

Title: Codimension two braneworlds, Episode 2: The cosmological constant strikes back

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Abstract:

The cosmological constant problem

QFT tells us the vacuum has a huge energy. GR tells us everything gravitates.

- ★ Fine-tuning problem:

- ★ Why is the observed CC so small (but not exactly zero)?
- ★ Even if it were small, why should its value be stable to quantum corrections?

- ★ Coincidence problem:

- ★ Why is its value so close to the matter density *today*?

We will be addressing only the first of these issues here.

The cosmological constant problem

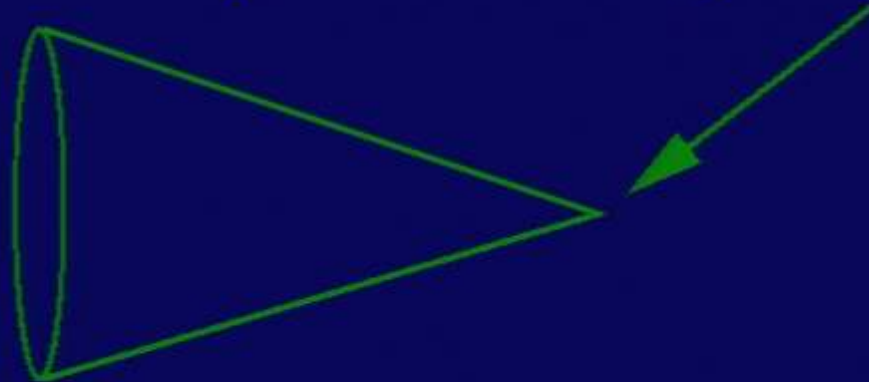
Theorists never run out of ideas:

- ★ Quintessence?
- ★ K-essence?
- ★ Modified gravity?
- ★ Anthropic principle???
- ★ Extra dimensions, **braneworlds**?
- ★ ...

Codimension-2 branes

What's the big deal anyway?

★ Codimension 2 gives conical singularity



deficit angle is $\Delta\theta = \kappa^2 T$

Codimension-2 branes

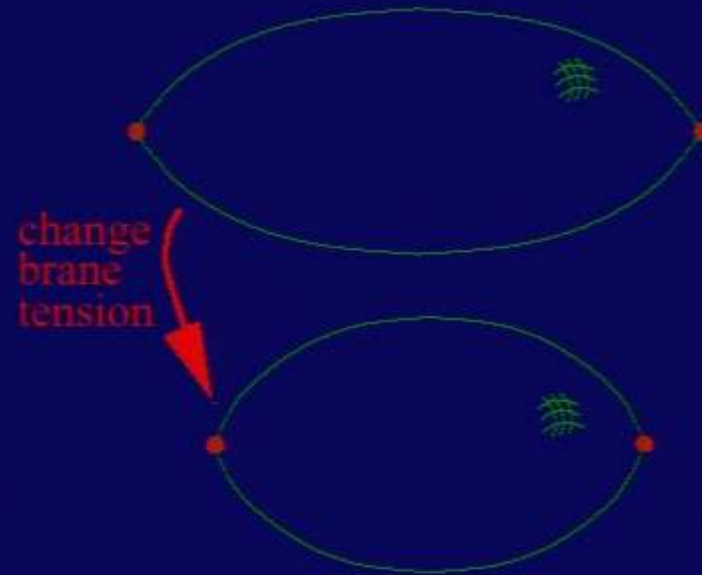
Suppose we find a static solution for braneworld with a given tension T :

$$ds^2 = a^2(r)(-dt^2 + d\vec{x}^2) + dr^2 + b^2(r)d\theta^2$$

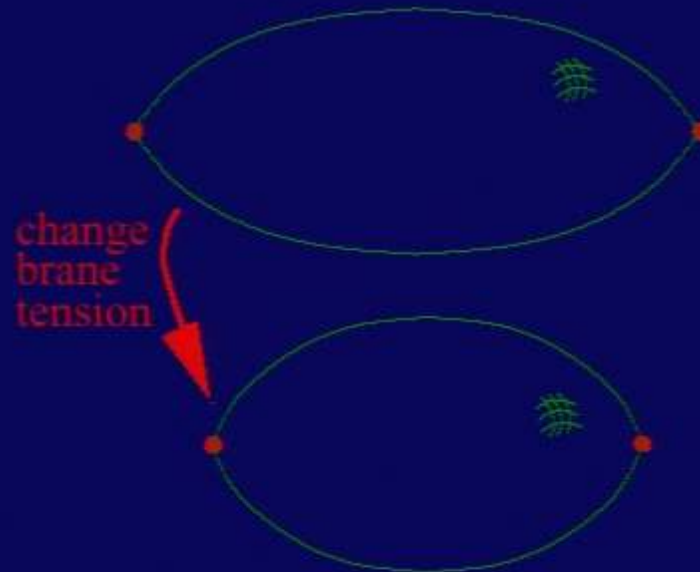
Can find new solution for tension T' by rescaling

$$b \rightarrow \frac{1 - T'/2\pi}{1 - T/2\pi} b$$

Codimension-2 branes



Codimension-2 branes



Static solution exists for any

tension!* Rubakov, Shapshnikov '84; Wetterich '85;
Sundrum '99; Chen, Luty, Ponton '00; Carroll, Guica '03;
Navarro '03

Codimension-2 branes

Actually...

