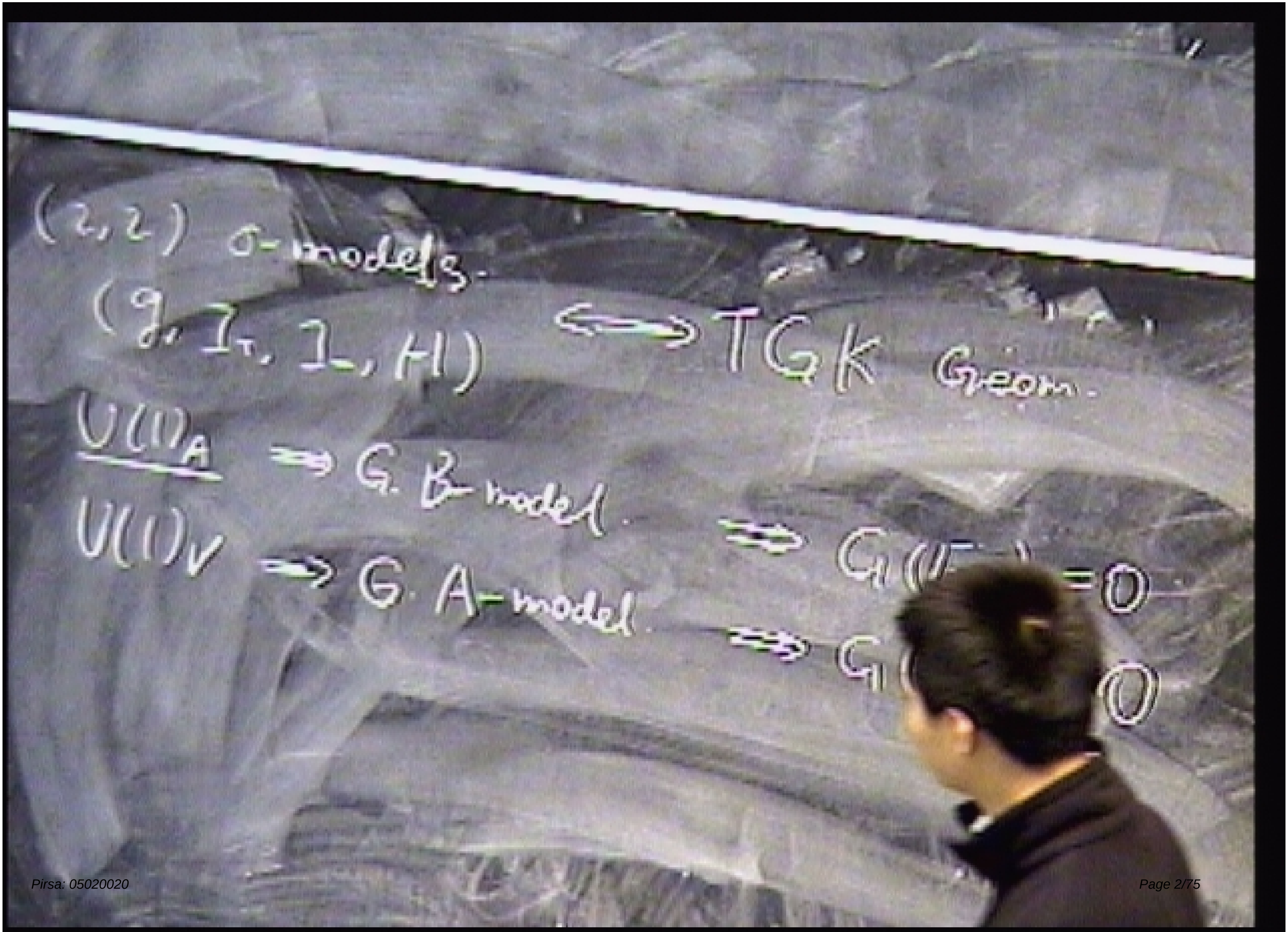
Title: Twisted Generalized Calabi-Yau Manifolds and Topological Sigma Models with Flux (Part 2)

Date: Feb 16, 2005 02:00 PM

URL: http://pirsa.org/05020020

Abstract: In these lectures, we examine how twisted generalized Calabi-Yau (GCY) manifolds arise in the construction of a general class of topological sigma models with non-trivial three-form flux. The topological sigma model defined on a twisted GCY can be regarded as a simultaneous generalization of the more familiar A-model and B-model. Emphasis will be given to the relation between topological observables of the sigma model and a Lie algebroid cohomology intrinsically associated with the twisted GCY. If time permits, we shall also discuss topological D-branes in this more general setting, and explain how the viewpoint from the Lie algebroid helps to elucidate certain subtleties even for the conventional A-branes and B-branes. The lectures will be physically motivated, although I will try to make the presentation self-contained for both mathematicians and physicists.



