Title: Codimension two braneworlds, Episode 2: The cosmological constant strikes back

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Abstract:

The cosmological constant problem

QFT tells us the vacuum has a huge energy. GR tells us everything gravitates.

- ★ Fine-tuning problem:
  - ★ Why is the observed CC so small (but not exactly zero)?
  - ★ Even if it were small, why should its value be stable to quantum corrections?
- ★ Coincidence problem:
  - ★ Why is its value so close to the matter density *today*?

We will be addressing only the first of these issues here.

The cosmological constant problem

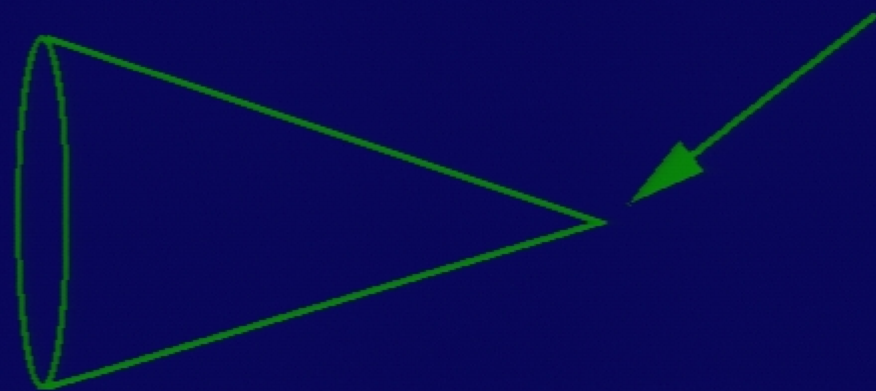
Theorists never run out of ideas:

- ★ Quintessence?
- ★ K-essence?
- ★ Modified gravity?
- ★ Anthropic principle???
- ★ Extra dimensions, braneworlds?
- ★ ...

Codimension-2 branes

What's the big deal anyway?

★ Codimension 2 gives conical singularity



deficit angle is $\Delta\theta = \kappa^2 T$

Codimension-2 branes

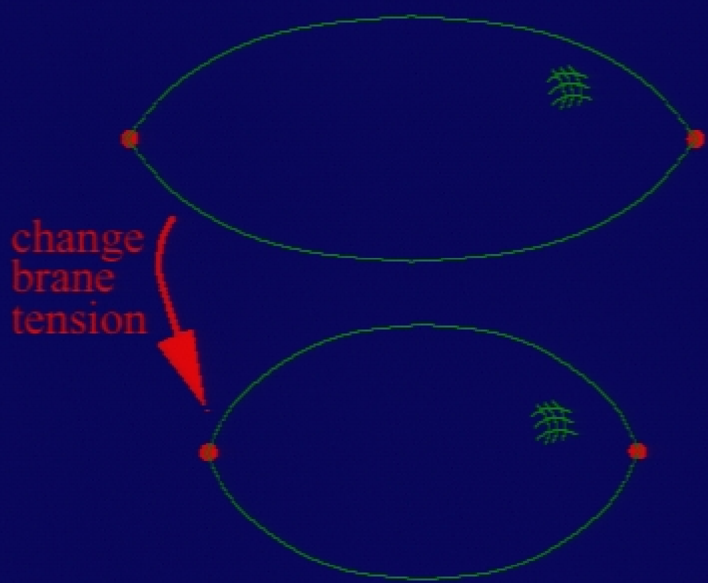
Suppose we find a static solution for braneworld with a given tension T:

$$ds^2 = a^2(r)(-dt^2 + d\vec{x}^2) + dr^2 + b^2(r)d\theta^2$$

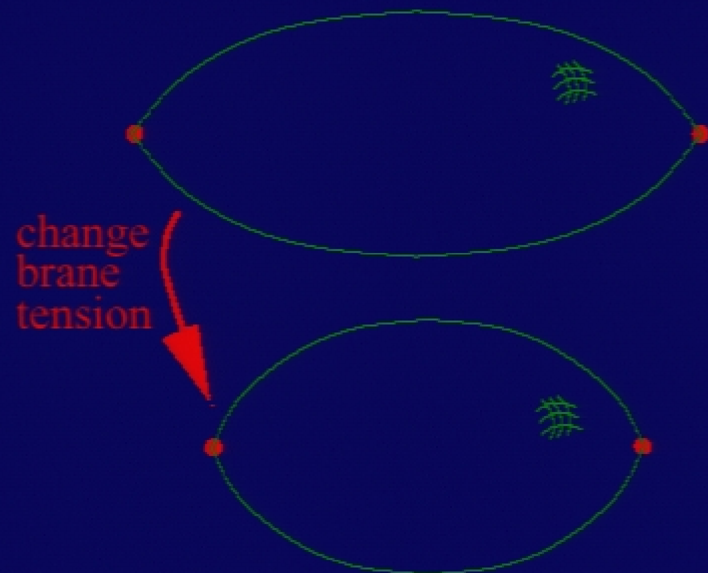
Can find new solution for tension T' by rescaling

$$b \to \frac{1 - T'/2\pi}{1 - T/2\pi} b$$

Codimension-2 branes



Codimension-2 branes



Static solution exists for any tension!* Rubakov, Shapshnikov '84; Wetterich '85; Sundrum '99; Chen, Luty, Ponton '00; Carroll, Guica '03; Navarro '03

Codimension-2 branes

Actually...