★ In Einstein-Hilbert gravity,

$$H^2 = \frac{1}{6M_6^4} \left(\Lambda_6 - \frac{\beta^2}{2} \right)$$

where β^2 is the field strength from a bulk two-form.

6D Supergravity

The relevant part of the action:

$$S = \int dx^6 \sqrt{-g} \left[\frac{M_6^4}{2} \left(\mathcal{R} - \frac{1}{2} \partial_M \phi \partial^M \phi \right) - \frac{1}{4} e^{-\phi} F^2 - \frac{1}{2} h(\Phi)_{ab} \partial_M \Phi^a \partial^M \Phi^b - e^\phi v(\Phi) \right] + S_b,$$

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If we have $\Phi^a = cst$ and $\phi = cst$, then this is just like the E-H action, with $v(\Phi)$ playing the role of the bulk CC. But... there is one more equation of motion for ϕ .

6D Supergravity

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$$S = \int dx^6 \sqrt{-g} \left[\frac{M_6^4}{2} \left(\mathcal{R} - \frac{1}{2} \partial_M \phi \partial^M \phi \right) - \frac{1}{4} e^{-\phi} F^2 - \frac{1}{2} h(\Phi)_{ab} \partial_M \Phi^a \partial^M \Phi^b - e^\phi v(\Phi) \right] + S_b,$$

This equation enforces the tuning between the field strength and bulk CC needed for a static solution!

(Aghababaie, Burgess, Parameswaran, Quevedo '03)

Einstein-Hilbert gravity

In E-H gravity, we can see easily that static solutions depend of a fine tuning of the brane tensions. (Vinet, Cline '04; Garriga, Porrati '04)

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Let's start with Λ_6 and F^2 tuned to give a static solution, for a given set of brane tensions.

Now let's change one of the tensions slightly...

Einstein-Hilbert gravity

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Let's start with Λ_6 and F^2 tuned to give a static solution, for a given set of brane tensions.

Now let's change one of the tensions slightly...

- ★ The deficit angle changes at that brane, so the volume of the internal space changes.

Six dimensional supergravity

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Six dimensional supergravity

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But wait... What happens to the enforced tuning $\Lambda_6 = \frac{\beta^2}{2}$???

$$V(\sigma_1, \sigma_2) = e^{-\sigma_2} (k e^{-2\sigma_1} - 2K e^{-\sigma_1} + 2\Lambda_6)$$

- ★ Static dilaton, radion $\Rightarrow V = 0, \partial V / \partial \sigma_i = 0$.
- ★ If we change the constants k and K , $\sigma_2 \rightarrow \infty$

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- ★ Gibbons, Güven, Pope '03 found all solutions to 6d supergravity with an axially symmetric static internal space, a static dilaton, a bulk two-form and a maximally symmetric external space.

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- ★ Gibbons, Güven, Pope '03 found all solutions to 6d supergravity with an axially symmetric static internal space, a static dilaton, a bulk two-form and a maximally symmetric external space.
- ★ They showed that the only solutions are ones where the external space is static.

General solution

$$ds^2 = W^2 dx^\mu dx_\mu + a^2 W^8 dr^2 + a^2 d\theta^2,$$

$$e^\phi = W^4 e^{2\lambda_3 r}$$

$$W^4 = \frac{q\lambda_2 \cosh \lambda_1 (r - r_1)}{4g\lambda_1 \cosh \lambda_2 (r - r_2)}$$

$$a^{-4} = \frac{gq^3}{\lambda_1^3 \lambda_2} e^{-2\lambda_3 r} \cosh^3 \lambda_1 (r - r_1) \cosh \lambda_2 (r - r_2)$$

$$\lambda_2^2 = \lambda_1^2 + \lambda_3^2.$$

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When $\lambda_3 \neq 0$, the singularities at the branes are not conical,
i.e. $ds_2^2 \sim dr^2 + kr^{2-k_1} d\theta^2$.

Burgess et. al. '04

The hope

Starting from a static solution with conical singularities how does the system react to perturbing the tensions?

