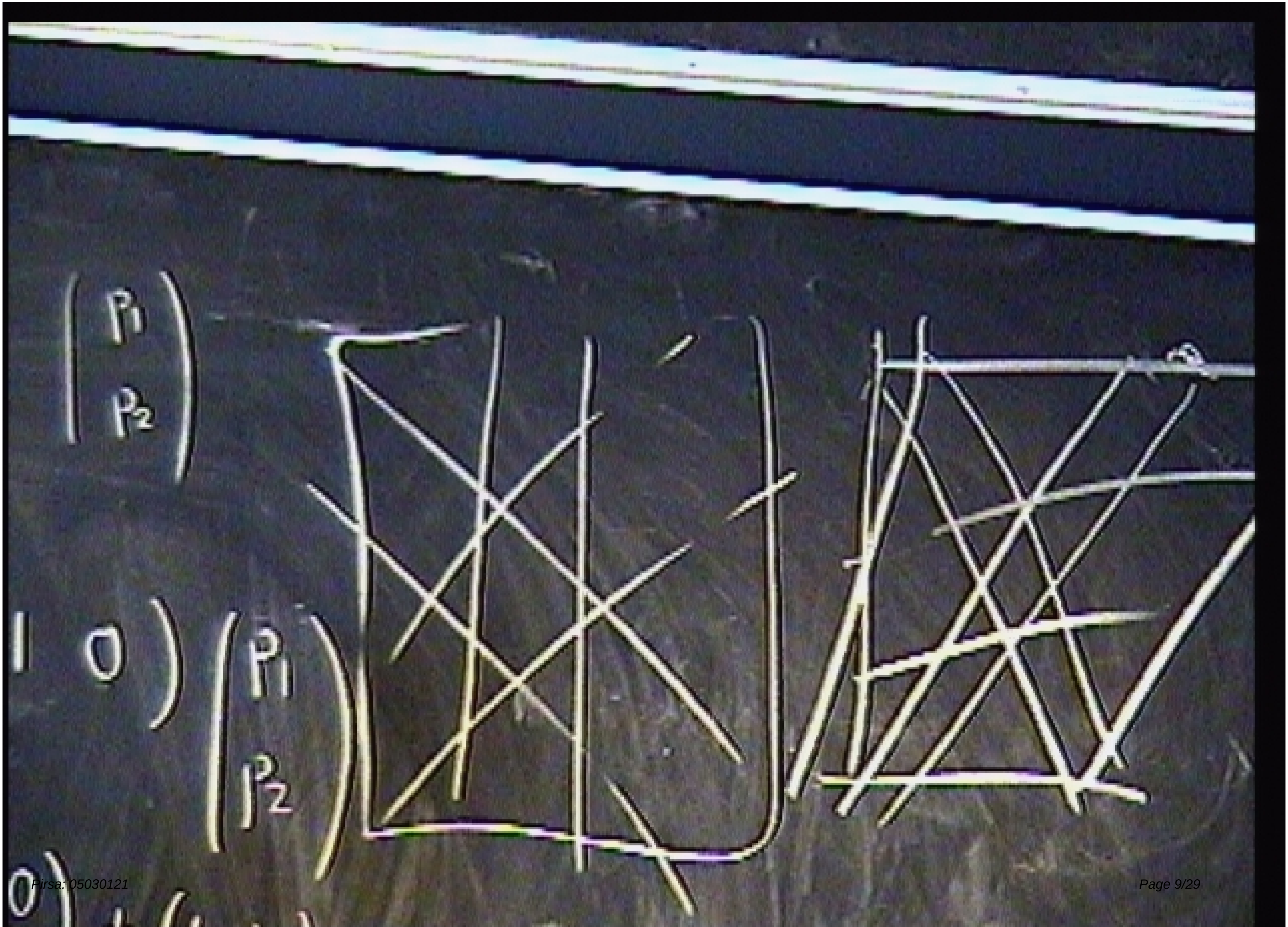
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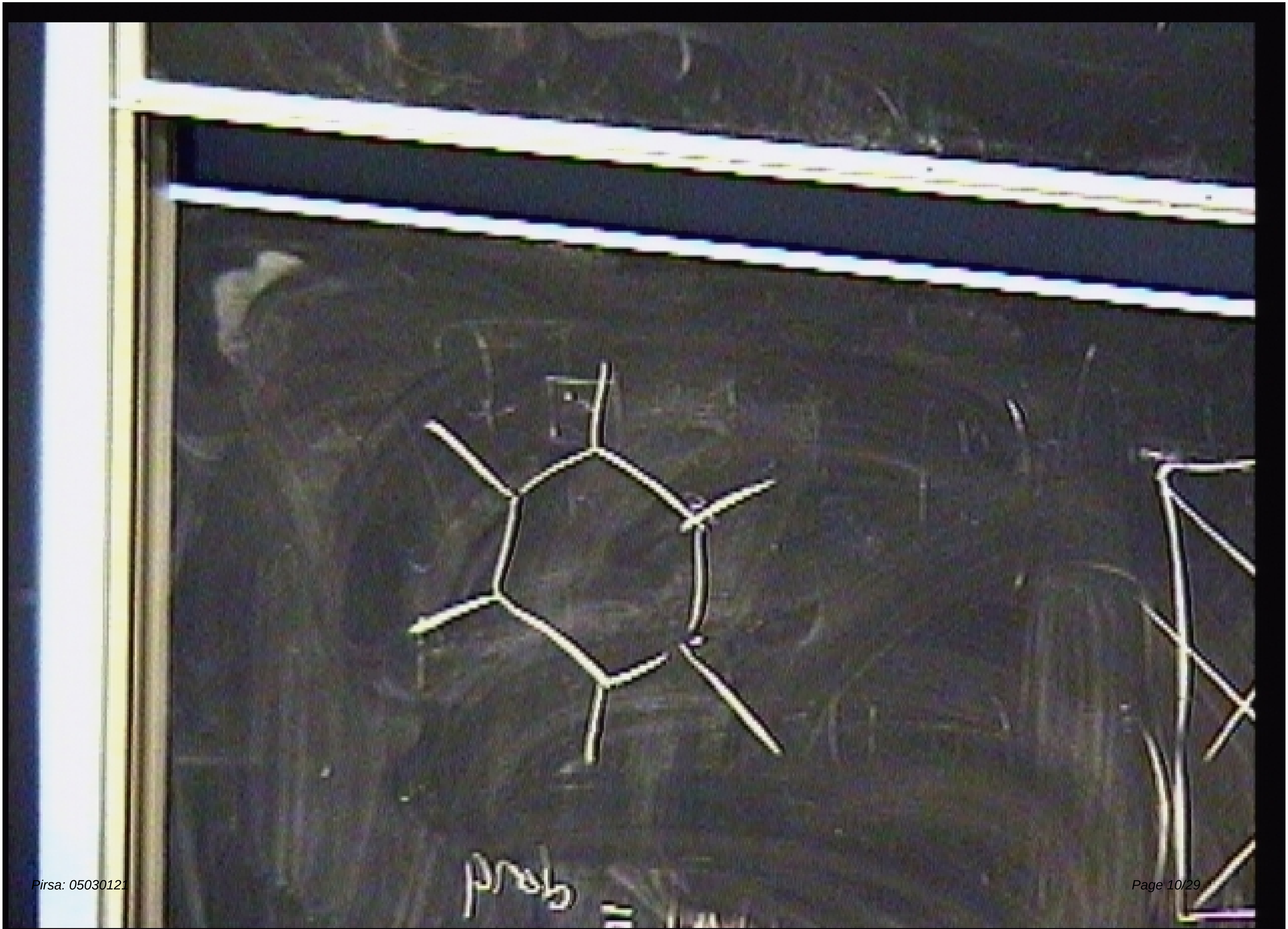

