Title: A Unified Picture of Decoherence Control

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Abstract:

A Unified Picture of Decoherence Control

(quant-ph/0501109)

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Unified point of view based on the energy conservation principle and "forbidden transitions" on :

a) minimal decoherence model,

b)"bang-bang" techniques,

c) Zeno effect,

d) decoherence-free subspaces.

Fermi Golden Rule

$$H = H_0 + V\ ,\ \ H_0|k>= E_k|k>$$

Transition probability per unit time $|m> \mapsto |n>$

$$P_{nm} = \frac{2\pi}{\hbar}| < n|V|m > |^2 \delta(E_n - E_m)\ .$$

No decoherence for $\{|m>\}$:

1)$E_m \neq E_n$ - energy conservation cannot be satisfied
or
2) $< n|V|m >= 0$ - forbidden transitions.

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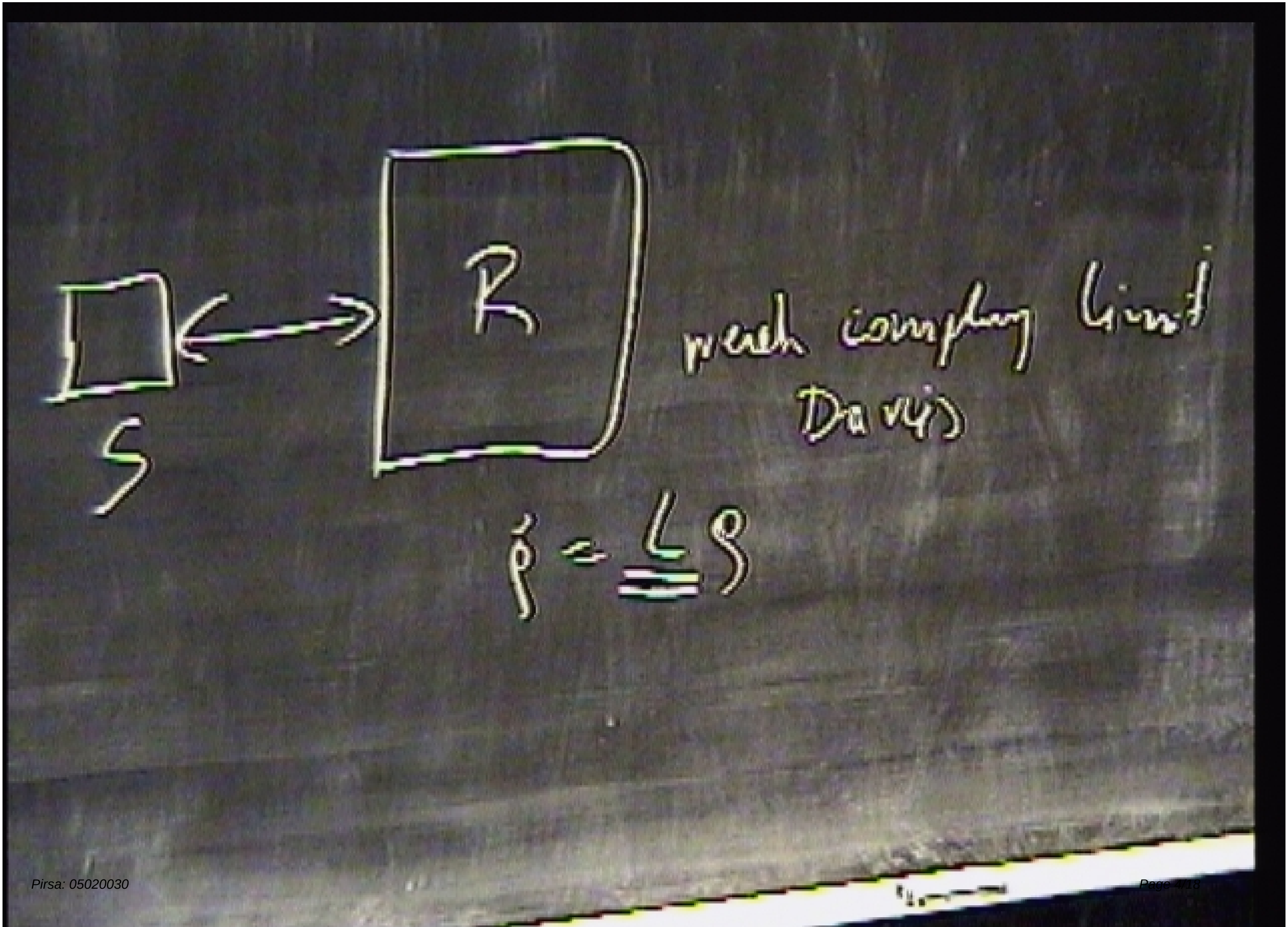
S
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weak coupling limit
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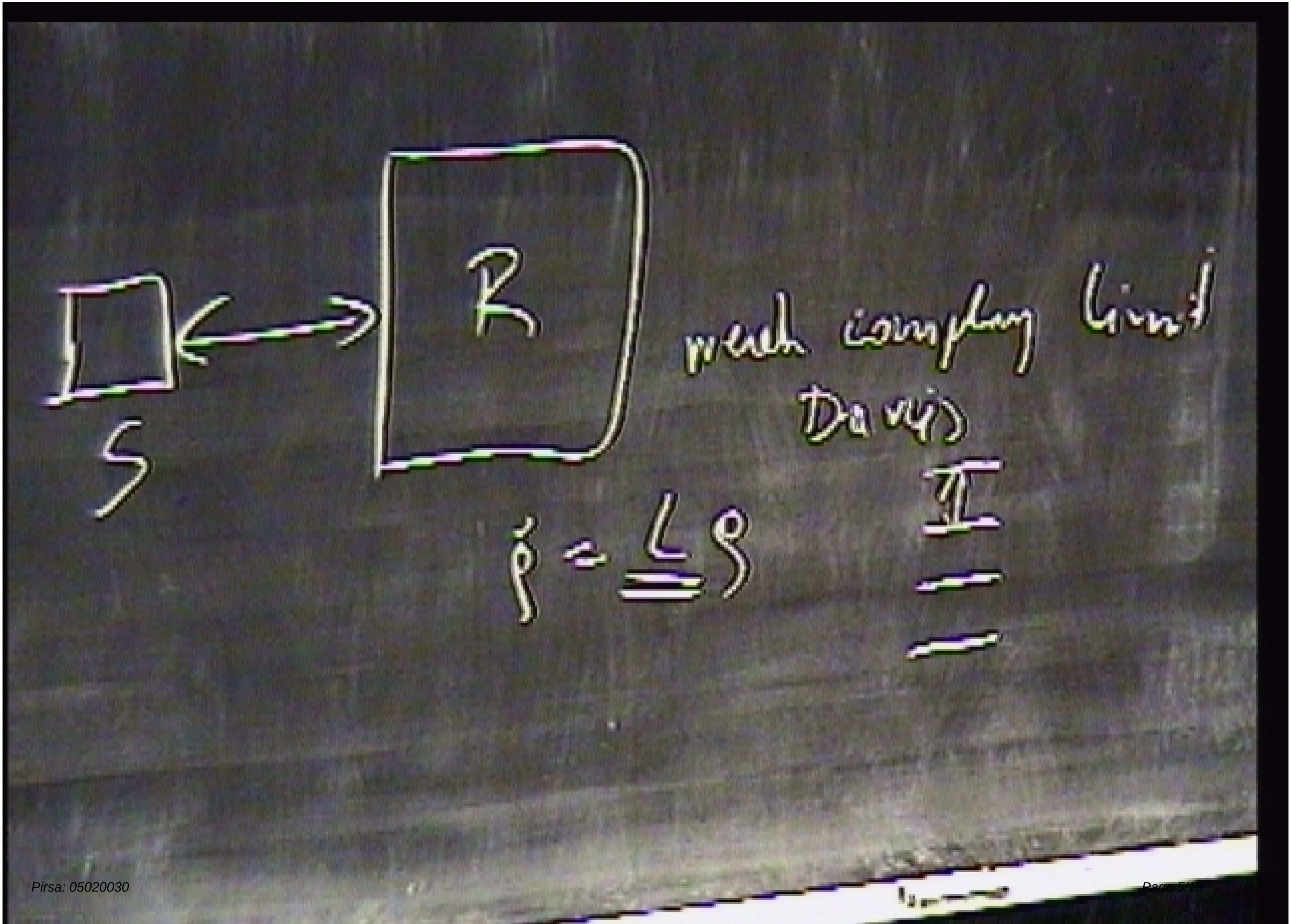
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Kubo-type formula