Codimension-2 branes

Questions we'd like to answer

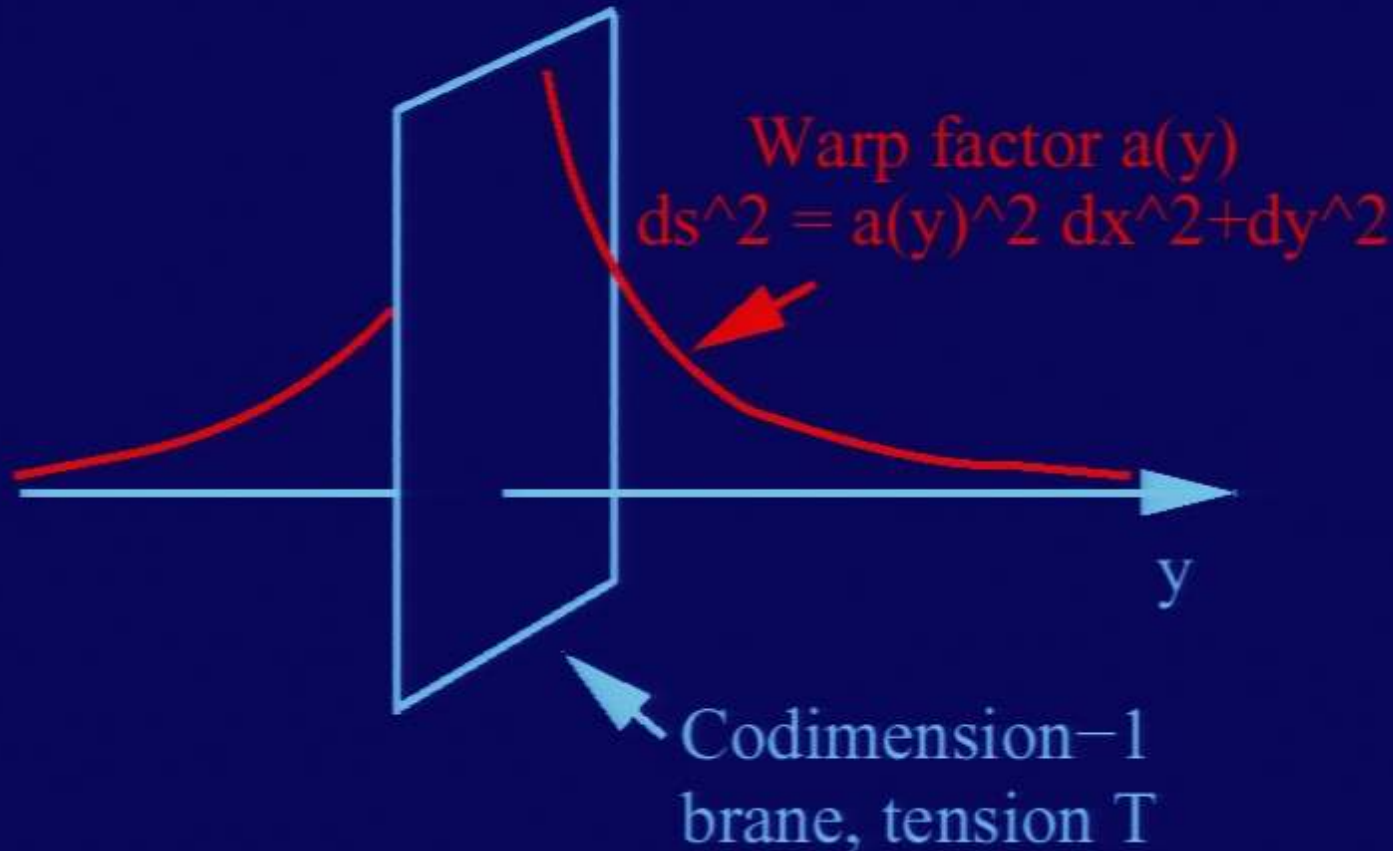
Codimension-2 branes

Questions we'd like to answer

- ★ What happens if we put more general matter on the brane? (i.e. $\rho \neq -p$)
- ★ Do we recover FRW cosmology?

Codimension-2 branes

For codimension-1 branes,



the discontinuities in g_{00}' and g_{ii}' allow us to have branes with arbitrary equations of state ($\rho \neq -p$).

Codimension-2 branes

Since in 2D

$$\nabla^2(\ln(r)) = \frac{\delta(r)}{2\pi r},$$

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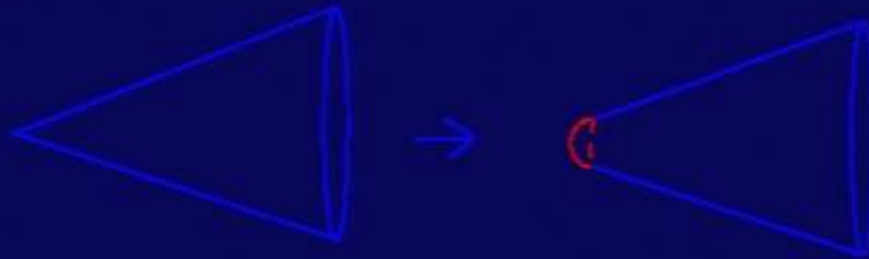
For codimension-2 branes, the 2D delta function in $T_{\mu\nu}$ must match with terms like $\nabla^2(\ln(g_{\mu\nu}))$ in $G_{\mu\nu}$.