★ In Einstein-Hilbert gravity,

$$H^2 = \frac{1}{6M_6^4}\left(\Lambda_6 - \frac{\beta^2}{2}\right)$$

where β^2 is the field strength from a bulk two-form.

6D Supergravity

The relevant part of the action:

$$S = \int dx^6 \sqrt{-g}\left[\frac{M_6^4}{2}\left(\mathcal{R} - \frac{1}{2}\partial_M\phi\partial^M\phi\right) - \frac{1}{4}e^{-\phi}F^2 \right.$$
$$\left. -\frac{1}{2}h(\Phi)_{ab}\partial_M\Phi^a\partial^M\Phi^b - e^{\phi}v(\Phi)\right] + S_b,$$

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If we have $\Phi^a = cst$ and $\phi = cst$, then this is just like the E-H action, with $v(\Phi)$ playing the role of the bulk CC. But... there is one more equation of motion for ϕ.

6D Supergravity

The relevant part of the action:

$$S = \int dx^6 \sqrt{-g} \left[\frac{M_6^4}{2} \left(\mathcal{R} - \frac{1}{2} \partial_M \phi \partial^M \phi \right) - \frac{1}{4} e^{-\phi} F^2 - \frac{1}{2} h(\Phi)_{ab} \partial_M \Phi^a \partial^M \Phi^b - e^{\phi} v(\Phi) \right] + S_b,$$

This equation enforces the tuning between the field strength and bulk CC needed for a static solution!

(Aghababaie, Burgess, Parameswaran, Quevedo '03)

Einstein-Hilbert gravity

In E-H gravity, we can see easily that static solutions depend of a fine tuning of the brane tensions. (Vinet, Cline '04; Garriga, Porrati '04)

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Let's start with Λ_6 and F^2 tuned to give a static solution, for a given set of brane tensions.
Now let's change one of the tensions slightly...

Einstein-Hilbert gravity

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Let's start with Λ_6 and F^2 tuned to give a static solution, for a given set of brane tensions.
Now let's change one of the tensions slightly...

- ★ The deficit angle changes at that brane, so the volume of the internal space changes.

Six dimensional supergravity

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Six dimensional supergravity

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But wait... What happens to the enforced tuning $\Lambda_6 = \frac{\beta^2}{2}$???

$$V(\sigma_1, \sigma_2) = e^{-\sigma_2}\left(ke^{-2\sigma_1} - 2Ke^{-\sigma_1} + 2\Lambda_6\right)$$

- ★ Static dilaton, radion $\Rightarrow V = 0, \partial V/\partial\sigma_i = 0.$
- ★ If we change the constants k and K, $\sigma_2 \to \infty$

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- ★ Gibbons, Güven, Pope ’03 found all solutions to 6d supergravity with an axially symmetric static internal space, a static dilaton, a bulk two-form and a maximally symmetric external space.

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- ★ Gibbons, Güven, Pope ’03 found all solutions to 6d supergravity with an axially symmetric static internal space, a static dilaton, a bulk two-form and a maximally symmetric external space.
- ★ They showed that the only solutions are ones where the external space is static.

General solution

$$
\begin{aligned}
ds^2 &= W^2 dx^\mu dx_\mu + a^2 W^8 dr^2 + a^2 d\theta^2, \\
e^\phi &= W^4 e^{2\lambda_3 r} \\
W^4 &= \frac{q\lambda_2}{4g\lambda_1} \frac{\cosh \lambda_1 (r - r_1)}{\cosh \lambda_2 (r - r_2)} \\
a^{-4} &= \frac{gq^3}{\lambda_1^3 \lambda_2} e^{-2\lambda_3 r} \cosh^3 \lambda_1 (r - r_1) \cosh \lambda_2 (r - r_2) \\
\lambda_2^2 &= \lambda_1^2 + \lambda_3^2.
\end{aligned}
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Gibbons, Güven, Pope '03

When $\lambda_3 \neq 0$, the singularities at the branes are not conical, i.e. $ds_2^2 \sim dr^2 + kr^{2-k_1} d\theta^2$.

Burgess et. al. '04

The hope

Starting from a static solution with conical singularities how does the system react to perturbing the tensions?

