Title: Interpretation of Quantum Theory: Lecture 17

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Abstract:

Consistent/Decoherent Histories

- Conceptual difficulties in QM come from introducing probabilities in the wrong way

- Histories approach: consistent introduction of probabilities eliminates difficulties and resolves (tames) paradoxes

- History of histories:
  - Griffiths 1984
  - Omnès 1987
  - Gell-Mann and Hartle 1990
  - Many subsequent papers, books
  - Griffiths, CONSISTENT QUANTUM THEORY (Cambridge 2002)

↕

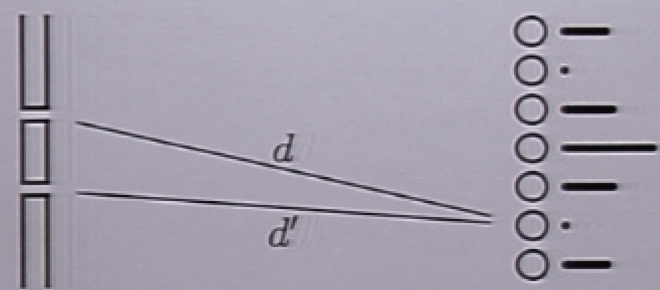
first 12 chapters at

http://quantum.phys.cmu.edu

Histories and Paradoxes

- Paradoxes that are resolved/tamed using histories:
  - Einstein, Podolsky, Rosen
  - Double slit
  - Bell, Kochen, Specker
  - Greenberger, Horne, Zeilinger
  - Hardy
  - Aharonov and Vaidman multiple box
  - Wheeler delayed choice
  - Elitzur and Vaidman noninteracting measurement
  - ...
- Paradoxes that are not resolved using histories:
  - Any suggestions?

Double Slit I

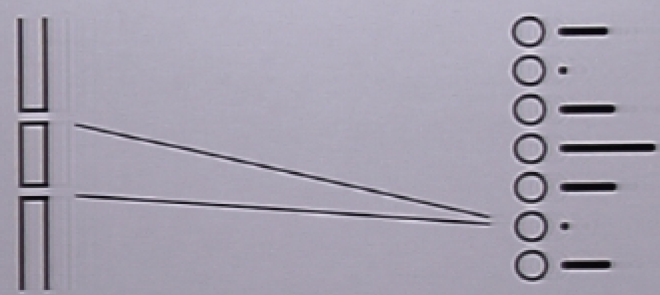


- Slit system, detectors in interference region
  - Horizontal bars: counting rates
  - Interference depends on *difference* $d - d'$, so
    – particles pass through slits *coherently*
  - Particles arrive *randomly* at detectors
- Consistent histories
  - Randomness an intrinsic part of nature
  - Anti-Einstein. There are no hidden variables

Histories and Measurements

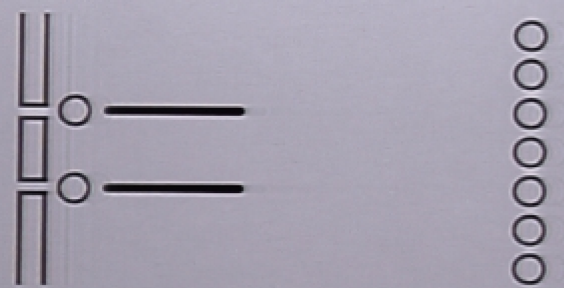
- Textbook QM:
  - Randomness arises through *measurements*
- Histories:
  - Randomness *intrinsic* in QM
  - Measurements are examples of physical processes
  - Same quantum principles govern *all* processes
  - There is *no* classical world, apparatus
  - *Sometimes* classical mechanics is a good approx
  - Quantum principles determine those circumstances

Double Slit II



- Experimentalist:
  - Detector triggers *because* particle arrives
  - Just *before* detection particle was near detector, on its way to detector
- Historian:
  - Good experimentalists know what they're doing
  - Triggered detector indicates arrival of particle
  - QM justifies this talk; indicates its limitations
  - Theorists should not bully competent people!