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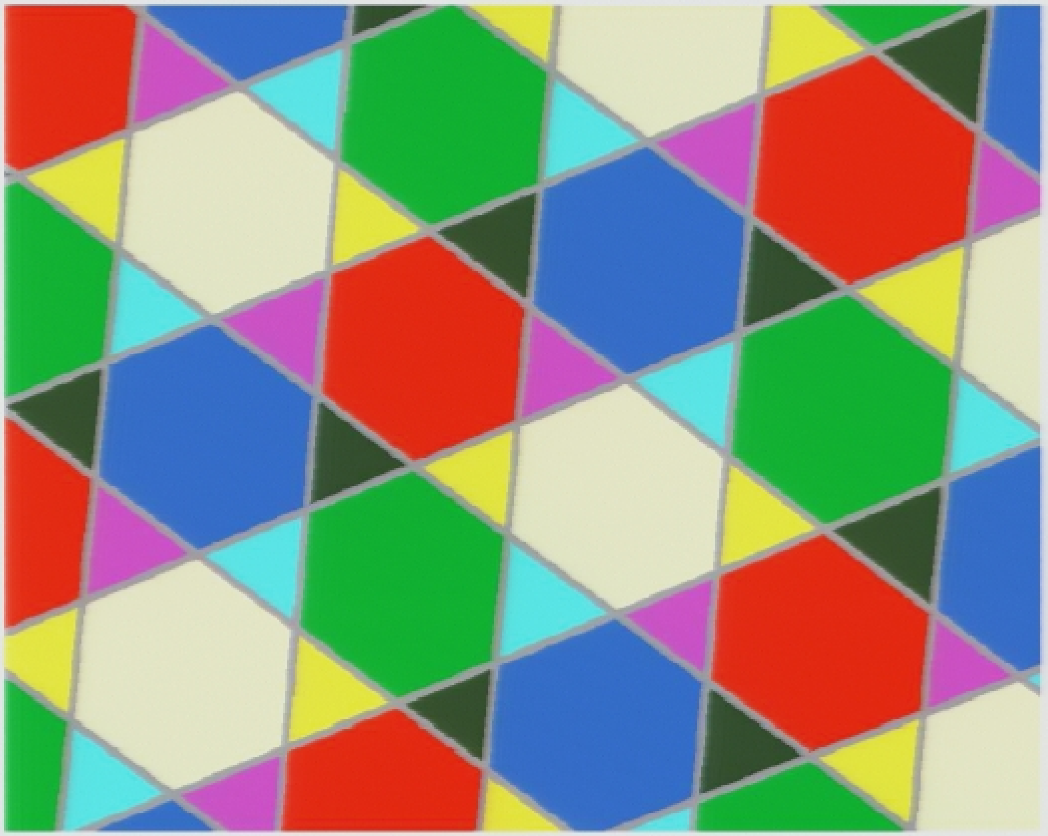


FIG. 1 – Illustration of 2 dimensional section through "even" (body centered cubic) lattice obtainable for system with eightfold vacuum as provided by broken O{ N } model with $\varepsilon > 0$.

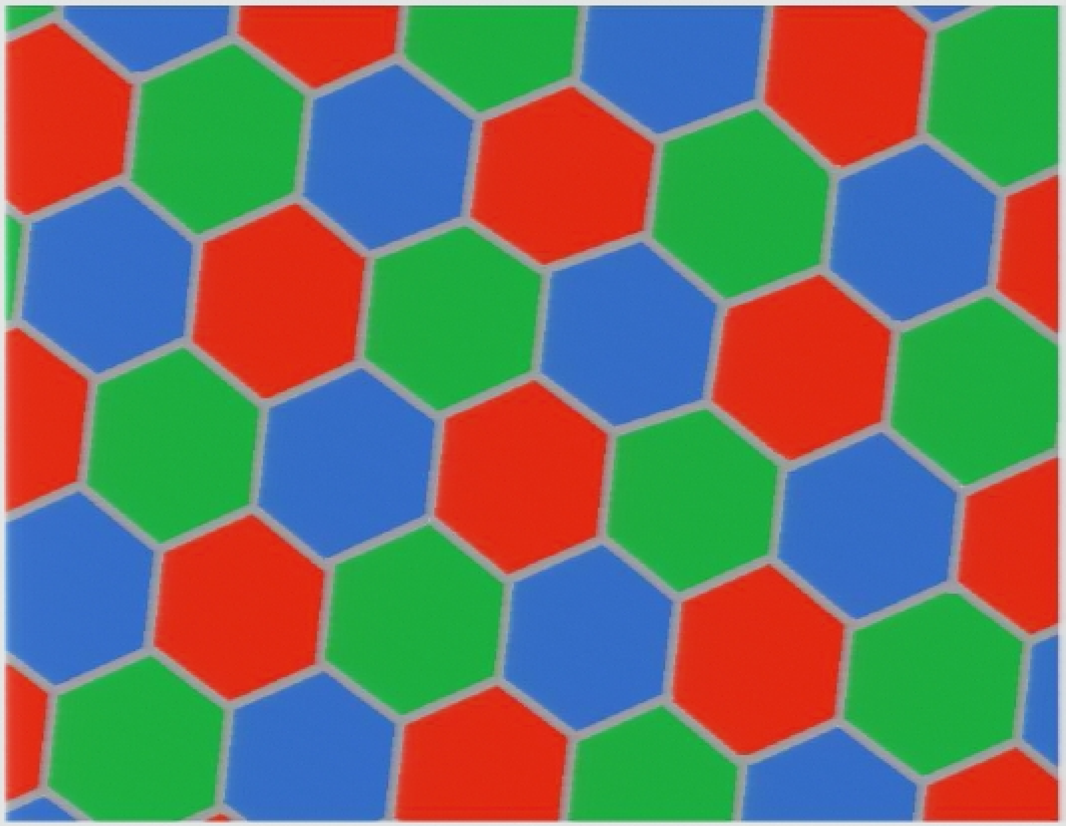


FIG. 2 – The simplest “odd” type (triply intersecting) example : hexagonal tiling for tricolored vacuum system in 2 dimensions.