Codimension-2 branes

Questions we'd like to answer

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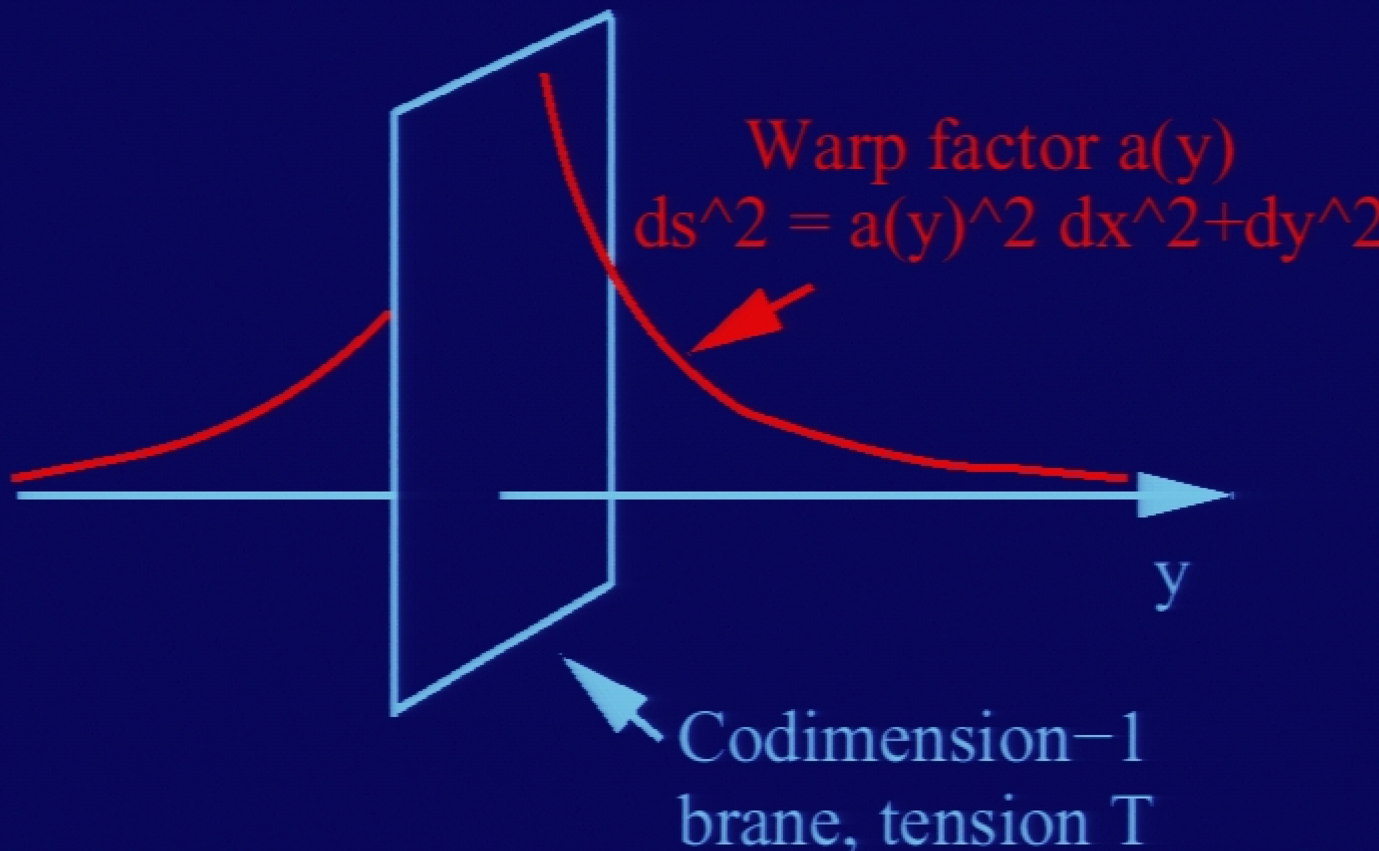
★ What happens if we put more general matter on the brane? (i.e. $\rho \neq -p$)

★ Do we recover FRW cosmology?

Codimension-2 branes

For codimension-1 branes,



the discontinuities in g_{00}' and g_{ii}' allow us to have branes with arbitrary equations of state $(\rho \neq -p)$.

Codimension-2 branes

Since in 2D

$$\nabla^2(\ln(r)) = \frac{\delta(r)}{2\pi r},$$

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For codimension-2 branes, the 2D delta function in $T_{\mu\nu}$ must match with terms like $\nabla^2(\ln(g_{\mu\nu}))$ in $G_{\mu\nu}$.

Thick branes

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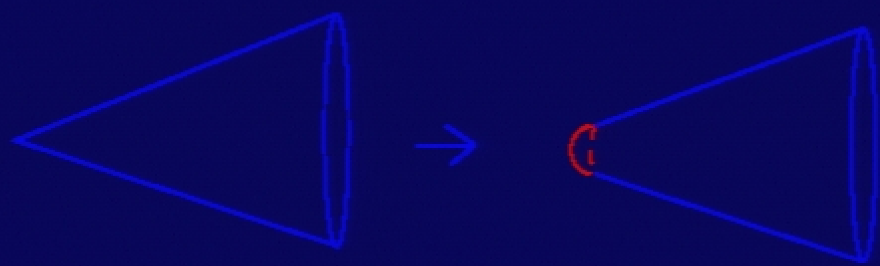

★ Reduces to the expected δ-function solution in the zero thickness limit;

Thick branes

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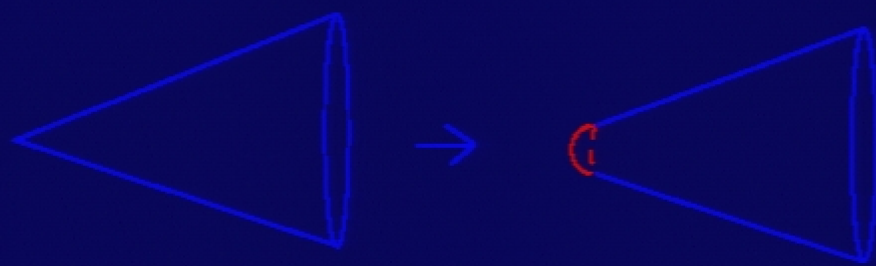

★ Allows freedom to detune the deficit angle and brane tension

Thick branes

We can treat the matter as a perturbation to the 3-brane tension

$$s_0^0 = -\theta(r - r_0(t))(\tau_3 + \rho(t))$$

$$s_i^i = \theta(r - r_0(t))(-\tau_3 + p(t)).$$

Treat the time dependence in the thickness as a perturbation

$$\theta(r - r_0 - \Delta r_0(t)) \approx \theta(r - r_0) - \delta(r - r0)\Delta r_0(t) + \ldots$$

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so

$$\begin{aligned} s_0^0 &= -\theta(r - r_0)\tau_3 + (-\theta(r - r_0)\rho(t) \\ &+ \tau_3\delta(r - r_0)\Delta r_0(t)) + \ldots \end{aligned}$$

Thick branes

To *greatly* simplify the calculations, we will expand around a braneless background ($\tau_3 = 0$) but we will leave in the 1D δ-function term that encodes the time varying thickness.

