Title: Generalized Geometric Structures (Part 3)

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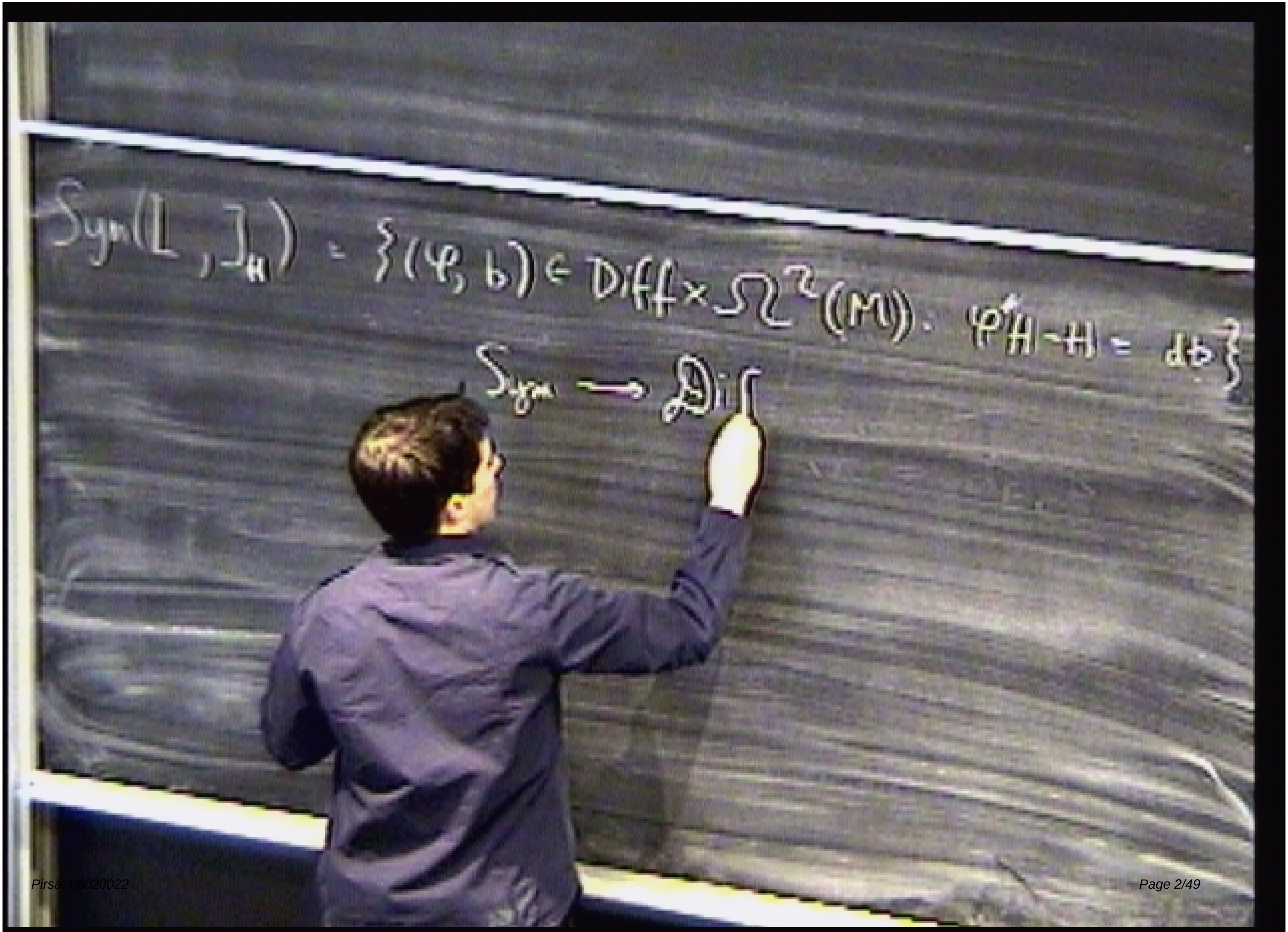
Abstract:


$\mathrm{Sym}(\mathbb{L}, \mathbb{J}_H) = \{(\varphi, b) \in \mathrm{Diff} \times \Omega^2(M) : \varphi^* H - H = db\}$
$\mathrm{Sym} \longrightarrow \mathrm{Dif}$

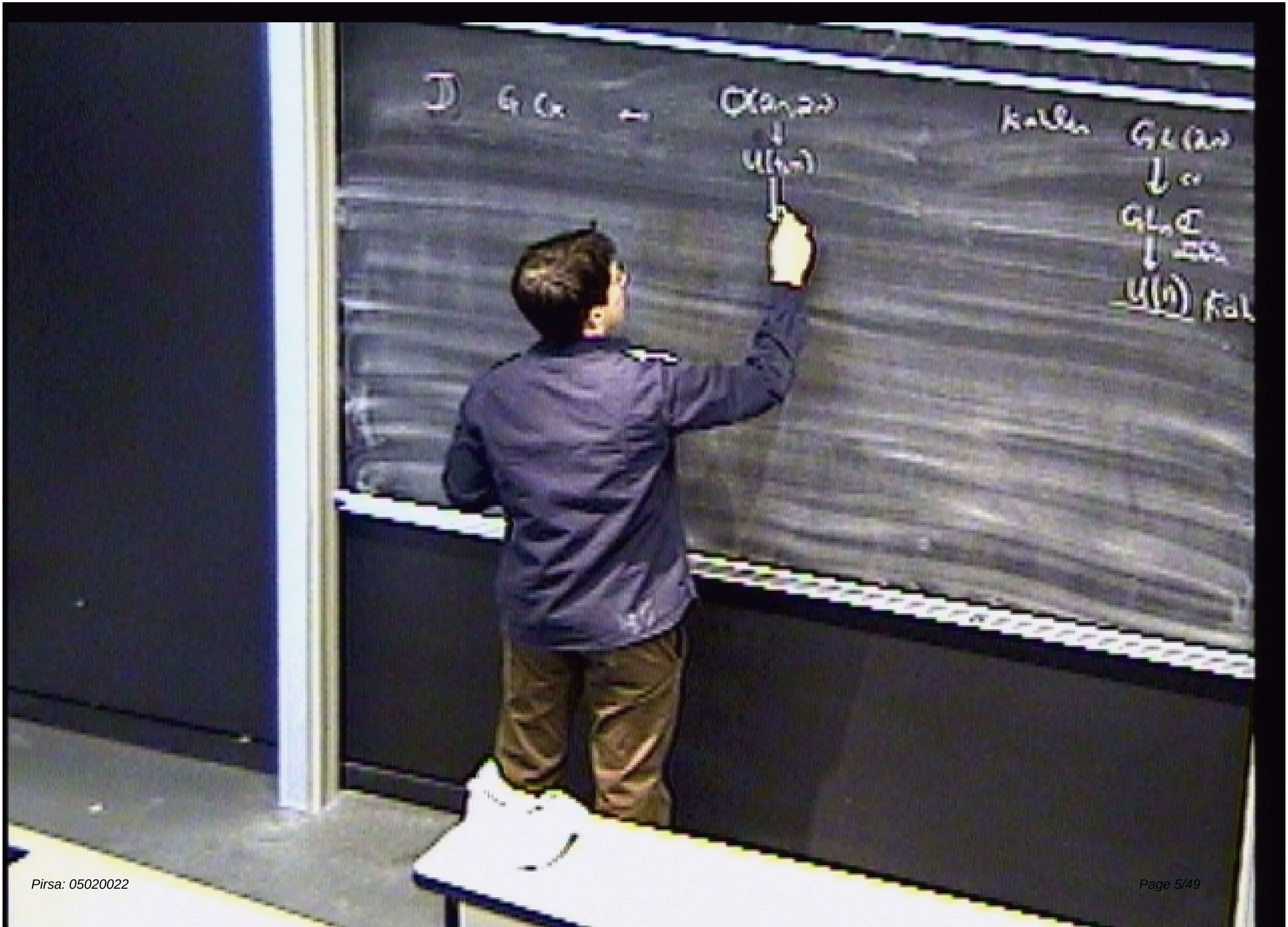
$$\mathrm{Syn}([\ ,\ ]_H) = \{(\varphi, b) \in \mathrm{Diff} \times \Omega^2(M) : \varphi^* H - H = db\}$$

