Title: Interpretations of Probability in Quantum Mechanics

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Abstract:

Introduction

Quantum Mechanics

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What is meant by the term 'probability' here?

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- objective property of the physical system?
- 'subjective' state of knowledge of the physical system?

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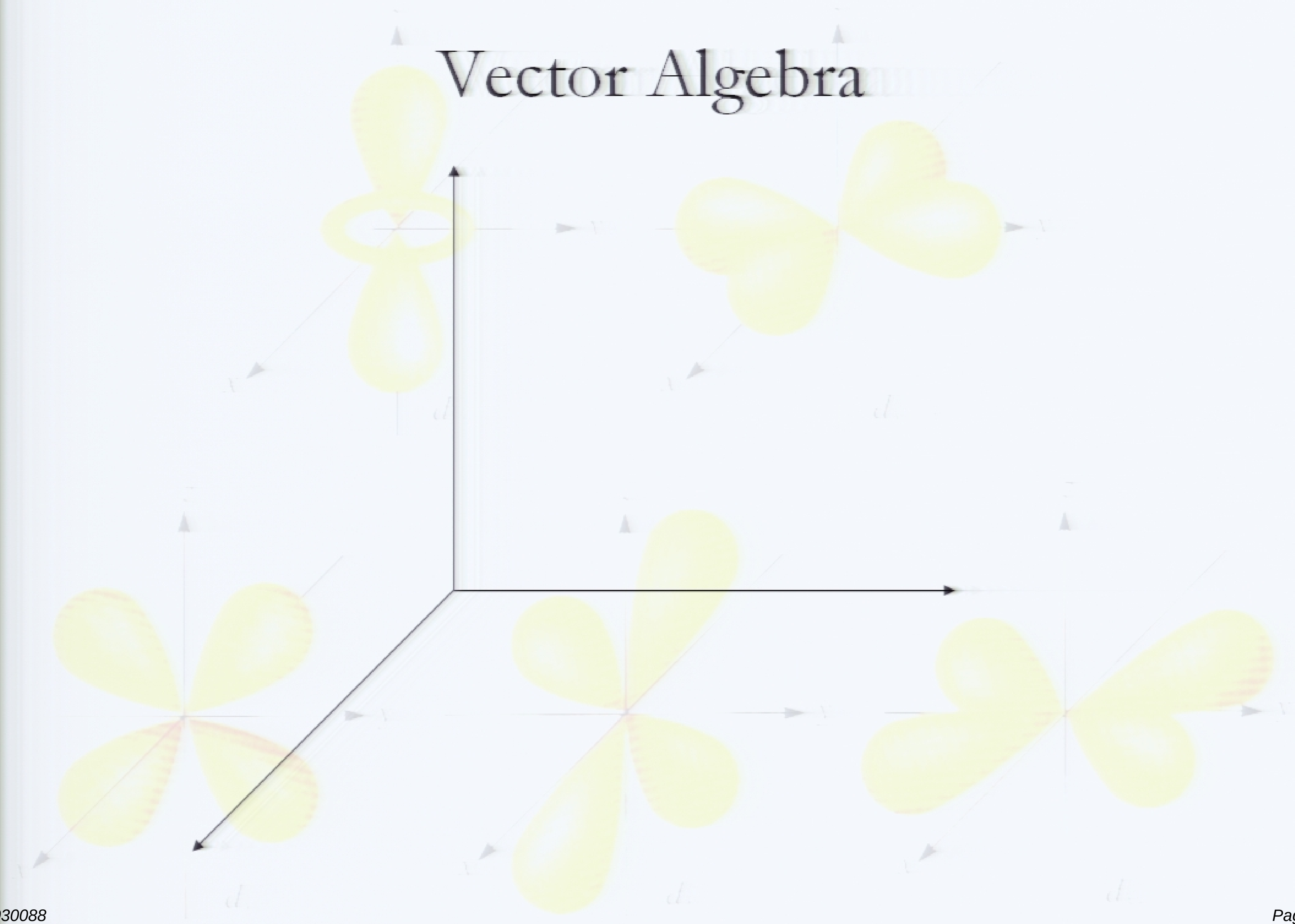
  - DBIs require a generalized theory of measurement (POVMs)

Vector Algebra



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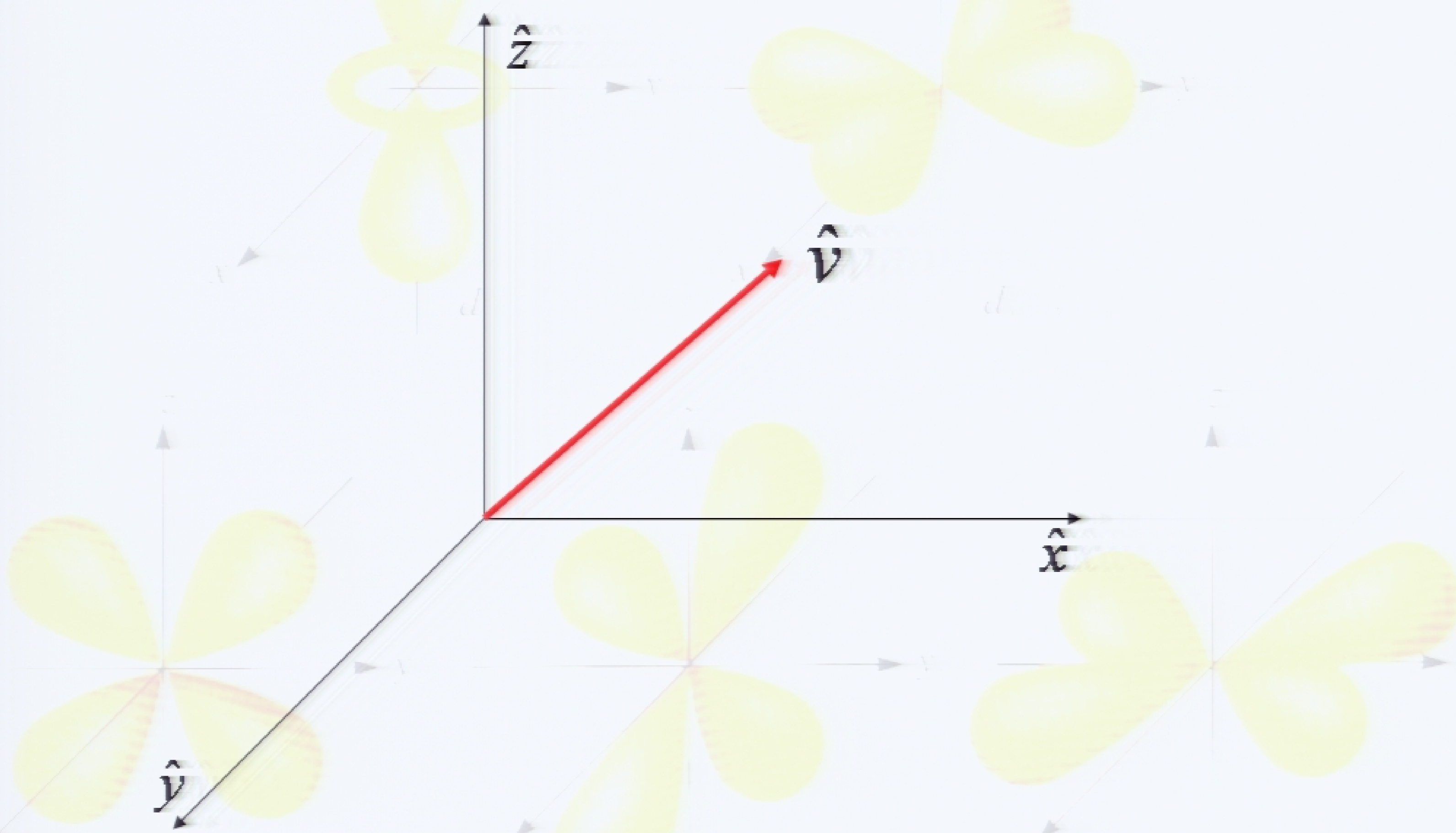