$$P_{nm} = \frac{1}{\hbar^2} \int_{-\infty}^{\infty} V_{nm}(t) V_{mn}(0)\, dt$$

where $V_{nm}(t) = < n|V(t)|m > = \exp\{i(E_n - E_m)t/\hbar\} < n|V|m >$

$$V(t) = e^{iH_0 t/\hbar} V e^{-iH_0 t/\hbar}$$

For a general time-dependent perturbation $V(t)$ (interaction picture)

$$\bar{P}_{nm} = \frac{1}{\hbar^2} \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} ds \int_{-\infty}^{\infty} V_{nm}(t+s) V_{mn}(s)\, dt$$

Open system in weak coupling regime

Open system S weakly coupled to a large reservoir R

$$H = H_S + H_R + H_{int}$$
$$H_{int} = S \otimes R \ \ (\text{or} \sum_{\alpha} S_\alpha \otimes R_\alpha)$$

Spectrum : discrete for H_S, continuous for H_R

$$H_S|k>=\epsilon_k|k> \ , \ H_R|E,\gamma>=E|E,\gamma> \ .$$

Transition probabilities (**Fermi Golden Rule**)

$$P_{kl} = \frac{2\pi}{\hbar}| < k|S|l > |^2$$
$$\times \sum_{\gamma',\gamma} \int dE' \int dE\, \sigma(E,\gamma)| < E',\gamma'|R|E,\gamma > |^2 \delta(\epsilon_l + E - \epsilon_k - E')$$

$\sigma(E,\gamma)$- describes the initial state of the reservoir.

Equivalent to Markovian Master Equation in weak coupling (van Hove) limit.

Minimal decoherence model

System S coupled to a quantum bosonic field (quantum harmonic oscillators)

$$H_{int} = S \otimes \sum_k (f_k a_k + \bar{f}_k a_k^\dagger)$$

$| < E, \gamma | R | E', \gamma' > |^2 > 0$ for states which differ by a single boson .

Energy conservation

$$|E - E'| = \hbar\omega = |\epsilon_k - \epsilon_l| \; .$$

$P_{kl} \sim$ density of bosonic states

$$n(\omega) \sim \omega^r, r > 0$$

at $\omega = |\epsilon_k - \epsilon_l|/\hbar$.

Minimal decoherence strategy:

Use almost degenerated energy levels and apply slow gates producing sufficiently small energy level splitting - $\max|\epsilon_k - \epsilon_l|$.

Natural restrictions:

slow gates ≡ low frequency external fields for the system's control ≡ long waves ≡ difficult individual control of "qubits"

scattering processes with different kinematics ($|\epsilon_k - \epsilon_l| = 0$*, but density of outcoming scattered states* $\neq 0$*)*

Bang-bang techniques

Cut-off at high energies

$$< E, \gamma | R | E', \gamma' > \simeq 0 \quad \text{if} \quad E' - E >> E_{cut}$$

Rapidly varying Hamiltonian $H_S(t) \to$ time-dependent effective Hamiltonian

$$H_{int}(t) = S(t) \otimes R(t)$$

where $S(t), R(t)$ - solutions of Heisenberg eqs

$$\frac{d}{dt} S(t) = \frac{i}{\hbar}[H_S(t), S(t)] \ , \ \frac{d}{dt} R(t) = \frac{i}{\hbar}[H_R, R(t)] \ .$$

Transition probability

$$\bar{P}_{kl} = \frac{2\pi}{\hbar^2} \sum_{\gamma', \gamma} \int dE' \int dE \, \sigma(E, \gamma) | < E', \gamma' | R | E, \gamma > |^2 \mathcal{S}_{kl}((E - E')/\hbar)$$

$\mathcal{S}_{kl}(\omega)$ - power spectrum of the autocorrelation function

$$S_{kl}(t) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} < k | S(t+s) | l >< l | S(s) | k > \, ds$$

$$= \int_{-\infty}^{\infty} \mathcal{S}_{kl}(\omega) e^{i\omega t} \, d\omega \ .$$