Double Slit III



- Detectors directly behind slits
  - Particles arrive at random
  - Total counting rate same as before
  - *One* detector, not both, detects each particle
- Explanations
  - Experimentalist:

Particle came through slit preceding detector—
Collimators work this way

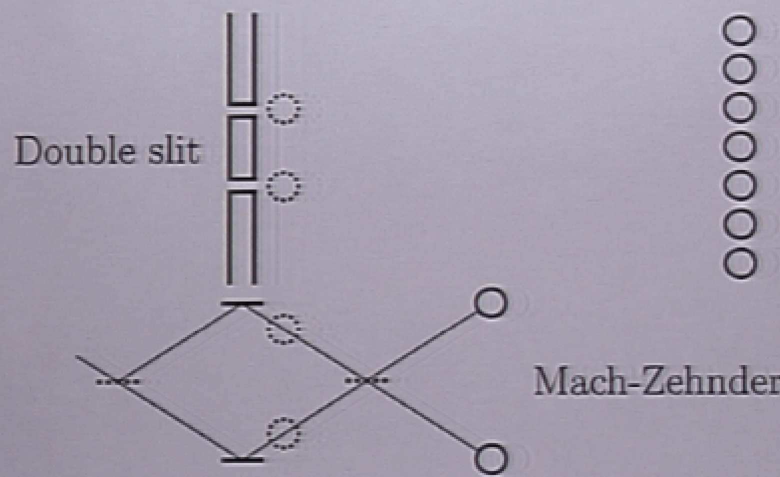
  - Textbook:

Cannot discuss what happened before measurement
"Great Smoky Dragon"

  - Historian:

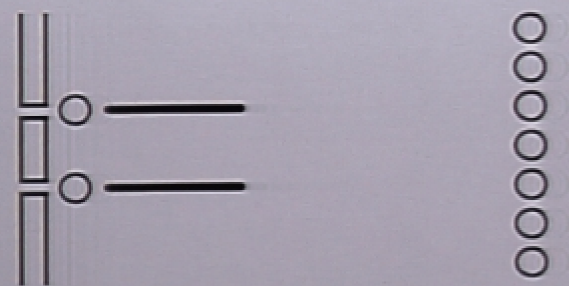
QM supports experimentalist account

Double Slit + Mach-Zehnder



- Correspondences:
  - Which slit? ↔ Which arm?
  - Detectors behind slits ↔ inside interferometer
  - In interference zone ↔ following 2d beam splitter
- For precise description, use Mach-Zehnder
  - Basic idea applies to double slit

Double Slit III



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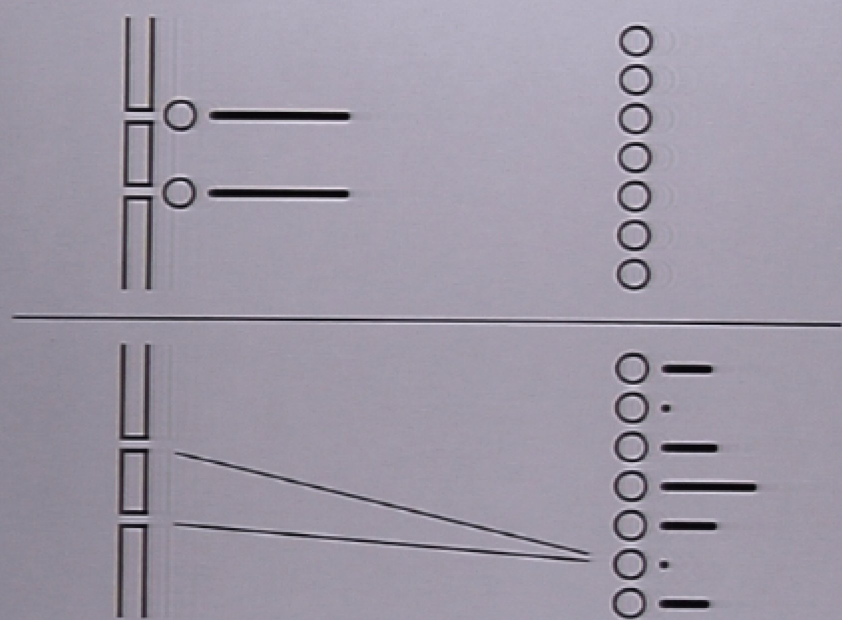
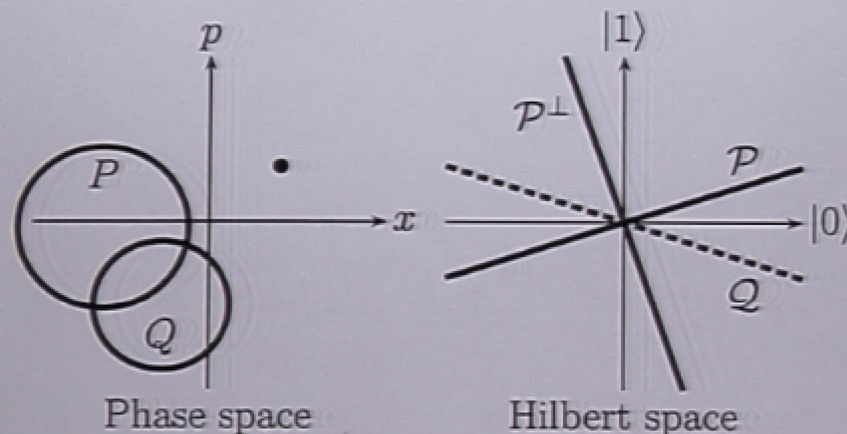
Double Slit IV