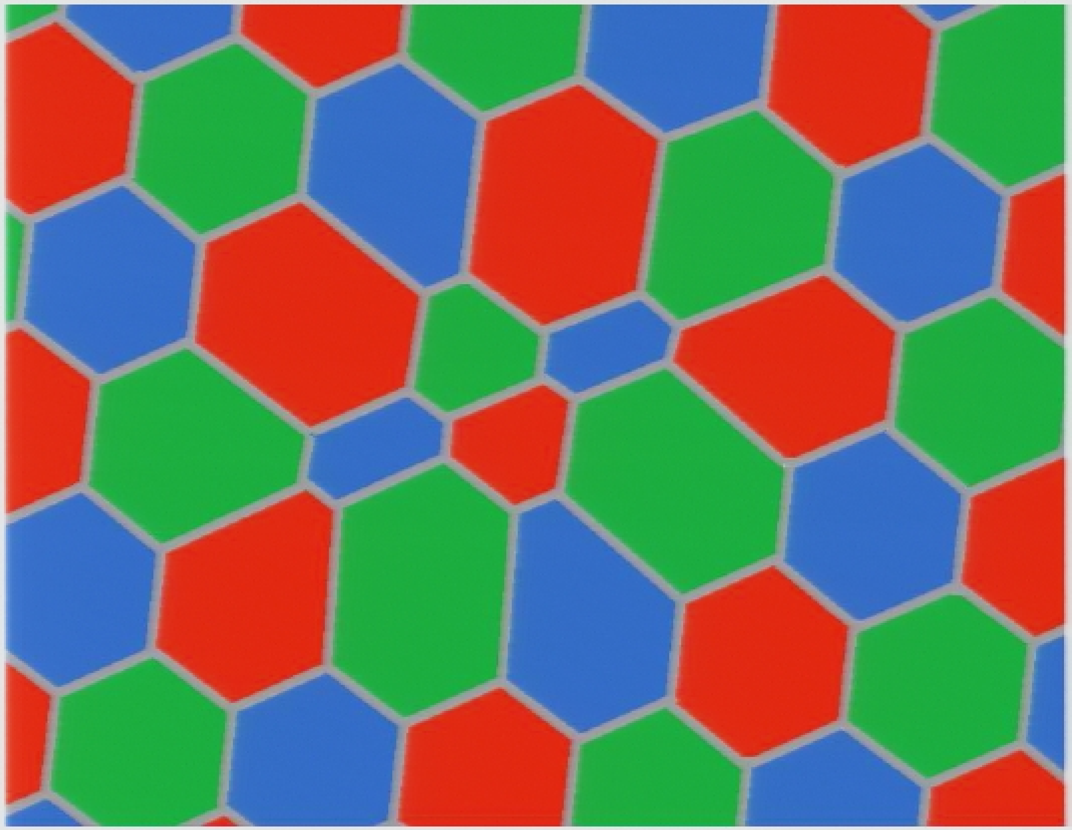
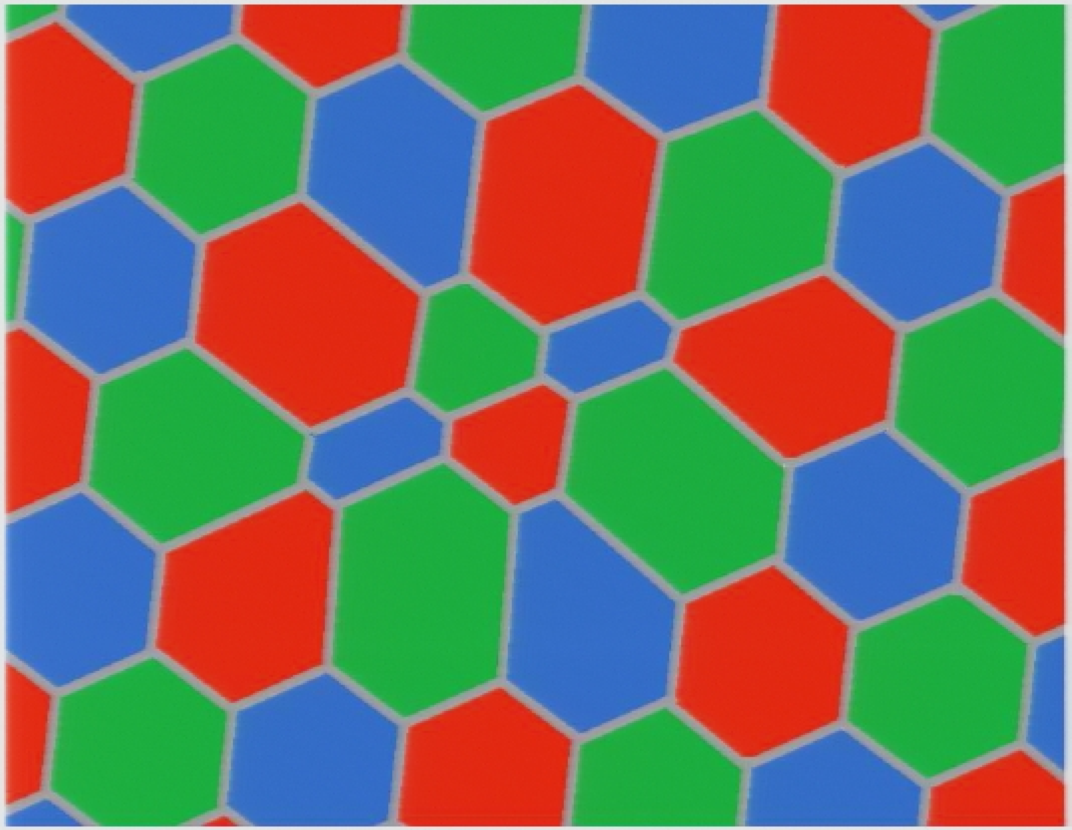


FIG. 3 – Local energy conserving deformation of "odd" type hexagonal tiling with Y form junctions in 2 dimensions.



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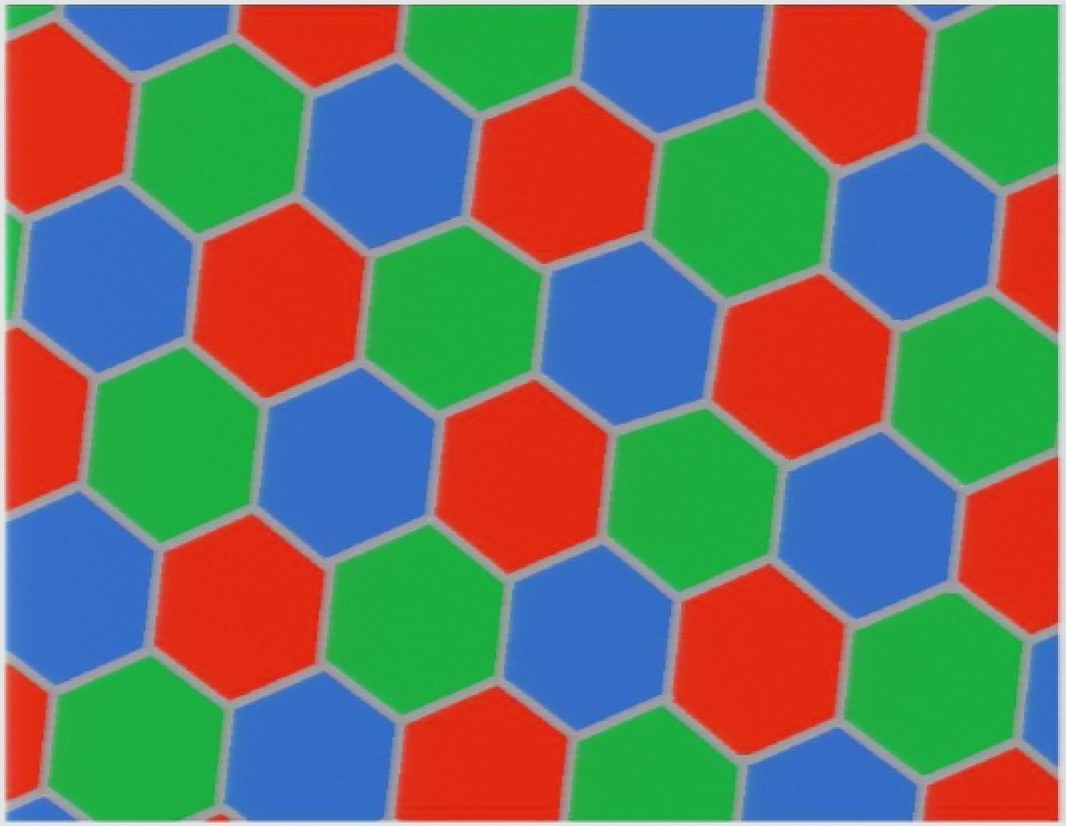

