Title: Supergravity Backgrounds from Generalized Calabi-Yau Manifolds (Part 1)

Date: Mar 01, 2005 11:00 AM

URL: http://pirsa.org/05030077

Abstract: We will see how generalized Calabi-Yau manifolds as defined by Hitchin emerge from supersymmetry equations in type II theories. In the first lecture, we will review the formalism of G-structures, which is central in the context of compactification with fluxes. In the second lecture we will see how (twisted) generalized Calabi-Yau manifolds emerge from supersymmetry equations using SU(3) structure. In the last lecture, we will discuss special features about compactifications on Generalized Calabi-Yau's.

Supersymmetric Backgrounds From Generalized Calabi-Yau Manifolds

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Perimeter Institute-March 2005

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$\mathcal{M}_6$ is CY $\xrightarrow{\text{Turn on fluxes}}$ $\mathcal{M}_6$ ~~CY~~

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SU(3) holonomy ⟶ SU(3) structure

$$\nabla_m \eta = 0$$

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CY $\longrightarrow$ **Twisted Generalized CY**

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CY ⟶ Twisted Generalized CY

↑ Not standard

↑ Hitchin`s

Outline

- Introduction / Motivation for flux compactifications
- G-structures (in particular SU(3) structure)
- Generalized CY from N=1 supergrsvity vacua
- Generalized structures and N=2 sugra vacua
- Generalized structures and N=2 effective actions

Introduction / Motivation

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Supersymmetric string/M-theory solutions with fluxes play important role in compactifications

- Break part or all of supersymmetry
- Fix some of the moduli of the compactification
- Have non conformal gauge theory duals
- Induce soft susy breaking terms on D-branes
- Generate warp factors – hierarchy problem?

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New compactification manifolds are not Calabi-Yau

- Characterization of allowed geometries
- Extension of properties of conventional compactifications on CY to the new geometries (moduli spaces, moduli fixing, mirror symmetry, D-branes, supersymmetry breaking…)

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Generalized complex geometry provides a natural framework to study flux backgrounds

Make use of the formalism of G-structures

- Led to progress in understanding properties of flux backgrounds: classification of allowed geometries

Supersymmetric solutions with fluxes $\leftrightarrow$ G-structures

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Sol to SUSY + Bianchi identity and EOM for the fluxes ($dF = d{*}F = 0$) → sol EOM

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Candelas, Horowitz, Strominger, Witten 85

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→ $\partial_\mu \theta = 0$, θ is a constant spinor in 4d

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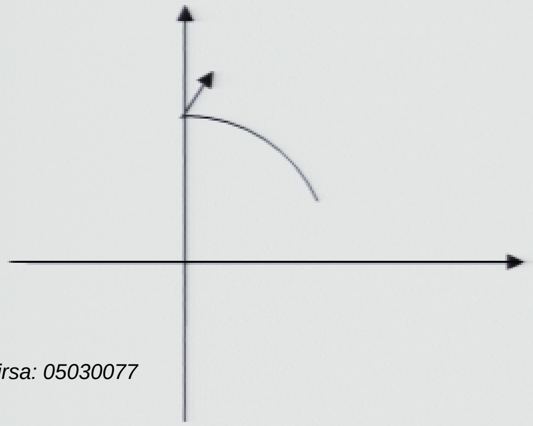
⬇

$\mathcal{M}_6$ has reduced holonomy



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SU(3) holonomy SO(6) → SU(3)

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spinor 4 → 3+1 $\swarrow \eta$

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Type II $\epsilon^1 = \theta^1 \otimes \eta$

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SU(3) holonomy

$$\begin{array}{llcl} & SO(6) & \rightarrow & SU(3) \\ \text{vector} & 6 & \rightarrow & 3+\bar{3} \\ \text{spinor} & 4 & \rightarrow & 3+1 \swarrow \eta \\ A_2 & 15 & \rightarrow & 8+3+\bar{3}+1 \swarrow J \\ A_3 & 10 & \rightarrow & 6+3+1 \swarrow \Omega \end{array}$$

Type II $\begin{matrix} \epsilon^1 = \theta^1 \otimes \eta \\ \epsilon^2 = \theta^2 \otimes \eta \end{matrix} \rightarrow \mathcal{M}_{10} = \mathcal{M}_4 \times CY_6 :\ \mathcal{N} = 2$ in 4d

$$\boxed{\mathcal{M}_{10} = \mathcal{M}_4 \times \mathcal{M}_6}$$

$$\boxed{\mathcal{M}_{10} = \mathcal{M}_4 \times \mathcal{M}_6}$$

- No flux $\Rightarrow \mathcal{M}_6$ is CY → SU(3) holonomy
- $\exists$ globally defined SU(3) invariant tensors J_{mn}, Ω_{mnp}

or

- $\exists$ SU(3) invt. spinor η $\begin{cases} \eta^\dagger \gamma_{mn} \gamma \eta = i\, J_{mn} \\ \eta^\dagger \gamma_{mnp} (1+\gamma) \eta = i\, \Omega_{mnp} \end{cases}$

$$J \wedge \Omega = 0$$

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Use $F_n = (-1)^{\mathrm{Int}[n/2]} *_{10} F_{10-n}$

to write for ex. $F_{\mu\nu\lambda\rho}$ in terms of F_{mnpqrs}
→ all fluxes are internal

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SU(3) structure in 6 dimensions

	SO(6)	→	SU(3)
Vector	6	→	$3 + \overline{3}$
A_2	15	→	$8 + 3 + \overline{3} + 1$
A_3	10	→	$6 + 3 + 1$
Spinor	4	→	$3 + 1$

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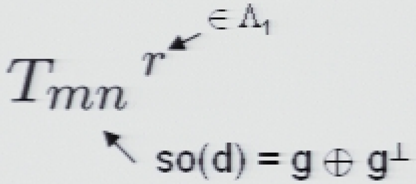

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$k^0{}_{[mn]}{}^p = T^0{}_{mn}{}^p$: intrinsic torsion

Intrinsic Torsion

- Given a metric compatible connection $\nabla`$ ($\nabla`g=0$) define $\begin{cases}\text{curvature}\\ \text{torsion}\end{cases}$

$$[\nabla`_m, \nabla`_n] V_p = -R_{mnp}{}^q V_q - 2\, T_{mn}{}^r\, \nabla`_r V_p$$