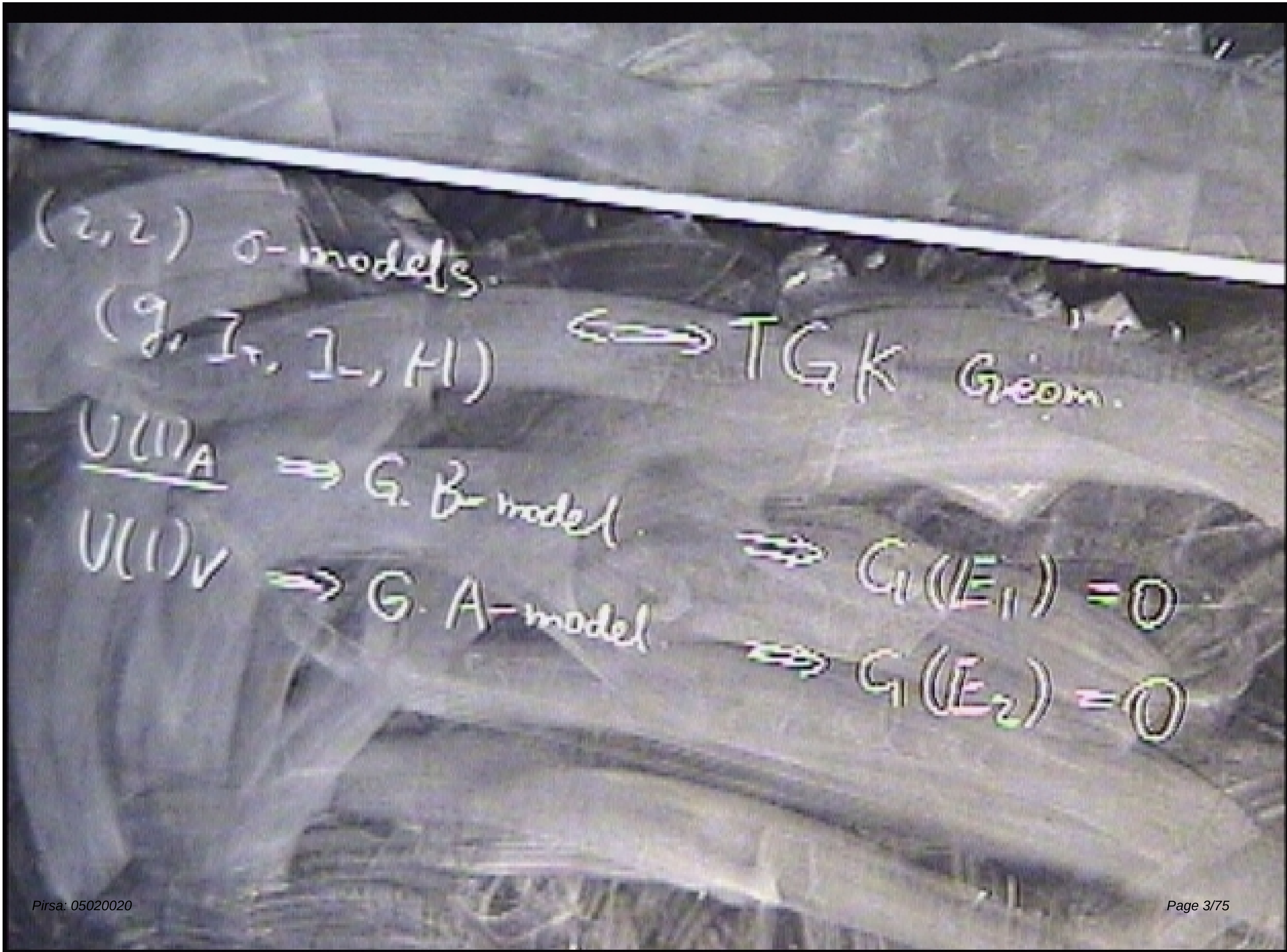
(2,2) σ-models.

$(g, I_+, I_-, H) \Longleftrightarrow$ TGK Geom.

