Title: Numerical Analysis of a Black String Horizon

Date: Mar 09, 2005 11:00 AM

URL: http://pirsa.org/05030101

Abstract: Using the numerical data of the UBC group simulation, an analysis is done of the properties of the horizon of an evolving black string. The results are consistent with pinch off in infinite affine parameter

Numerical treatment of a black string horizon

(DG, L Lehner, F Pretorius)
gr-qc/0414014, PRD in press

(1) black strings and GL instability

(2) pinch off, HM theorem
and HM argument

(3) Numerical simulation of black strings

(4) Numerical treatment of the horizon

(5) results

(6) conclusion

black strings and GL instability

(R. Gregory and R. Laflamme)
PRL **70**, 2837 (1993)

$$ds^2 = dz^2 + ds^2{}_{\rm Schw}$$

$$ds^2{}_{\rm Schw} = -F dt^2 + F^{-1} dr^2 + r^2 d\Omega^2$$

$$F = 1 - (2M/r)$$

to check for instability
perturb and look for
an exponentially growing mode

Black holes are stable

Black strings are unstable

black strings and GL instability

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what does an unstable black string
evolve to?

black holes have more entropy
than a black string.

perhaps a black string
pinches off into black holes

proof uses Raychaudhuri equation

$$d\theta/d\lambda + \tfrac{1}{3}\theta^2 + \sigma_{ab}\sigma^{ab} = 0 \ .$$

θ must be positive
so shrinking in the r direction
must be compensated by expansion
in the z direction.
This gives rise to shear which
tends to drive θ negative

HM argument

pinch off in infinite affine parameter is allowed

However, θ and σ_{ab} must be very small and very slowly varying functions of λ

HM conclude that pinch off does not happen.

Black string numerical simulation

(M Choptuik, L Lehner, I Olabarrieta,
R Petryk, F Pretorius, H Villegas)
PRD **68**, 044001 (2003)

$$ds^2 = (-\alpha^2 + \beta_A \beta^A) dt^2 + 2\beta_A dx^A dt$$

$$+\gamma_{AB} dx^A dx^B + \gamma_\Omega d\Omega^2$$

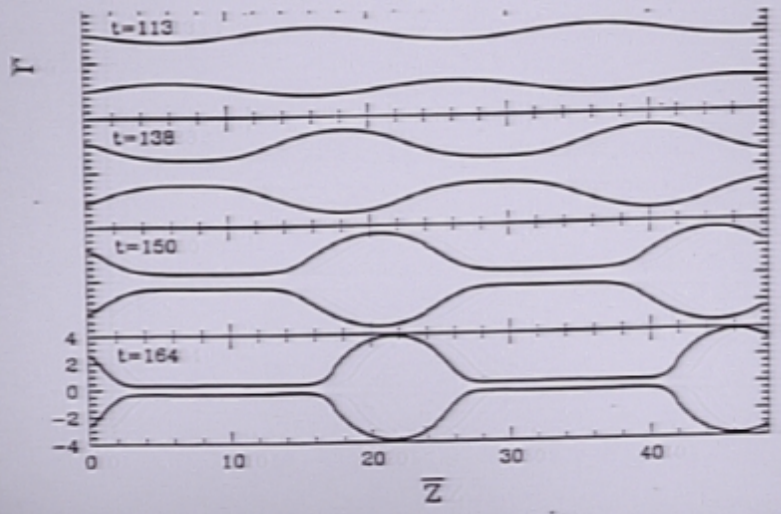
$x^A = (r, z)$
$\beta_A = \gamma_{AB} \beta^B$

ADM equations
α and β^A freely specified
second order evolution equations
for γ_{AB} and γ_Ω

Evolution starts from
a slightly perturbed straight string

The evolution goes well into the
nonlinear regime; but evenutally
the code crashes
(punchbowl effect)

At the end, the string looks like
black holes connected by
long thin strings
(i.e. it looks like it is in the
process of pinching off)



numerical examination of horizon

Find horizon as $r = R(t, z)$
Trace light rays, but horizon
is well approximated by
apparent horizon