T: torsion

- Levi Civita ∇ : unique torsionless connection

- Any $\nabla`$ can be written : $\nabla` = \nabla + k$

$k_{mn}{}^p$: contorsion

Metric compatibility $\Rightarrow$ $k_{mnp} = -k_{mpn}$

Inserting in $[\nabla`_m, \nabla`_n]$: 1-1 correspondence between T and k

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$$T_{mn}{}^{\overset{\in \Lambda_1}{\swarrow}\, r} \qquad \in \Lambda_1 \otimes (g \oplus g^\perp)$$

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(drop 0 from now on)

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Measure of the failure of a G-structure to become G-holonomy

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$\swarrow \in \Lambda_1$

$$T_{mn}{}^{r} \in \Lambda_1 \otimes su(3)^{\perp} = (3+\bar{3}) \otimes (1+3+\bar{3})$$

$\nwarrow$ $so(d) = \cancel{g} \oplus g^{\perp}$

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$$\begin{array}{ccccc} W_1 & W_2 & W_3 & W_4 & W_5 \\ \text{scalar} & (1,1) & (2,1)+(1,2) & \multicolumn{2}{c}{(1,0)+(0,1)} \\ & \text{primitive} & \text{primitive} & & \end{array}$$

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Torsion versus fluxes

Torsion: $$dJ = \mathrm{Im}\,\underset{1 \oplus 1}{(W_1\, \Omega)} + \underset{3 \oplus \bar{3}}{W_4 \wedge J} + \underset{6 \oplus \bar{6}}{W_3}$$

$$d\,\Omega = \underset{1 \oplus 1}{W_1\, J^2} + \underset{3 \oplus \bar{3}}{W_5 \wedge \Omega} + \underset{8 \oplus 8}{W_2 \wedge J}$$

	$1 \oplus 1$	$3 \oplus \bar{3}$	$6 \oplus \bar{6}$	$8 \oplus 8$
Torsion	1 (W_1)	2 (W_4, W_5)	1 (W_3)	1 (W_2)
H_3	1	1	1	
IIA: F_{2n}	2 (F_0, F_2, F_4)	2 (F_2, F_4)		1 (F_2, F_4)
IIB: F_{2n+1}	1 (F_3)	3 (F_1, F_3, F_5)	1 (F_3)	

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IIA: F_{2n}	2 (F_0,F_2,F_4)	2 (F_2, F_4)		1 (F_2, F_4)
IIB: F_{2n+1}	1 (F_3)	3 (F_1,F_3,F_5)	1 (F_3)	

Torsion versus fluxes

Torsion:
$$dJ = \mathrm{Im}\,(\underset{1\oplus 1}{W_1}\,\Omega) + \underset{3\oplus\bar{3}}{W_4} \wedge J + \underset{6\oplus\bar{6}}{W_3}$$

$$d\,\Omega = \underset{1\oplus 1}{W_1\, J^2} + \underset{3\oplus\bar{3}}{W_5} \wedge \Omega + \underset{8\oplus 8}{W_2} \wedge J$$

	$1\oplus 1$	$3\oplus\bar{3}$	$6\oplus\bar{6}$	$8\oplus 8$
Torsion	1 (W_1)	2 (W_4,W_5)	1 (W_3)	1 (W_2)
H_3	1	1	1	
IIA: F_{2n}	2 (F_0,F_2,F_4)	2 (F_2, F_4)		1 (F_2, F_4)
IIB: F_{2n+1}	1 (F_3)	3 (F_1,F_3,F_5)	1 (F_3)	

Torsion versus fluxes

Torsion: $$dJ = \mathrm{Im}\,(\underset{1 \oplus 1}{W_1}\,\Omega) + \underset{3 \oplus \bar{3}}{W_4} \wedge J + \underset{6 \oplus \bar{6}}{W_3}$$

$$d\,\Omega = \underset{1 \oplus 1}{W_1}\, J^2 \;\; + \underset{3 \oplus \bar{3}}{W_5} \wedge \Omega + \underset{8 \oplus 8}{W_2} \wedge J$$

	$1 \oplus 1$	$3 \oplus \bar{3}$	$6 \oplus \bar{6}$	$8 \oplus 8$
Torsion	1 (W_1)	2 (W_4, W_5)	1 (W_3)	1 (W_2)
H_3	1	1	1	
IIA: F_{2n}	2 (F_0, F_2, F_4)	2 (F_2, F_4)		1 (F_2, F_4)
IIB: F_{2n+1}	1 (F_3)	3 (F_1, F_3, F_5)	1 (F_3)	

Torsion versus fluxes

Torsion: $dJ = \text{Im}\,(\underset{1\oplus 1}{W_1}\,\Omega) + \underset{3\oplus\bar{3}}{W_4} \wedge J + \underset{6\oplus\bar{6}}{W_3}$

$$d\,\Omega = \underset{1\oplus 1}{W_1}\,J^2 \; + \underset{3\oplus\bar{3}}{W_5} \wedge \Omega + \underset{8\oplus 8}{W_2} \wedge J$$

	$1 \oplus 1$	$3 \oplus \bar{3}$	$6 \oplus \bar{6}$	$8 \oplus 8$
Torsion	1 (W_1)	2 (W_4, W_5)	1 (W_3)	1 (W_2)
H_3	1	1	1	0
IIA: F_{2n}	2 (F_0, F_2, F_4)	2 (F_2, F_4)		1 (F_2, F_4)
IIB: F_{2n+1}	1 (F_3)	3 (F_1, F_3, F_5)	1 (F_3)	0

Torsion versus fluxes

Torsion: $$dJ = \mathrm{Im}\,(\underset{1 \oplus 1}{W_1}\,\Omega) + \underset{3 \oplus \bar{3}}{W_4} \wedge J + \underset{6 \oplus \bar{6}}{W_3}$$

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	$1 \oplus 1$	$3 \oplus \bar{3}$	$6 \oplus \bar{6}$	$8 \oplus 8$
Torsion	1 (W_1)	2 (W_4, W_5)	1 (W_3)	1 (W_2)
H_3	1	1	1	0
IIA: F_{2n}	2 (F_0, F_2, F_4)	2 (F_2, F_4)		1 (F_2, F_4)
IIB: F_{2n+1}	1 (F_3)	3 (F_1, F_3, F_5)	1 (F_3)	0