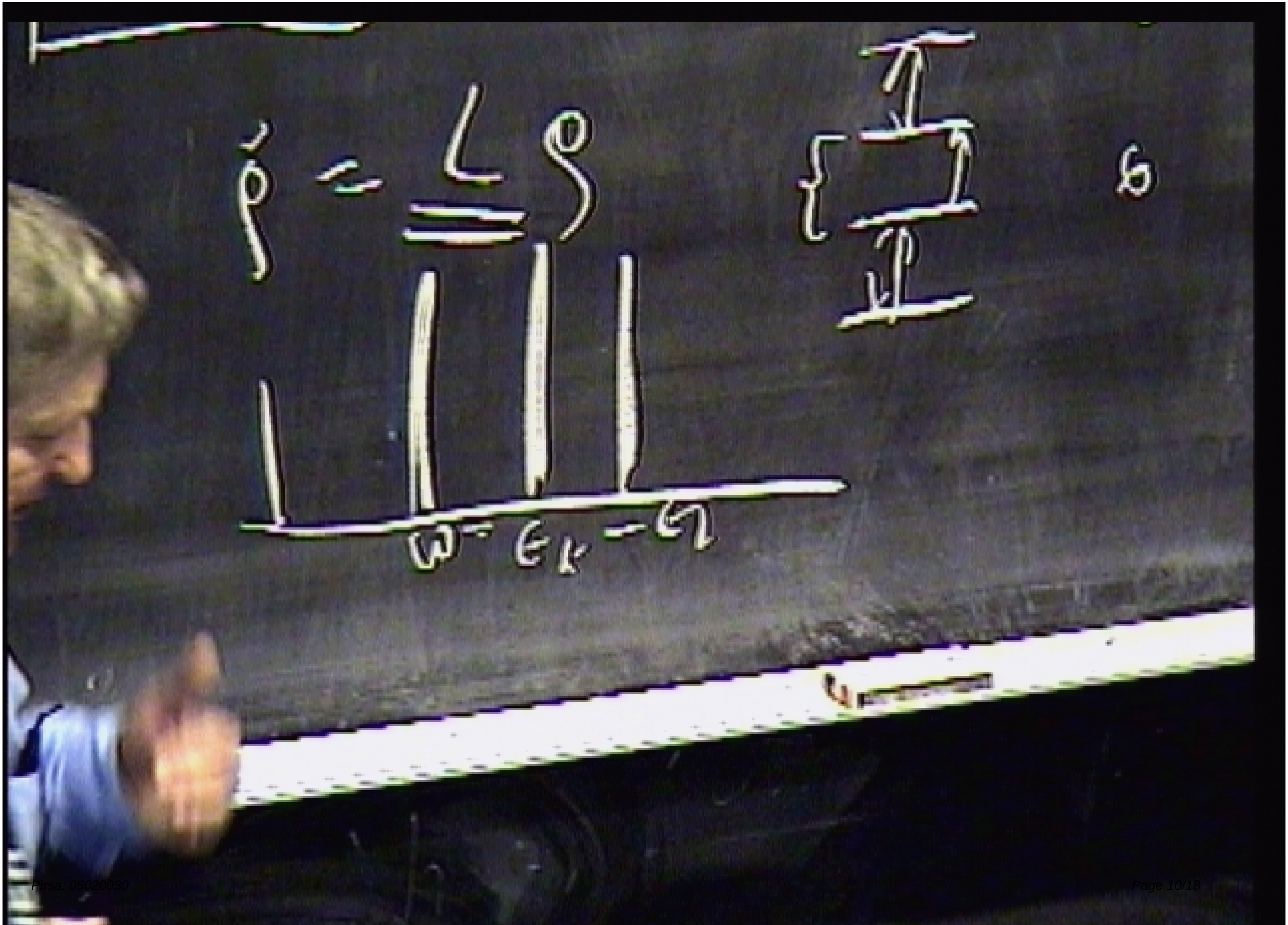
Hamiltonian periodic in time $H_S(t + \tau) = H_S(t)$.