Non-affinely parameterized geodesic
$k_a = \nabla_a (r - R)$

Find affinely parameterized geodesic
$l^a = e^{-\nu} k^a$
and affine parameter λ

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use x^a and y^a unit vectors
in 2-sphere and orthogonal directions.
Find A and B given by

$$A = 2x^a x^b \nabla_a l_b$$

$$B = -y^a y^b \nabla_a l_b$$

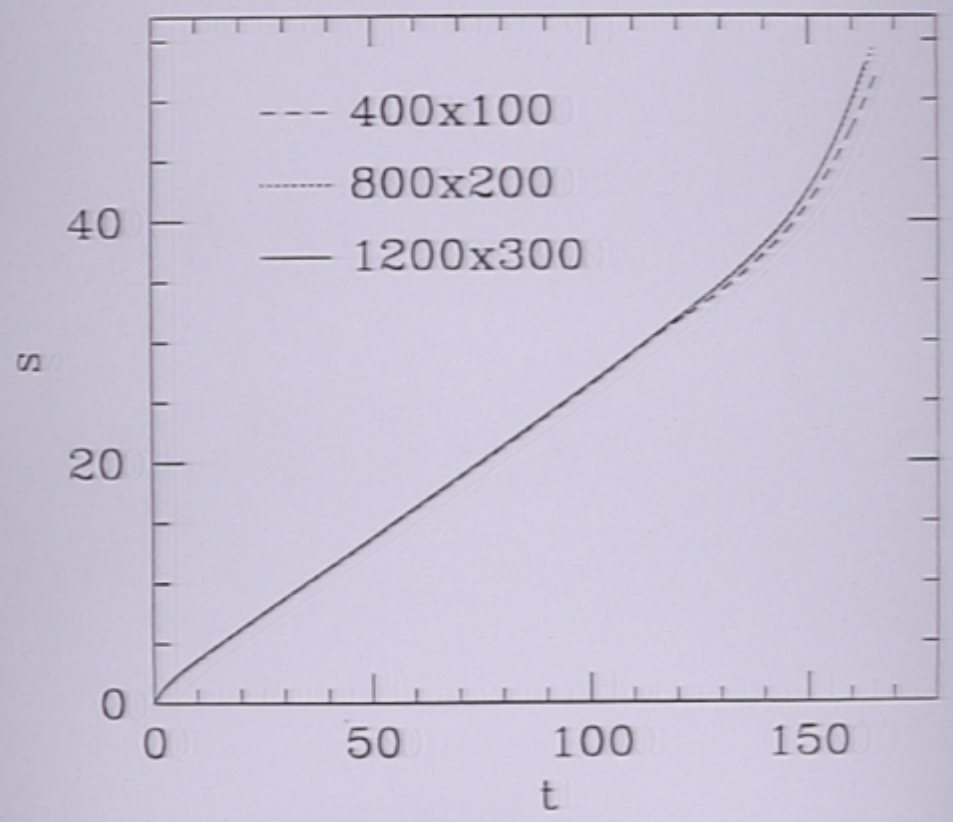