Torsion versus fluxes

Torsion: $$dJ = \mathrm{Im}\,(\underset{1\oplus 1}{W_1}\,\Omega) + \underset{3\oplus\bar{3}}{W_4} \wedge J + \underset{6\oplus\bar{6}}{W_3}$$

$$d\,\Omega = \underset{1\oplus 1}{W_1}\,J^2 \;+ \underset{3\oplus\bar{3}}{W_5} \wedge \Omega + \underset{8\oplus 8}{W_2} \wedge J$$

	$1\oplus 1$	$3\oplus\bar{3}$	$6\oplus\bar{6}$	$8\oplus 8$
Torsion	1 (W_1)	2 (W_4,W_5)	1 (W_3)	1 (W_2)
H_3	1	1	1	0
IIA: F_{2n}	2 (F_0,F_2,F_4)	2 (F_2, F_4)		1 (F_2, F_4)
IIB: F_{2n+1}	1 (F_3)	3 (F_1,F_3,F_5)	1 (F_3)	0

Torsion versus fluxes

Torsion: $$dJ = \mathrm{Im}\,(\underset{1\oplus 1}{W_1}\,\Omega) + \underset{3\oplus\bar{3}}{W_4} \wedge J + \underset{6\oplus\bar{6}}{W_3}$$

$$d\,\Omega = \underset{1\oplus 1}{W_1}\,J^2 + \underset{3\oplus\bar{3}}{W_5} \wedge \Omega + \underset{8\oplus 8}{W_2} \wedge J$$

	$1 \oplus 1$	$3 \oplus \bar{3}$	$6 \oplus \bar{6}$	$8 \oplus 8$
Torsion	1 (W_1)	2 (W_4, W_5)	1 (W_3)	1 (W_2)
H_3	1	1	1	0
IIA: F_{2n}	2 (F_0, F_2, F_4)	2 (F_2, F_4)		1 (F_2, F_4)
IIB: F_{2n+1}	1 (F_3)	3 (F_1, F_3, F_5)	1 (F_3)	0

Torsion versus fluxes

Torsion: $$dJ = \mathrm{Im}\,(\underset{1 \oplus 1}{W_1}\,\Omega) + \underset{3 \oplus \bar{3}}{W_4} \wedge J + \underset{6 \oplus \bar{6}}{W_3}$$

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Torsion	1 (W_1)	2 (W_4, W_5)	1 (W_3)	1 (W_2)
H_3	1	1	1	0
IIA: F_{2n}	2 (F_0, F_2, F_4)	2 (F_2, F_4)		1 (F_2, F_4)
IIB: F_{2n+1}	1 (F_3)	3 (F_1, F_3, F_5)	1 (F_3)	0

Torsion versus fluxes

Torsion: $$dJ = \mathrm{Im}\,(\underset{1 \oplus 1}{W_1\,\Omega}) + \underset{3 \oplus \bar{3}}{W_4 \wedge J} + \underset{6 \oplus \bar{6}}{W_3}$$

$$d\,\Omega = \underset{1 \oplus 1}{W_1\, J^2} + \underset{3 \oplus \bar{3}}{W_5 \wedge \Omega} + \underset{8 \oplus 8}{W_2 \wedge J}$$

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Torsion	1 (W_1)	2 (W_4, W_5)	1 (W_3)	1 (W_2)
H_3	1	1	1	0
IIA: F_{2n}	2 (F_0, F_2, F_4)	2 (F_2, F_4)		1 (F_2, F_4)
IIB: F_{2n+1}	1 (F_3)	3 (F_1, F_3, F_5)	1 (F_3)	0

↓

In IIB $W_2 = 0$ (integrability of complex structure)

Torsion versus fluxes

Torsion: $$dJ = \mathrm{Im}\,\underset{1\oplus 1}{(W_1\,\Omega)} + \underset{3\oplus\bar{3}}{W_4} \wedge J + \underset{6\oplus\bar{6}}{W_3}$$

$$d\,\Omega = \underset{1\oplus 1}{W_1}\, J^2 + \underset{3\oplus\bar{3}}{W_5} \wedge \Omega + \underset{8\oplus 8}{W_2} \wedge J$$

	$1\oplus 1$	$3\oplus\bar{3}$	$6\oplus\bar{6}$	$8\oplus 8$
Torsion	1 (W_1)	2 (W_4, W_5)	1 (W_3)	1 (W_2)
H_3	1	1	1	0
IIA: F_{2n}	2 (F_0, F_2, F_4)	2 (F_2, F_4)		1 (F_2, F_4)
IIB: F_{2n+1}	1 (F_3)	3 (F_1, F_3, F_5)	1 (F_3)	0

↓

In IIB $W_2 = 0$ (~~integrability~~ of complex structure)

Torsion versus fluxes

Torsion:
$$dJ = \underset{1 \oplus 1}{\mathrm{Im}\,(W_1\, \Omega)} + \underset{3 \oplus \bar{3}}{W_4 \wedge J} + \underset{6 \oplus \bar{6}}{W_3}$$

$$d\,\Omega = \underset{1 \oplus 1}{W_1\, J^2} + \underset{3 \oplus \bar{3}}{W_5 \wedge \Omega} + \underset{8 \oplus 8}{W_2 \wedge J}$$

	$1 \oplus 1$	$3 \oplus \bar{3}$	$6 \oplus \bar{6}$	$8 \oplus 8$
Torsion	1 (W_1)	2 (W_4, W_5)	1 (W_3)	1 (W_2)
H_3	1	1	1	0
IIA: F_{2n}	2 (F_0, F_2, F_4)	2 (F_2, F_4)		1 (F_2, F_4)
IIB: F_{2n+1}	1 (F_3)	3 (F_1, F_3, F_5)	1 (F_3)	0

↓

In IIB $W_2 = 0$ (~~integrability~~ of complex structure)

Torsion versus fluxes

Torsion: $$dJ = \mathrm{Im}\,(\underbrace{W_1}_{1\oplus 1}\,\Omega) + \underbrace{W_4}_{3\oplus\bar{3}} \wedge J + \underbrace{W_3}_{6\oplus\bar{6}}$$

$$d\,\Omega = \underbrace{W_1}_{1\oplus 1}\, J^2 + \underbrace{W_5}_{3\oplus\bar{3}} \wedge \Omega + \underbrace{W_2}_{8\oplus 8} \wedge J$$

	$1 \oplus 1$	$3 \oplus \bar{3}$	$6 \oplus \bar{6}$	$8 \oplus 8$
Torsion	1 (W_1)	2 (W_4, W_5)	1 (W_3)	1 (W_2)
H_3	1	1	1	0
IIA: F_{2n}	2 (F_0, F_2, F_4)	2 (F_2, F_4)		1 (F_2, F_4)
IIB: F_{2n+1}	1 (F_3)	3 (F_1, F_3, F_5)	1 (F_3)	0