$\dfrac{U(1)_A}{U(1)_V}$

$U(1)_A \Longrightarrow$ G. B-model $\Longrightarrow G($ [illegible] $) = 0$

$U(1)_V \Longrightarrow$ G. A-model $\Longrightarrow G$ [illegible] 0

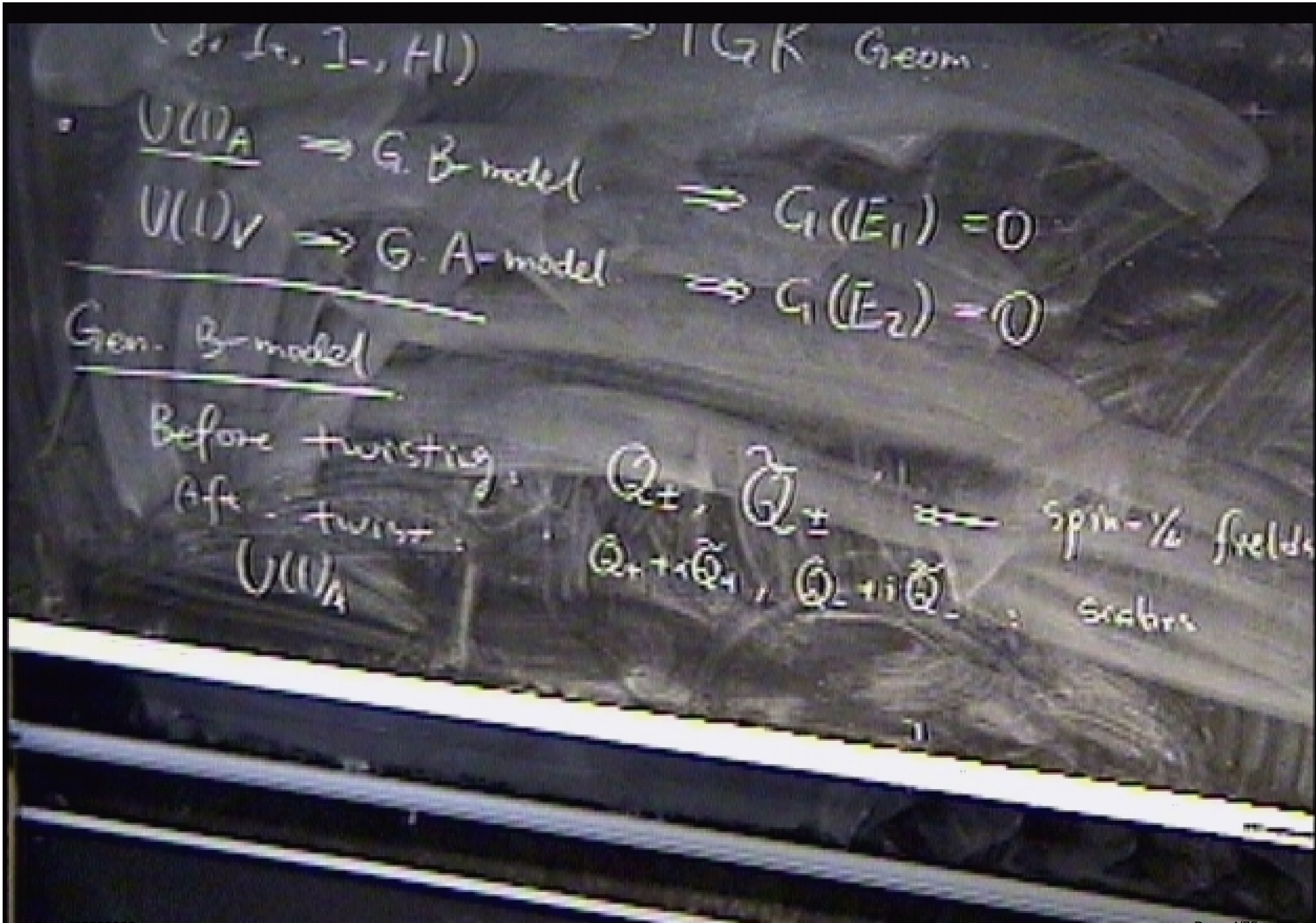


(2,2) σ-models.

$(g, I_+, I_-, H) \Longleftrightarrow$ TGK Geom.