Thick branes

Simple regularization scheme: replace δ -function brane by a step function.

Thick branes

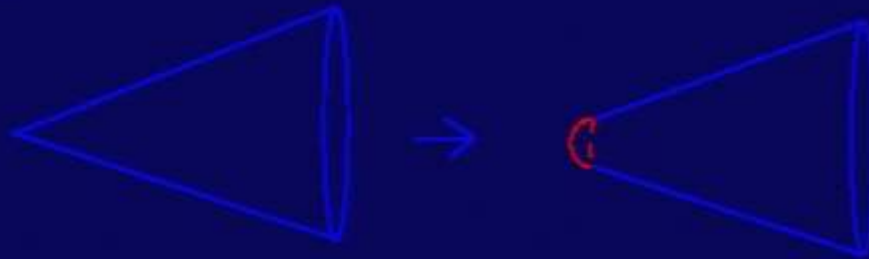
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- ★ Reduces to the expected δ -function solution in the zero thickness limit;

Thick branes

Simple regularization scheme: replace δ -function brane by a step function.



- ★ Allows freedom to detune the deficit angle and brane tension

Thick branes

We can treat the matter as a perturbation to the 3-brane tension

$$s_0^0 = -\theta(r - r_0(t))(\tau_3 + \rho(t))$$

$$s_i^i = \theta(r - r_0(t))(-\tau_3 + p(t)).$$

Treat the time dependence in the thickness as a perturbation

$$\theta(r - r_0 - \Delta r_0(t)) \approx \theta(r - r_0) - \delta(r - r_0)\Delta r_0(t) + \dots$$

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SO

$$\begin{aligned} s_0^0 &= -\theta(r - r_0)\tau_3 + (-\theta(r - r_0)\rho(t) \\ &+ \tau_3\delta(r - r_0)\Delta r_0(t)) + \dots \end{aligned}$$

Thick branes

To *greatly* simplify the calculations, we will expand around a braneless background ($\tau_3 = 0$) but we will leave in the 1D δ -function term that encodes the time varying thickness.

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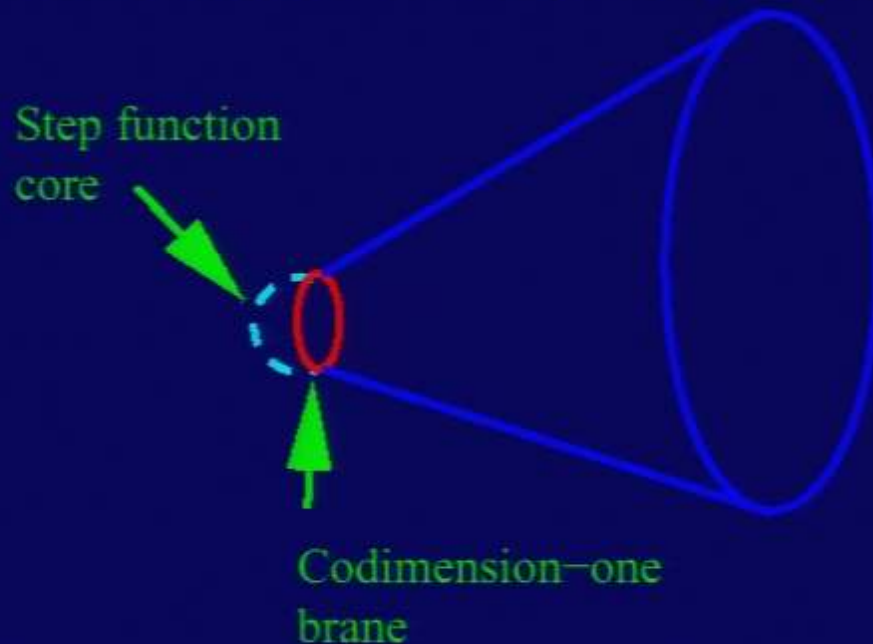
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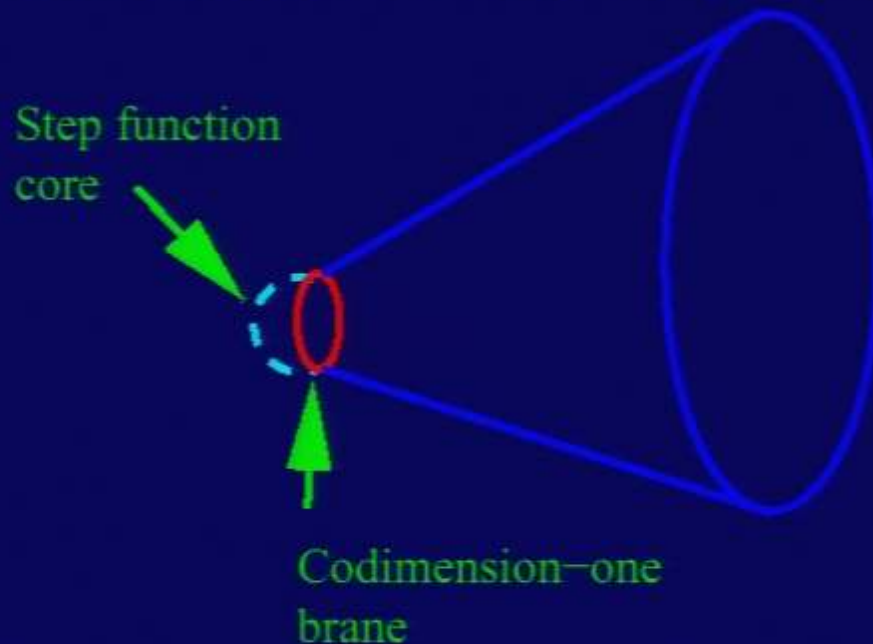
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(Kanno, Soda '04; Navarro, Santiago '04)