↓

In IIB $W_2 = 0$ (~~integrability~~ of complex structure)

Torsion versus fluxes

Torsion: $$dJ = \mathrm{Im}\,(\underset{1\oplus 1}{W_1}\,\Omega) + \underset{3\oplus\bar{3}}{W_4} \wedge J + \underset{6\oplus\bar{6}}{W_3}$$

$$d\,\Omega = \underset{1\oplus 1}{W_1}\,J^2 + \underset{3\oplus\bar{3}}{W_5} \wedge \Omega + \underset{8\oplus 8}{W_2} \wedge J$$

	$1 \oplus 1$	$3 \oplus \bar{3}$	$6 \oplus \bar{6}$	$8 \oplus 8$
Torsion	1 (W_1)	2 (W_4,W_5)	1 (W_3)	1 (W_2)
H_3	1	1	1	0
IIA: F_{2n}	2 (F_0,F_2,F_4)	2 (F_2, F_4)		1 (F_2, F_4)
IIB: F_{2n+1}	1 (F_3)	3 (F_1,F_3,F_5)	1 (F_3)	0

↓

In IIB $W_2 = 0$ (in~~tegrab~~ility of complex structure)

Torsion versus fluxes

Torsion:
$$dJ = \underset{1 \oplus 1}{\mathrm{Im}\,(W_1\,\Omega)} + \underset{3 \oplus \bar{3}}{W_4 \wedge J} + \underset{6 \oplus \bar{6}}{W_3}$$

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	$1 \oplus 1$	$3 \oplus \bar{3}$	$6 \oplus \bar{6}$	$8 \oplus 8$
Torsion	1 (W_1)	2 (W_4, W_5)	1 (W_3)	1 (W_2)
H_3	1	1	1	0
IIA: F_{2n}	2 (F_0, F_2, F_4)	2 (F_2, F_4)	0	1 (F_2, F_4)
IIB: F_{2n+1}	1 (F_3)	3 (F_1, F_3, F_5)	1 (F_3)	0

↓

In IIB $W_2 = 0$ (~~integrability~~ of complex structure)

Torsion versus fluxes

Torsion: $$dJ = \mathrm{Im}\,\underset{1\oplus 1}{(W_1\,\Omega)} + \underset{3\oplus\bar{3}}{W_4} \wedge J + \underset{6\oplus\bar{6}}{W_3}$$

$$d\,\Omega = \underset{1\oplus 1}{W_1\,J^2} + \underset{3\oplus\bar{3}}{W_5} \wedge \Omega + \underset{8\oplus 8}{W_2} \wedge J$$

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Torsion	1 (W_1)	2 (W_4, W_5)	1 (W_3)	1 (W_2)
H_3	1	1	1	0
IIA: F_{2n}	2 (F_0, F_2, F_4)	2 (F_2, F_4)	0	1 (F_2, F_4)
IIB: F_{2n+1}	1 (F_3)	3 (F_1, F_3, F_5)	1 (F_3)	0

$\downarrow$

In IIB $W_2 = 0$ (~~integrability~~ of complex structure)

In IIA $W_3 \sim H^{(6)}$ (~~symplectic geometry~~)

Torsion versus fluxes

Torsion:
$$dJ = \mathrm{Im}\,(\underset{1 \oplus 1}{W_1\,\Omega}) + \underset{3 \oplus \bar{3}}{W_4} \wedge J + \underset{6 \oplus \bar{6}}{W_3}$$

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	$1 \oplus 1$	$3 \oplus \bar{3}$	$6 \oplus \bar{6}$	$8 \oplus 8$
Torsion	1 (W_1)	2 (W_4,W_5)	1 (W_3)	1 (W_2)
H_3	1	1	1	0
IIA: F_{2n}	2 (F_0,F_2,F_4)	2 (F_2, F_4)	0	1 (F_2, F_4)
IIB: F_{2n+1}	1 (F_3)	3 (F_1,F_3,F_5)	1 (F_3)	0

↓

In IIB $W_2 = 0$ (~~integrability~~ of complex structure)

In IIA $W_3 \sim H^{(6)}$ (~~symplectic~~ ~~geometry~~)

Torsion versus fluxes

Torsion:

$$dJ = \underset{1 \oplus 1}{\mathrm{Im}\,(W_1\,\Omega)} + \underset{3 \oplus \bar{3}}{W_4 \wedge J} + \underset{6 \oplus \bar{6}}{W_3}$$

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Torsion	1 (W_1)	2 (W_4, W_5)	1 (W_3)	1 (W_2)
H_3	1	1	1	0
IIA: F_{2n}	2 (F_0, F_2, F_4)	2 (F_2, F_4)	0	1 (F_2, F_4)
IIB: F_{2n+1}	1 (F_3)	3 (F_1, F_3, F_5)	1 (F_3)	0

↓

In IIB $W_2 = 0$ (~~integrability~~ of complex structure)

In IIA $W_3 \sim H^{(6)}$ (~~symplectic~~ geometry)

Torsion versus fluxes

Torsion:
$$dJ = \mathrm{Im}\,(\underbrace{W_1}_{1\oplus 1}\,\Omega) + \underbrace{W_4}_{3\oplus\bar{3}}\wedge J + \underbrace{W_3}_{6\oplus\bar{6}}$$

$$d\,\Omega = \underbrace{W_1}_{1\oplus 1}\,J^2 + \underbrace{W_5}_{3\oplus\bar{3}}\wedge\Omega + \underbrace{W_2}_{8\oplus 8}\wedge J$$

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Torsion	1 (W_1)	2 (W_4, W_5)	1 (W_3)	1 (W_2)
H_3	1	1	1	0
IIA: F_{2n}	2 (F_0, F_2, F_4)	2 (F_2, F_4)	0	1 (F_2, F_4)
IIB: F_{2n+1}	1 (F_3)	3 (F_1, F_3, F_5)	1 (F_3)	0

In IIB $W_2 = 0$ (~~integrability~~ of complex structure)

↓

In IIA $W_3 \sim H^{(6)}$ (~~symplectic geometry~~)