Expansion and squared shear given by

$$\theta = A - B$$

$$\sigma^{ab}\sigma_{ab} = \tfrac{1}{6}(A + 2B)^2$$

Introduce rescaled quantities
s, $\tilde{\theta}$, $\tilde{\sigma}_{ab}$

$$s = \ln \lambda$$
$$\tilde{\theta} = \lambda\theta$$
$$\tilde{\sigma}_{ab} = \lambda\sigma_{ab}$$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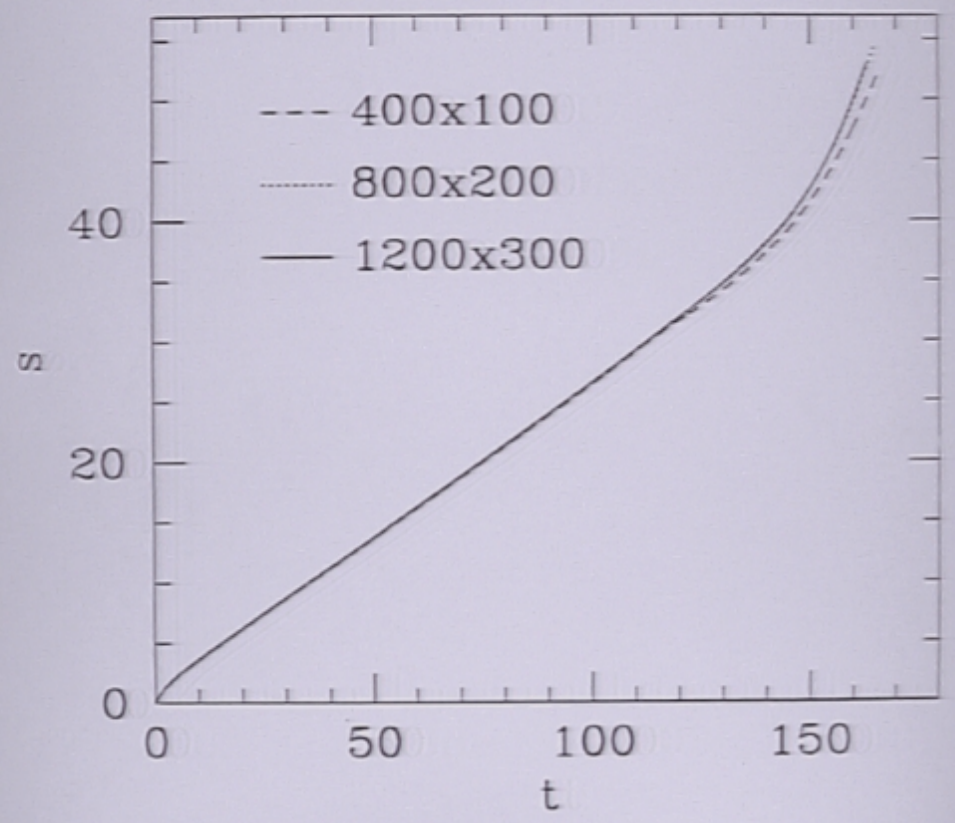