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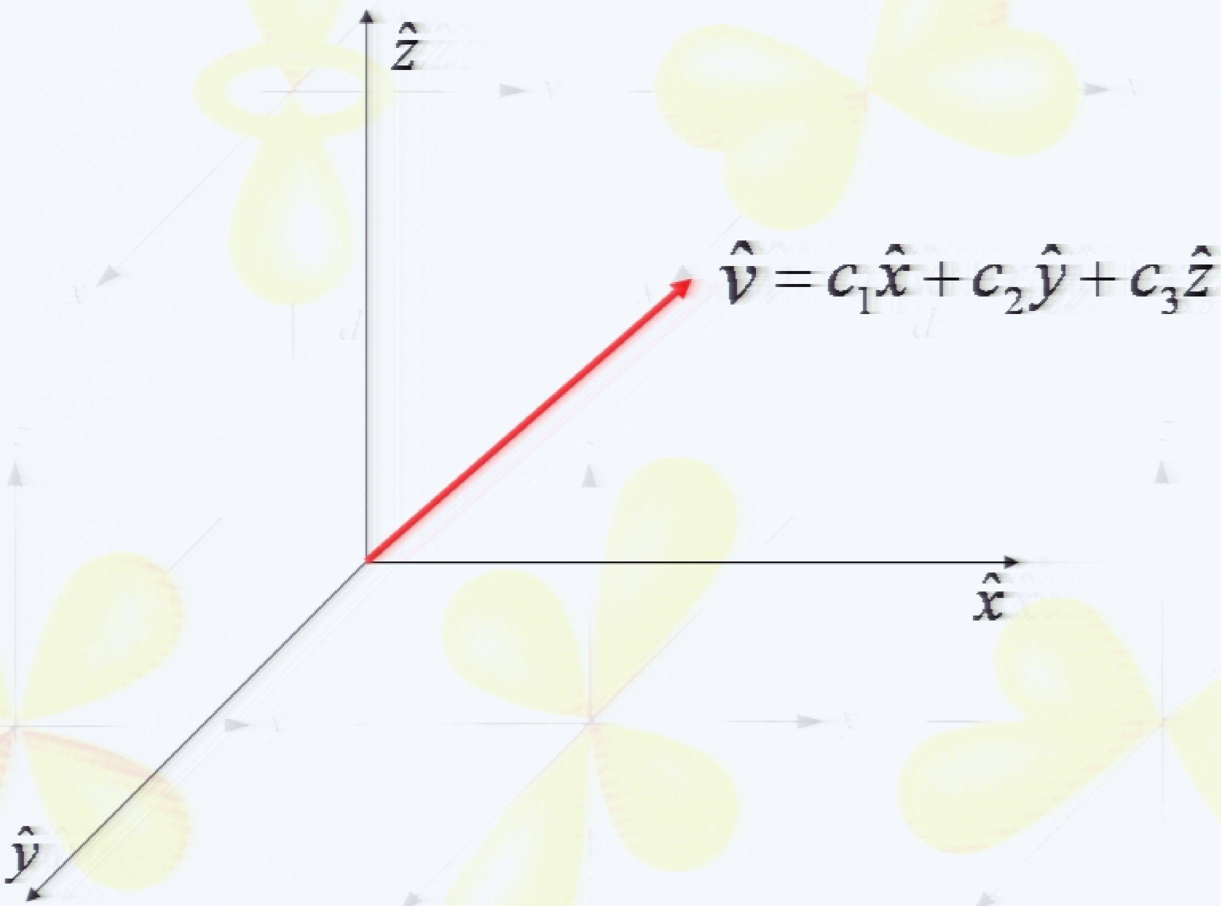