Torsion versus fluxes

Torsion:
$$dJ = \mathrm{Im}\,(\underset{1 \oplus 1}{W_1}\,\Omega) + \underset{3 \oplus \bar{3}}{W_4} \wedge J + \underset{6 \oplus \bar{6}}{W_3}$$

$$d\,\Omega = \underset{1 \oplus 1}{W_1}\,J^2 + \underset{3 \oplus \bar{3}}{W_5} \wedge \Omega + \underset{8 \oplus 8}{W_2} \wedge J$$

	$1 \oplus 1$	$3 \oplus \bar{3}$	$6 \oplus \bar{6}$	$8 \oplus 8$
Torsion	1 (W_1)	2 (W_4, W_5)	1 (W_3)	1 (W_2)
H_3	1	1	1	0
IIA: F_{2n}	2 (F_0, F_2, F_4)	2 (F_2, F_4)	0	1 (F_2, F_4)
IIB: F_{2n+1}	1 (F_3)	3 (F_1, F_3, F_5)	1 (F_3)	0

↓

In IIB $W_2 = 0$ (~~integrability~~ of complex structure) In IIA $W_3 \sim H^{(6)}$ (~~symplectic geometry~~)

Torsion versus fluxes

Torsion:
$$dJ = \mathrm{Im}\,(\underbrace{W_1}_{1\oplus 1}\,\Omega) + \underbrace{W_4}_{3\oplus\bar{3}} \wedge J + \underbrace{W_3}_{6\oplus\bar{6}}$$

$$d\,\Omega = \underbrace{W_1}_{1\oplus 1}\, J^2 + \underbrace{W_5}_{3\oplus\bar{3}} \wedge \Omega + \underbrace{W_2}_{8\oplus 8} \wedge J$$

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H_3	1	1	1	0
IIA: F_{2n}	2 (F_0,F_2,F_4)	2 (F_2, F_4)	0	1 (F_2, F_4)
IIB: F_{2n+1}	1 (F_3)	3 (F_1,F_3,F_5)	1 (F_3)	0

↓

In IIB $W_2 = 0$ (~~integrability~~ of complex structure)

In IIA $W_3 \sim H^{(6)}$ (~~symplectic~~ geometry)

Torsion versus fluxes

Torsion: $$dJ = \mathrm{Im}\,(\underset{1 \oplus 1}{W_1}\,\Omega) + \underset{3 \oplus \bar{3}}{W_4} \wedge J + \underset{6 \oplus \bar{6}}{W_3}$$

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H_3	1	1	1	0
IIA: F_{2n}	2 (F_0, F_2, F_4)	2 (F_2, F_4)	0	1 (F_2, F_4)
IIB: F_{2n+1}	1 (F_3)	3 (F_1, F_3, F_5)	1 (F_3)	0

$\downarrow$

In IIB $W_2 = 0$ (~~integrability~~ of complex structure)

In IIA $W_3 \sim H^{(6)}$ (~~symplectic geometry~~)

If also W_1=0 $\rightarrow$ IIB: $d\Omega = W_5 \wedge \Omega$

(true in all susy vacua)

IIA: $dJ = W_4 \wedge J + H^{(6)}$

Torsion versus fluxes

Torsion: $$dJ = \mathrm{Im}\,(\underset{1\oplus 1}{W_1}\,\Omega) + \underset{3\oplus\bar{3}}{W_4} \wedge J + \underset{6\oplus\bar{6}}{W_3}$$

$$d\,\Omega = \underset{1\oplus 1}{W_1 J^2} + \underset{3\oplus\bar{3}}{W_5 \wedge \Omega} + \underset{8\oplus 8}{W_2 \wedge J}$$

	$1\oplus 1$	$3\oplus\bar{3}$	$6\oplus\bar{6}$	$8\oplus 8$
Torsion	1 (W_1)	2 (W_4, W_5)	1 (W_3)	1 (W_2)
H_3	1	1	1	0
IIA: F_{2n}	2 (F_0, F_2, F_4)	2 (F_2, F_4)	0	1 (F_2, F_4)
IIB: F_{2n+1}	1 (F_3)	3 (F_1, F_3, F_5)	1 (F_3)	0

In IIB $W_2 = 0$ (~~integrability~~ of complex structure) ↓ In IIA $W_3 \sim H^{(6)}$ (~~symplectic~~ geometry)

If also W_1=0 → IIB: $d\Omega = W_5 \wedge \Omega$

(true in all susy vacua)

IIA: $dJ = W_4 \wedge J + H^{(6)}$

Torsion versus fluxes

Torsion: $dJ = \mathrm{Im}\,(\underset{1\oplus 1}{W_1}\,\Omega) + \underset{3\oplus\bar{3}}{W_4}\wedge J + \underset{6\oplus\bar{6}}{W_3}$

$$d\Omega = \underset{1\oplus 1}{W_1}\,J^2 + \underset{3\oplus\bar{3}}{W_5}\wedge\Omega + \underset{8\oplus 8}{W_2}\wedge J$$

	$1\oplus 1$	$3\oplus\bar{3}$	$6\oplus\bar{6}$	$8\oplus 8$
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IIA: F_{2n}	2 (F_0, F_2, F_4)	2 (F_2, F_4)	0	1 (F_2, F_4)
IIB: F_{2n+1}	1 (F_3)	3 (F_1, F_3, F_5)	1 (F_3)	0

↓

In IIB $W_2 = 0$ (~~integrability~~ of complex structure)

In IIA $W_3 \sim H^{(6)}$ (~~symplectic geometry~~)

If also $W_1=0$ → IIB: $d\Omega = W_5\wedge\Omega$

(true in all susy vacua)

IIA: $dJ = W_4\wedge J + H^{(6)}$

Torsion versus fluxes

Torsion: $$dJ = \mathrm{Im}\,(\underset{1 \oplus 1}{W_1\,\Omega}) + \underset{3 \oplus \bar{3}}{W_4} \wedge J + \underset{6 \oplus \bar{6}}{W_3}$$

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	$1 \oplus 1$	$3 \oplus \bar{3}$	$6 \oplus \bar{6}$	$8 \oplus 8$
Torsion	1 (W_1)	2 (W_4, W_5)	1 (W_3)	1 (W_2)
H_3	1	1	1	0
IIA: F_{2n}	2 (F_0, F_2, F_4)	2 (F_2, F_4)	0	1 (F_2, F_4)
IIB: F_{2n+1}	1 (F_3)	3 (F_1, F_3, F_5)	1 (F_3)	0

↓

In IIB $W_2 = 0$ (~~integrability~~ of complex structure)