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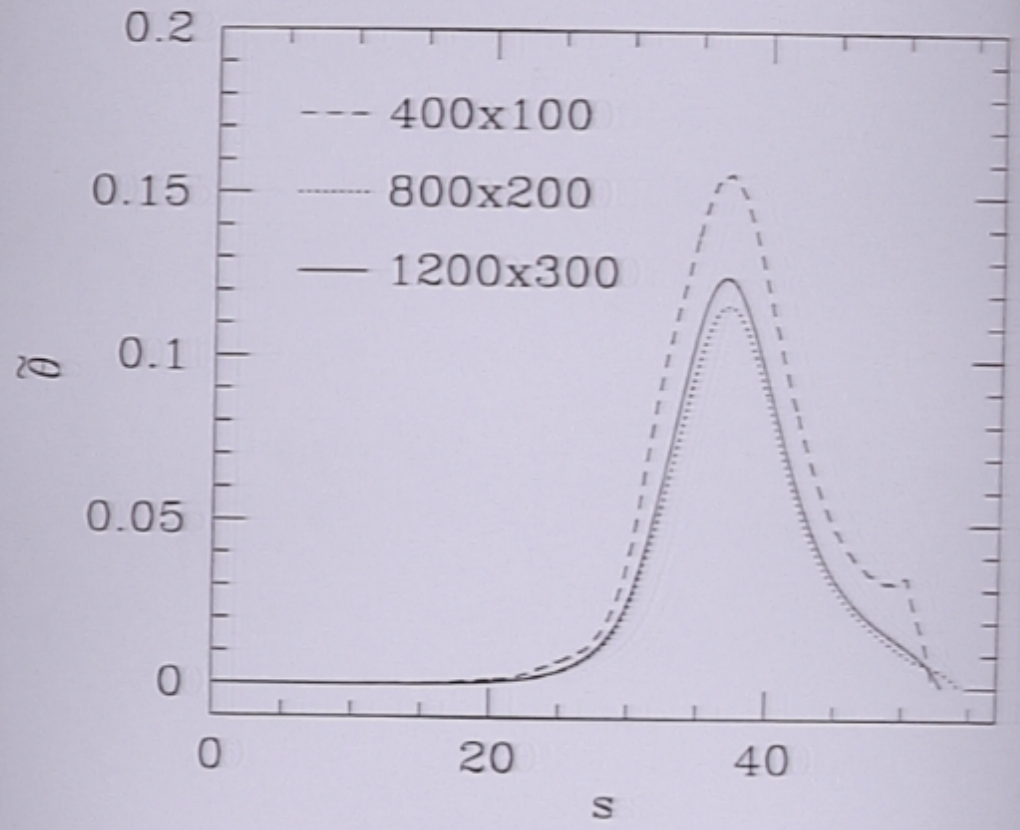
At the end of the simulation
the affine parameter has reached 10^{21}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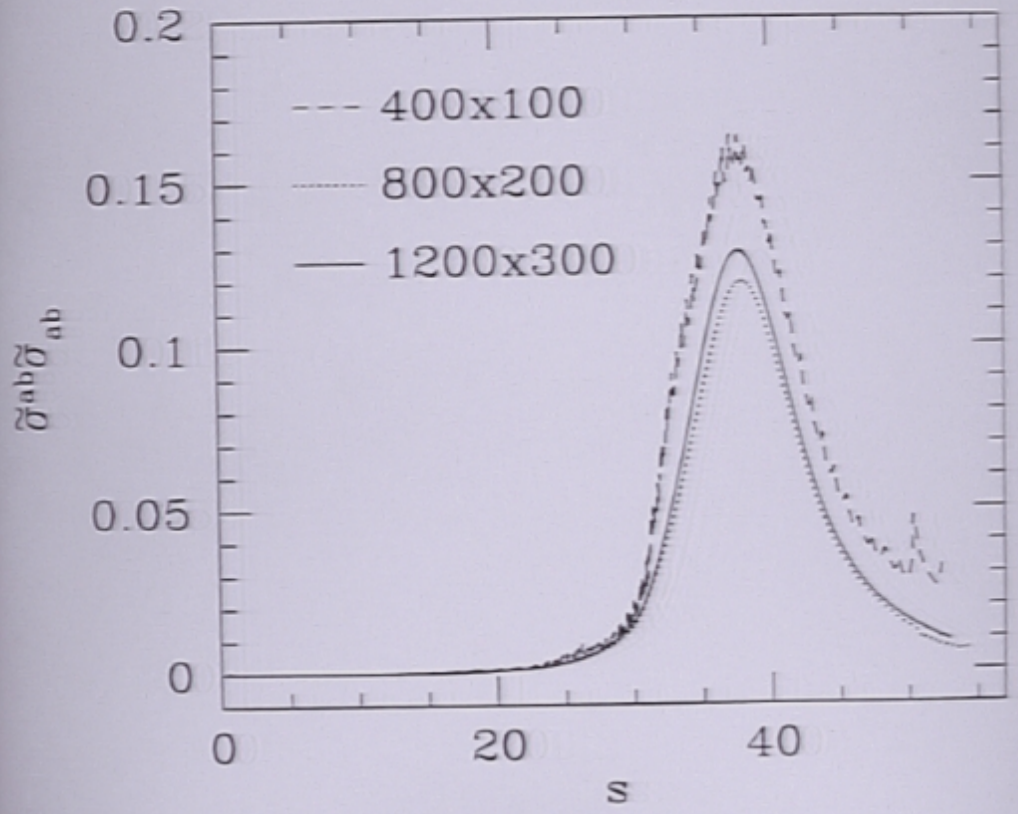
0.2
0.15
0.1
0.05
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40
s
$\tilde{\sigma}^{ab}\tilde{\sigma}_{ab}$
400x100
800x200
1200x300