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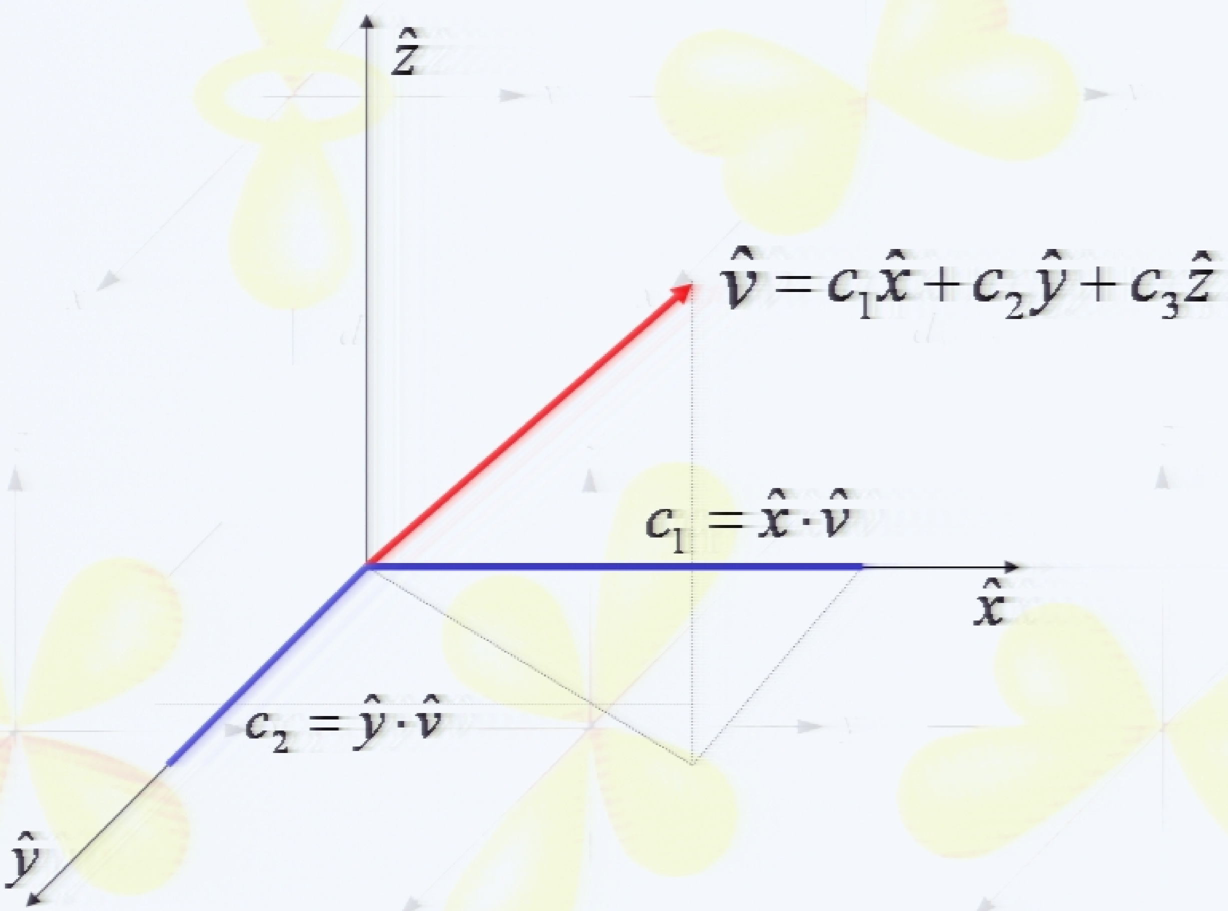

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- physical observables are represented by Hermitian operators A that act on H

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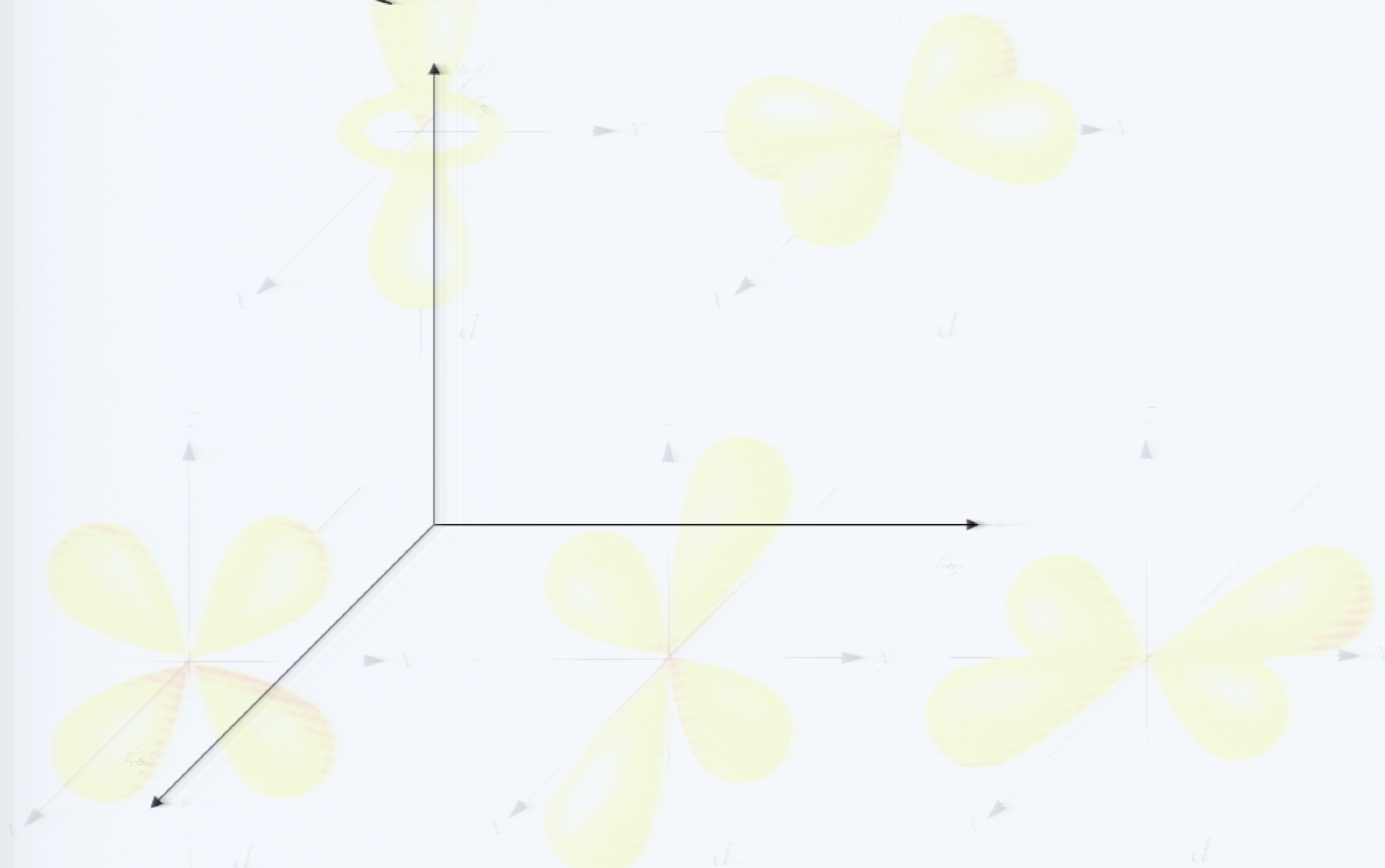
– these projection operators are in **1-1** correspondence with (pure) states and so we can use projection operators to represent quantum states