In IIA $W_3 \sim H^{(6)}$ (~~symplectic~~ geometry)

If also $W_1 = 0$ → IIB: $d\Omega = W_5 \wedge \Omega$

(true in all susy vacua)

IIA: $dJ = W_4 \wedge J + H^{(6)}$

Torsion versus fluxes

Torsion: $$dJ = \mathrm{Im}\,(\underset{1\oplus 1}{W_1}\,\Omega) + \underset{3\oplus\bar{3}}{W_4}\wedge J + \underset{6\oplus\bar{6}}{W_3}$$

$$d\,\Omega = \underset{1\oplus 1}{W_1\,J^2} + \underset{3\oplus\bar{3}}{W_5}\wedge\Omega + \underset{8\oplus 8}{W_2}\wedge J$$

	$1\oplus 1$	$3\oplus\bar{3}$	$6\oplus\bar{6}$	$8\oplus 8$
Torsion	1 (W_1)	2 (W_4,W_5)	1 (W_3)	1 (W_2)
H_3	1	1	1	0
IIA: F_{2n}	2 (F_0,F_2,F_4)	2 (F_2, F_4)	0	1 (F_2, F_4)
IIB: F_{2n+1}	1 (F_3)	3 (F_1,F_3,F_5)	1 (F_3)	0

↓

In IIB $W_2 = 0$ (~~integrability~~ of complex structure)

In IIA $W_3 \sim H^{(6)}$ (~~symplectic~~ geometry)

If also W_1=0 → IIB: $d\Omega = W_5\wedge\Omega$

(true in all susy vacua)

IIA: $dJ = W_4\wedge J + H^{(6)}$

Torsion versus fluxes

Torsion:
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Torsion	1 (W_1)	2 (W_4, W_5)	1 (W_3)	1 (W_2)
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IIA: F_{2n}	2 (F_0, F_2, F_4)	2 (F_2, F_4)	0	1 (F_2, F_4)
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IIA: F_{2n}	2 (F_0, F_2, F_4)	2 (F_2, F_4)	0	1 (F_2, F_4)
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If also W_1=0 → IIB: $d\Omega = W_5 \wedge \Omega$ $\mathcal{M}_6$ is complex

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Torsion versus fluxes

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$\downarrow$

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IIA: $dJ = W_4 \wedge J + H^{(6)}$ $\mathcal{M}_6$ is "twisted symplectic"

Torsion versus fluxes

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Torsion versus fluxes

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Torsion versus fluxes

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In IIB $W_2 = 0$ (~~integrability~~ of complex structure)

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Is there a mathematical construction that contains complex and symplectic geometry?

Torsion versus fluxes

Torsion: $$dJ = \mathrm{Im}\,(\underset{1\oplus 1}{W_1}\,\Omega) + \underset{3\oplus\bar{3}}{W_4}\wedge J + \underset{6\oplus\bar{6}}{W_3}$$

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Is there a mathematical construction that extends complex and symplectic geometry?

GENERALIZED COMPLEX GEOMETRY

Torsion versus fluxes

Torsion: $$dJ = \mathrm{Im}\,(\underset{1\oplus 1}{W_1\,\Omega}) + \underset{3\oplus\bar{3}}{W_4 \wedge J} + \underset{6\oplus\bar{6}}{W_3}$$

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IIA: F_{2n}	2 (F_0, F_2, F_4)	2 (F_2, F_4)	0	1 (F_2, F_4)
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Is there a mathematical construction that extends complex and symplectic geometry?

GENERALIZED COMPLEX GEOMETRY

Torsion versus fluxes

Torsion: $$dJ = \mathrm{Im}\,(\underset{1\oplus 1}{W_1}\,\Omega) + \underset{3\oplus\bar{3}}{W_4} \wedge J + \underset{6\oplus\bar{6}}{W_3}$$

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GENERALIZED COMPLEX GEOMETRY

Torsion versus fluxes

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Is there a mathematical construction that extends complex and symplectic geometry?

GENERALIZED COMPLEX GEOMETRY

Torsion versus fluxes

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GENERALIZED COMPLEX GEOMETRY

Torsion versus fluxes

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Is there a mathematical construction that extends complex and symplectic geometry?

GENERALIZED COMPLEX GEOMETRY

Torsion versus fluxes

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If also W_1=0 → IIB: $d\Omega = W_5 \wedge \Omega$ $\mathcal{M}_6$ is complex

(true in all susy vacua)

IIA: $dJ = W_4 \wedge J + H^{(6)}$ $\mathcal{M}_6$ is "twisted symplectic"

Is there a mathematical construction that extends complex and symplectic geometry?

GENERALIZED COMPLEX GEOMETRY

Torsion versus fluxes

Torsion: $$dJ = \mathrm{Im}\,(\underset{1\oplus 1}{W_1}\,\Omega) + \underset{3\oplus\bar{3}}{W_4} \wedge J + \underset{6\oplus\bar{6}}{W_3}$$

$$d\,\Omega = \underset{1\oplus 1}{W_1}\, J^2 + \underset{3\oplus\bar{3}}{W_5} \wedge \Omega + \underset{8\oplus 8}{W_2} \wedge J$$

	$1\oplus 1$	$3\oplus\bar{3}$	$6\oplus\bar{6}$	$8\oplus 8$
Torsion	1 (W_1)	2 (W_4, W_5)	1 (W_3)	1 (W_2)
H_3	1	1	1	0
IIA: F_{2n}	2 (F_0, F_2, F_4)	2 (F_2, F_4)	0	1 (F_2, F_4)
IIB: F_{2n+1}	1 (F_3)	3 (F_1, F_3, F_5)	1 (F_3)	0