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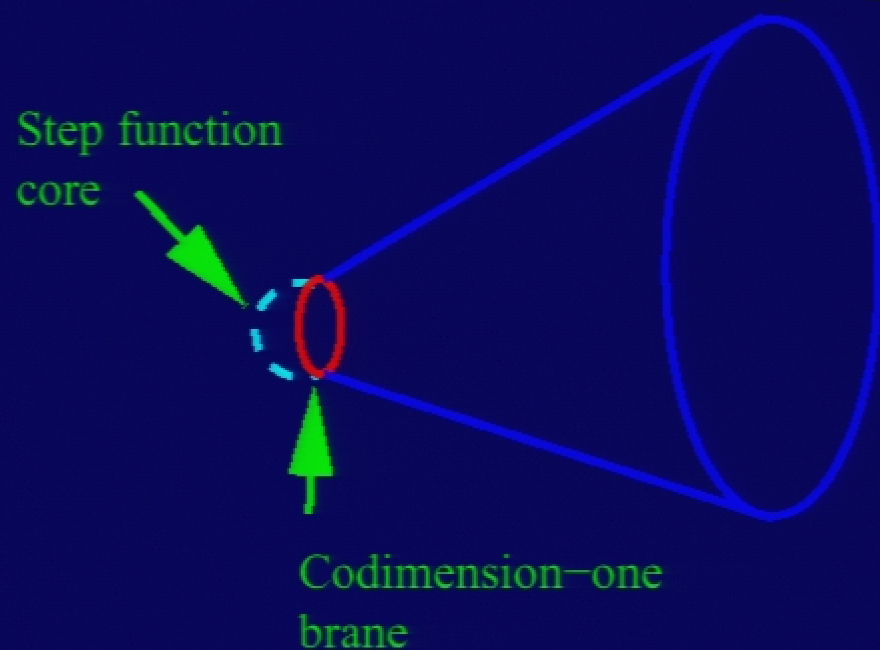

(Kanno,Soda '04; Navarro, Santiago '04)

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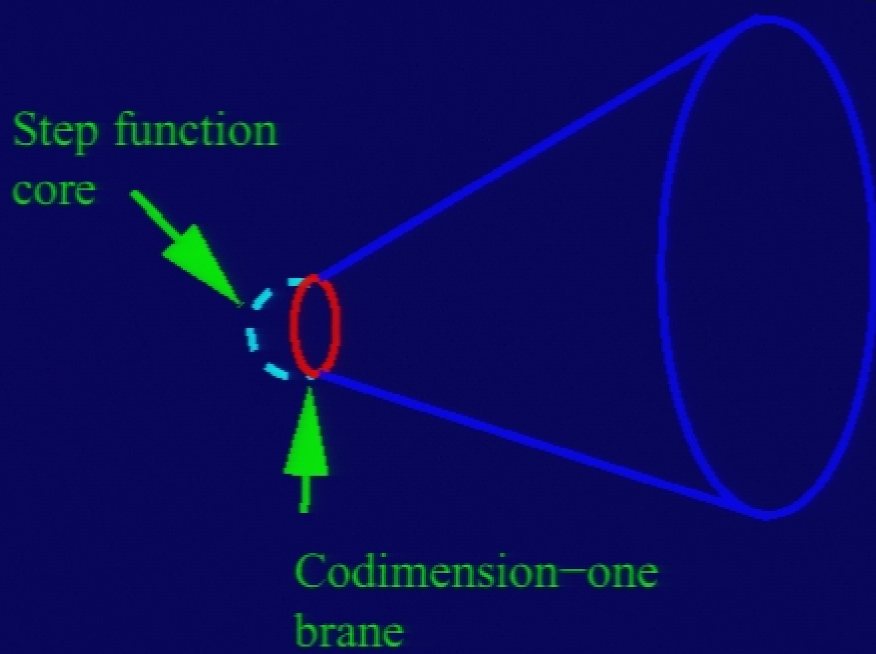


(Kanno,Soda '04; Navarro, Santiago '04)