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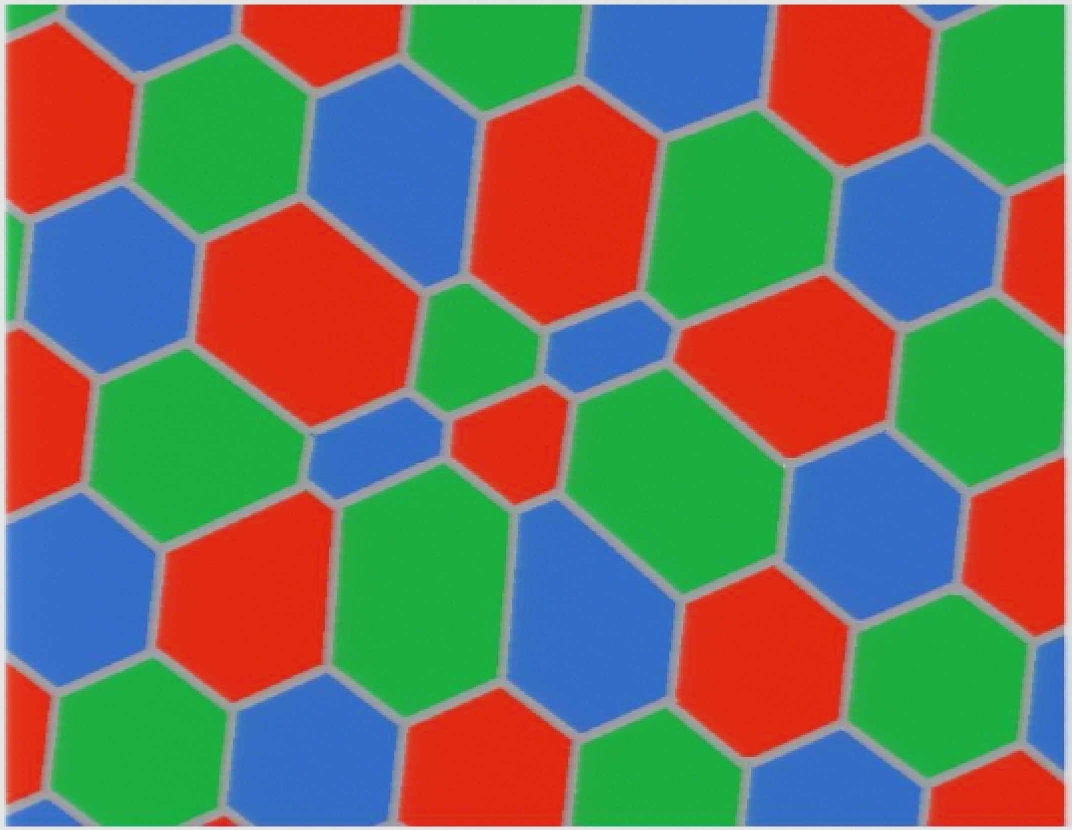

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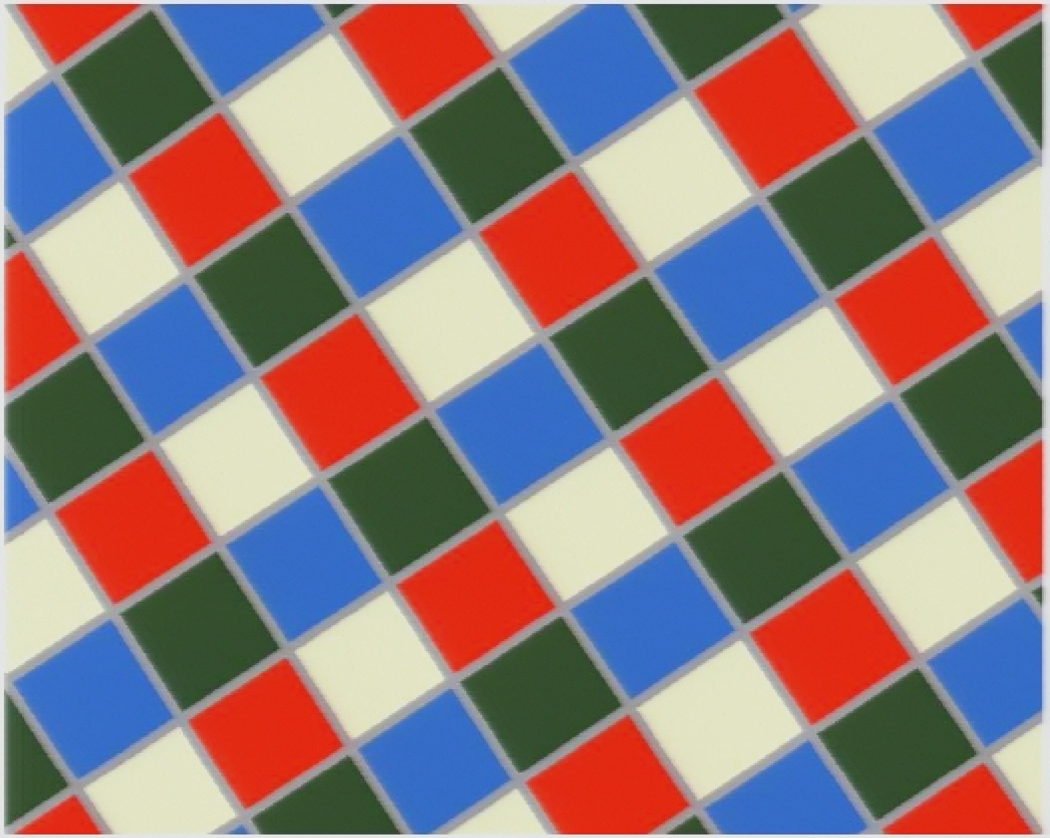


FIG. 4 – The simplest "even" type (straight crossing) example : "tartan" tiling for a system with fourfold vacuum in 2 dimensions.