$$\rho = |\alpha\rangle\langle\alpha|$$

– this is the **density operator** representation of (pure) states

Quantum State Vectors



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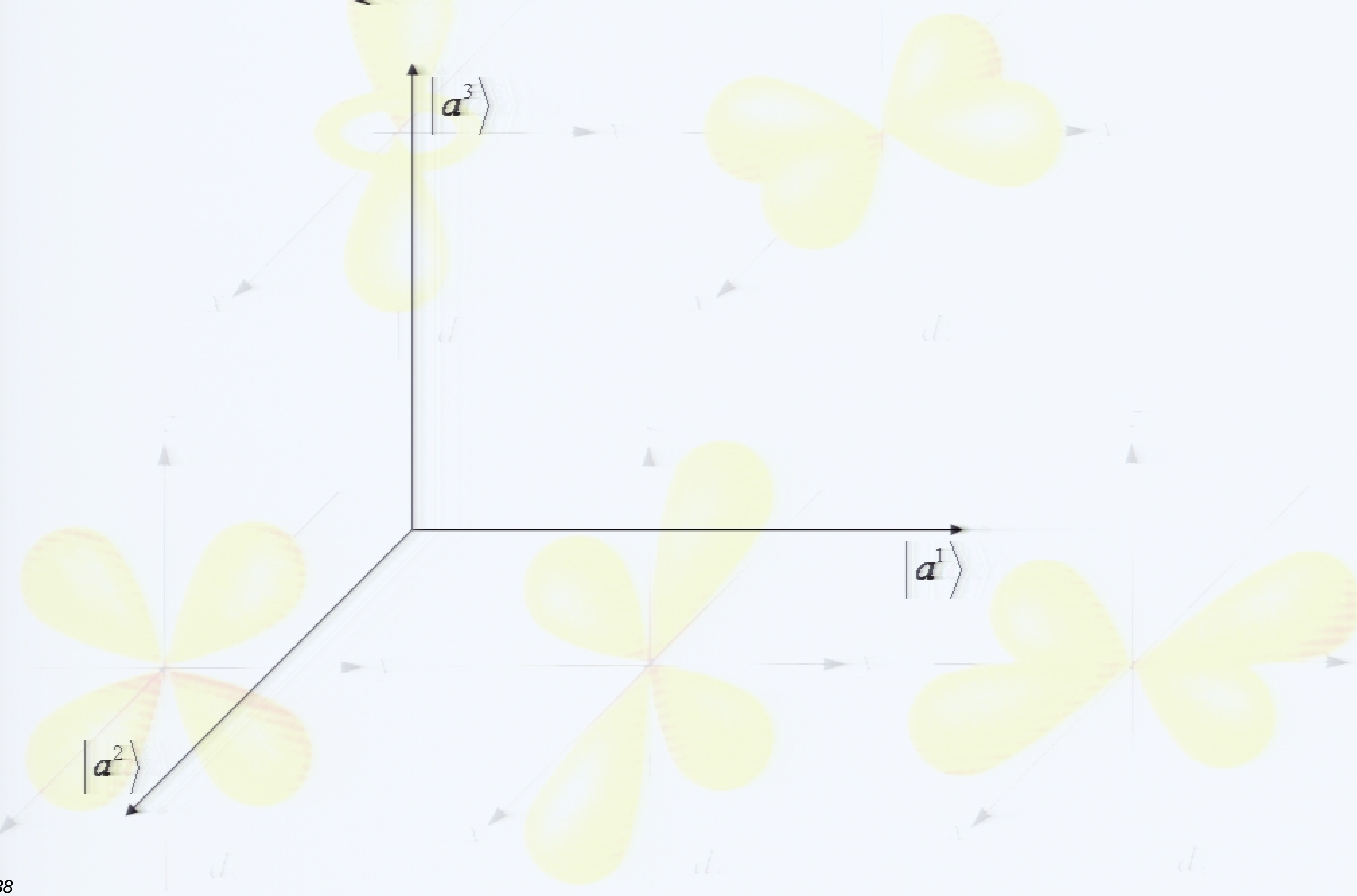

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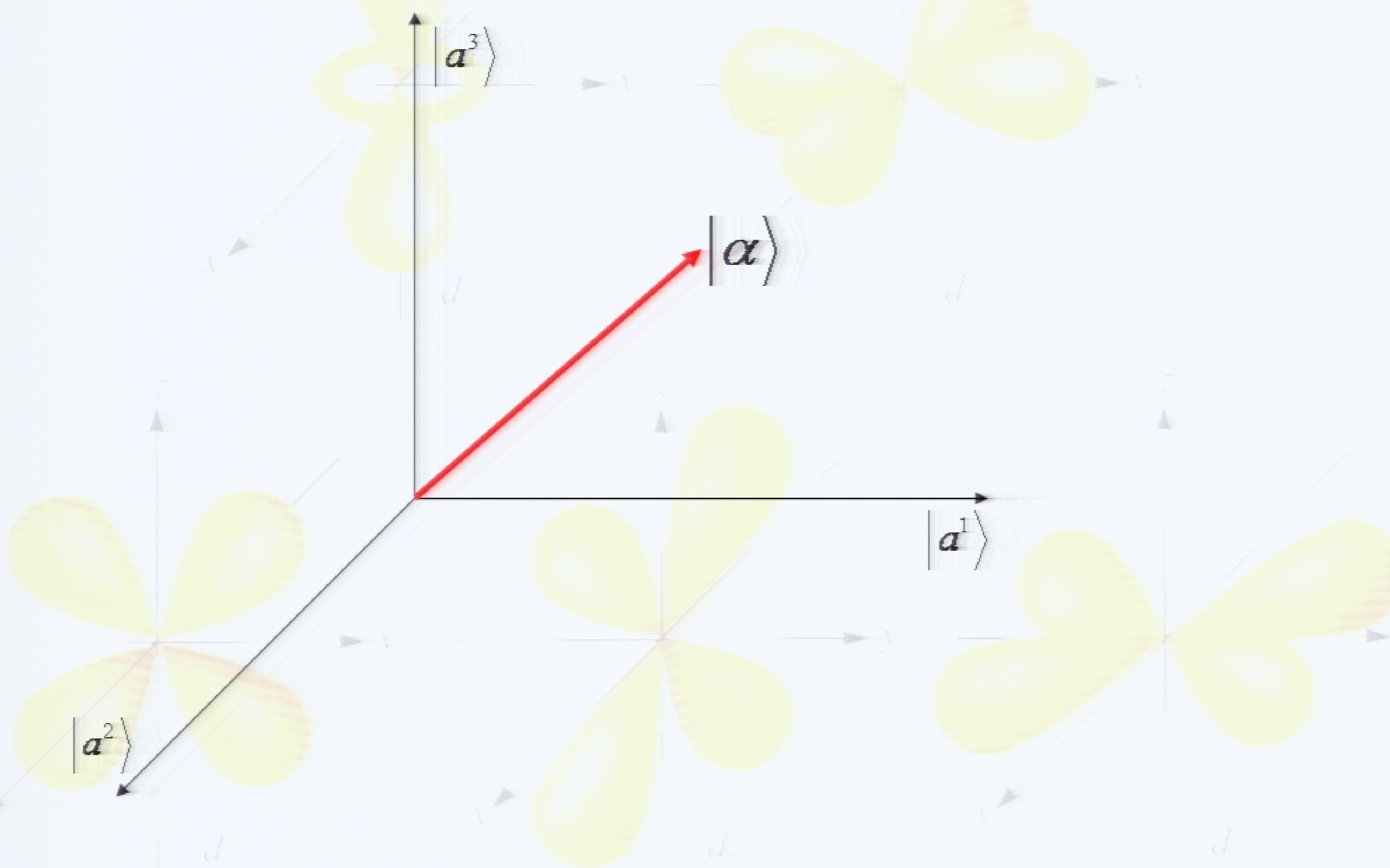