↓

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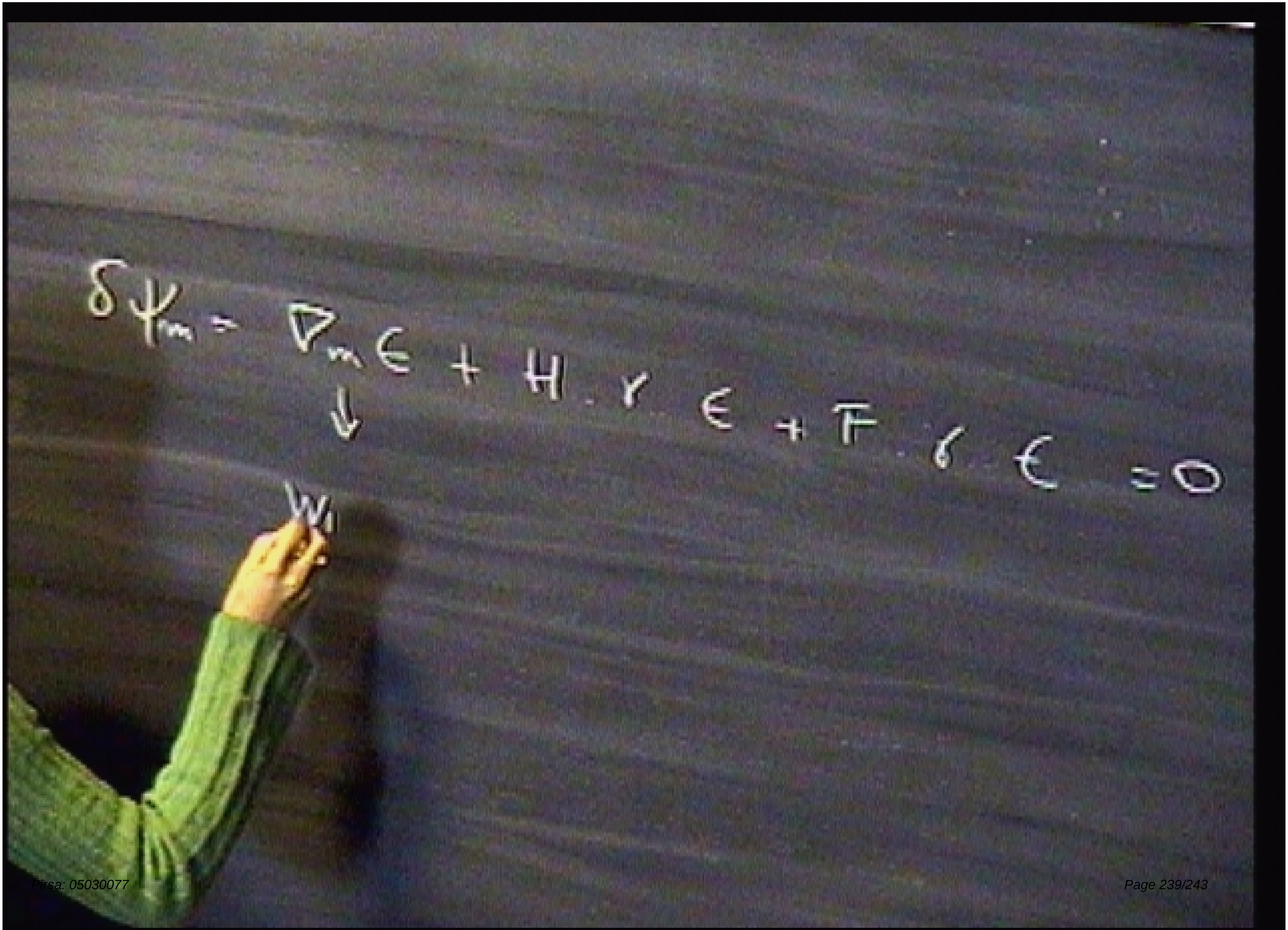
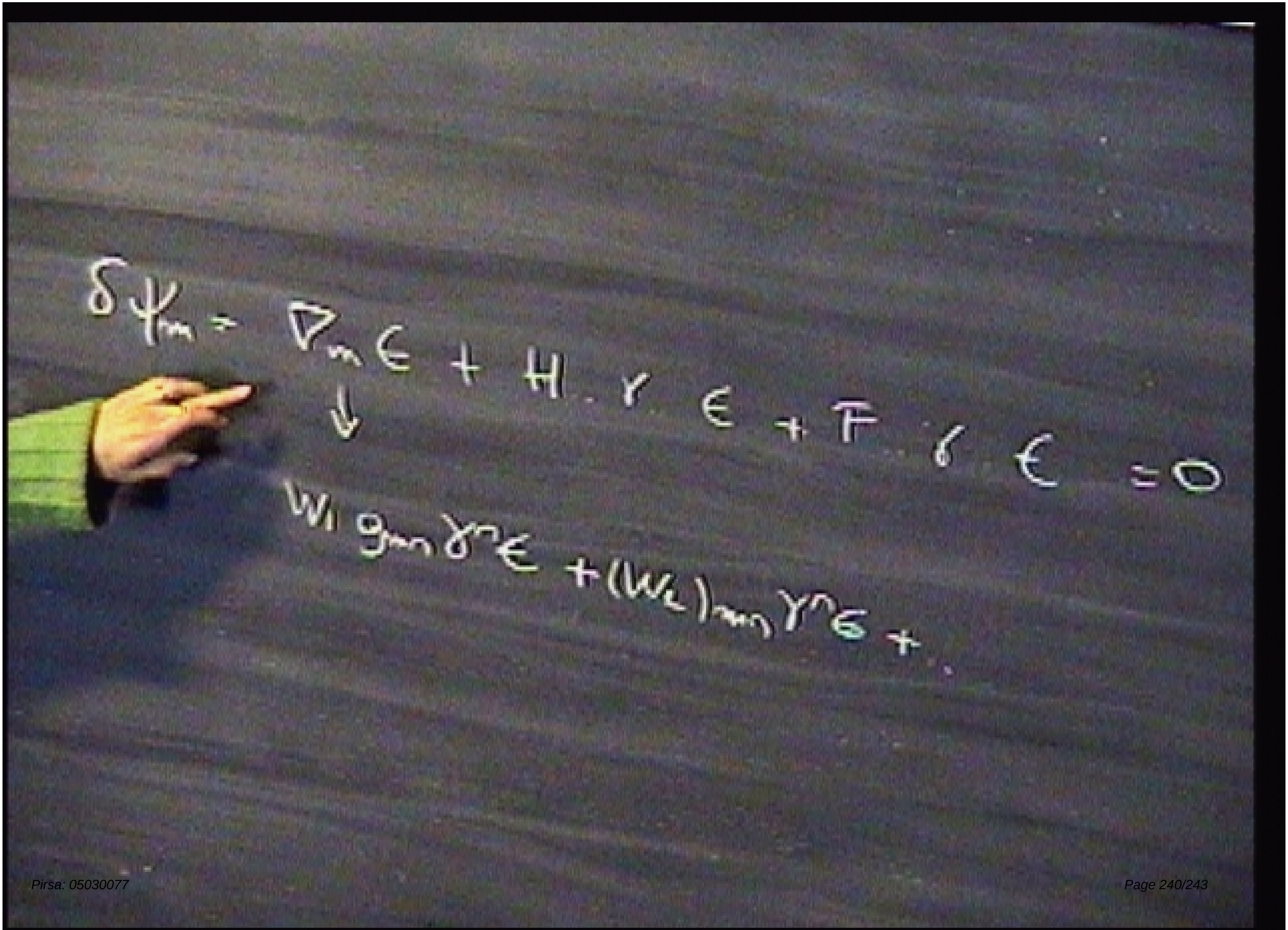
GENERALIZED COMPLEX GEOMETRY