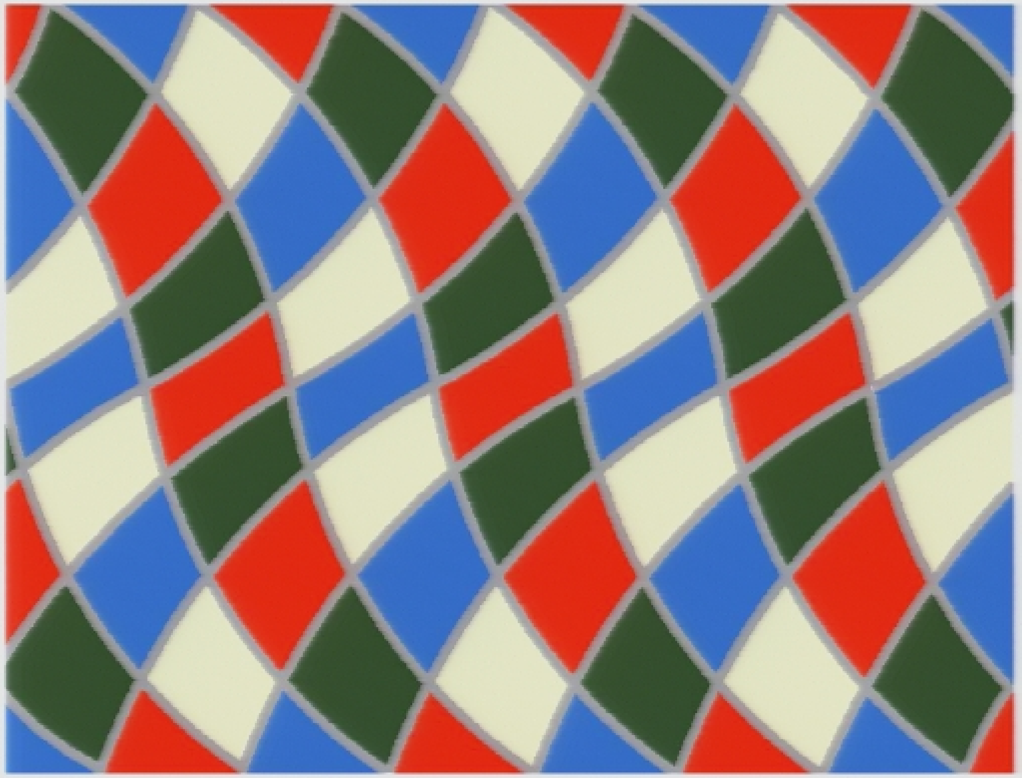


FIG. 5 – Effect of (vertically propagating) shear wave in "even" type system with X form junctions in 2 dimensions.

The preceding "even" examples involved an even number of vacua, but an "even" example with an odd number of vacua is obtainable from ordinary $U\{1\} \times U\{1\}$ model involving 4 real scalars that combine to form a pair of complex fields $\Phi_1 + i\Phi_2 = |\Phi| e^{i\phi}, \qquad \Psi_1 + i\Psi_2 = |\Psi| e^{i\psi}$, by adding a symmetry breaking term with coefficient $0 < \varepsilon < 1$, to the usual quartic potential to get $V \propto (|\Phi|^2 - \eta^2)^2 + (|\Psi|^2 - \eta^2)^2 + \varepsilon |\Phi|^2 |\Psi|^2 \Big(\cos(\psi + 2\phi) + \cos(2\psi - \phi) \Big)$. This has 5 degenerate minima where $|\Phi|^2 = |\Psi|^2 = \eta^2/(1-\varepsilon)$. Not only are these all equivalent but so are all 10 of their pair combinations. However only 15 of their 45 pairs of pairs are allowed to cross each other diagonally as shown in following regular tiling illustration.

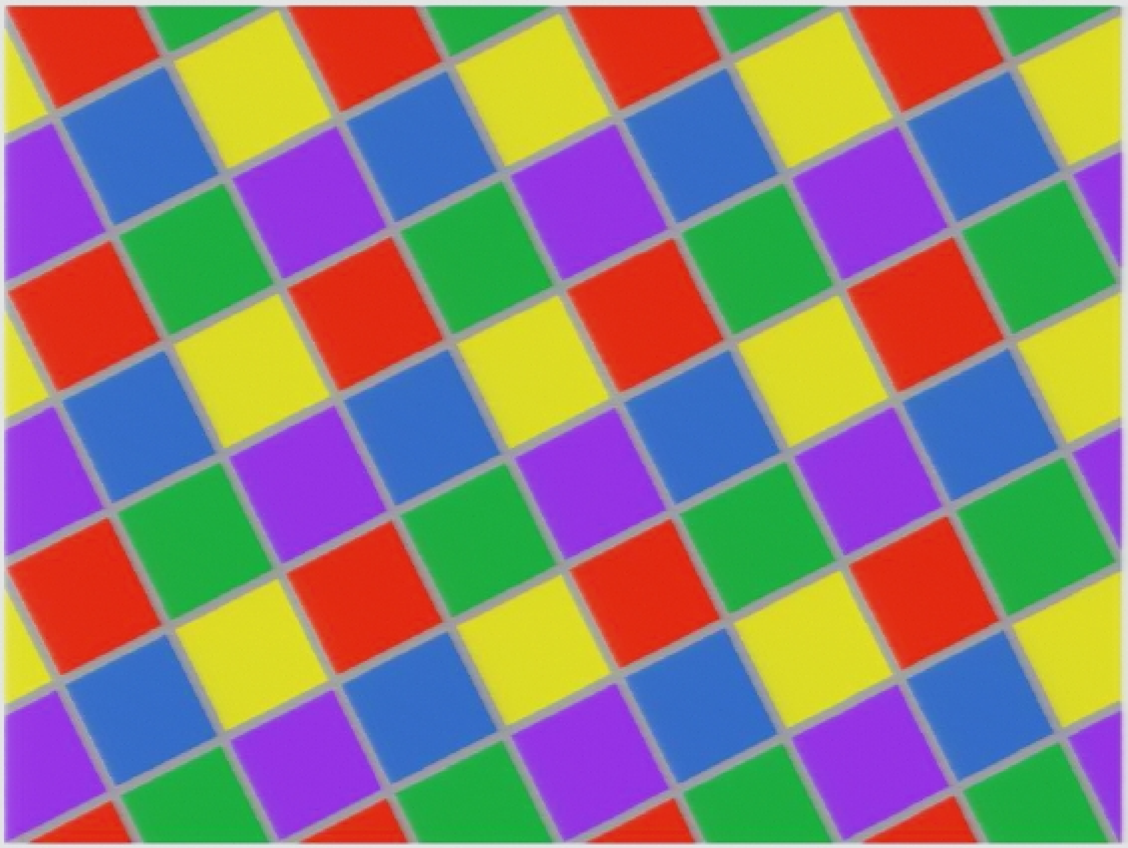


FIG. 6 – Periodic “even” type (straight crossing) example : tiling for a system with fivefold vacuum in 2 dimensions.