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Perturbative ansatz

$$ds^2 = n^2 dt^2 + a^2 d\vec{x}^2 + b^2 dr^2 + c^2 d\theta^2 + 2E dr dt$$

$$n(r, t) = e^{N_0(r) + N_1(r, t)}; \quad a(r, t) = a_0(t) e^{A_0(r) + A_1(r, t)};$$

$$b(r, t) = b_0(t) e^{B_0(r) + B_1(r, t)}; \quad c(r, t) = c_0(t) e^{C_0(r) + C_1(r, t)};$$

$$E(r, t) = E_1(r, t); \quad A_\theta(r, t) = A_\theta^{(0)}(r) + A_\theta^{(1)}(r, t)$$

$$e^{\phi(r, t)} = \varphi_0(t) e^{\phi_0(r) + \phi_1(r, t)}$$

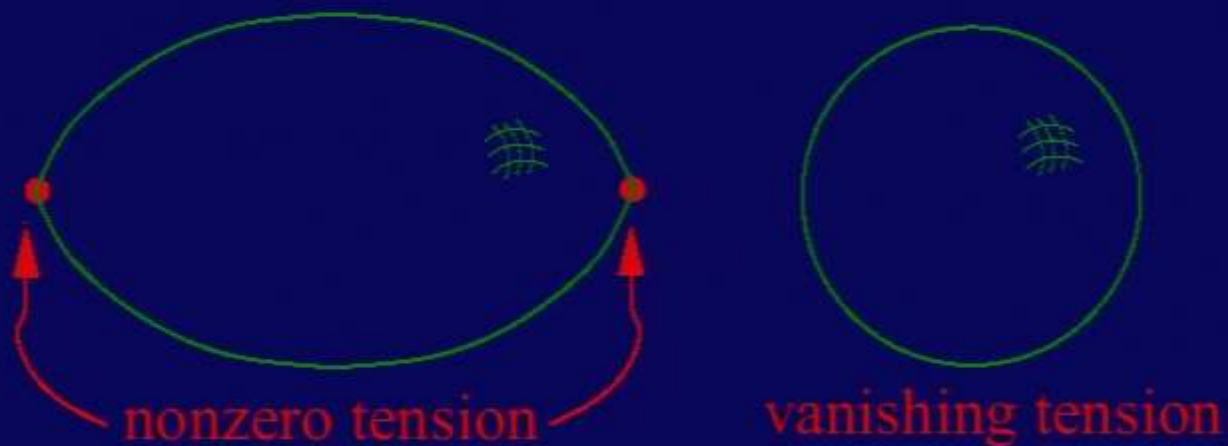
Background

For simplicity, we will choose a **braneless, static, unwarped** background:



Background

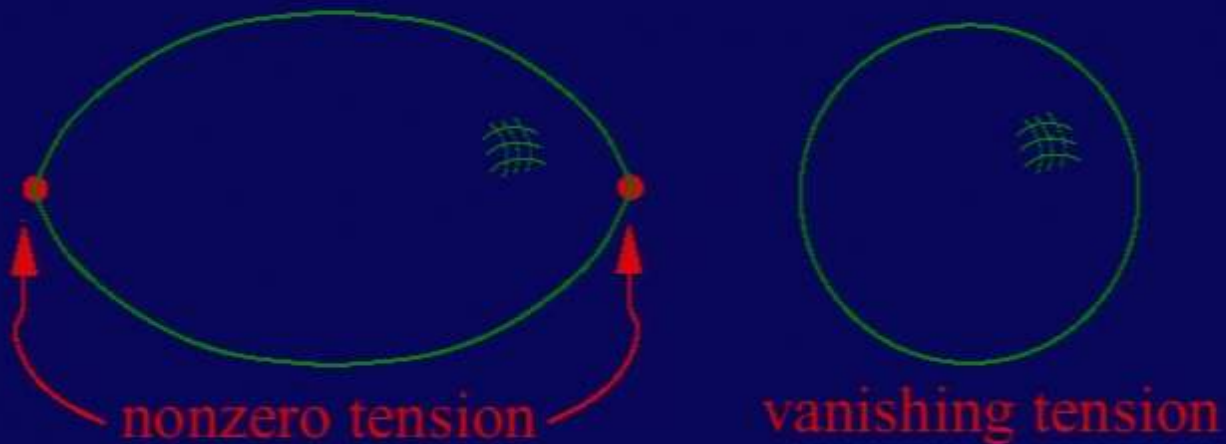
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Choosing the **braneless** solution makes the equations much simpler to solve, and does not take away from the generality of the conclusions.