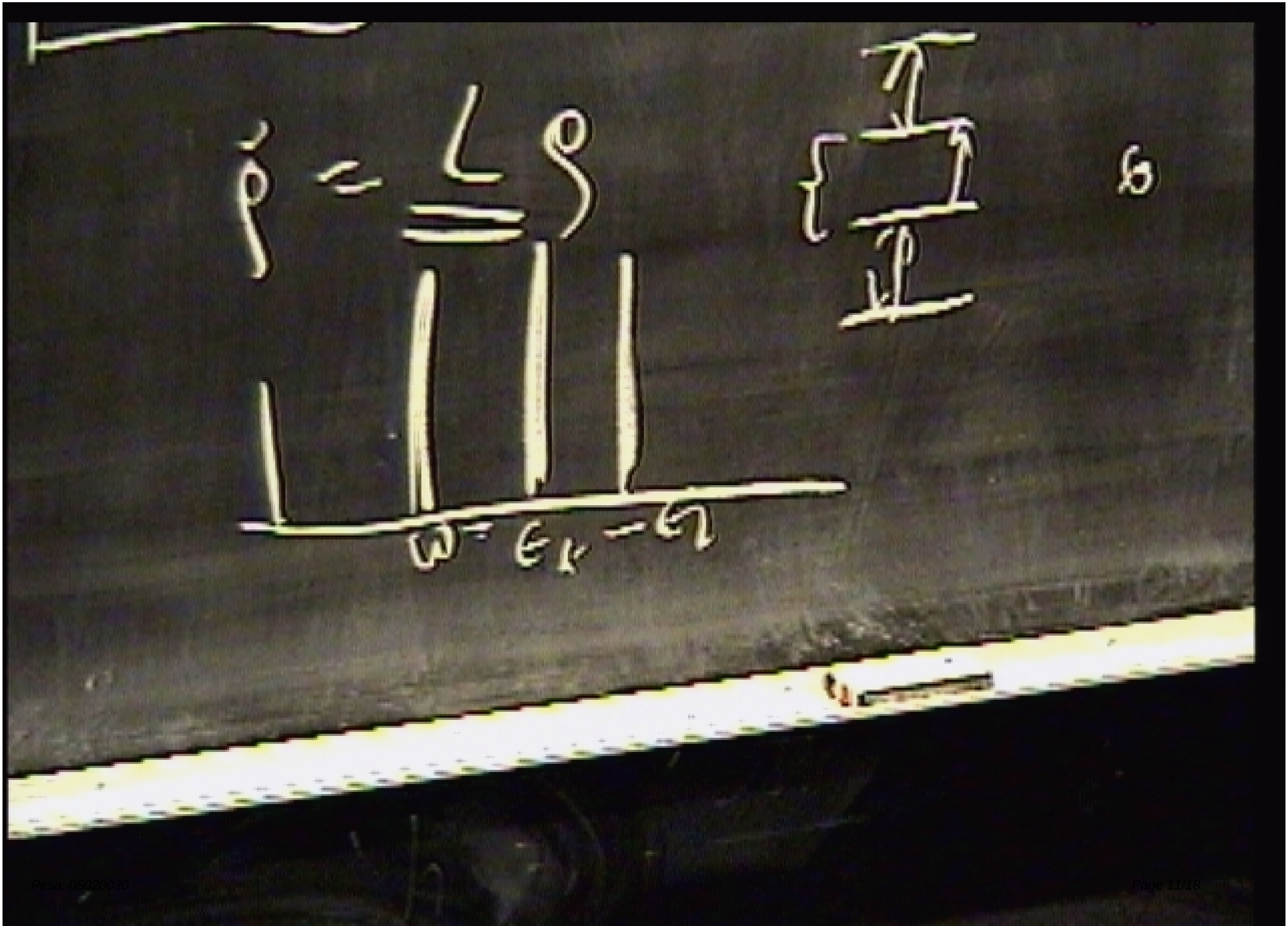
$\{|k>\}$- eigenvectors of Floquet operator

$$U(\tau) \equiv U(\tau, 0) \ , \ U(\tau)|k> = \exp(-\frac{i\epsilon_k \tau}{\hbar})|k>$$

where

$$U(t, s) = \mathbf{T} \exp\Big(-\frac{i}{\hbar} \int_s^t H_S(u) du\Big)$$





Representation of the evolution

$$U(t,0)|k>=\exp(-\frac{i\epsilon_k t}{\hbar})|\phi_k(t)> \ .$$

$\phi_k(t)$ - periodic function of t

$$S_{kl}(t) = \exp\{-\frac{i}{\hbar}(\epsilon_k - \epsilon_k)t\}\times$$

$$\times\frac{1}{\tau}\int_0^\tau <\phi_k(t+s)|S|\phi_l(t+s)><\phi_l(s)|S|\phi_k(s)> ds \ .$$