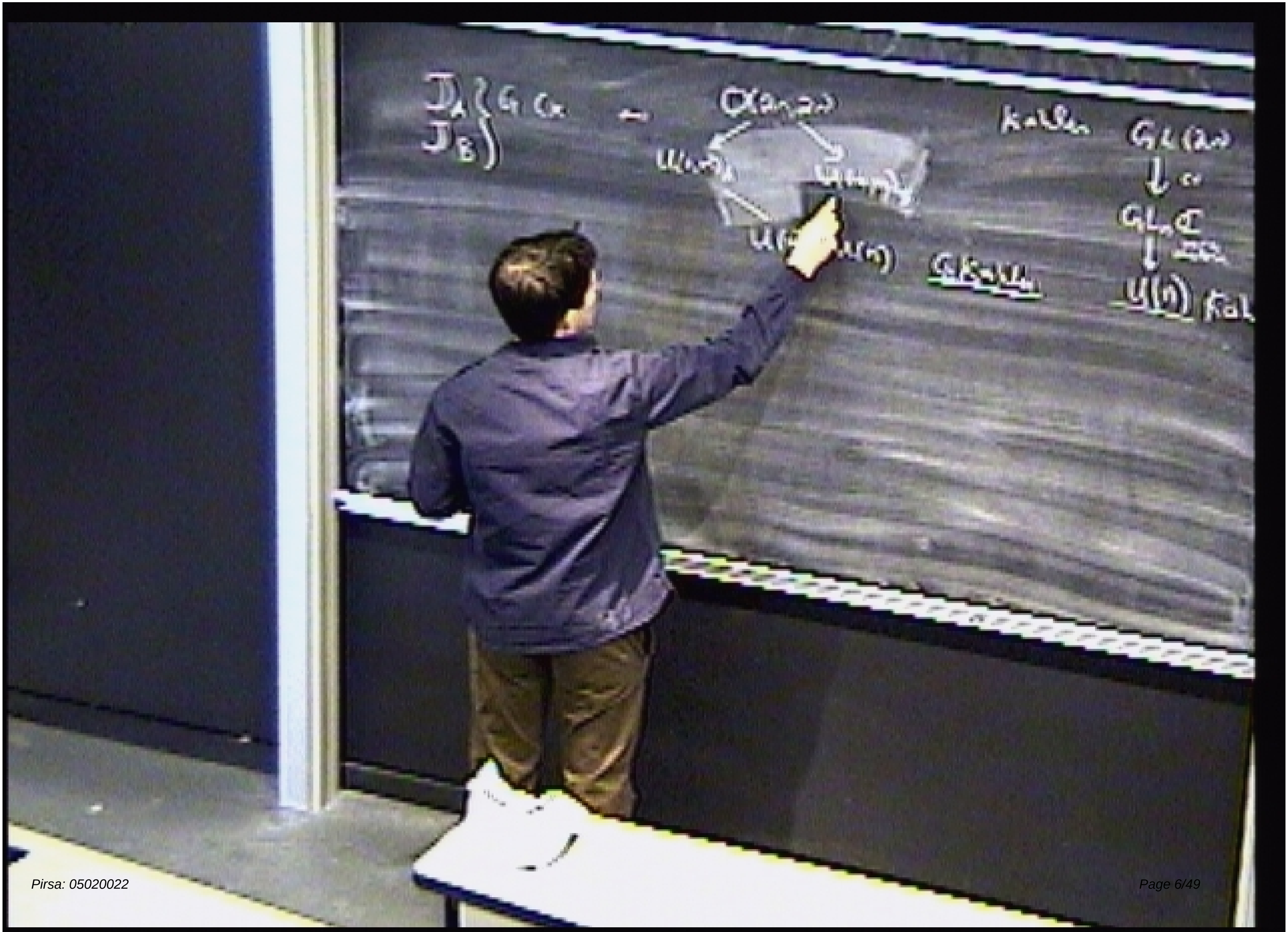
$$0 \to \Omega^2_{cl} \to S_{syn} \to \mathcal{D}iff_{[H]} \to 0$$

$$\mathrm{Sym}(L, J_H) = \{ (\varphi, b) \in \mathrm{Diff} \times \Omega^2(M) : \varphi^* H - H = db \}$$

$$0 \to \Omega^2_{cl} \to \mathrm{Sym} \to \mathcal{D}\mathrm{iff}_{[H]} \to 0$$

if $H = 0$, $\mathrm{Sym} \quad \mathcal{D}\mathrm{iff} \ltimes \Omega^2_{cl}$





• $\mathbb{J}_A$, $\mathbb{J}_B$ $\}$ G Cx ← $O(2n,2n)$ Kähler $GL(2n)$

$U(n)_A$ $\quad$ $U(n)_B$ $\quad \downarrow \cong$

$U(n)\times U(n)$ $\quad$ Gen Kähler $\quad$ $GL_n\mathbb{C}$ $\downarrow$ $U(n)$ Kähler