Background

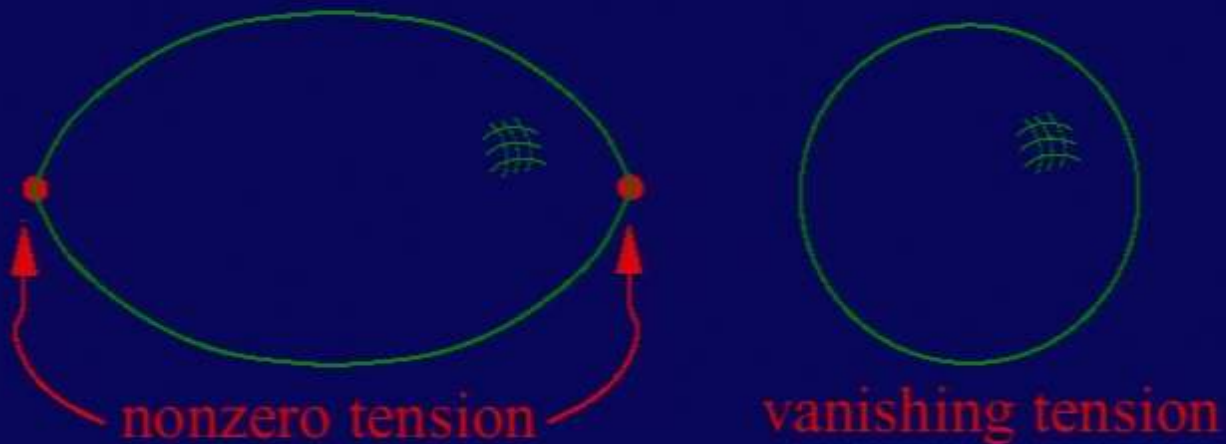
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Unwarped solutions require both branes to have *same tension*. This is not general, but our perturbations will allow us to relax this.

Background

For simplicity, we will choose a **braneless, static, unwarped** background:



As we have already seen, the static solution is singled out in 6D supergravity.

Background

$$N_0(r) = A_0(r) = B_0(r) = 0$$

$$\phi_0(r) = \phi_0$$

$$e^{C_0(r)} = \frac{\sin(kr)}{k}$$

$$A_\theta^{(0)'}(r) = -\beta e^{\phi_0} e^{C_0(r)}$$

$$b_0(t) = c_0(t)$$

$$\varphi_0(t) = \frac{\beta}{\sqrt{2v(\Phi_0)c_0(t)^2}}$$

$$\beta^2 = \frac{k^4 M_6^8}{2v(\Phi_0)}$$

Gauge invariant variables

The following variables are invariant under
 $t \rightarrow t + \Delta t(r, t), r \rightarrow r + \Delta r(r, t)$:

$$Z = N'_1 - A'_1; \quad W = 3A'_1 + N'_1; \quad X = \frac{C'_1}{C'_0} - B_1 - \frac{C''_0}{C'^2_0} C_1;$$

$$Y = A_{\theta}^{(1)'} - A_{\theta}^{(0)'} (B_1 + C_1); \quad U = \dot{A}_{\theta}^{(1)} - \frac{A_{\theta}^{(0)'}}{C'_0} \dot{C}_1;$$

$$\tilde{p} = -s_t^t - s_{*t}^t; \quad \tilde{p} = s_i^i + s_{*i}^i; \quad \tilde{p}_t^r = s_t^r + s_{*t}^r;$$

$$\tilde{p}_r^t = s_r^t + s_t^r + s_{*r}^t + s_{*t}^r; \quad \tilde{p}_5 = s_r^r + s_{*r}^r;$$

$$\tilde{p}_6 = s_{\theta}^{\theta} - s_r^r + s_{*\theta}^{\theta} - s_{*r}^r$$

$\mathcal{O}(\rho)$ equations of motion

$$W' - C_0' W = c_0(t)^2 \frac{\tilde{p}_6}{M_6^4}$$

$$\frac{Z' + C_0' Z}{c_0(t)^2} = 2 \left[\frac{\ddot{a}_0}{a_0} - \left(\frac{\dot{a}_0}{a_0} \right)^2 - \frac{\dot{a}_0 \dot{c}_0}{a_0 c_0} + 2 \left(\frac{\dot{c}_0}{c_0} \right)^2 + \frac{\ddot{c}_0}{c_0} \right] + \frac{1}{M_6^4} (\tilde{\rho} + \tilde{p})$$