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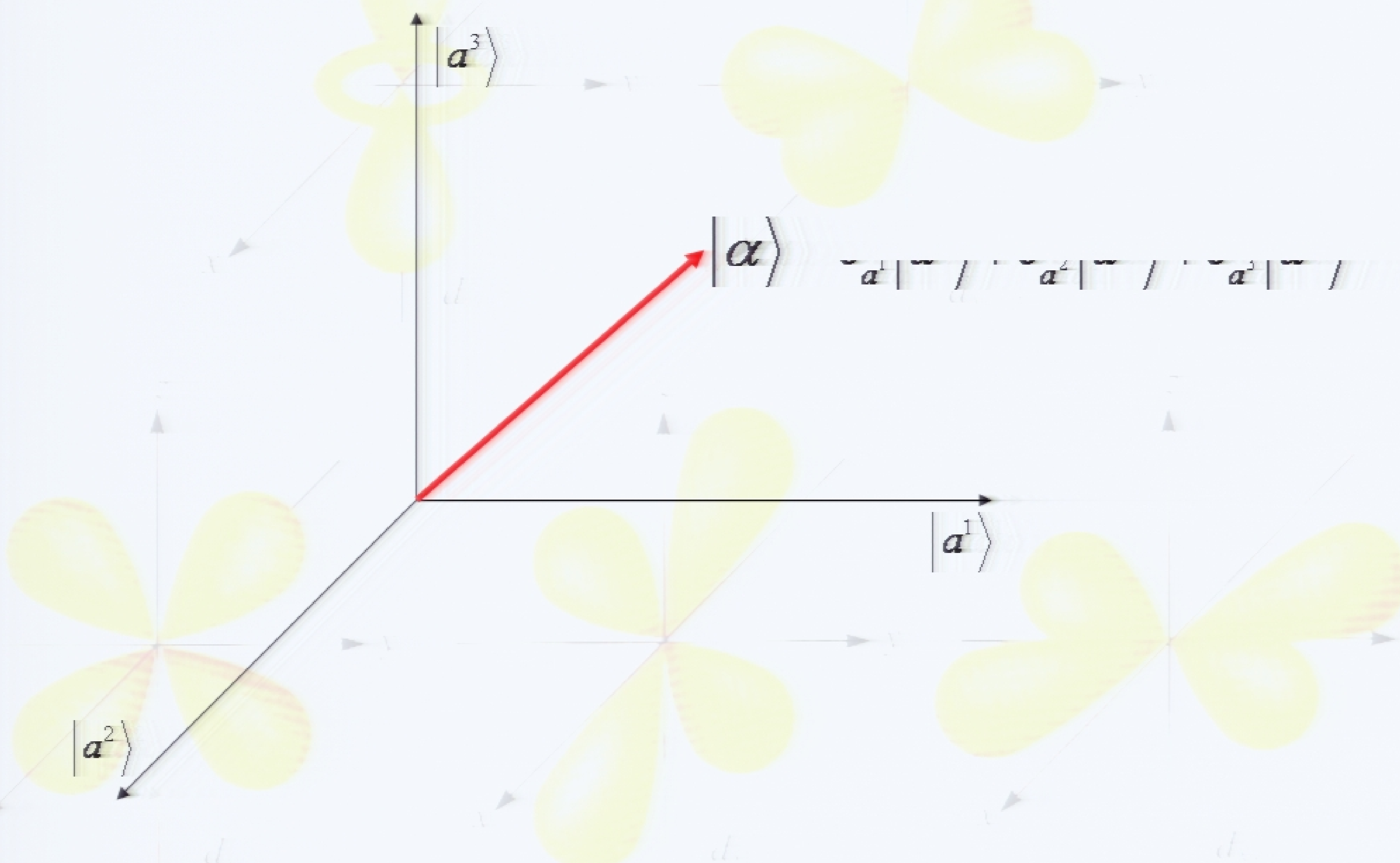