• $\mathbb{J}_A, \mathbb{J}_B$ commute, each integrable

$$\mathbb{J}_J = \begin{pmatrix} J & \\ & -J^* \end{pmatrix} \qquad \mathbb{J}_\omega = \begin{pmatrix} & -\omega^{-1} \\ \omega & \end{pmatrix} \quad J \quad \omega$$

⇒ type $(1,1) \Leftrightarrow [\mathbb{J}_J, \mathbb{J}_\omega] = 0$

- $\left.\begin{matrix}\mathbb{J}_A \\ \mathbb{J}_B\end{matrix}\right\}$ $G\,C_x$ $\sim$ $O(2n,2n)$ Kahler $GL(2n)$

$U(n,n)_A$ $\quad$ $U(n,n)_B$ $\quad$ $\downarrow \simeq$

$GL_n\mathbb{C}$

$\downarrow$ metric

- $\mathbb{J}_A, \mathbb{J}_B$ commute, each integrable $\quad U(n)\times U(n)$ $\quad$ G Kahler $\quad$ $U(n)$ Kahler

$$\mathbb{J}_J = \begin{pmatrix} J & \\ & -J^* \end{pmatrix} \qquad \mathbb{J}_\omega = \begin{pmatrix} & -\omega^{-1} \\ \omega & \end{pmatrix}$$

$\sim$ type $(1,1) \Leftrightarrow [\mathbb{J}_J, \mathbb{J}_\omega] = 0$

ωJ pos def

$-\mathbb{J}_A \mathbb{J}_B = G$ pos def $\quad G$ Gen^l Riem metric

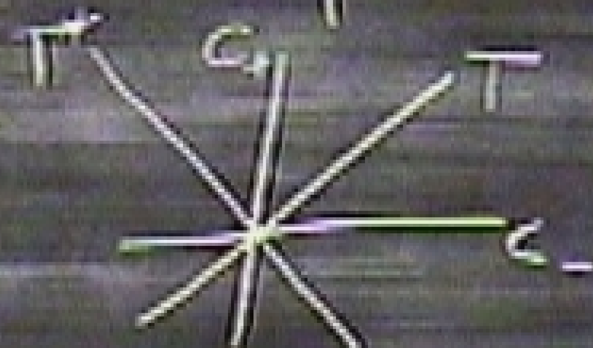
$\langle -\mathbb{J}_A \mathbb{J}_B X, Y \rangle$ pos def metric on

$\langle \mathbb{J}_A X, \mathbb{J}_B Y \rangle$

$-\mathbb{J}_A \mathbb{J}_B = G$ pos def G Gen^d Riem metric

$\langle -\mathbb{J}_A \mathbb{J}_B X, Y \rangle$ pos def metric on T

$\langle \mathbb{J}_A X, \mathbb{J}_B Y \rangle$



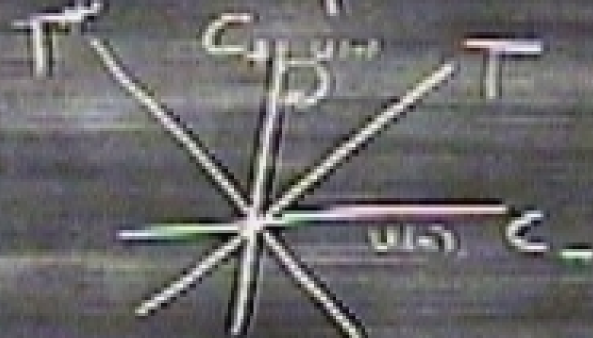
$G^2 = 1$, ± 1 eigenbundles $C_\pm$

$-\mathbb{J}_A \mathbb{J}_B = G$ pos def

G Gen^d Riem metric

$\langle -\mathbb{J}_A \mathbb{J}_B X, Y \rangle$ pos def metric on T

$\langle \mathbb{J}_A X, \mathbb{J}_B Y \rangle$



$G^2 = \underline{1}$, ± 1 eigenbundles $C_\pm$

$-\mathbb{J}_A \mathbb{J}_B = G$ pos def G Gen^d Riem metric

$\langle -\mathbb{J}_A \mathbb{J}_B X , Y \rangle$ pos def metric on T

$\quad \parallel$

$\langle \mathbb{J}_A X , \mathbb{J}_B Y \rangle$

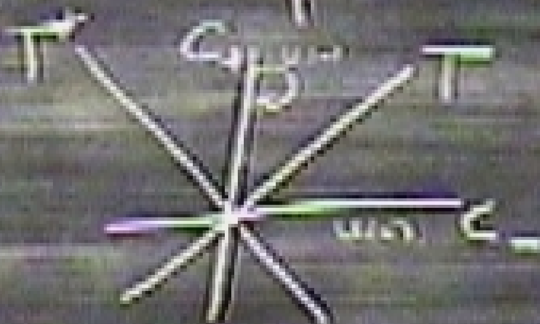
$G^2 = 1$, ± 1 eigenbundles $C_\pm$

$-\mathbb{J}_A \mathbb{J}_B = G$ pos def $\quad G$ Gen^d Riem. metric

$\langle -\mathbb{J}_A \mathbb{J}_B X, Y \rangle$ pos def metric on $T \oplus T^*$

$\parallel$

$\langle \mathbb{J}_A X, \mathbb{J}_B Y \rangle$



$G^2 = 1$, ± 1 eigenbundles $C_\pm$ are complex for $\mathbb{J}_A, \mathbb{J}_B$

$G \longleftrightarrow (g + b,$ almost cx structures $J_\pm$ determined by $\pi(C_\pm)$.

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2 sets of Chern classes $c_i^\pm$

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G $\longleftrightarrow$ ($g + b$, almost cx structures $J_\pm$ determined by $\pi(C_\pm)$, both herm wrt g.