$U(1)_A \Longrightarrow$ G. B-model $\Longrightarrow c_1(\mathbb{E}_1)=0$

$U(1)_V \Longrightarrow$ G. A-model $\Longrightarrow c_1(\mathbb{E}_2)=0$



(g, I_+, I_-, H) ⟹ GK Geom.

- $\underline{U(1)_A}$ ⟹ G. B-model ⟹ $G(E_1) = 0$

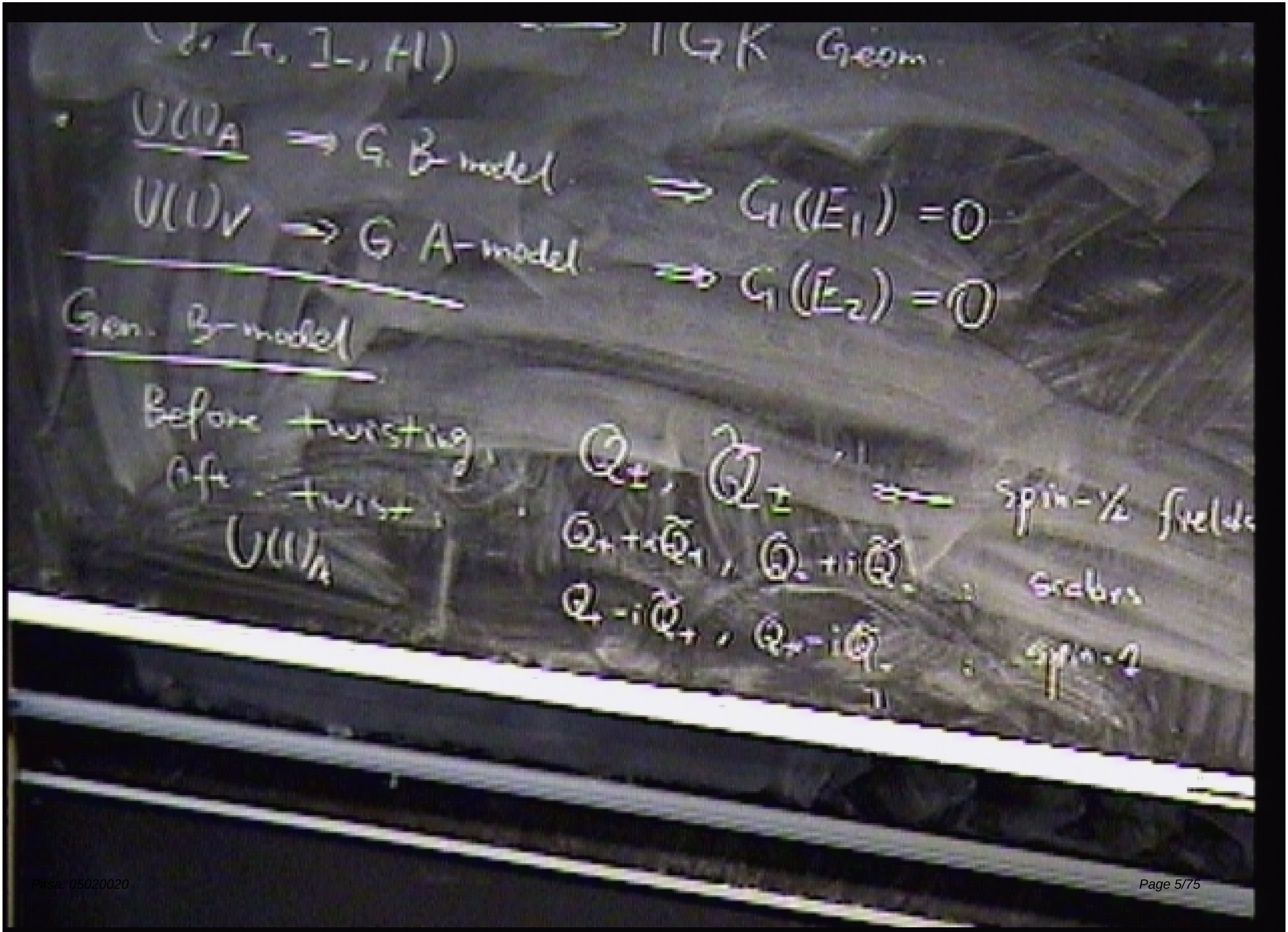
  $U(1)_V$ ⟹ G. A-model ⟹ $G(E_2) = 0$

Gen. B-model

Before twisting: $Q_\pm, \tilde{Q}_\pm$ ← spin-½ fields

After twist: $Q_+ + i\tilde{Q}_+$, $Q_- + i\tilde{Q}_-$: scalars

$U(1)_A$



$(g, I_+, I_-, H) \Longleftrightarrow$ GK Geom.