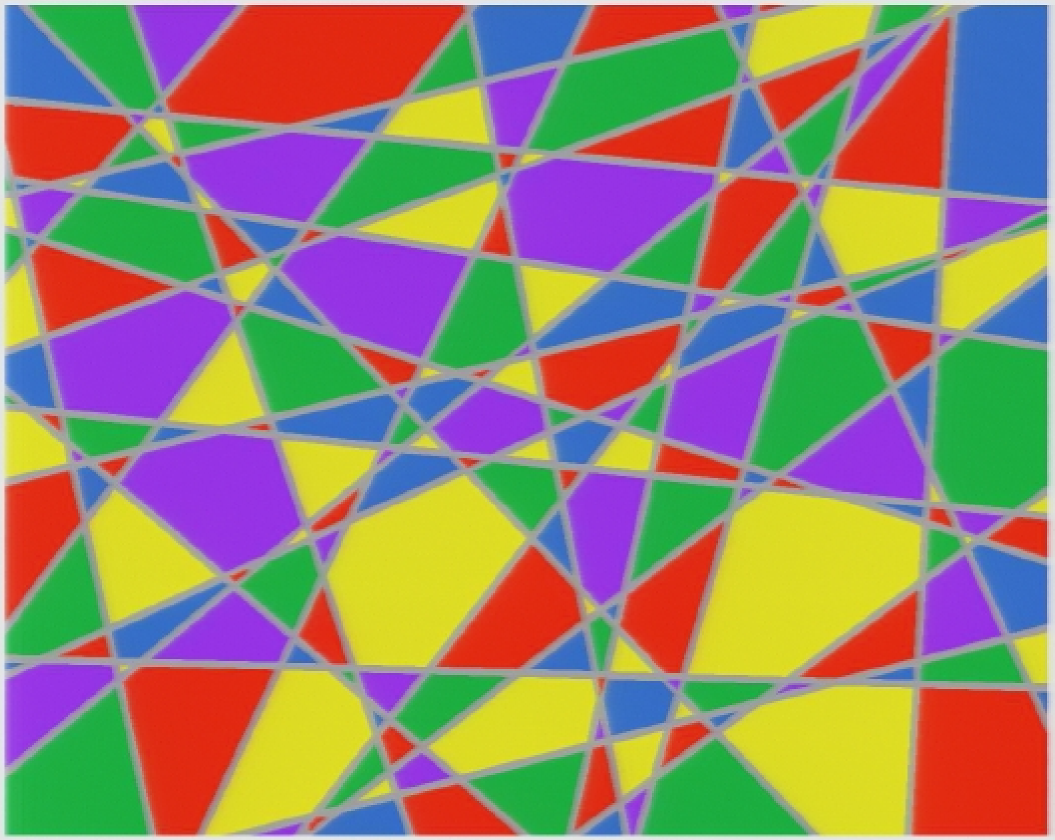
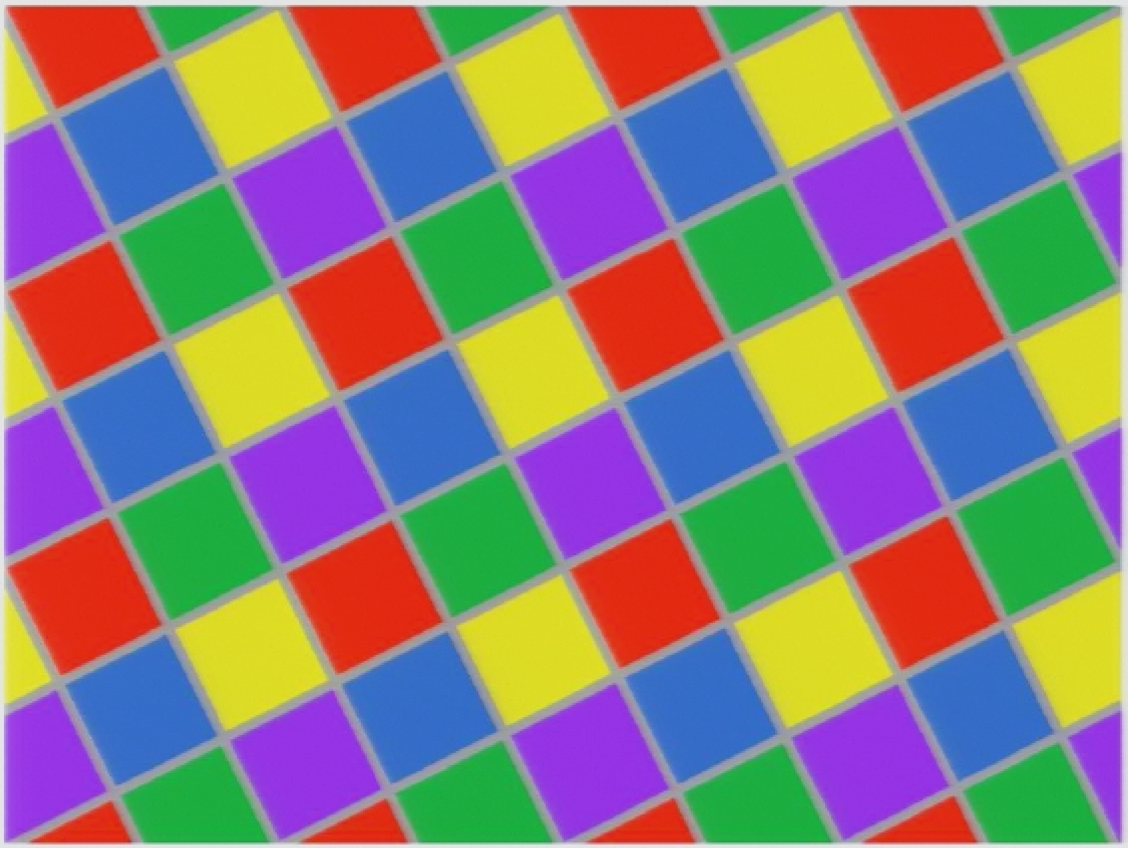


FIG. 7 – Disordered “even” type tiling satisfying conditions of simple 5 color conjecture for systems with X form junctions, but violating supplementary pair crossing restrictions.



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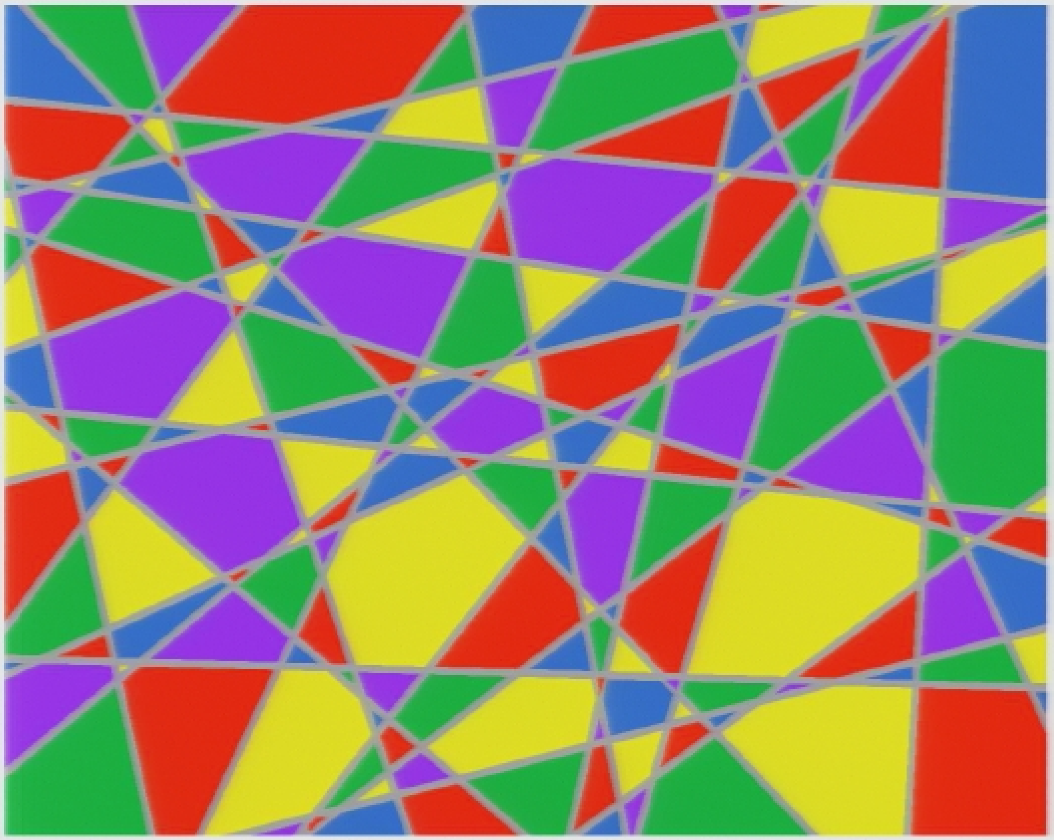