2 sets of Chern classes $c_i^\pm$

$(G) \longleftrightarrow \left(g + b, \ \text{almost cx structures } J_\pm \text{ determined by } \pi(C_\pm), \text{ both herm wrt } g \right)$

2 sets of Chern classes $c_i^\pm$

$$(\mathbb{J}_A, \mathbb{J}_B) \longleftrightarrow (g, b, J_\pm) \quad \text{bihermitian} \quad \omega_\pm = g J_\pm$$

$$\mathbb{J}_{A/B} = \frac{1}{2}\begin{pmatrix} 1 & \\ b & 1 \end{pmatrix}\begin{pmatrix} J_+ \pm J_- & -(\omega_+^{-1} \mp \omega_-^{-1}) \\ \omega_+ \mp \omega_- & -(J_+^* \pm J_-^*) \end{pmatrix}\begin{pmatrix} 1 & \\ -b & 1 \end{pmatrix}$$

J_A, J_B are $[\ ,\ ]_H$ integ.

- $J_\pm$ integ on str
- $\nabla^\pm J_\pm = 0$, $\nabla^\pm = \nabla^{LC} \pm \frac{1}{2} g^{-1}\underbrace{(H - db)}_{\text{g-equiv}}$

(Gates, Hull, Roček) $b \mapsto b + b'$

$H \mapsto H - db'$

$T \otimes T^* \simeq C_+ \oplus C_-$

$(T \otimes T^*) \otimes \mathbb{C} \simeq L_A \oplus \bar{L}_A$ defin with $L_A = +i$ of $\mathbb{J}_A$.

	L^+_A	$\bar{L}_A$	L^+_A	$\bar{L}^-_A$
$\mathbb{J}_A$	$+i$	$+i$	$-i$	$-i$
$\mathbb{J}_B$	$+i$	$-i$	$-i$	$+i$
G	$+i$	$-i$	$+i$	$-i$

$T\oplus T^* = C_L \oplus C_R$

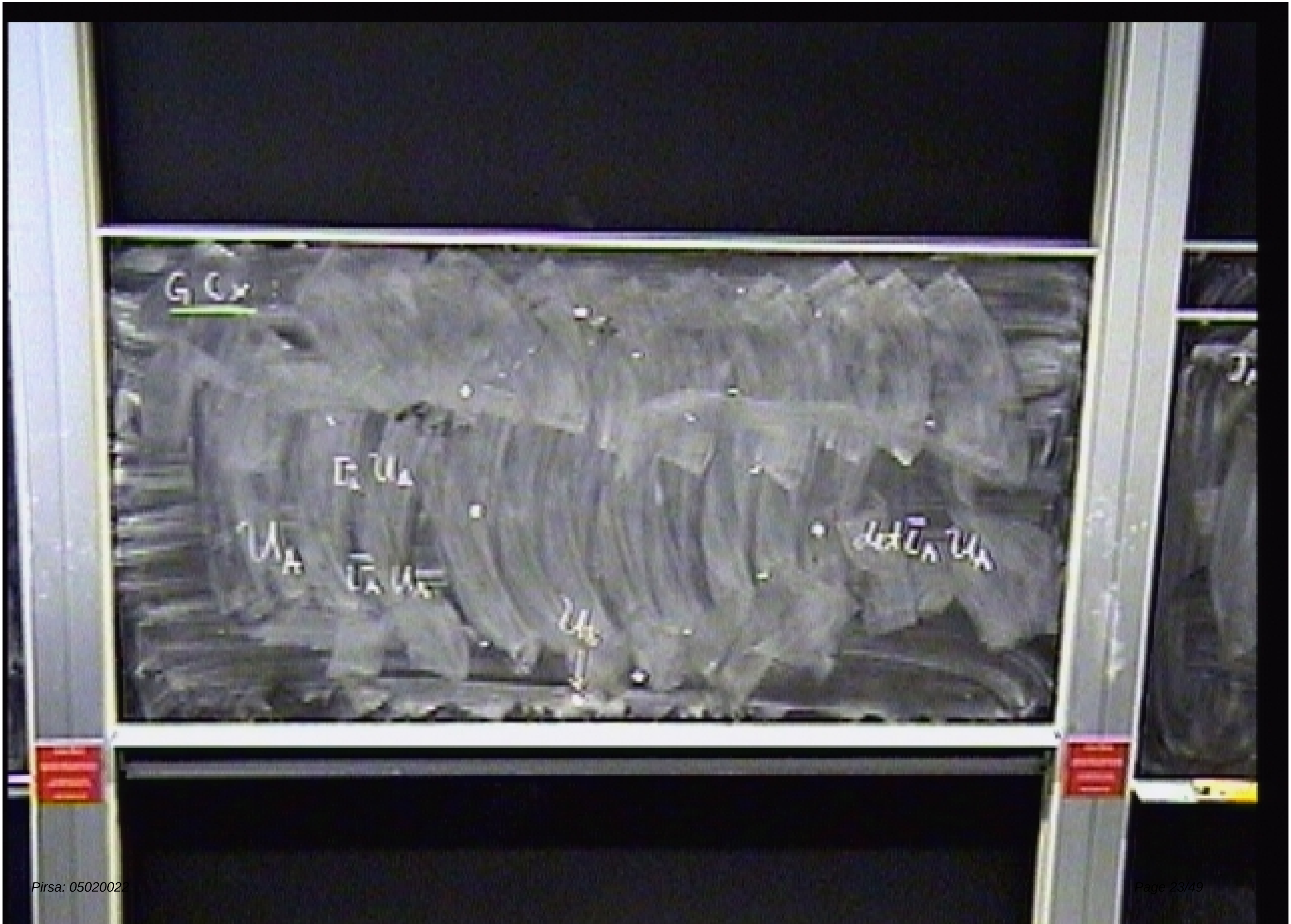
$(T\oplus T^*)\otimes\mathbb{C} = L_A \oplus \bar{L}_A$ definition

L_A +i of $\mathbb{J}_A$.