At the end of the simulation
θ and σ_{ab}
are of order 10^{-22}.

This sort of behavior is
just what HM say is needed
for pinch off in infinite affine parameter

In retrospect this is not surprising.
In Schwarzschild, the affine parameter
is $\lambda \propto \exp(v/(4M))$
and the black string starts out as
Schwarzschild times a circle.

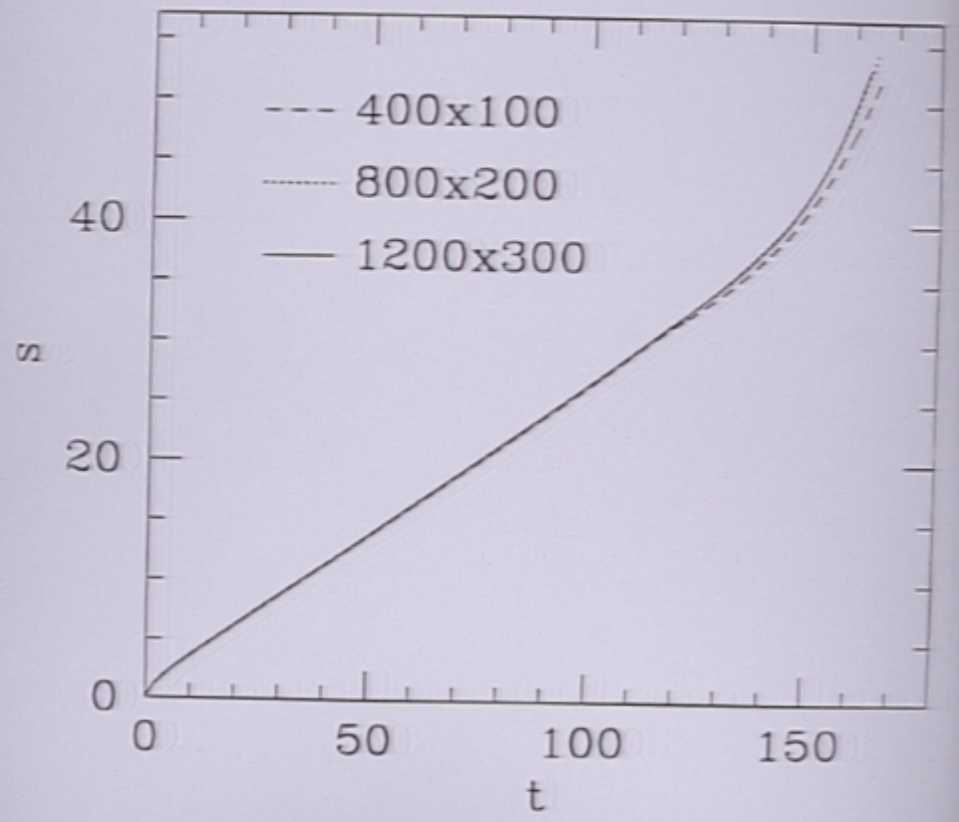
Thus, log of affine parameter is
a more natural dynamical time
than affine parameter, and rescaled
expansion and shear are more natural
dynamical quantities than
expansion and shear

Conclusions

Pinch off in infinite affine parameter
is possible and may be
the most likely outcome

The pinch off could occur at finite
simulation time even if it occurs
at infinite affine parameter

But better numerical simulations
are needed to resolve the issue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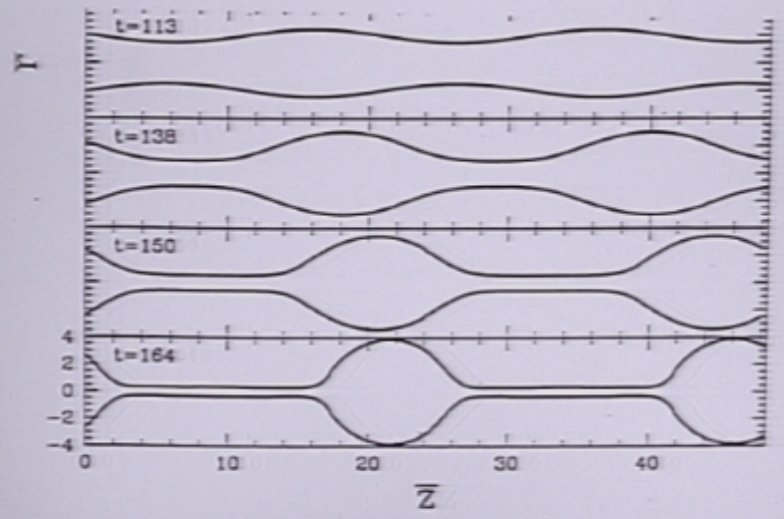