- Detectors behind slits removed at the last moment
  - Detectors remain:
    - particle came through definite slit
  - Detectors removed:
    - particle passed through slits coherently
- Particle could enter slit system before decision to remove detectors was made! (Wheeler delayed choice)
  - Does the future influence the past?

Phase Space and Hilbert Space



	Classical	Quantum
Physical state	Point	Ray
Property P	Subset P	Subspace $\mathcal{P}$
NOT P	Compl. $\sim P$	Orthog. compl. $\mathcal{P}^\perp$
P AND Q	$P \cap Q$	?

Spin Half Particle

- $S_z = +1/2$ is a physical state
  - Ray in Hilbert space. Point on Bloch sphere
- $S_z = -1/2$ is negation of $S_z = +1/2$
  - Orthogonal ray. Antipode on Bloch sphere
- For any spin-half particle,
  - Either $S_z = +1/2$ or $S_z = -1/2$, not both
  - Stern-Gerlach measurement shows which is the case

- Nothing special about z. The x axis is just as good.
- For any spin-half particle,
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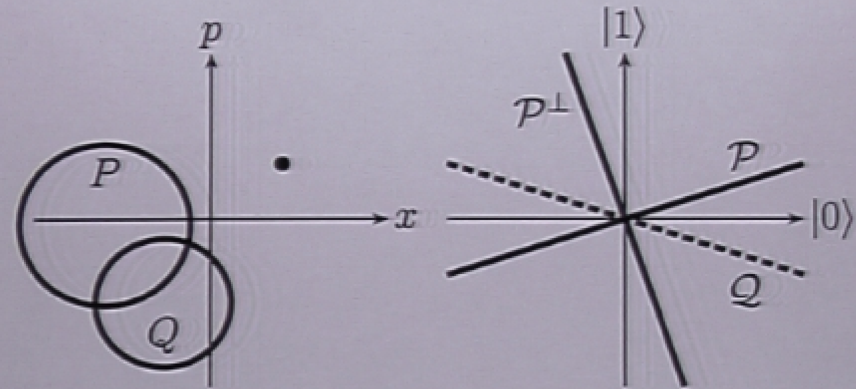
- $S_z = +1/2$ AND $S_x = +1/2$ is *meaningless*:
  - Hilbert space QM assigns it no meaning
    - No corresponding ray in the Hilbert space
  - No experiment which can measure it
    - Because it is not there!

Phase Space and Hilbert Space



Phase space   Hilbert space

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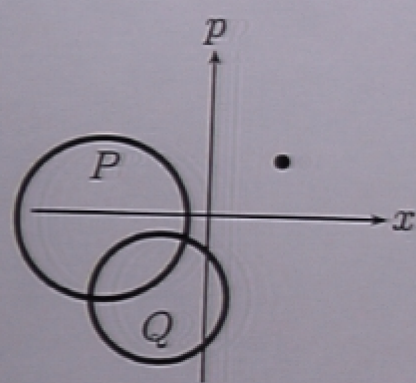
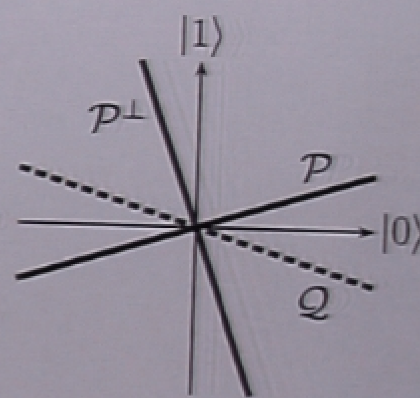


Phase space



Hilbert space

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Logic of Quantum Properties

- False statement is one whose negation is true
  - "Pennsylvania is a Canadian province"

- Meaningless statement: not formed according to rules governing proper use of the language
  - Example: "$P \wedge \vee Q$"
  - Negation of meaningless statement is meaningless

- Classical physical system:
  - Meaningful to combine two properties with AND
  - "The position is..." AND "The momentum is..."