	L_A^+	L_A^-	$\bar{L}_A^+$	$\bar{L}_A^-$
$\mathbb{J}_A$	$-i$	$+i$	$-i$	$-i$
$\mathbb{J}_B$	$+i$	$-i$	$-i$	$-i$
G	$+1$	-1	-1	$+1$
			$+1$	-1

$\oplus$ 4 bundles $T\oplus T^*$

$\underline{GC_x}$: $\begin{cases} L & \text{Lie alg}, \quad d : \Lambda^\bullet L^* \circlearrowleft \\ U, & \Lambda^\bullet T^* \otimes \mathbb{C} = \bigoplus \Lambda^k \overline{L} \, U . \end{cases}$


GCx :

G, C_x :

$\Gamma_L\, U_A$

U_A

$\bar{\Gamma}_A\, U_A$

L_L^- $\quad \bar{\Gamma}_L^-$

L_A^- $\quad \bar{\Gamma}_A^-$

$\det \bar{\Gamma}_A\, U_A$

U_B

$\underline{G\,\mathbb{C}_x}$:

$\mathcal{U}_{0,n}$

$$\Lambda^* T^* \otimes \mathbb{C} = \bigoplus_{\substack{|p-q| \le n \\ p+q = n(\bmod 2)}} \mathcal{U}_{p,q}$$

$\bar{L}_A\, \mathcal{U}_A$

L_A $\bar{L}_A^*$ $\bar{L}_A$ L_A^*

$\mathcal{U}_{-n,0} = \mathcal{U}_A$

$\bar{L}_A\, \mathcal{U}_A$

$\mathcal{U}_{0,-n}$

$\mathcal{U}_E$

$\det \bar{L}_A\, \mathcal{U}_A$

$\parallel$

$\mathcal{U}_{n,0}$

$g_m \ni *$ = "reflection" in $C_+ Y$ = ...

on basis for C_+

$O(n,n) \ni G = J_A J_B$

$J_{A/B} \in SO(T \oplus T^*)$

$G = \begin{cases} +1 & \text{on } C_+ \\ -1 & \text{on } C_- \end{cases}$

$$\boxed{* = e^{\frac{\pi}{2} J_A} e^{\frac{\pi}{2} J_B}}$$

$\rho_n = \pi$ "reflection in C_n" $= a_1 \cdots a_n$

on ... for C_n

$O(n,n)$ $-G = J_n J_n$

$J_{n/G} \in SO(T \oplus T^*)$ $G =$

$$\boxed{* = e^{\frac{\pi}{2} J_n} e^{\frac{\pi}{2} J_G}}$$

$\Rightarrow$ preserves ...

hyperkähler, ... anything in BI

$\mathbb{I}_m = *$ "reflection" in $C_+ \check{Y} = a_1 \cdots a_n$

on basis for C_+

$O(n,n) - G = J_A J_B$

$J_{A/B} \in SO(T \oplus T^*)$ $\qquad G = \begin{matrix} +1 & \text{on } C_+ \\ -1 & \text{on } C_- \end{matrix}$

$$\boxed{* = e^{\frac{\pi}{2} J_A} e^{\frac{\pi}{2} J_B}} \Rightarrow \text{preserves decomp.}$$

hyperdiculia, $\oplus U_{p,q}$ orthog. in BI metric

Kähler identity I

$\mathbb{J}_A \in \mathfrak{so}(T \oplus T^*)$

U_n can bundle $\mathbb{J}_A \cdot \rho \in \Lambda^{\cdot} T^*$

$$\begin{pmatrix} i & \\ & -i \end{pmatrix} \begin{matrix} L \\ \oplus \\ \bar{L} \end{matrix}$$

$-A^* + \frac{1}{2} \operatorname{Tr} A$

$-2ni + ni$

$$d = \bar{\delta}_+ + \bar{\delta}_- + \delta_+ + \delta$$

$$d = \bar\delta_+ + \bar\delta_- + \delta_+ + \delta_-$$

Kahler identity, II: $\bar\delta_+^* = -\bar\delta_+$, $\bar\delta_-^* = \delta_-$

$\mathcal{U}_8$ $\qquad$ $\mathcal{U}_{n,0}$

$\delta_+ = 0$

$\{\delta_+, \bar\delta_-\} = 0$

$d = \delta_+ + \delta_- + \bar\delta_+ + \bar\delta_-$

$D^+ \quad + D_-^{-1}$

Kahler identity II: $\bar\delta_+^* = -\bar\delta_-$, $\bar\delta_-^* = \delta_-$

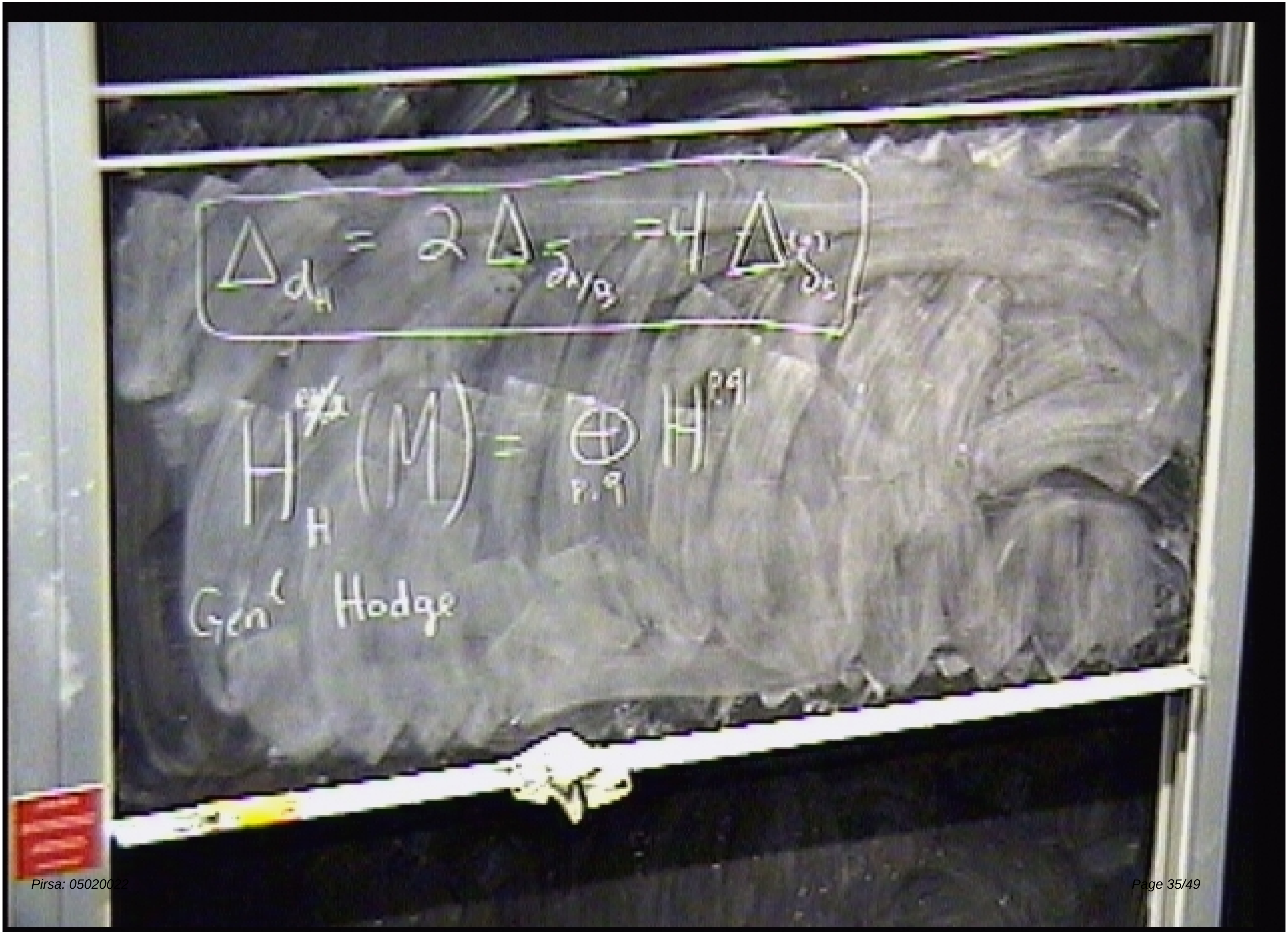
$$\Delta_d = 2\Delta_{\bar{\partial}_{n/B}} = 4\Delta^{(s)}_{\bar{S}}$$