$$Y' - C_0' Y - e^{\phi_0} \beta e^{C_0(r)} (W - \phi_1') = 0$$

$$\frac{C_0' W}{c_0(t)^2} + \frac{\sqrt{2v(\Phi_0)} e^{-C_0(r)}}{c_0(t)^2 M_6^4} Y + \beta \frac{\sqrt{2v(\Phi_0)} e^{\phi_0}}{c_0(t)^2 M_6^4} \phi_1 = 3 \left[\frac{\ddot{a}_0}{a_0} + \left(\frac{\dot{a}_0}{a_0} \right)^2 \right] + \frac{\ddot{c}_0}{c_0} + 2 \left(\frac{\dot{c}_0}{c_0} \right)^2 + \frac{1}{M_6^4} \tilde{p}_5$$

$\mathcal{O}(\rho)$ equations of motion

$$\tilde{p}'_5 - C'_0 \tilde{p}_6 = 0$$

$$U' - \dot{Y} - \beta e^{\phi_0} e^{C_0(r)} \dot{X} = 0$$

$$\tilde{p}'_r = -c_0(t)^2 \tilde{p}'_t$$

$$3 \frac{\dot{a}_0}{a_0} Z - C'_0 \dot{X} + \frac{3}{4} (\dot{Z} - \dot{W}) = \frac{c_0(t)^2}{M_6^4} \tilde{p}'_t - \frac{\sqrt{2v(\Phi_0)} e^{-C_0(r)}}{M_6^4} U - \frac{\dot{c}_0}{c_0} (W + 2\phi'_1)$$

$$\ddot{\rho} + 3 \frac{\dot{a}_0}{a_0} (\dot{\rho} + \dot{p}) + \frac{\dot{c}_0}{c_0} (2\dot{\rho} + 2\dot{p}_5 + \dot{p}_6) = \tilde{p}'_t + C'_0 \tilde{p}_t$$

$\mathcal{O}(\rho)$ equations of motion

$$\frac{\phi_1'' + C_0' \phi_1' + C_0' W}{3c_0(t)^2} = \left[\frac{\ddot{a}_0}{a_0} + \left(\frac{\dot{a}_0}{a_0} \right)^2 - \frac{1}{3} \frac{\ddot{c}_0}{c_0} - \frac{\dot{a}_0 \dot{c}_0}{a_0 c_0} + \frac{\tilde{p}_5}{3M_6^4} \right]$$

$$\begin{aligned} & \frac{C_0' X'}{c_0(t)^2} - \frac{2X\beta\sqrt{2v(\Phi_0)}e^{\phi_0}}{M_6^4 c_0(t)^2} + \frac{3C_0' W}{2c_0(t)^2} - \frac{\sqrt{2v(\Phi_0)}e^{-C_0(r)}}{c_0(t)^2 M_6^4} Y \\ & = -\frac{1}{4M_6^4} (\tilde{\rho} - 3\tilde{p} + 3\tilde{p}_6) \\ & \quad + \frac{3}{2} \left[\frac{\ddot{a}_0}{a_0} + \left(\frac{\dot{a}_0}{a_0} \right)^2 + \frac{\ddot{c}_0}{c_0} + 3 \frac{\dot{a}_0 \dot{c}_0}{a_0 c_0} + \frac{4}{3} \left(\frac{\dot{c}_0}{c_0} \right)^2 \right] \end{aligned}$$

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$$\tilde{p}_6(r, t) = \theta(r - r_0)e^{2C_0(r)}\mathcal{P}_6(t) + \theta(r_* - r)e^{2C_0(r)}\mathcal{P}_{*6}(t)$$

- ★ Assume that $\rho(r, t)$, $p(r, t)$, $\rho_*(r, t)$ and $p_*(r, t)$ are functions of time only.

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★ Assume that $\rho(r, t)$, $p(r, t)$, $\rho_*(r, t)$ and $p_*(r, t)$ are functions of time only.

With these assumptions, perturbed EOM's are straightforward (if somewhat tedious) to solve.