- $\underline{U(1)_A} \Rightarrow$ G. B-model $\Rightarrow G(\mathbb{E}_1) = 0$

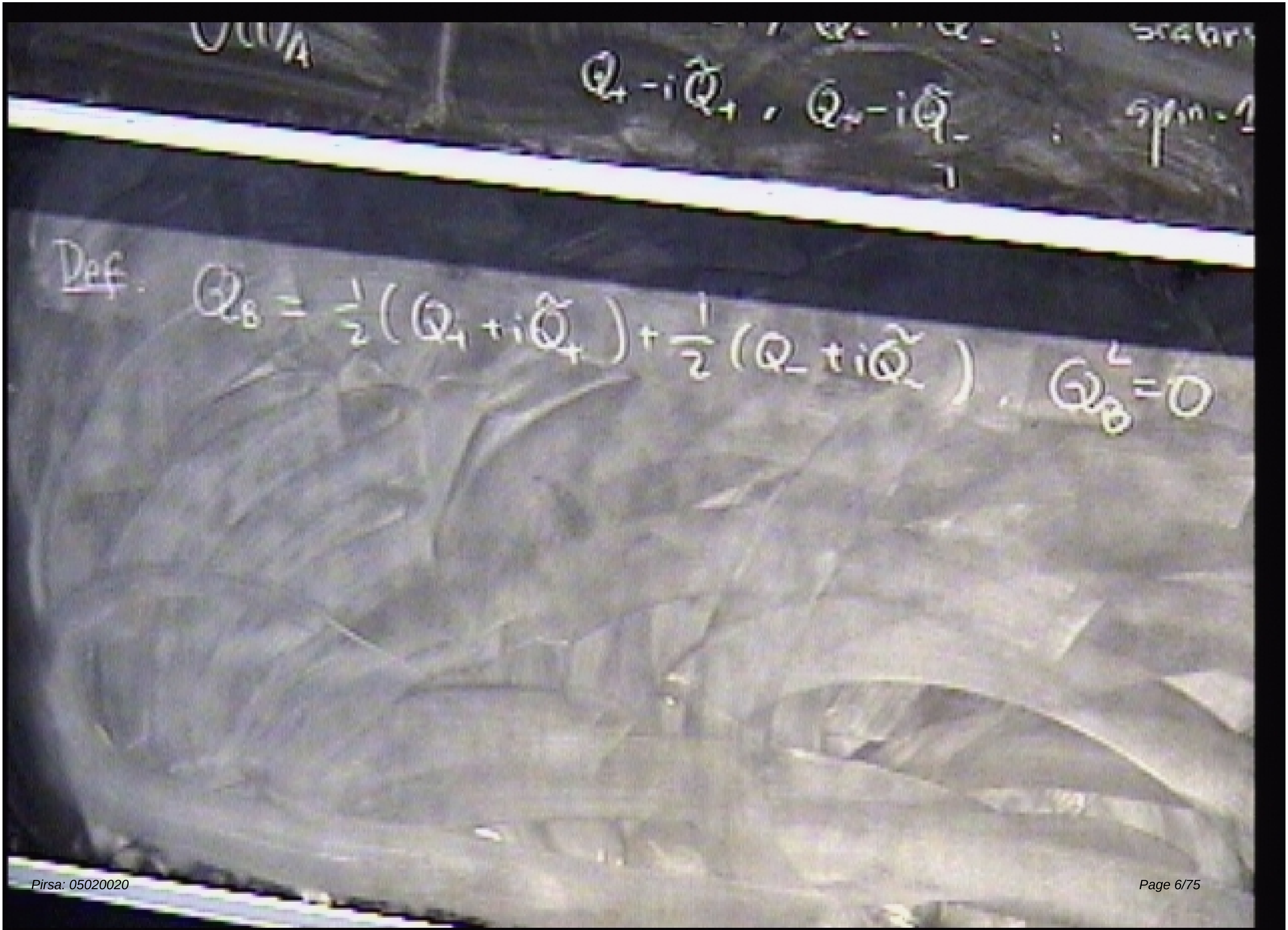
$U(1)_V \Rightarrow$ G. A-model $\Rightarrow G(\mathbb{E}_2) = 0$