$$\delta \psi_m = \nabla_m \epsilon + H \cdot \gamma \cdot \epsilon + F \cdot \sigma \cdot \epsilon = 0$$

$$\Downarrow$$

$$W$$



$$\delta \psi_m = \nabla_m \epsilon + H \cdot \gamma \, \epsilon + F \cdot \gamma \, \epsilon = 0$$

$$\Downarrow$$

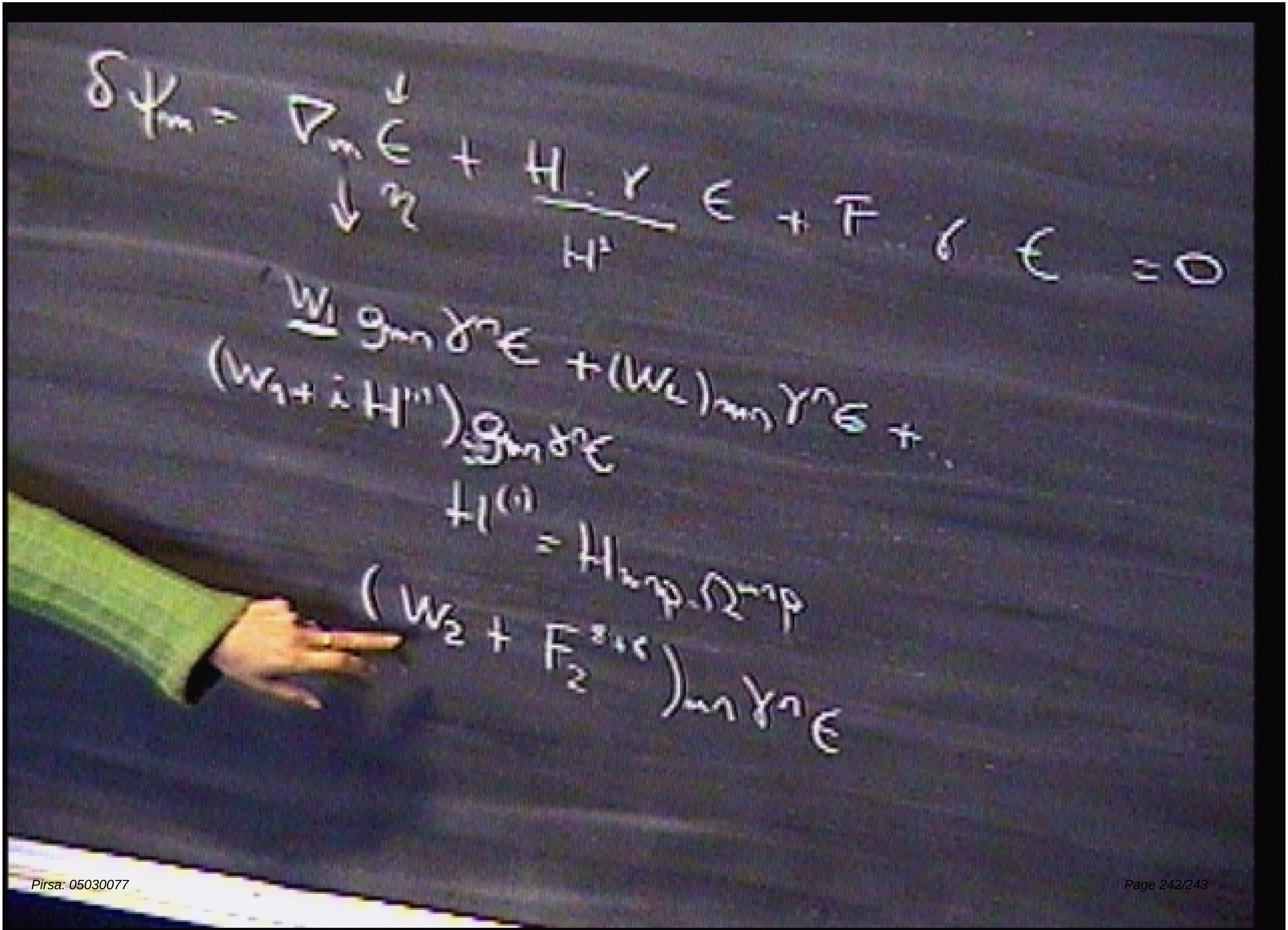
$$W_1 \, g_{mn} \gamma^n \epsilon + (W_2)_{mn} \gamma^n \epsilon +$$

$$\delta\psi_m = \underset{\downarrow\,\eta}{\nabla_m}\overset{\downarrow}{\epsilon} + \frac{H \cdot \gamma}{H^2}\,\epsilon + F \cdot \sigma \cdot \epsilon = 0$$

$$\underline{W_1}\, g_{mn}\gamma^n\epsilon + (W_2)_{mn}\gamma^n\epsilon + \ldots$$

$$(W_1 + i H^{(1)})\, g_{mn}\gamma^n\epsilon$$

$$H^{(1)} = H_{mnp}\Omega^{mnp}$$



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$$(W_1 + iH^{(1)})\, g_{mn}\gamma^n\epsilon$$

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$$(W_2 + F_2^{8,0})_{mn}\gamma^n\epsilon$$

$$\delta \psi_m = \nabla_m \overset{\downarrow}{\epsilon} + \frac{H \cdot \gamma}{H^2} \epsilon + F \gamma \epsilon = 0$$

$$\downarrow \eta$$

$$\underline{W_1} g_{mn} \gamma^n \epsilon + (W_2)_{mn} \gamma^n \epsilon +$$

$$(W_1 + i H^{(1)}) g_{mn} \gamma^n \epsilon$$

$$H^{(1)} = H_{mnp} \Omega^{mnp}$$

$$(W_2 + F_2^{2+6})_{mn} \gamma^n \epsilon$$

$$W_2^+$$