$\delta_{\pm} = 0$

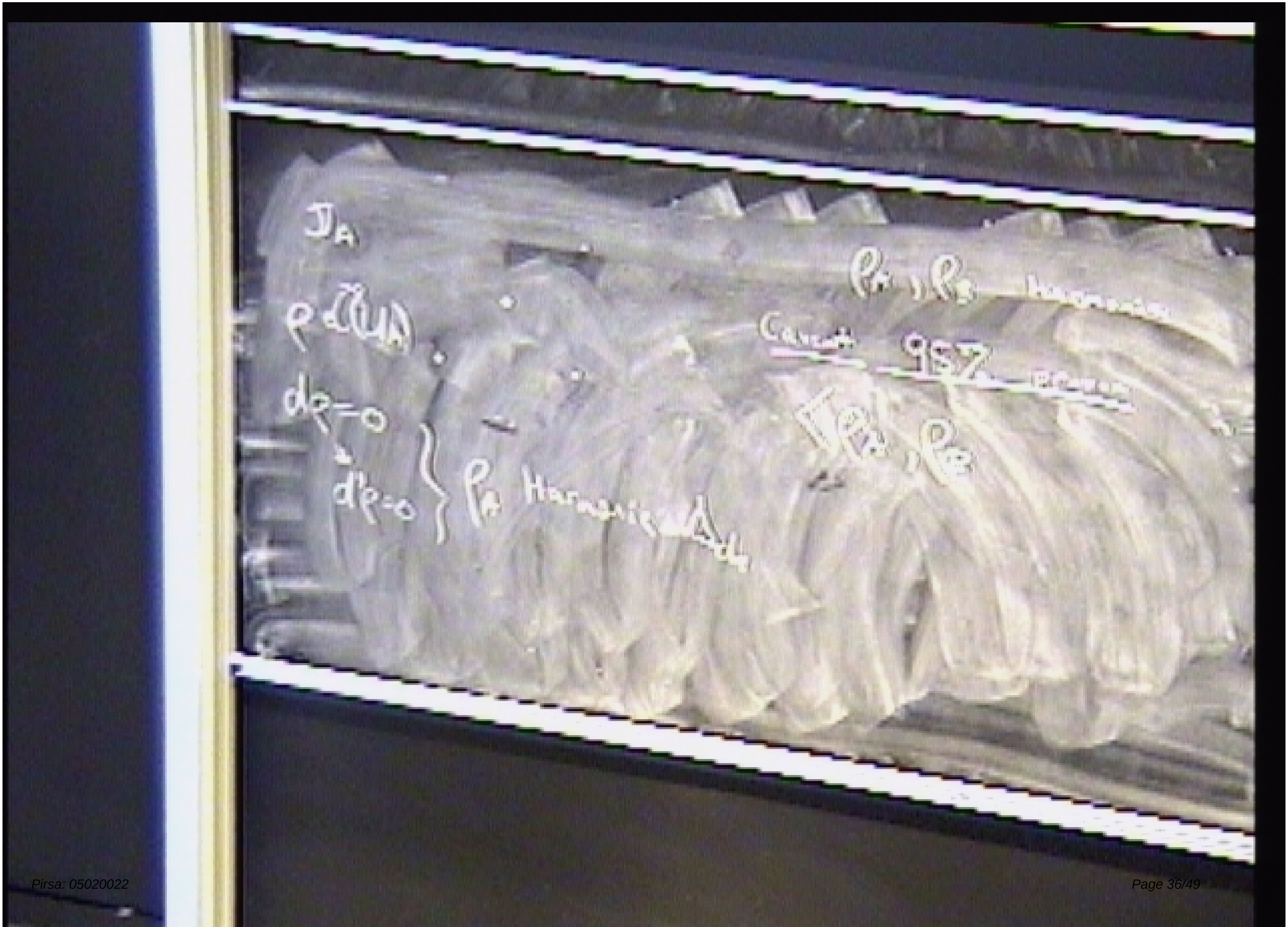
$\{\bar\delta_+, \delta_-\} = 0$

$d_H = \delta_+ + \delta_- + \bar\delta_+ + \bar\delta_-$

Kahler identities, II: $\bar\delta_+^* = -\bar\delta_+$, $\bar\delta_-^* = \delta_-$



$\Delta_{d_H} = 2\Delta_{\bar{\partial}_{J_2}} = 4\Delta_{\delta_+}$
$H_H(M) = \bigoplus_{p,q} H^{p,q}$
Gen^l Hodge