Gen. B-model

Before twisting, $Q_\pm, \tilde{Q}_\pm$ — spin-½ fields

After twist, $U(1)_A$: $Q_+ + i\tilde{Q}_+$, $Q_- + i\tilde{Q}_-$: scalars

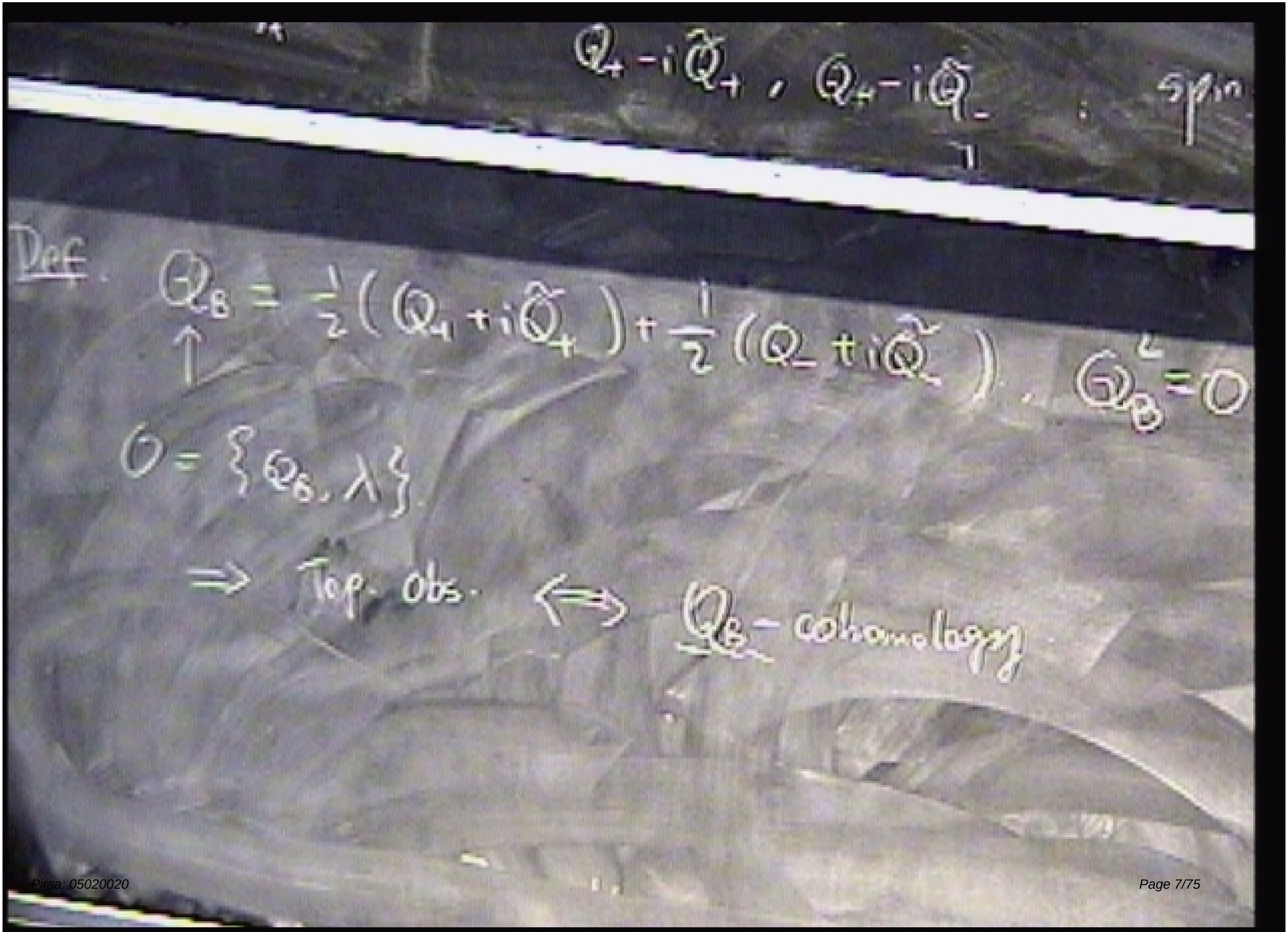
$Q_+ - i\tilde{Q}_+$, $Q_- - i\tilde{Q}_-$: spin-1



$U(1)_A$

$Q_+ - i\tilde{Q}_+$, $Q_+ - i\tilde{Q}_-$: spin-1

Def. $Q_B = \frac{1}{2}(Q_+ + i\tilde{Q}_+) + \frac{1}{2}(Q_- + i\tilde{Q}_-)$, $Q_B^2 = 0$