Power spectrum

$$S_{kl}(\omega) = \sum_{n=-\infty}^{\infty} \nu_n \, \delta[\omega - \frac{i}{\hbar}(\epsilon_l - \epsilon_k) - \frac{2\pi n}{\tau}]$$

Bang-bang technique

Select $\tau << \hbar/E_{cut}$ and the proper shape of $H_S(t)$,
seems to work for NMR and "engineered noises"

However:

the similar effect for fast gates - $t_g << \hbar/E_{cut}$,

frequencies satisfying $\hbar\omega >> E_{cut}$ often inconsistent with the model.

Zeno effect

Zeno effect - von Neumann measurements combined with unitary evolution → effective dynamics

$$\rho(t) = \sum_j W_j(t) \rho W_j^{\dagger}(t)$$

where

$$W_j(t) = \lim_{n\to\infty} [P_j U(t/n) P_j]^n = P_j \exp\{-(i/\hbar) P_j H P_j t\} \ .$$

Stabilization of a quantum state $\psi \in P_j(\mathcal{H})$.

Predictive power of "Zeno effect" $\simeq 0$

Example *Stability or instability of a nucleon in a nucleus*

Open system in strong coupling regime

$$H = H_S + H_R + H_{int} \ , \ H_{int} = S \otimes R$$

New basis from spectral resolution of S

$$S = \sum_j s_j P_j \ , P_j = |j><j| \ .$$

Decomposition of H

$$H_R^{(j)} = H_R + s_j R \ , \ H_S = \sum_j \epsilon_j P_j + V$$

$$H = H_0 + V \ , \ \ H_0 = \sum_j P_j \otimes (\epsilon_j \mathbf{1}_R + H_R^{(j)})$$

Eigenvectors and eigenvalues

$$H_R^{(j)}|E,\gamma;j>=E|E,\gamma;j> \ .$$

For any j , $E \geq E_g^{(j)}$ - ground state energy of $H_R^{(j)}$.

$$H_0|j>\otimes|E,\gamma;j>=(\epsilon_j+E)|j>\otimes|E,\gamma;j> \ , \ E\geq E_g^{(j)} \ .$$

Initial (ground) state $|E_g^{(l)}>$) (zero temperature reservoir)

Transition probability

$$P_{kl}=\frac{2\pi}{\hbar}|<k|V|l>|^2\sum_\gamma\int_{E_g^{(k)}}^{\infty}dE\,|<E,\gamma|R|E_g^{(l)}>|^2\delta(\epsilon_k+E-\epsilon_l-E_g^{(l)}) \ .$$

Threshold condition for $P_{kl}=0$ ("Zeno effect")

$$E_g^{(k)}+\epsilon_k>E_g^{(l)}+\epsilon_l$$

Energy conservation forbids transitions $|l>\mapsto|k>$

Remark

The system S becomes "dressed system" due to strong interaction with R!

A model of Zeno effect

Experimental test of Zeno effect
(W.M. Itano et.al.: Phys.Rev,A **41**, 2295 (1990))

3-level atom (1,2,3),
1 $\Longleftrightarrow$ 3- strong laser field (F_k),
1 $\Longleftrightarrow$ 2- Rabi oscillations with a frequency Ω

Model Hamiltonian

$$H = \hbar\omega_{13}|3><3| + \frac{\hbar}{2}\Omega(|1><2| + |2><1|) + \hbar\sum_k \omega_k[a^\dagger - \bar{F}_k][a_k - F_k]$$

$$+g_{13}(|1><3| + |3><1|) \otimes \sum_k (f_k a_k + \bar{f}_k a_k^\dagger) \ .$$