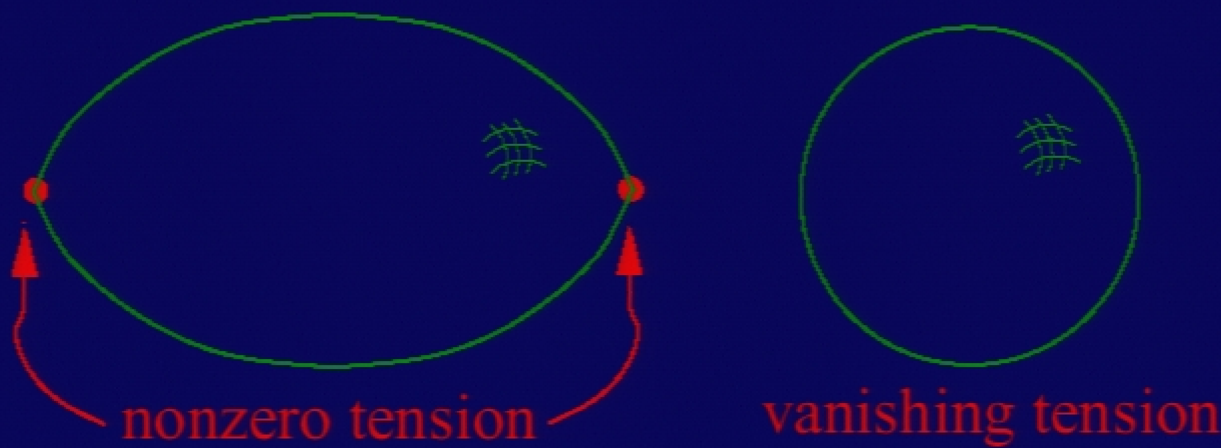
Perturbative ansatz

$$ds^2 = n^2 dt^2 + a^2 d\vec{x}^2 + b^2 dr^2 + c^2 d\theta^2 + 2E dr dt$$

$$\begin{aligned}
n(r,t) &= e^{N_0(r)+N_1(r,t)}; \qquad a(r,t) = a_0(t) e^{A_0(r)+A_1(r,t)}; \\
b(r,t) &= b_0(t) e^{B_0(r)+B_1(r,t)}; \qquad c(r,t) = c_0(t) e^{C_0(r)+C_1(r,t)}; \\
E(r,t) &= E_1(r,t); \qquad A_\theta(r,t) = A_\theta^{(0)}(r) + A_\theta^{(1)}(r,t) \\
e^{\phi(r,t)} &= \varphi_0(t) e^{\phi_0(r)+\phi_1(r,t)}
\end{aligned}$$

Background

For simplicity, we will choose a braneless, static, unwarped background:



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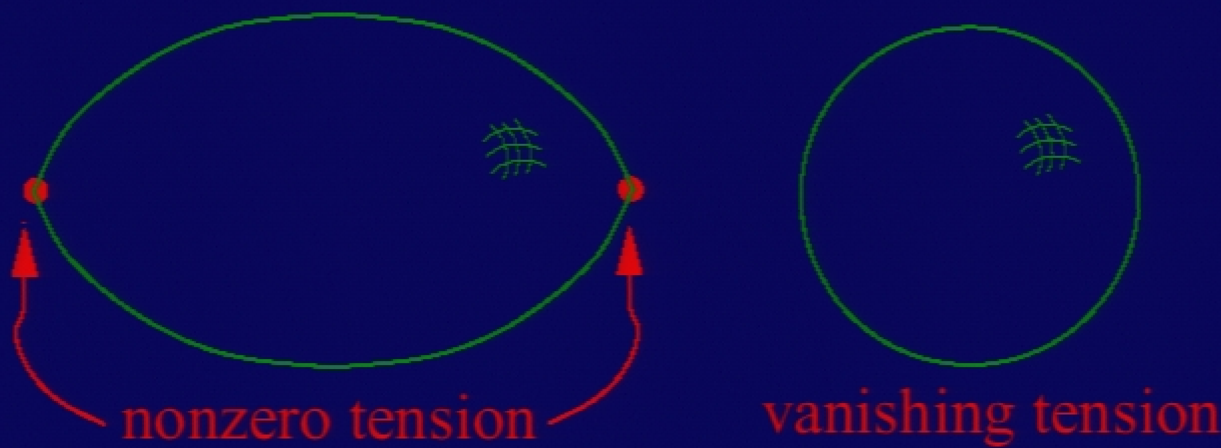

Choosing the braneless solution makes the equations much simpler to solve, and does not take away from the generality of the conclusions.

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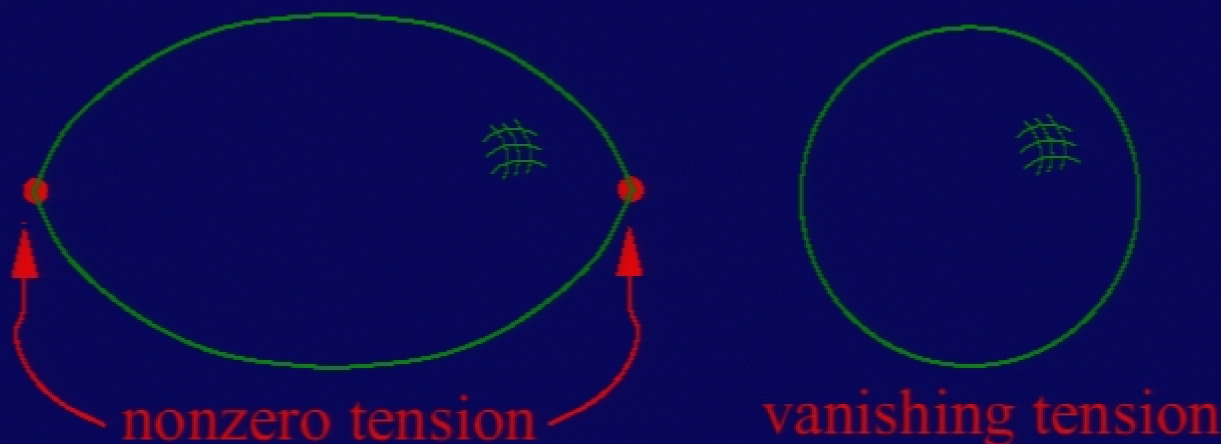

Unwarped solutions require both branes to have *same tension*. This is not general, but our perturbations will allow us to relax this.