$Q_+ - i\tilde{Q}_+ \,,\; Q_+ - i\tilde{Q}_-$: spin

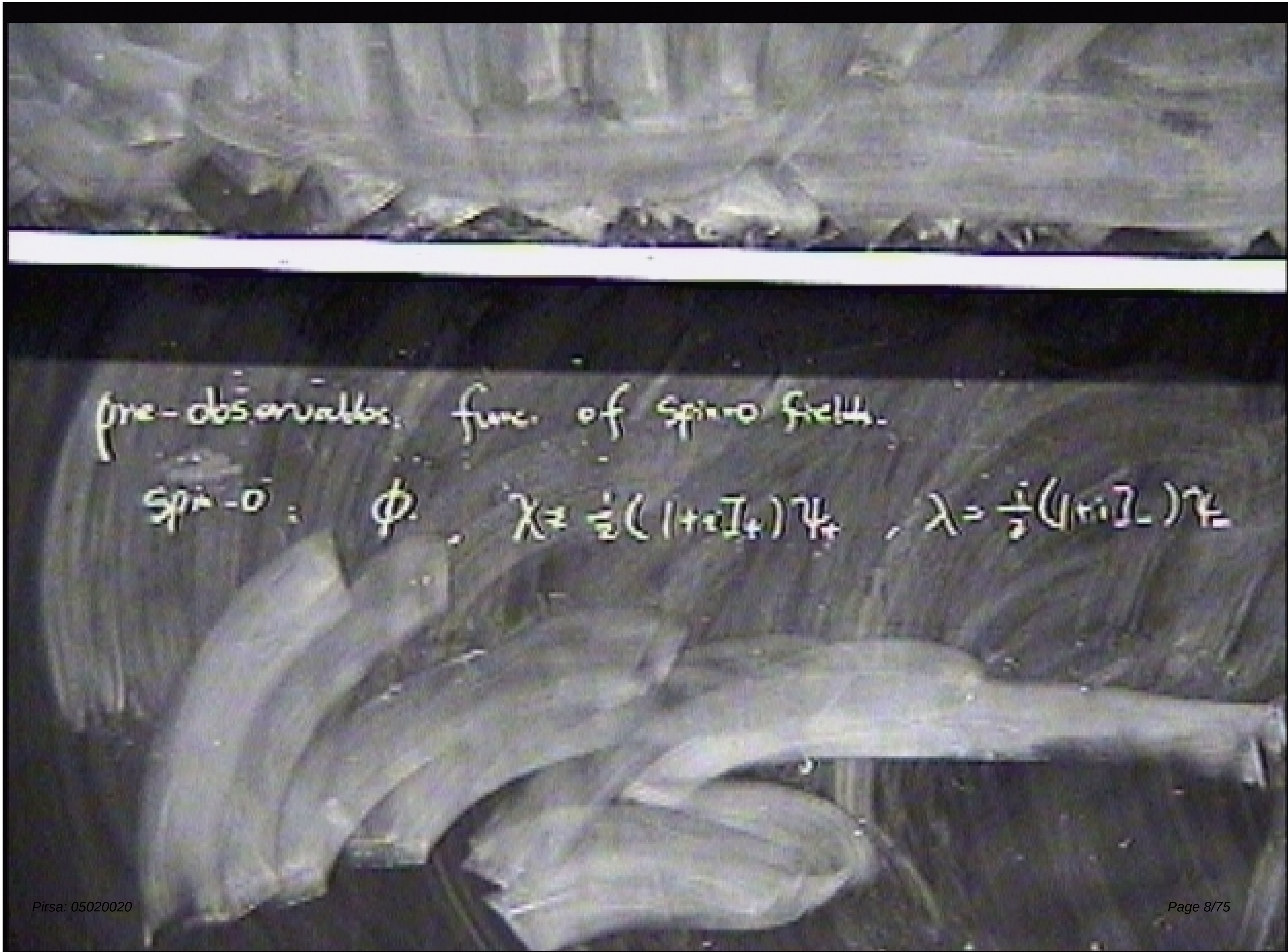
1

Def. $Q_B = \frac{1}{2}(Q_+ + i\tilde{Q}_+) + \frac{1}{2}(Q_- + i\tilde{Q}_-)$, $Q_B^2 = 0$