New basis

$$|\pm> = \frac{1}{\sqrt{2}}(|1> \pm |3>) \ , \ |2> \ , \ b_k = a_k - F_k$$

$$H = H_0 + V$$

$$H_0 = P_- \otimes (H_-^{(em)} + \epsilon_-) + P_+ \otimes (H_+^{(em)} + \epsilon_+) + P_2 \otimes H^{(em)}$$

$$H^{(em)} = \sum_k \omega_k b_k^\dagger b_k$$

$$H_\pm^{(em)} = H^{(em)} \pm g_{13}\sum_k (f_k b_k + \bar{f}_k b_k^\dagger) \ ,$$

$$\epsilon_\pm = \frac{\hbar\omega_{13}}{2} \pm g_{13}\sum_k (f_k F_k + \bar{f}_k \bar{F}_k) \ ,$$

$$V = -\frac{\hbar\omega_{13}}{2}(|+><-| + |-><+|)$$

$$+\frac{\hbar\Omega}{2\sqrt{2}}(|+><2| + |-><2| + |2><-| + |2><+|) \ .$$

Ground state energies of $H^{(em)}, H^{(em)}_{\pm}$

$$E_g = 0\ ,\ E_g^{(-)} = E_g^{(+)} = -\sum_k \frac{|f_k|^2}{\hbar^2 \omega_k^2}$$

Corresponding coherent ground states $|F>, |F^{(\pm)}>$ satisfying

$$b_k|F> = 0\ , b_k|F^{(\pm)}> = \mp \frac{\bar{f}_k}{\hbar\omega_k}|F^{(\pm)}>\ .$$

For $g_{13}\sum_k (f_k F_k + \bar{f}_k \bar{F}_k) > 0$ and large $|F_k|$

(energy $|- > \otimes |F^{(-)}$) << (energy of $|2> \otimes |F>$)

Hence the transition $|- > \mapsto |2>$ driven by the Rabi term is forbidden by the energy conservation principle.

"Zeno effect" caused by "continuous measurements" performed on the levels 1 and 3 by strong laser field.

Remark
The main difficulty in applications: S becomes a **dressed system,** *"individual qubits" with their own controlled unitary dynamics are not well-defined.*

Decoherence free subspaces

Open system S weakly interacting with a reservoir R.

Decoherence-free subspace $\mathcal{H}^{DFS}$:

$< k|S|l >= 0$ for all $|l >\in \mathcal{H}^{DFS}$ and $|k >; < k|l >= 0$ (forbidden transitions).

$\Longleftrightarrow$ $S|l >= s|l >$ for all $|l >\in \mathcal{H}^{DFS}$.

Symmetries and forbidden transitions

Symmetries $\Rightarrow$ representation of semi-simple Lie algebra (group) given by a basis of operators $X_\mu = X_\mu^\dagger$ with the Casimir operator C

$$C = \sum_\mu X_\mu^2 \,.$$

$\mathcal{H}_0$ - eigenspace of C to the eigenvalue zero

$$\text{for all} \phi \in \mathcal{H}_0 \;,\; 0 =< \phi|C|\phi >= \sum_\mu \|X_\mu \phi\|^2 \Leftrightarrow X_\mu \phi = 0 \,.$$

Therefore, for any

$$H_{int} = \sum_\mu X_\mu \otimes R_\mu$$

$\mathcal{H}_0$ is DFS.

Typical examples:

systems with rotational (or isospin) symmetry

$$[J_k, J_l] = i\epsilon_{klm} J_m \;, k, l, m = 1, 2, 3$$

$$C = \mathbf{J}^2 = J_1^2 + J_2^2 + J_3^2, eigenvalues - j(j+1) \;, j = 0, 1/2, 1, ...$$

Singlet states $(j = 0)$ span DFS for $H_{int} = \sum_k J_k \otimes R_k$.

Collective decoherence

Collective interaction for 2N qubits (superradiance and subradiance phenomena)

$$H_{int} = \sum_{j=1}^{2N} \sum_{k=1}^{3} \sigma_k^{(j)} \otimes R_k$$

Collective spin operators

$$J_k = \frac{1}{2} \sum_{j=1}^{2N} \sigma_k^{(j)} \ , \ k = 1, 2, 3$$

generate a **reducible** representation of $SU(2)$.

$2N$-qubit singlet states span the DFS of the dimension $(2N!)/(N+1)!N!$.

DSF $\Longleftrightarrow$ symmetry of H_{int} with respect to qubit permutations

Remarks

The engineering of collective H_{int} in real systems is a very difficult task.

The permutation invariance can be only approximative.

All qubits should be placed within a space region of a small linear dimension.

A dense packing of qubits leads to unwanted interactions - not permutation invariant.