Harmonic
Caveat

Π_A

$\rho \in \tilde{\Pi}(\mathcal{U}_A)$

$d\rho = 0$

$d^*\rho = 0$ $\Big\}$ ρ_A Harmonic $\Delta_{\mathcal{U}_A}$

ρ_A, ρ_B harmonic

Cannot 9SZ across

$\langle \rho_A, \rho_B$

G Kähler manifold, U_A, U_B can bundles

U_A, U_B both hol trivial

$\boxed{\text{CY metric}}$

ρ_A, ρ_B triv. of U_A, U_B

$\langle \rho_A, \bar{\rho}_A \rangle = \langle \rho_B, \bar{\rho}_B \rangle$

(ρ_A) is harmonic

Caveat $\boxed{957}$ proven

$\rho \in \tilde{\Omega}(U_h)$

$$\left.\begin{array}{l} d\rho = 0 \\ d^{*}\rho = 0 \end{array}\right\} \rho_A \text{ Harmonic } \Delta_{dH}$$

$$\langle \rho_A, \bar{\rho}_A \rangle = e^{2f} \text{ vol}$$

$$[\rho_A \wedge \sigma(\rho_A)]_{top}$$

$$\Delta e^{2f} = \|H + db\|^2 e^{2f}$$

$$\nabla^{\pm} = \nabla^{LC} \Longleftarrow 0 = \int_{M_{cpt}} \Delta e^{2f} \Rightarrow H + db = 0.$$

- $\mathbb{J}_A, \mathbb{J}_B$ $\quad \mathcal{U}_A, \mathcal{U}_B$ trivial
- $J_\pm \ \exists\ \Omega_\pm$ torsion $\to SU(n)$ struct.

  Riem spinor ϕ, ψ
- $(g, b, \phi, \psi$ Riem spinor$) \Rightarrow$ SUGRA sol^n

$\mathcal{D}_A, \mathcal{J}_B$ $\mathcal{U}_A, \mathcal{U}_B$ trivial

$J_\pm \ \exists \ \Omega_\pm$ torsion $SU(n)$ structures

Riem spin ϕ, ψ

$(g, b, \phi, \psi$ Riem spins$) \Rightarrow$ SUGRA sol^n $\left(\begin{array}{c} \text{NO GO} \\ \text{compact} \end{array} \right)$

Left + Right invariant cx struct

J_L, J_R, $g =$ kil), H - (Cartan 3-form)

$\nabla^{\pm} J_{L/R} = 0.$

$$\Rightarrow \begin{pmatrix} J_L \pm J_R & * \\ \omega_L^{-1} \pm \omega_R^{-1} & * \end{pmatrix}$$

GKähler $\quad G$ even-dim^l cpct Lie group

Left + Right - invariant cx structure

$J_L, J_R \quad g = \text{kill}$), H - (Cartan 3-form)

$\nabla^{\pm} J_{L/R} = 0.$

$\Rightarrow \begin{pmatrix} J_L \pm J_R & * \\ \omega_L \mp \omega_R & * \end{pmatrix}$


ρ_A, ρ_B triv. of U_A, U_B
⟨ρ_A, ρ̄_A⟩ = ⟨ρ_B, ρ̄_B⟩

type J_A + type J_B = $\frac{1}{2}\dim_{\mathbb{R}} \ker [J_+, J_-]$

J_+, J_- cx st.

also $f_0(H)=0$, $g[J_+, J_-]$ is holomorphic (p) bivector

ρ_A, ρ_B triv. of U_A, U_B

$\langle \rho_A, \bar{\rho}_A \rangle = \langle \rho_B, \bar{\rho}_B \rangle$

$\mathcal{U}_S$ $\qquad$ $U_{n,0}$

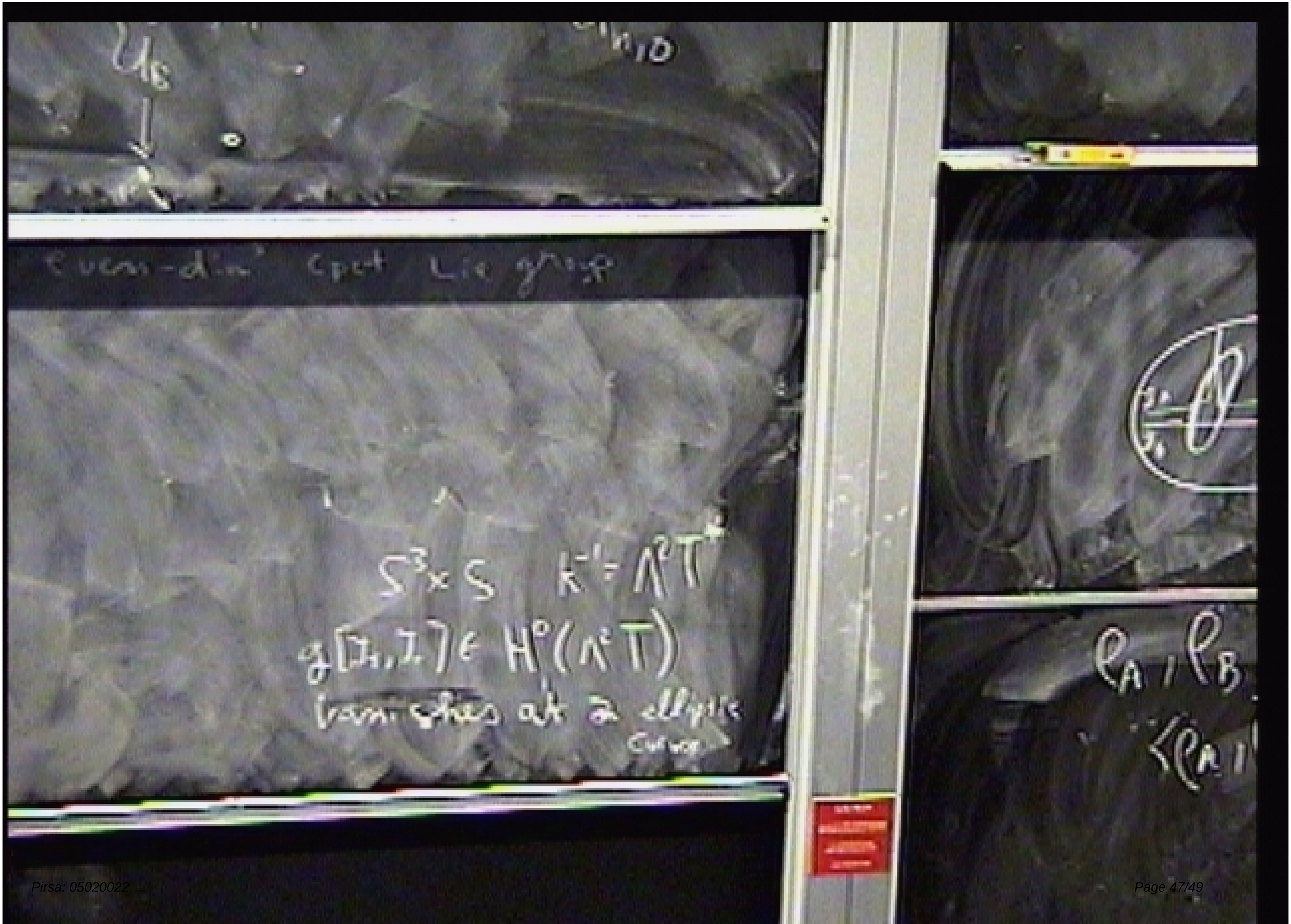
<u>GKahler</u> $\quad G$ even-dim' cpct Lie group