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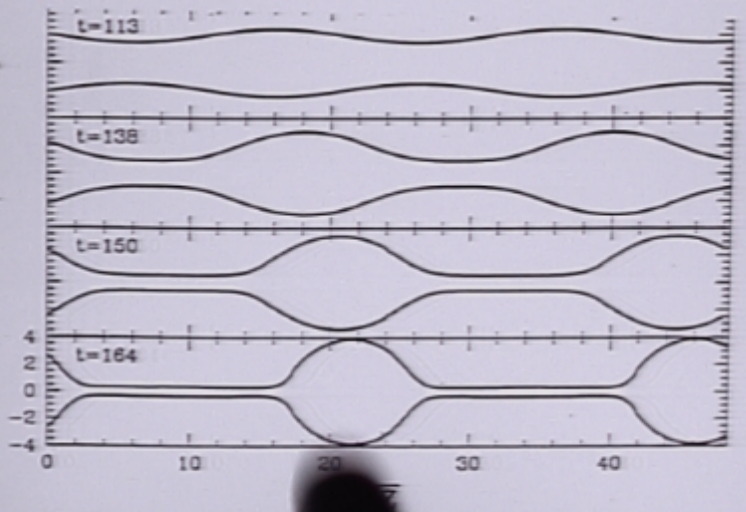

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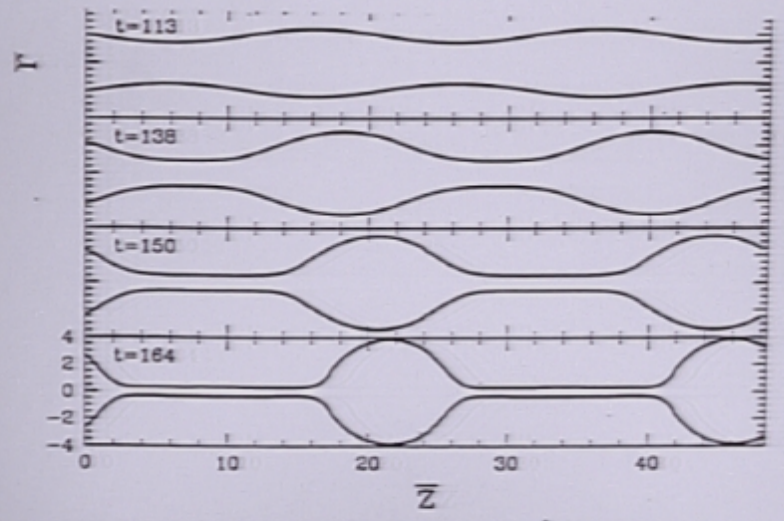