- Quantum physical system:
  - Use AND only with *compatible* properties
  - Compatible: projectors commute: $PQ = QP$
  - $S_z = +1/2$, $S_x = +1/2$ are *incompatible*

Quantum Logic

- George Birkhoff and John von Neumann, Ann. Math. 37 (1936) 823, "The Logic of Quantum Mechanics"
  - $S_z = +1/2$ AND $S_x = +1/2$ is *meaningful, false*
  - Must modify rules of logic:

$$A \vee (B \wedge C) \neq (A \vee B) \wedge (A \vee C)$$
$$A \wedge (B \vee C) \neq (A \wedge B) \vee (A \wedge C)$$

- Consistent histories recognizes logical problem
  - But solves it in a different way
  - $S_z = +1/2$ AND $S_x = +1/2$ is *meaningless*

- Rules of logic remain unchanged, but one must
  - Recognize and exclude meaningless statements

- Single framework rule:
  - Meaningful quantum descriptions use a *single collection* of mutually compatible properties
  - Incompatible descriptions cannot be combined!

- Spin half
  - Can discuss S_z, which is $+1/2$ or $-1/2$
  - Can discuss S_x, which is $+1/2$ or $-1/2$
  - Cannot *combine* these discussions
    - Doing so makes no sense in Hilbert space QM

Probabilities I

- Standard (textbook) probability theory: $(\mathcal{S}, \mathcal{E}, \mathrm{Pr})$
- Sample space $\mathcal{S}$ of *mutually-exclusive possibilities*
  - One and only one occurs in a given experiment
  - Examples:
  - $\{H, T\}$ for coin toss
  - $\{1, 2, 3, 4, 5, 6\}$ for roll of die
- Event algebra $\mathcal{E}$.
  - Assume $\mathcal{S}$ discrete; $\mathcal{E} =$ all subsets of $\mathcal{S}$
- Probability distribution Pr
  - To each s_i in $\mathcal{S}$ assign $p_i = \mathrm{Pr}(s_i) \geq 0$; $\sum_i p_i = 1$.

- Quantum mechanics: three options for probabilities

(i) Use standard theory; (ii) Invent new one;
(iii) Become confused (very popular option)

- Consistent histories uses *standard* probability theory
  - There are two tasks:
  - Define quantum sample space $\mathcal{S}$
  - Introduce probabilities Pr

Probabilities II

- Example of spin half
- $S_z = +1/2, -1/2$ are mutually exclusive possibilities
  - If one is true, the other is false
  - One, only one occurs in Stern-Gerlach experiment
  - They constitute the S_z sample space
- Likewise, $S_x = +1/2, -1/2$ constitute S_x sample space
- S_z and S_x sample spaces are *incompatible*
  - Events cannot be combined
  - Probabilistic inference cannot be combined
- WARNING!
  - *Incompatible* is a *quantum* concept
  - *Mutually Exclusive* is classical or quantum
  - Do not confuse the two!

Probabilities III

- General structure of quantum sample spaces
- Decomposition of the identity in projectors $\{P^j\}$
  - (Superscript is label, not power)
  - $P^j = (P^j)^\dagger, \quad P^j P^k = \delta_{jk} P^j, \quad I = \sum_j P^j$
  - Each $P^j \leftrightarrow$ physical property (Hilbert subspace)
  - $j \neq k \Rightarrow P^j P^k = 0$: mutually exclusive properties
  - $\sum_j P^j = 1$: at least one property is true.
- Event algebra $\mathcal{E}$ consists of all projectors of type
  $P = \sum_j \pi_j P^j, \quad \pi_j = 0 \text{ or } 1$
- Example: Orthonormal basis $\{|\phi^j\rangle\}$; $P^j = |\phi^j\rangle\langle\phi^j|$.
- S_z sample space for spin half: $I = [z^+] + [z^-]$
  - Use $[\psi]$ as abbreviation for dyad $|\psi\rangle\langle\psi|$.
- Before discussing quantum probabilities, make sure sample space exists! Many quantum paradoxes and other confusion can be traced to nonexistent sample spaces!