Effective four dimensional quantities

$$\rho^{(4)}(t) = 2\pi \int_0^{r_0} c_0(t)^2 e^{C_0(r)} \rho(t) dr + 2\pi c_0(t)^2 e^{C_0(r_0)} \mathcal{F}_0(t)$$

$$p^{(4)}(t) = 2\pi \int_0^{r_0} c_0(t)^2 e^{C_0(r)} p(t) dr - 2\pi c_0(t)^2 e^{C_0(r_0)} \mathcal{F}_0(t)$$

$$\rho_*^{(4)}(t) = 2\pi \int_{\pi/k - r_0}^{\pi/k} c_0(t)^2 e^{C_0(r)} \rho_*(t) dr + 2\pi c_0(t)^2 e^{C_0(r_*)} \mathcal{F}_*(t)$$

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Effective four dimensional quantities

$$\frac{1}{8\pi G_4(t)} = M_4^2 = 2\pi \int_0^{\pi/k} c_0(t)^2 e^{C_0(r)} M_6^4 dr$$
$$\Rightarrow G_4(t) = \frac{k^2}{32\pi^2 M_6^4 c_0(t)^2}$$

Non-conical solutions?

GGP '03 showed that with the metric

$$ds^2 = n(r)^2 dx^\mu dx_\mu + dr^2 + c(r)^2 d\theta^2$$

$$\lambda_3 = \frac{1}{2}c(r) (2n(r)^4 \phi(r)' + 4n(r)^3 n(r)') .$$

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In our perturbative language, $\lambda_3 = 0$ at the level of the background. But the above can be written perturbatively as

$$\delta\lambda_3 = \frac{1}{2} e^{C_0(r)} (2\phi_1(r)' + W(r)) .$$

Non-conical solutions?

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Dependence on extra dimensional part of brane stress-energy tensor, consistent with Burgess et.al. '04; Navarro, Santiago '04.

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- ★ All functions smooth at the poles $r = 0, \pi/k$;
- ★ All functions (but not their radial derivatives!) continuous across core/bulk boundary, except where warranted by 1D δ -function.

Friedmann equations

$$\left(\frac{\dot{a}_0}{a_0}\right)^2 = \frac{8\pi G_4(t)}{3}(\rho^{(4)} + \rho_*^{(4)}) - \frac{16\pi^2 \sin(kr_0)}{3k} G_4(t) \mathcal{Q}_1$$

$$+ \frac{1}{3} \left(\frac{\dot{c}_0}{c_0}\right)^2 - 2 \frac{\dot{a}_0 \dot{c}_0}{a_0 c_0}$$

$$\frac{\ddot{a}_0}{a_0} - \left(\frac{\dot{a}_0}{a_0}\right)^2 = -4\pi G_4(t)(\rho^{(4)} + p^{(4)} + \rho_*^{(4)} + p_*^{(4)}) - \frac{\ddot{c}_0}{c_0}$$

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- ★ $S = \int d^4x \sqrt{-g} \left(\frac{1}{16\pi G} \left[\zeta \mathcal{R} - \frac{1}{2\zeta} \partial_\mu \zeta \partial^\mu \zeta - V(\zeta) \right] + \mathcal{L}_m \right)$
with $c_0(t) = \sqrt{\zeta}$, $V(\zeta) = -32\pi^2 \sin(kr_0) \bar{G} Q_1 / k$ and
 $\bar{G} / \zeta(t) = G_4(t)$
- ★ Ok, but what about self-tuning???

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- ★ starting from static solution with conical singularities, perturb brane stress energy;
- ★ rather than going to runaway solution, system would evolve to static solution with non-conical singularities;
- ★ we saw that $\delta\lambda_3 \sim \mathcal{P}_6 - \mathcal{P}_{*6}$
- ★ can \mathcal{P}_6 's evolve so as to keep solution static?

Absence of self-tuning

Furthermore \mathcal{P}_6 's do not appear in

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However \mathcal{P}_6 's allow us to keep internal space static, albeit not in a natural way by forcing the RHS of

$$\begin{aligned} \frac{\ddot{c}_0}{c_0} + \left(\frac{\dot{c}_0}{c_0} \right)^2 + 3 \frac{\dot{a}_0 \dot{c}_0}{a_0 c_0} &= \pi G_4(t) \left(\rho^{(4)} - 3p^{(4)} + \rho_*^{(4)} - 3p_*^{(4)} \right) \\ &+ \frac{2\pi^2 c_0(t)^2 (1 - \cos(kr_0))^3}{3k^4} G_4(t) (\mathcal{P}_6 + \mathcal{P}_{*6}) \\ &- \frac{8\pi^2 \sin(kr_0)}{k} G_4(t) \mathcal{Q}_1. \end{aligned}$$

to vanish.

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- ★ Codimension-two branes \Rightarrow unconventional relation between H^2 and vacuum energy;
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- ★ Codimension-two branes \Rightarrow unconventional relation between H^2 and vacuum energy;
- ★ Implicit relation between expansion and vacuum energy \Rightarrow no self-tuning;
- ★ Effective description apparently dependent on details of brane structure;
- ★ Possibility of getting stabilized internal space/dilaton;
- ★ Fine-tuning still needed, but SLEDs might still help explain smallness of quantum corrections;
- ★ Not mentionned here, but could breaking axial symmetry help? (Redi '04)

No Signal

VGA-1