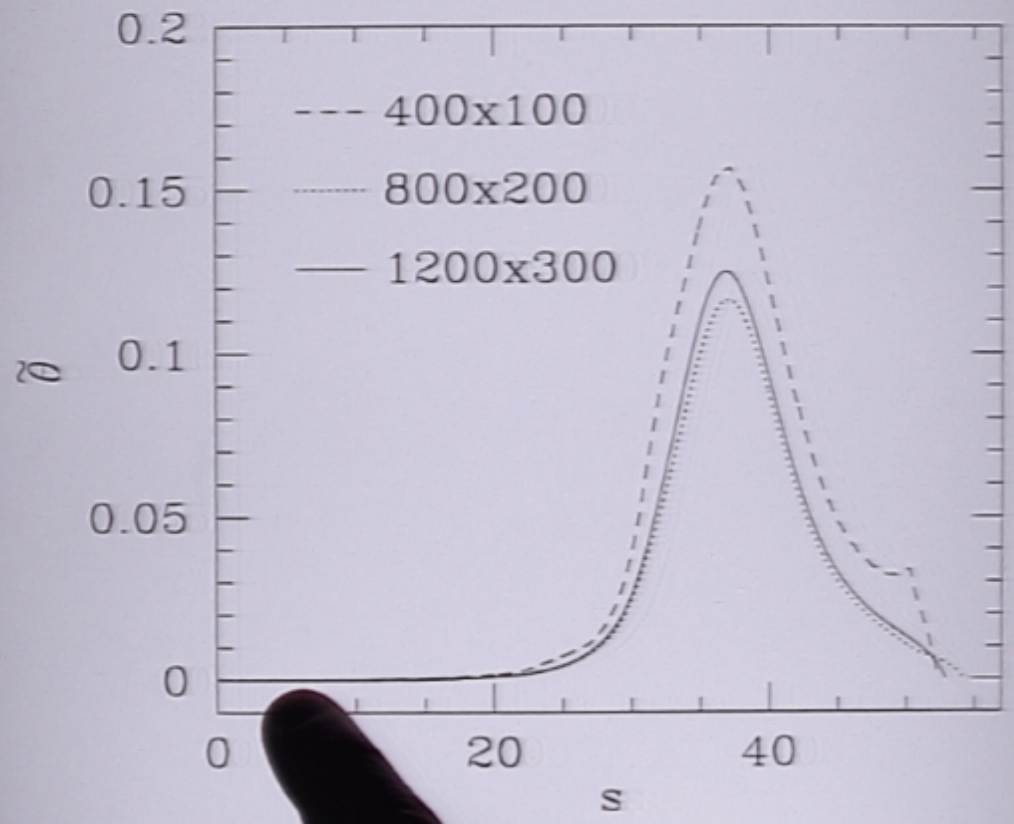
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θ
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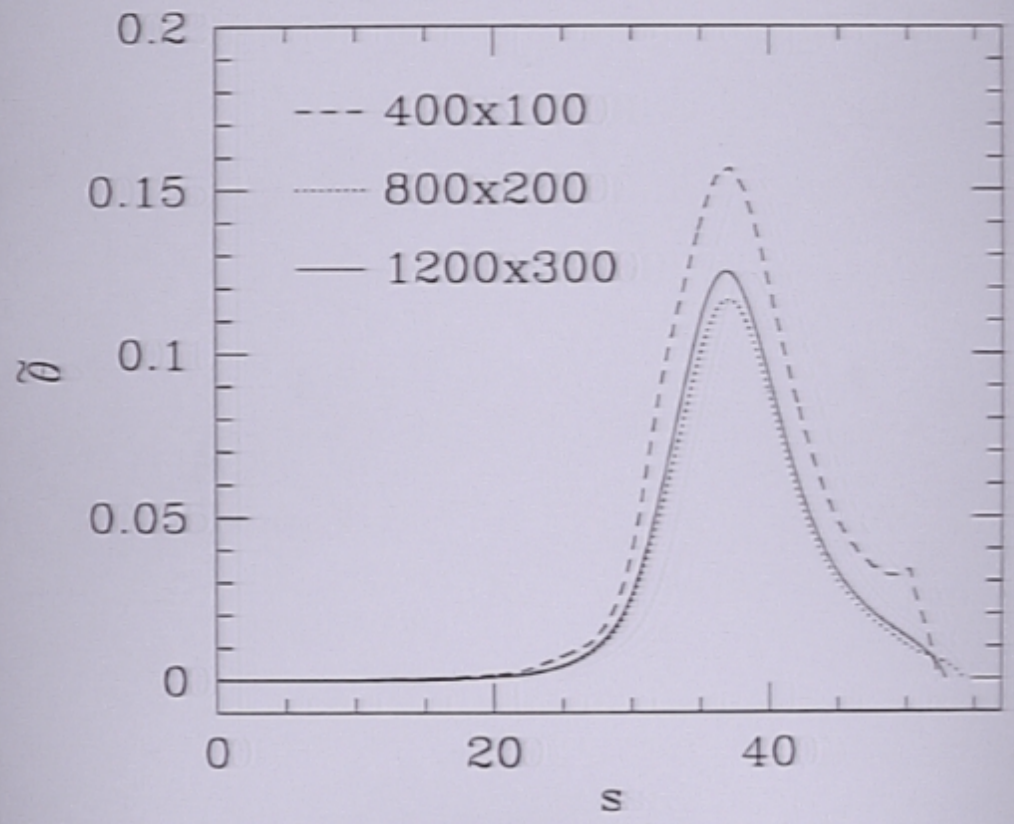
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