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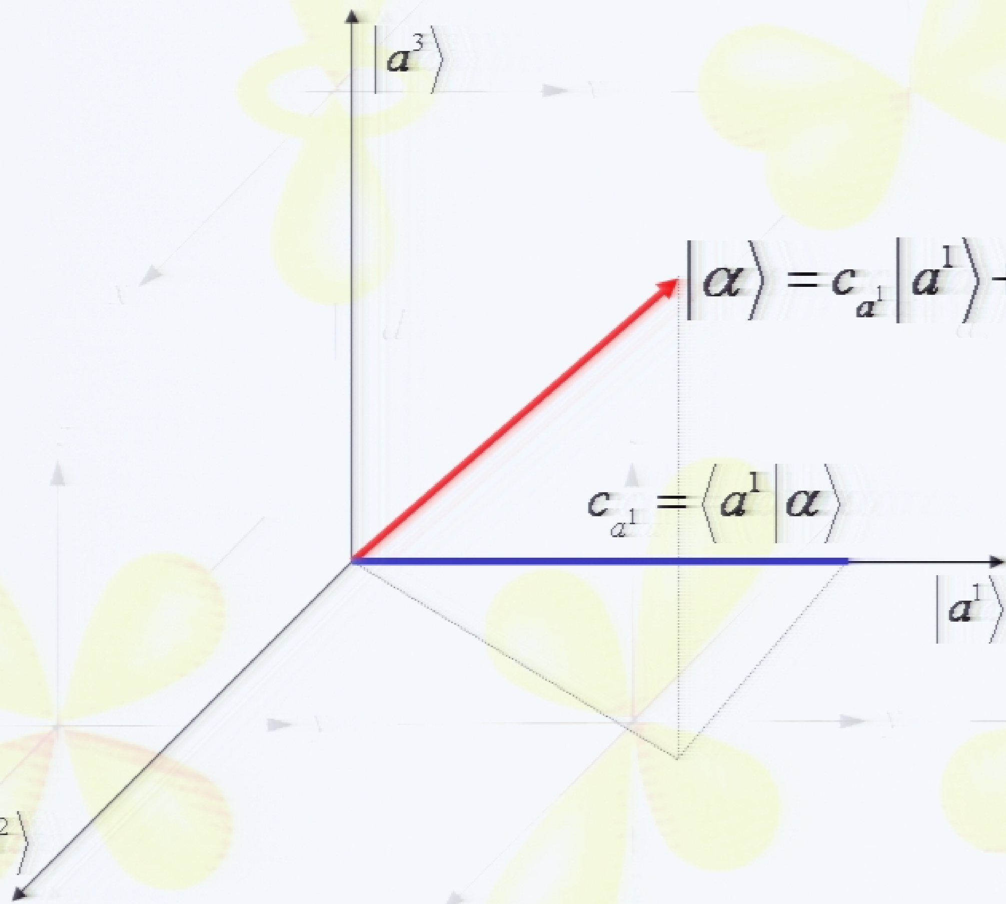

$$|\alpha\rangle = c_{a^1}|a^1\rangle + c_{a^2}|a^2\rangle + c_{a^3}|a^3\rangle$$

$$c_{a^1} = \langle a^1|\alpha\rangle$$

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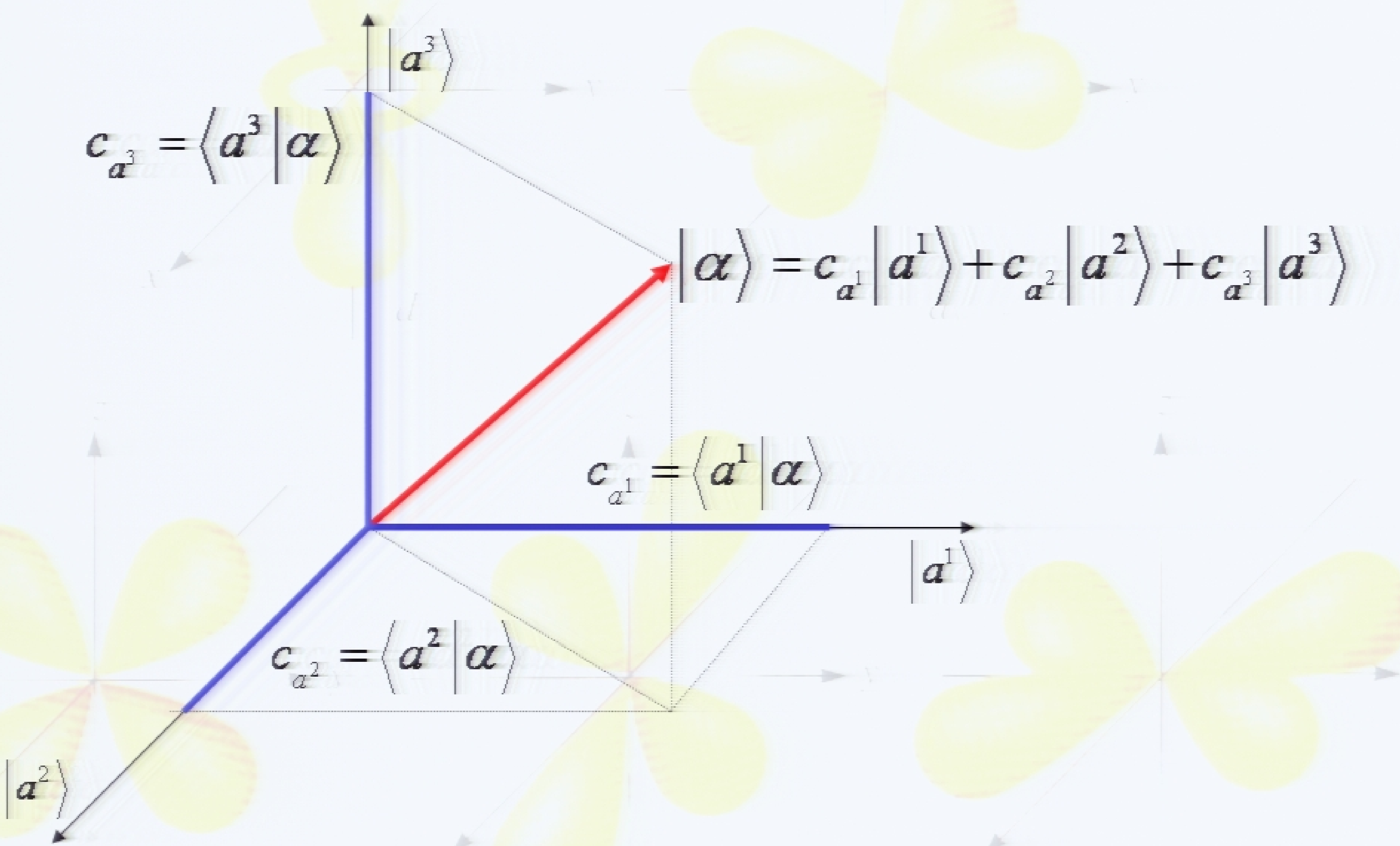

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$$|\alpha\rangle = \sum_i c_{a^i} |a^i\rangle \rightarrow |a^k\rangle$$