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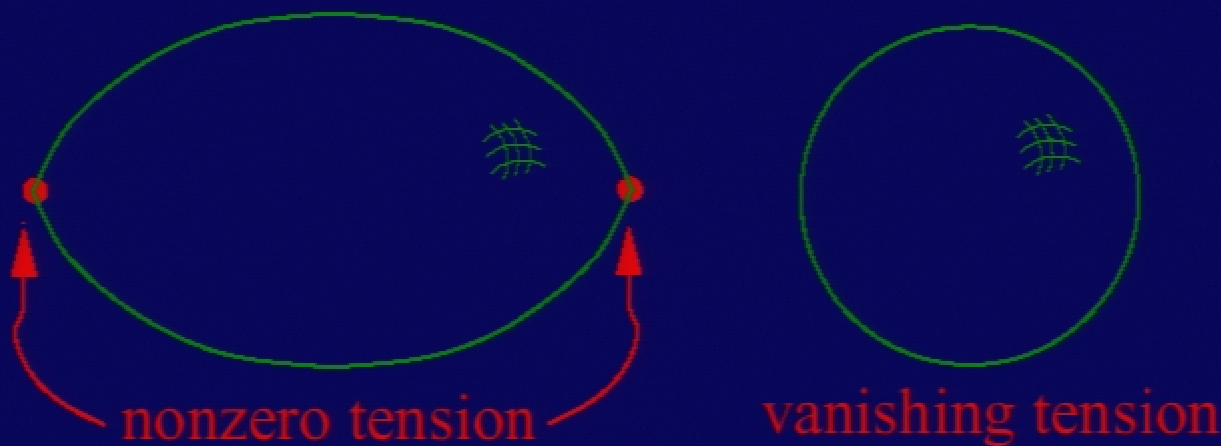

As we have already seen, the static solution is singled out in 6D supergravity.

Background

$$
\begin{aligned}
N_0(r) &= A_0(r) = B_0(r) = 0 \\
\phi_0(r) &= \phi_0 \\
e^{C_0(r)} &= \frac{\sin(kr)}{k} \\
A_\theta^{(0)\prime}(r) &= -\beta e^{\phi_0} e^{C_0(r)} \\
b_0(t) &= c_0(t) \\
\varphi_0(t) &= \frac{\beta}{\sqrt{2v(\Phi_0)}c_0(t)^2} \\
\beta^2 &= \frac{k^4 M_6^8}{2v(\Phi_0)}
\end{aligned}
$$

Gauge invariant variables

The following variable are invariant under
$t \to t + \Delta t(r,t), r \to r + \Delta r(r,t)$:

$$Z = N_1' - A_1'; \quad W = 3A_1' + N_1'; \quad X = \frac{C_1'}{C_0'} - B_1 - \frac{C_0''}{C_0'^2} C_1;$$

$$Y = A_\theta^{(1)'} - A_\theta^{(0)'}(B_1 + C_1); \quad U = \dot{A}_\theta^{(1)} - \frac{A_\theta^{(0)'}}{C_0'} \dot{C}_1;$$

$$\tilde{\rho} = -s_t^t - {s_*}_t^t; \quad \tilde{p} = s_i^i + {s_*}_i^i; \quad \tilde{p}_t^r = s_t^r + {s_*}_t^r;$$

$$\tilde{p}_r^t = s_r^t + s_t^r + {s_*}_r^t + {s_*}_t^r; \quad \tilde{p}_5 = s_r^r + {s_*}_r^r;$$

$$\tilde{p}_6 = s_\theta^\theta - s_r^r + {s_*}_\theta^\theta - {s_*}_r^r$$

$\mathcal{O}(\rho)$ equations of motion

$$W' - C_0' W = c_0(t)^2 \frac{\tilde{p}_6}{M_6^4}$$

$$\frac{Z' + C_0' Z}{c_0(t)^2} = 2\left[\frac{\ddot{a}_0}{a_0} - \left(\frac{\dot{a}_0}{a_0}\right)^2 - \frac{\dot{a}_0}{a_0}\frac{\dot{c}_0}{c_0} + 2\left(\frac{\dot{c}_0}{c_0}\right)^2 + \frac{\ddot{c}_0}{c_0}\right] + \frac{1}{M_6^4}(\tilde{\rho} + \tilde{p})$$

$$Y' - C_0' Y - e^{\phi_0}\beta e^{C_0(r)}\left(W - \phi_1'\right) = 0$$

$$\frac{C_0' W}{c_0(t)^2} + \frac{\sqrt{2v(\Phi_0)}e^{-C_0(r)}}{c_0(t)^2 M_6^4} Y + \beta\frac{\sqrt{2v(\Phi_0)}e^{\phi_0}}{c_0(t)^2 M_6^4}\phi_1 = 3\left[\frac{\ddot{a}_0}{a_0} + \left(\frac{\dot{a}_0}{a_0}\right)^2\right] + \frac{\ddot{c}_0}{c_0} + 2\left(\frac{\dot{c}_0}{c_0}\right)^2 + \frac{1}{M_6^4}\tilde{p}_5$$