Born Rule I

- Time development of *closed* or *isolated* physical system
  - Open system: make it part of larger closed system
  - Use Schrödinger Eqn to compute probabilities
  - Born rule is first (but not last!) step
- Unitary time development operator $T(t, t')$
  - Comes from solving Schrödinger's equation
  - Time-independent H: $T(t, t') = e^{-i(t-t')H/\hbar}$
- Assume $|\psi_0\rangle$ at t_0
  - Sample space $\mathcal{S}$: basis $\{|\phi_1^k\rangle\}$ at t_1
- Born probabilities:

$$\Pr(\phi_1^k) = \Pr(\phi_1^k \,|\, \psi_0) = |\langle\phi_1^k|T(t_1, t_0)|\psi_0\rangle|^2$$

- $\Pr(\phi_1^k)$ = prob of $[\phi_1^k]$, *not measurement* of ϕ_1^k.
  - Good measurements reveal pre-existing properties.
  - Use *quantum* description *including* apparatus to discuss measurements

Born Rule II

- Born probabilities *depend on basis* $\{|\phi_1^k\rangle\}$
- Example. Spin half, $|\psi_0\rangle = |z^+\rangle$, $H = 0$, $T(t, t') = I$
- S_z basis $\{|z^+\rangle, |z^-\rangle\}$ at t_1:
  - $\Pr(z_1^+) = 1$, $\Pr(z_1^-) = 0$
  - Subscript 1 indicates time t_1.
- S_x basis $\{|x^+\rangle, |x^-\rangle\}$ at t_1:
  - $\Pr(x_1^+) = 1/2 = \Pr(x_1^-)$
- Probabilities refer to properties of particle!
  - Bases incompatible; *cannot* assign probability to $S_z = +1/2$ AND $S_x = -1/2$ at time t_1
- "Gyroscope with axis in z direction" is misleading
  - $S_z = +1/2$ at t_0, $S_x = -1/2$ at t_1 does *not* mean change in direction of axis!
  - Better picture: gyroscope axis in random direction
  - Given z component at t_0, what is probability of x component at t_1?

Pre-Probability $|\psi_t\rangle$

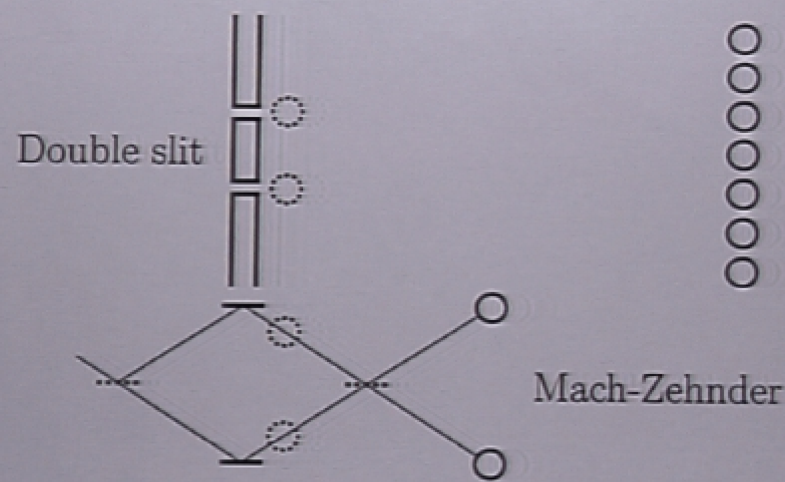
- Born probability

$$\Pr(\phi_1^k) = \Pr(\phi_1^k \,|\, \psi_0) = |\langle\phi_1^k|T(t_1,t_0)|\psi_0\rangle|^2$$

can be calculated in different ways.

- 1. Integrate Schrödinger Eqn from t_0 to t_1
  - $|\psi_1\rangle = T(t_1,t_0)|\psi_0\rangle$
  - $\Pr(\phi_1^k \,|\, \psi_0) = |\langle\phi_1^k|\psi_1\rangle|^2$
- 2. Integrate Schrödinger Eqn from t_1 to t_0
  - $|\phi_0^k\rangle = T(t_0,t_1)|\phi_1^k\rangle$
  - $\Pr(\phi_1^k \,|\, \psi_0) = |\langle\phi_0^k|\psi_0\rangle|^2$
- Approaches 1 and 2 equally good
  - Compare E&M: same result using different gauge
- Physical reality: $|\psi_0\rangle$ and the $\{|\phi_1^k\rangle\}$;
  however, $|\psi_1\rangle$ and $\{|\phi_0^k\rangle\}$ are *pre-probabilities*: tools for computing probabilities, *not* physical reality!
- "Wave function of universe" $|\psi_t\rangle = T(t,t_0)|\psi_0\rangle$
  - Everett: $|\psi_t\rangle$ represents physical reality
  - Histories: $|\psi_t\rangle$ is pre-probability: useful for finding Born probabilities; inadequate for others

Double Slit + Mach-Zehnder



- Correspondences:
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  - Detectors behind slits ↔ inside interferometer
  - In interference zone ↔ following 2d beam splitter
- For precise description, use Mach-Zehnder
  - Basic idea applies to double slit