- the modulus squared of the coefficients c_{a^i} is the probability that the system will be found in the state $|a^i\rangle$

$$P_\alpha(|a^i\rangle) = |\langle a^i|\alpha\rangle|^2$$

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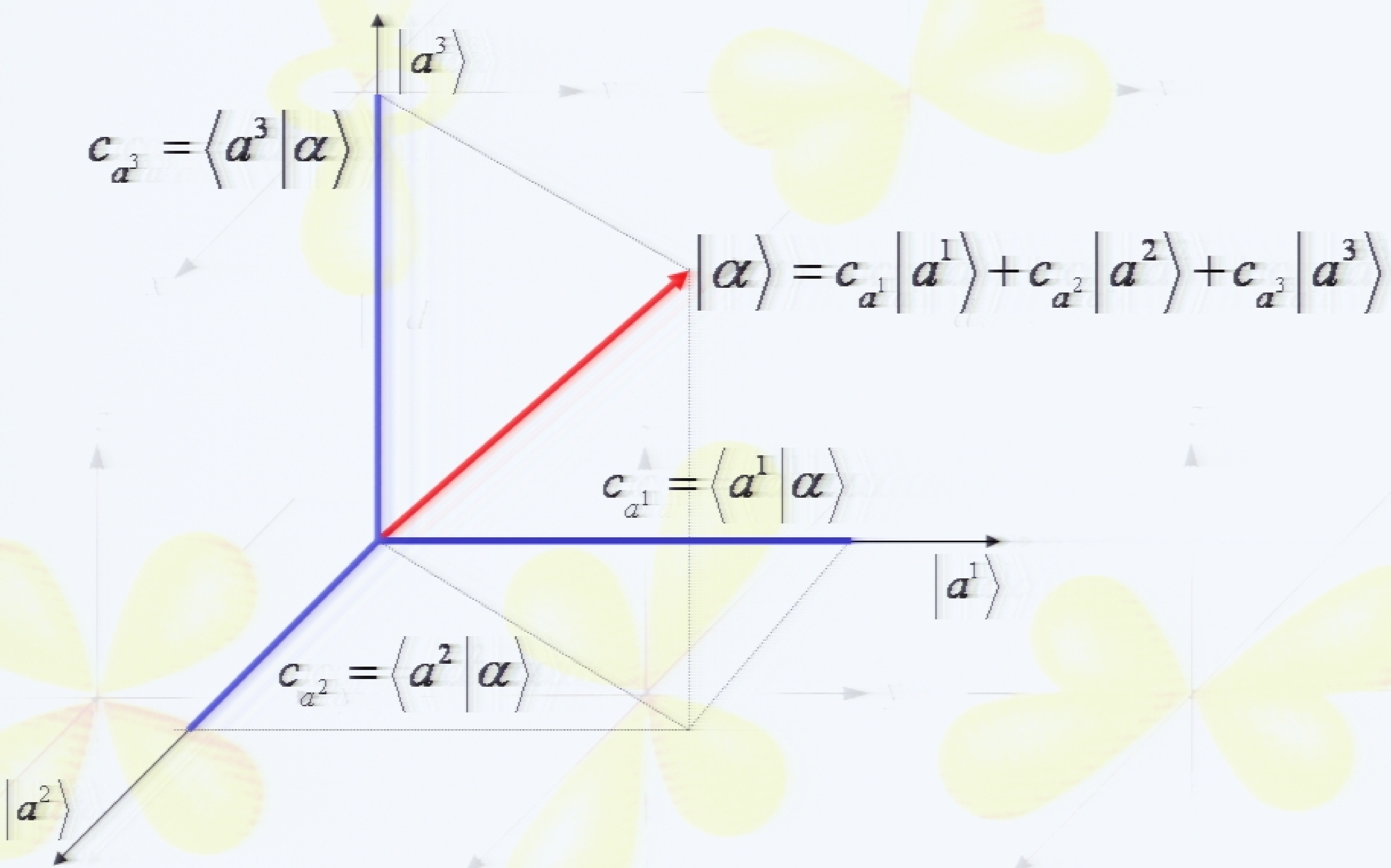
- in terms of the density operator we have that

$$\langle A \rangle = \mathrm{tr}(\rho A)$$

Quantum Measurement



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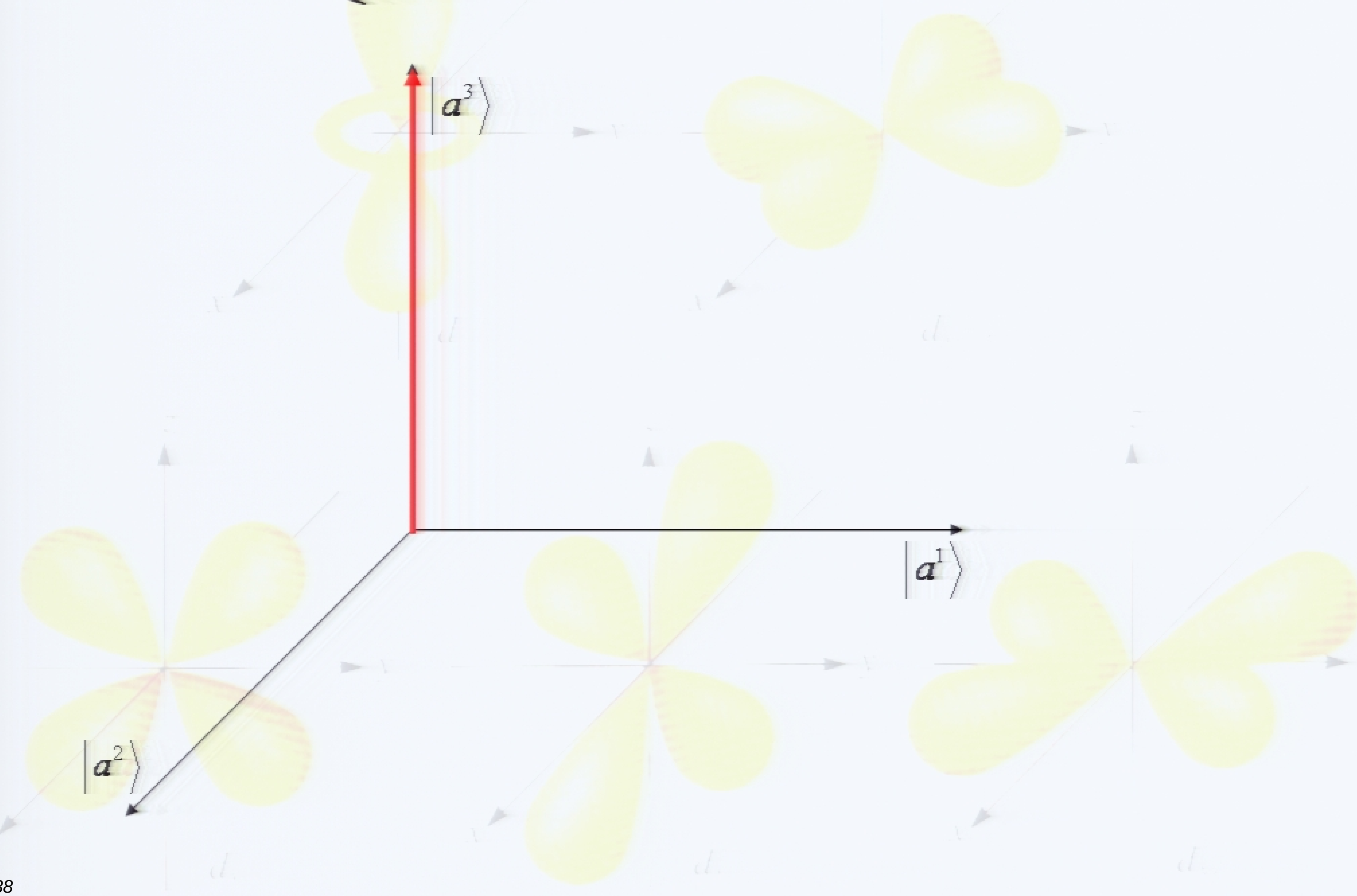