$\mathcal{O}(\rho)$ equations of motion

$$\tilde{p}_5' - C_0'\tilde{p}_6 = 0$$

$$U' - \dot{Y} - \beta e^{\phi_0} e^{C_0(r)} \dot{X} = 0$$

$$\tilde{p}_r^t = -c_0(t)^2 \tilde{p}_t^r$$

$$3\frac{\dot{a}_0}{a_0} Z - C_0'\dot{X} + \frac{3}{4}(\dot{Z} - \dot{W}) = \frac{c_0(t)^2}{M_6^4}\tilde{p}_t^r - \frac{\sqrt{2v(\Phi_0)}e^{-C_0(r)}}{M_6^4} U - \frac{\dot{c}_0}{c_0}(W + 2\phi_1')$$

$$\dot{\tilde{\rho}} + 3\frac{\dot{a}_0}{a_0}(\tilde{\rho} + \tilde{p}) + \frac{\dot{c}_0}{c_0}(2\tilde{\rho} + 2\tilde{p}_5 + \tilde{p}_6) = \tilde{p}_t^{r\prime} + C_0'\tilde{p}_t^r$$

$\mathcal{O}(\rho)$ equations of motion

$$\frac{\phi_1'' + C_0'\phi_1' + C_0'W}{3c_0(t)^2} = \left[\frac{\ddot{a}_0}{a_0} + \left(\frac{\dot{a}_0}{a_0}\right)^2 - \frac{1}{3}\frac{\ddot{c}_0}{c_0} - \frac{\dot{a}_0}{a_0}\frac{\dot{c}_0}{c_0} + \frac{\tilde{p}_5}{3M_6^4}\right]$$

$$\frac{C_0'X'}{c_0(t)^2} - \frac{2X\beta\sqrt{2v(\Phi_0)}e^{\phi_0}}{M_6^4 c_0(t)^2} + \frac{3C_0'W}{2c_0(t)^2} - \frac{\sqrt{2v(\Phi_0)}e^{-C_0(r)}}{c_0(t)^2 M_6^4}Y$$

$$= -\frac{1}{4M_6^4}(\tilde{\rho} - 3\tilde{p} + 3\tilde{p}_6)$$

$$+\frac{3}{2}\left[\frac{\ddot{a}_0}{a_0} + \left(\frac{\dot{a}_0}{a_0}\right)^2 + \frac{\ddot{c}_0}{c_0} + 3\frac{\dot{a}_0}{a_0}\frac{\dot{c}_0}{c_0} + \frac{4}{3}\left(\frac{\dot{c}_0}{c_0}\right)^2\right]$$

Simplifying assumptions

★ No dynamics for $\tilde{p}_6(r,t)$ in the EOM's, choose:

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$$3\frac{\dot{a}_0}{a_0} Z - C_0' \dot{X} + \frac{3}{4}(\dot{Z} - \dot{W}) = \frac{c_0(t)^2}{M_6^4} \tilde{p}_t^r - \frac{\sqrt{2v(\Phi_0)} e^{-C_0(r)}}{M_6^4} U - \frac{\dot{c}_0}{c_0}(W + 2\phi_1')$$

$$\dot{\tilde{\rho}} + 3\frac{\dot{a}_0}{a_0}(\tilde{\rho} + \tilde{p}) + \frac{\dot{c}_0}{c_0}(2\tilde{\rho} + 2\tilde{p}_5 + \tilde{p}_6) = \tilde{p}_t^{r\prime} + C_0' \tilde{p}_t^r$$

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The following variable are invariant under
$t \to t + \Delta t(r,t), r \to r + \Delta r(r,t)$:

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$$Y = A_\theta^{(1)\prime} - A_\theta^{(0)\prime}(B_1 + C_1); \quad U = \dot{A}_\theta^{(1)} - \frac{A_\theta^{(0)\prime}}{C_0'} \dot{C}_1;$$

$$\tilde{\rho} = -s_t^t - s_{*t}^{\ t}; \quad \tilde{p} = s_i^i + s_{*i}^{\ i}; \quad \tilde{p}_t^r = s_t^r + s_{*t}^{\ r};$$

$$\tilde{p}_r^t = s_r^t + s_t^r + s_{*r}^{\ t} + s_{*t}^{\ r}; \quad \tilde{p}_5 = s_r^r + s_{*r}^{\ r};$$

$$\tilde{p}_6 = s_\theta^\theta - s_r^r + s_{*\theta}^{\ \theta} - s_{*r}^{\ r}$$

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$$\tilde{p}_6(r,t) = \theta(r - r_0)e^{2C_0(r)}\mathcal{P}_6(t) + \theta(r_* - r)e^{2C_0(r)}\mathcal{P}_{*6}(t)$$

★ Assume that $\rho(r,t), p(r,t), \rho_*(r,t)$ and $p_*(r,t)$ are functions of time only.

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★ Assume that $\rho(r,t), p(r,t), \rho_*(r,t)$ and $p_*(r,t)$ are functions of time only.

With these assumptions, perturbed EOM's are straightforward (if somewhat tedious) to solve.

Effective four dimensional quantities

$$\rho^{(4)}(t) = 2\pi\int_0^{r_0} c_0(t)^2 e^{C_0(r)}\rho(t)dr + 2\pi c_0(t)^2 e^{C_0(r_0)}\mathcal{F}_0(t)$$

$$p^{(4)}(t) = 2\pi\int_0^{r_0} c_0(t)^2 e^{C_0(r)}p(t)dr - 2\pi c_0(t)^2 e^{C_0(r_0)}\mathcal{F}_0(t)$$

$$\rho_*^{(4)}(t) = 2\pi\int_{\pi/k-r_0}^{\pi/k} c_0(t)^2 e^{C_0(r)}\rho_*(t)dr + 2\pi c_0(t)^2 e^{C_0(r_*)}\mathcal{F}_*(t)$$

$$p_*^{(4)}(t) = 2\pi\int_{\pi/k-r_0}^{\pi/k} c_0(t)^2 e^{C_0(r)}p_*(t)dr - 2\pi c_0(t)^2 e^{C_0(r_*)}\mathcal{F}_*(t).$$

Effective four dimensional quantities

$$\frac{1}{8\pi G_4(t)} = M_4^2 = 2\pi \int_0^{\pi/k} c_0(t)^2 e^{C_0(r)} M_6^4 dr$$

$$\Rightarrow G_4(t) = \frac{k^2}{32\pi^2 M_6^4 c_0(t)^2}.$$

Non-conical solutions?