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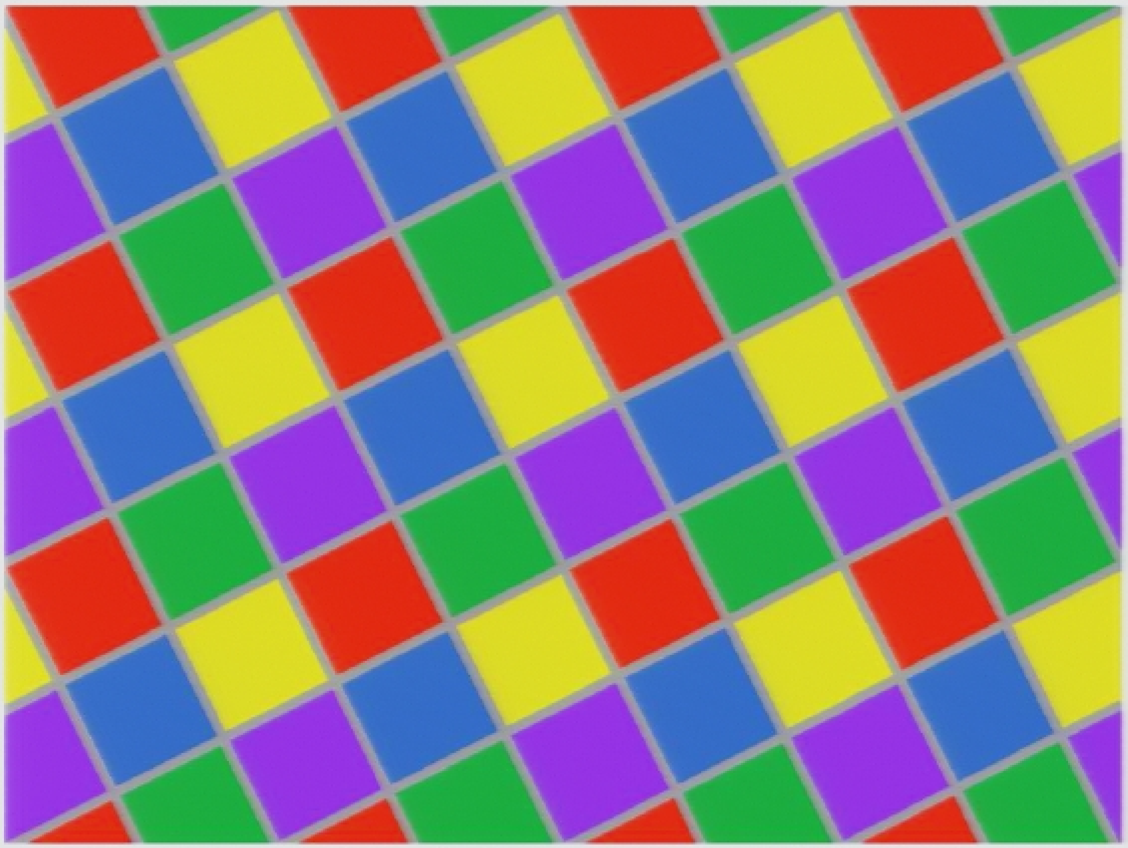

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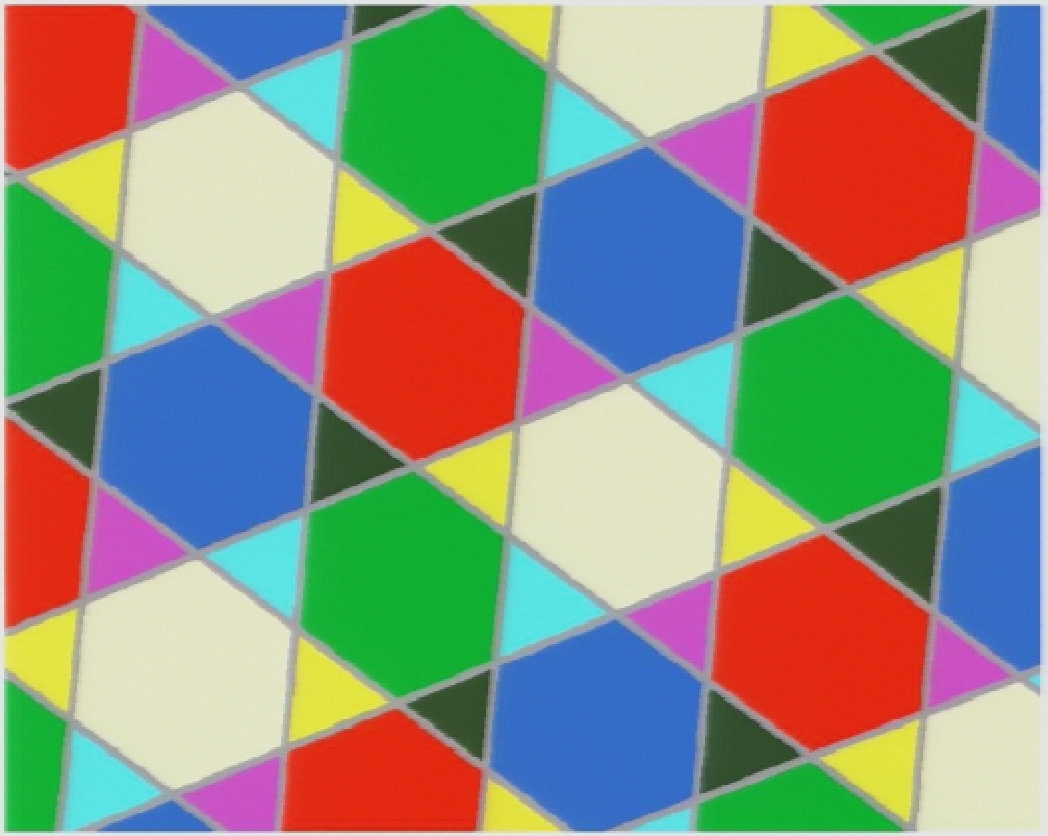