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1: fundamental object is a probability space (Ω, F, P) composed of:

- a set Ω, the sample space
- a σ-field F of subsets of Ω
- a real-valued set σ-additive set function P on F

2: the function P takes values in $[0,1]$

3: $P(\Omega) = 1$

Probability Theory

Kolmogorov's Axioms for Probability

– to add conditional probabilities there is the additional axiom:

4: $$P(B \mid A) = \frac{P(A \cap B)}{P(A)}$$

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- P assigns probabilities to eigenvectors $|a^i\rangle$ and to subsets of Ω such that

$$P_\alpha(\Omega) = P_\alpha\left(\bigcup_i |a^i\rangle\right) = \sum_i P_\alpha(|a^i\rangle) = \sum_i |\langle a^i|\alpha\rangle|^2 = 1$$

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– a propensity interpretation would, on top of this, seek some property of the physical system that is responsible for the observed frequency, but we do not have access to any such properties

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- to see how this works formally, consider a partition H_i of the sample space, then we have

$$P(E) = \sum_i P(H_i)P(E|H_i)$$

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- the updating of the probabilities is called **Bayesian conditionalization**

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- how QM is interpreted turns on the interpretation of the state vector

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  - the most pressing task for such interpretations is to explain what happens in the measurement process

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    - collapse interpretations, such as GRW and CSL provide dynamical reduction mechanisms

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  - for (I) the probability is a relative frequency, albeit an imaginary relative frequency

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- conclusion:
  - answering YES to Q1 leads to an objective relative frequency interpretation of the probability $P_\alpha(|a^i\rangle)$

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- knowledge of what sort of physical system?

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  - answering YES to this question leads to the following interpretations
    - the **Copenhagen Interpretation** (CI) takes the state vector to represent our state of knowledge of a physical system, which naturally changes in a discontinuous way upon measurement
    - the **Information Theoretic Interpretation** (ITI) takes the state vector to represent our state of knowledge of the combination of a physical system and the ancillary measurement apparatus

Interpretations of Quantum Probability

- QP: what does $P_\alpha(|a^i\rangle)$ represent?
  - the natural response is that the probability represents a rational degree of belief given that the updating of our knowledge is highly suggestive of Bayesian conditionalization

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  - the 'reduction of the state vector'

$$|\alpha\rangle = \sum_i c_{a^i} |a^i\rangle \rightarrow |a^k\rangle$$

is so similar to Bayesian conditionalization

$$P(E) = \sum_i P(H_i)P(E|H_i) \rightarrow P(E|H_k)$$

Implications…

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  - I will only compare the individual RFIs and DBIs, so we will not consider (SI) here

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- Relative Frequency Interpretation of (I)
  - as we saw, for an infinite sequence of measurements on quantum systems in the same state $|\alpha\rangle$, the relative frequency of measurements of the system to be in the state $|a^i\rangle$ is

$$P_\alpha\left(\left|a^i\right\rangle\right) = f_{a^i} = \lim_{n\to\infty}\frac{N(a^i)}{n}$$

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  - this makes $P_\alpha\left(|a^i\rangle\right)$ the proportion of the members of an infinite ensemble of systems in the same state measured to be $|a^i\rangle$
  - thus, $P_\alpha\left(|a^i\rangle\right)$ is an *ideally* observable quantity associated with the operator Π_{a^i}, or more properly, it is the expectation value of Π_{a^i}

$$P_\alpha\left(|a^i\rangle\right) = \langle\alpha|\Pi_{a^i}|\alpha\rangle = tr(\rho\Pi_{a^i})$$

Implications…

- Bayesian Degree of Belief Interpretations
  - as we saw, state reduction

$$|\alpha\rangle = \sum_i c_{a^i} \left|a^i\right\rangle \rightarrow \left|a^k\right\rangle$$

is suggestive of Bayesian conditionalization

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but this does not work because the state vector, the eigenvectors and the coefficients are not a probabilities

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  - this suggests the move to the density operator representation, but we have

$$\rho = \sum_i |c_{a^i}|^2 \, \Pi_{a^i} + \sum_{i \neq j} c_{a^i} c_j^* |a^i\rangle\langle a^j|$$

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- thus, QM formally does not accommodate a DBI
- this is a problem for (C) since there is nothing more to the measurement formalism than this

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  - there is a way out for the (ITI), however, since this interpretation considerers measurements on an ancillary quantum state (measurement device) entangled with the quantum state
  - treating this required a generalization of the measurement formalism from projection operators to general positive operator-valued measures (POVMs)
  - POVMs satisfy a generalized version of the probability rule

$$P(d) = \mathrm{tr}(\rho E_d)$$

Implications…

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  - using this formalism, as is shown in (Fuchs 2002) we have

$$\rho = \sum_{d} P(d)\tilde{\rho}_d \rightarrow \tilde{\rho}_{d'}$$

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which is formally analogous to conditionalization but at this point it does not agree with QM, though the further (unitary) adjustment

$$\tilde{\rho}_{d'} \rightarrow V_{d'}\tilde{\rho}_{d'}V_{d'}^{*}$$

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