GGP '03 showed that with the metric

$ds^2 = n(r)^2 dx^\mu dx_\mu + dr^2 + c(r)^2 d\theta^2$

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In our perturbative language, $\lambda_3 = 0$ at the level of the background. But the above can be written perturbatively as

$$\delta\lambda_3 = \frac{1}{2} e^{C_0(r)} \left(2\phi_1(r)' + W(r)\right).$$

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Substituting our bulk solutions into this expression and assuming staticity (i.e. $a_0(t) \equiv 1, c_0(t) \equiv 1, \rho = -p$ and $\rho_* = -p_*$), we find that

$$\delta\lambda_3 = \frac{1}{12k^4 M_6^4} \left(1 - \cos(kr_0)\right)^3 \left(\mathcal{P}_6(t) - \mathcal{P}_{*6}(t)\right).$$

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Dependence on extra dimensional part of brane stress-energy tensor, consistent with Burgess et.al. '04; Navarro, Santiago '04.

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- ★ All functions smooth at the poles $r = 0, \pi/k$;
- ★ All functions (but not their radial derivatives!) continuous across core/bulk boundary, except where warranted by 1D δ-function.

Friedmann equations

$$\left(\frac{\dot{a}_0}{a_0}\right)^2 = \frac{8\pi G_4(t)}{3}(\rho^{(4)} + \rho_*^{(4)}) - \frac{16\pi^2 \sin(kr_0)}{3k} G_4(t) Q_1$$
$$+ \frac{1}{3}\left(\frac{\dot{c}_0}{c_0}\right)^2 - 2\frac{\dot{a}_0}{a_0}\frac{\dot{c}_0}{c_0}$$

$$\frac{\ddot{a}_0}{a_0} - \left(\frac{\dot{a}_0}{a_0}\right)^2 = -4\pi G_4(t)(\rho^{(4)} + p^{(4)} + \rho_*^{(4)} + p_*^{(4)}) - \frac{\ddot{c}_0}{c_0}$$
$$- 2\left(\frac{\dot{c}_0}{c_0}\right)^2 + \frac{\dot{a}_0}{a_0}\frac{\dot{c}_0}{c_0}$$

Friedmann equations

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$$V(\sigma_1, \sigma_2) = e^{-\sigma_2}\left(ke^{-2\sigma_1} - 2Ke^{-\sigma_1} + 2\Lambda_6\right)$$

- ★ $S = \int d^4x\sqrt{-g}\left(\frac{1}{16\pi G}\left[\zeta\mathcal{R} - \frac{1}{2\zeta}\partial_\mu\zeta\partial^\mu\zeta - V(\zeta)\right] + \mathcal{L}_m\right)$ with $c_0(t) = \sqrt{\zeta}$, $V(\zeta) = -32\pi^2\sin(kr_0)\bar{G}Q_1/k$ and $\bar{G}/\zeta(t) = G_4(t)$
- ★ Ok, but what about self-tuning???

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Remember what the hope was:

- ★ starting from static solution with conical singularities, perturb brane stress energy;
- ★ rather than going to runaway solution, system would evolve to static solution with non-conical singularities;
- ★ we saw that $\delta\lambda_3 \sim \mathcal{P}_6 - \mathcal{P}_{*6}$
- ★ can $\mathcal{P}_6$'s evolve so as to keep solution static?

Absence of self-tuning

Furthermore $\mathcal{P}_6$'s do not appear in

$$\left(\frac{\dot{a}_0}{a_0}\right)^2 = \frac{8\pi G_4(t)}{3}(\rho^{(4)} + \rho_*^{(4)}) - \frac{16\pi^2 \sin(kr_0)}{3k} G_4(t) Q_1 + \frac{1}{3}\left(\frac{\dot{c}_0}{c_0}\right)^2 - 2\frac{\dot{a}_0}{a_0}\frac{\dot{c}_0}{c_0}$$

Absence of self-tuning

However $\mathcal{P}_6$'s allow us to keep internal space static, albeit not in a natural way by forcing the RHS of

$$\frac{\ddot{c}_0}{c_0} + \left(\frac{\dot{c}_0}{c_0}\right)^2 + 3\frac{\dot{a}_0}{a_0}\frac{\dot{c}_0}{c_0} = \pi G_4(t)\left(\rho^{(4)} - 3p^{(4)} + \rho_*^{(4)} - 3p_*^{(4)}\right)$$
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to vanish.

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- ★ Codimension-two branes $\Rightarrow$ unconventional relation between H^2 and vacuum energy;
- ★ Implicit relation between expansion and vacuum energy $\Rightarrow$ no self-tuning;
- ★ Effective description apparently dependent on details of brane structure;
- ★ Possibility of getting stabilized internal space/dilaton;
- ★ Fine-tuning still needed, but SLEDs might still help explain smallness of quantum corrections;
- ★ Not mentionned here, but could breaking axial symmetry help? (Redi '04)


No Signal
VGA-1