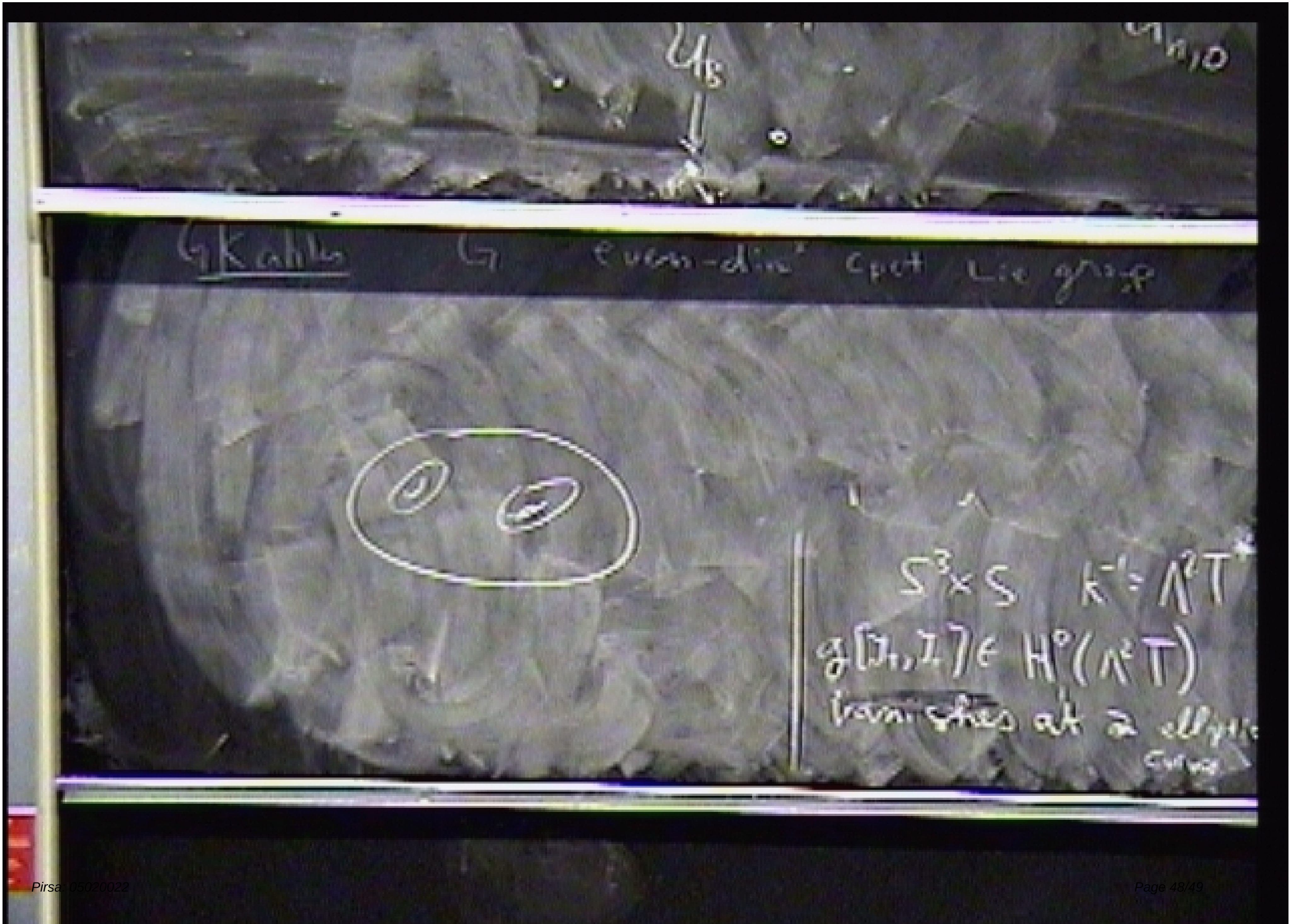
Left + Right - invariant cx structure

J_L, J_R $\quad g = \text{kil}$, H - (Cartan 3-form)

$\nabla^{\pm} J_{L/R} = 0.$ $\qquad S^3 \times S^1 \quad K^{-1} = \Lambda^2 T^*$

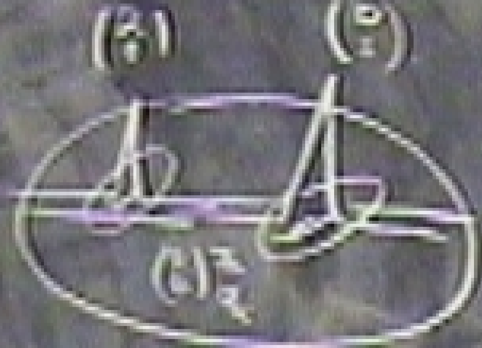
$$\Rightarrow \begin{pmatrix} J_L \pm J_R & * \\ \omega_L^{-1} \mp \omega_R^{-1} & * \end{pmatrix}$$


even-dim' cpct Lie group
$S^3 \times S$ $K^{-1} = \Lambda^2 T^*$
$g[\Sigma_1, \Sigma_2] \in H^0(\Lambda^2 T)$
vanishes at an elliptic curve
ρ_A, ρ_B



Kahler
even-dim' cpct Lie group
$S^3 \times S$ $K^{-1} = \Lambda^2 T$
$g[I_1, I_2] \in H^0(\Lambda^2 T)$
vanishes at a

G Kähler   G even-dim' cpct Lie group



$S^2 \times S$ $K^{-1} = \Lambda^2 T^*$

$g[I_+, I_-] \in H^0(\Lambda^2 T)$

vanishes at 2 elliptic curves