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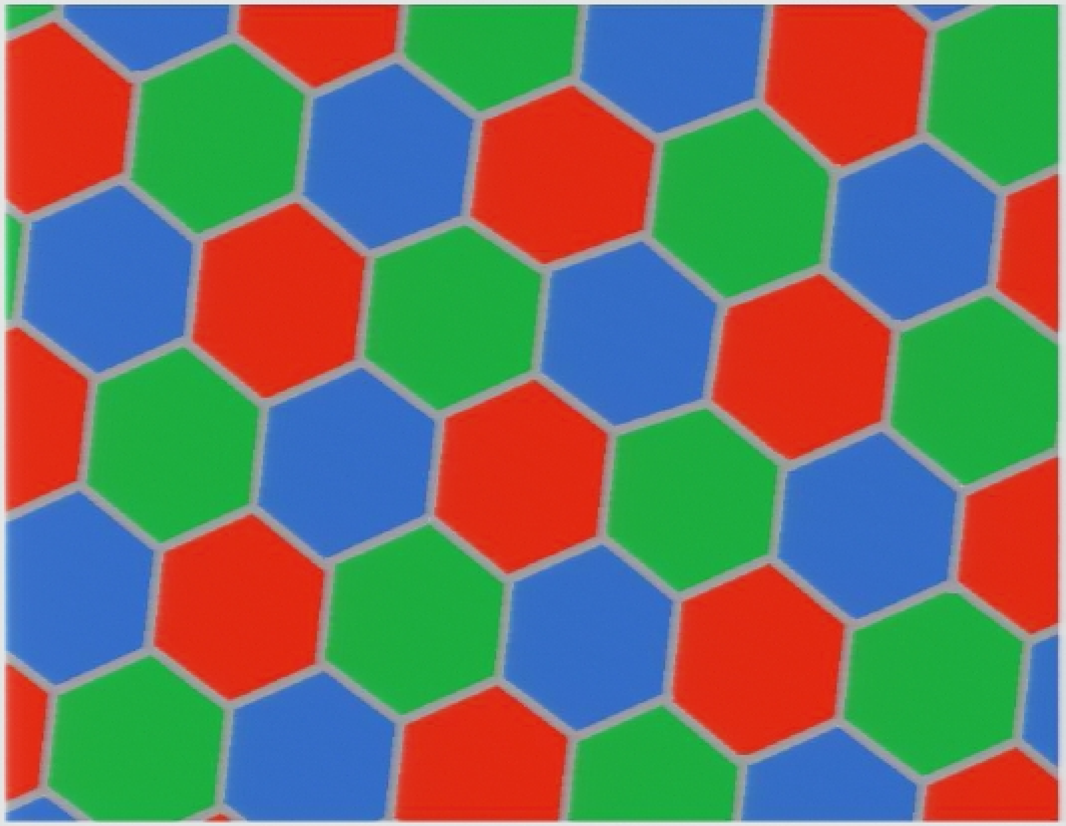

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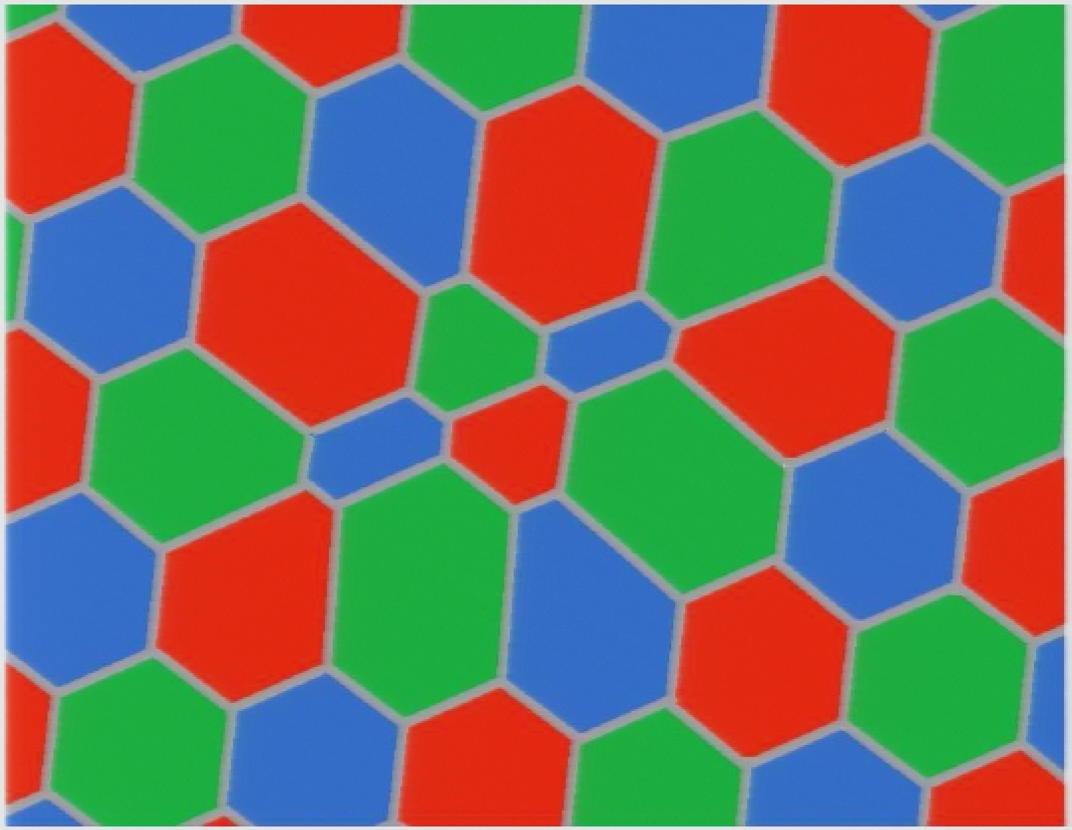

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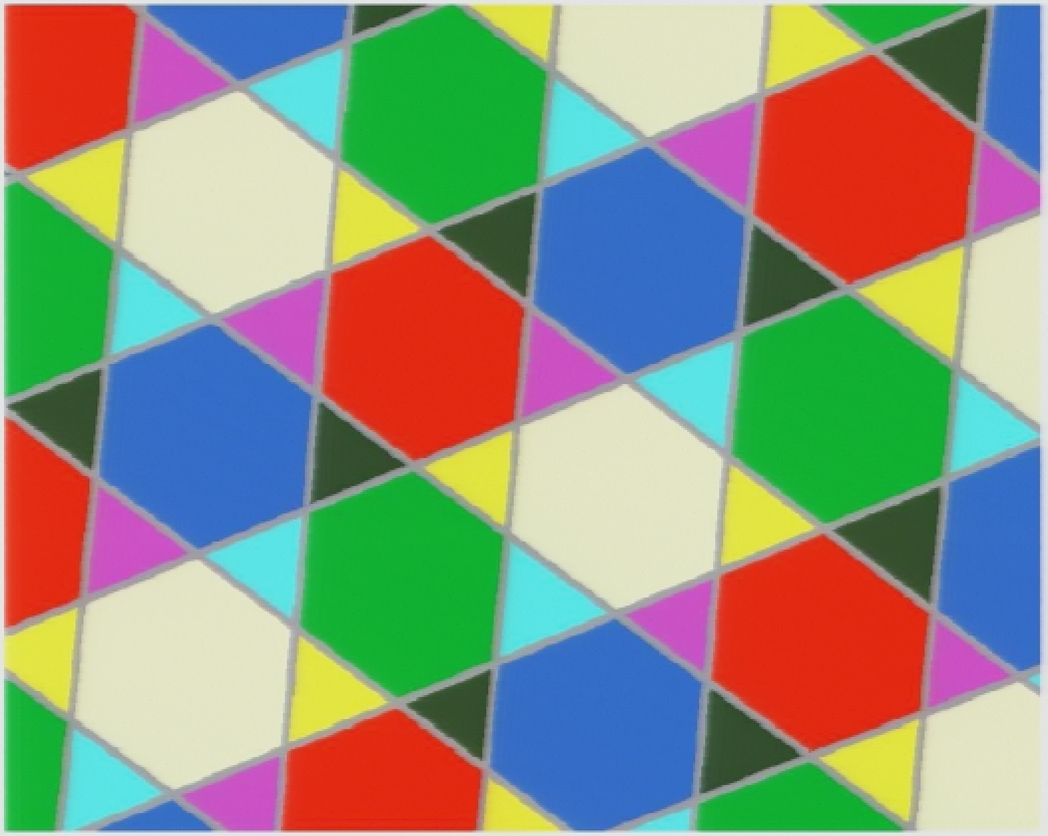

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