$\uparrow$

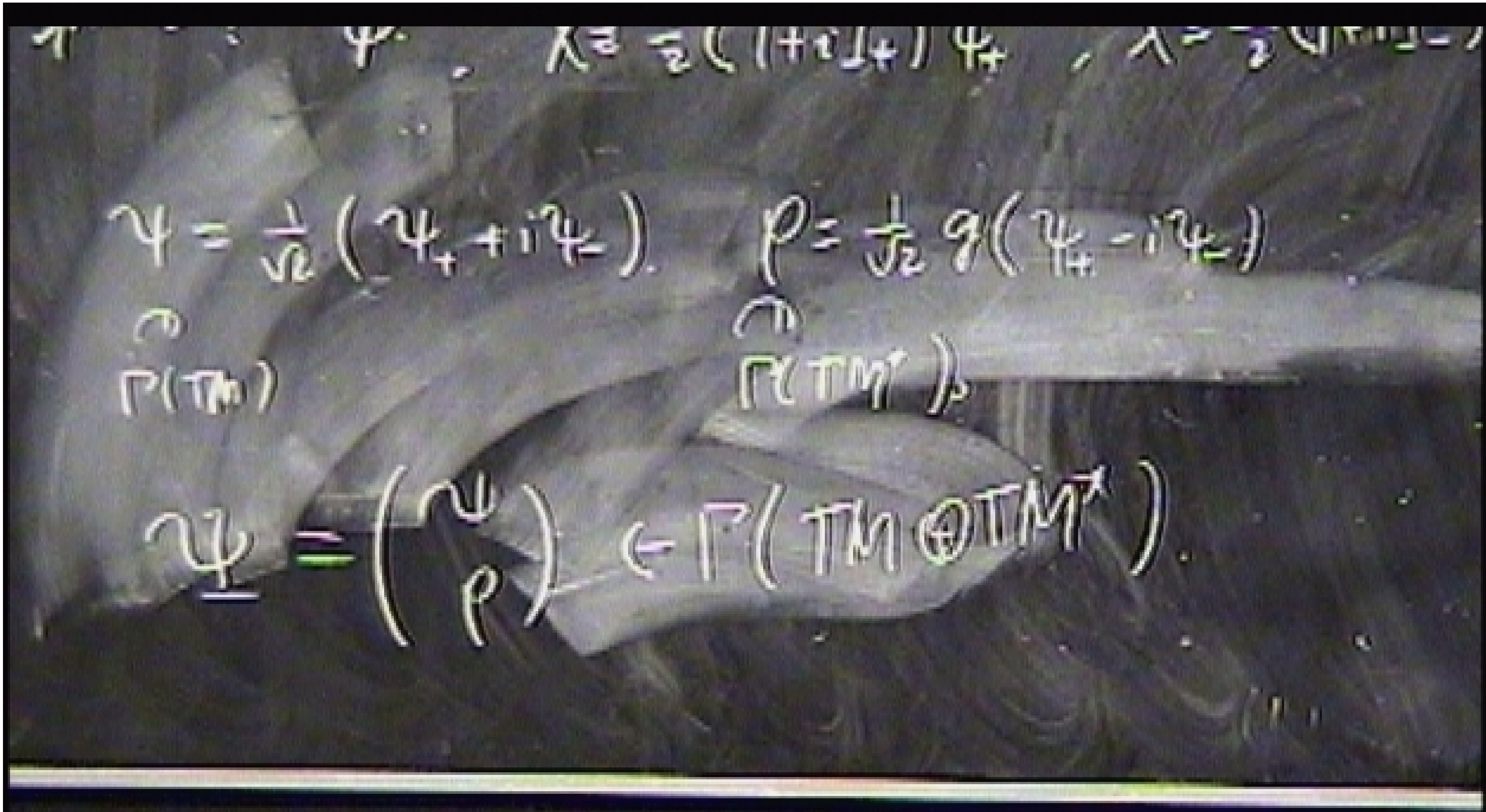
$O = \{Q_B, \lambda\}$

$\Rightarrow$ Top. obs. $\Longleftrightarrow$ $\underline{Q_B}$-cohomology.



pre-observables: func. of spin-0 fields.

spin-0: ϕ, $\chi = \frac{1}{2}(1+iJ_+)\psi_+$, $\lambda = \frac{1}{2}(1+iJ_-)\psi_-$



$\lambda = \frac{1}{2}(1+iJ_+)\psi_+$, $\lambda = \frac{1}{2}(1+iJ_-)$

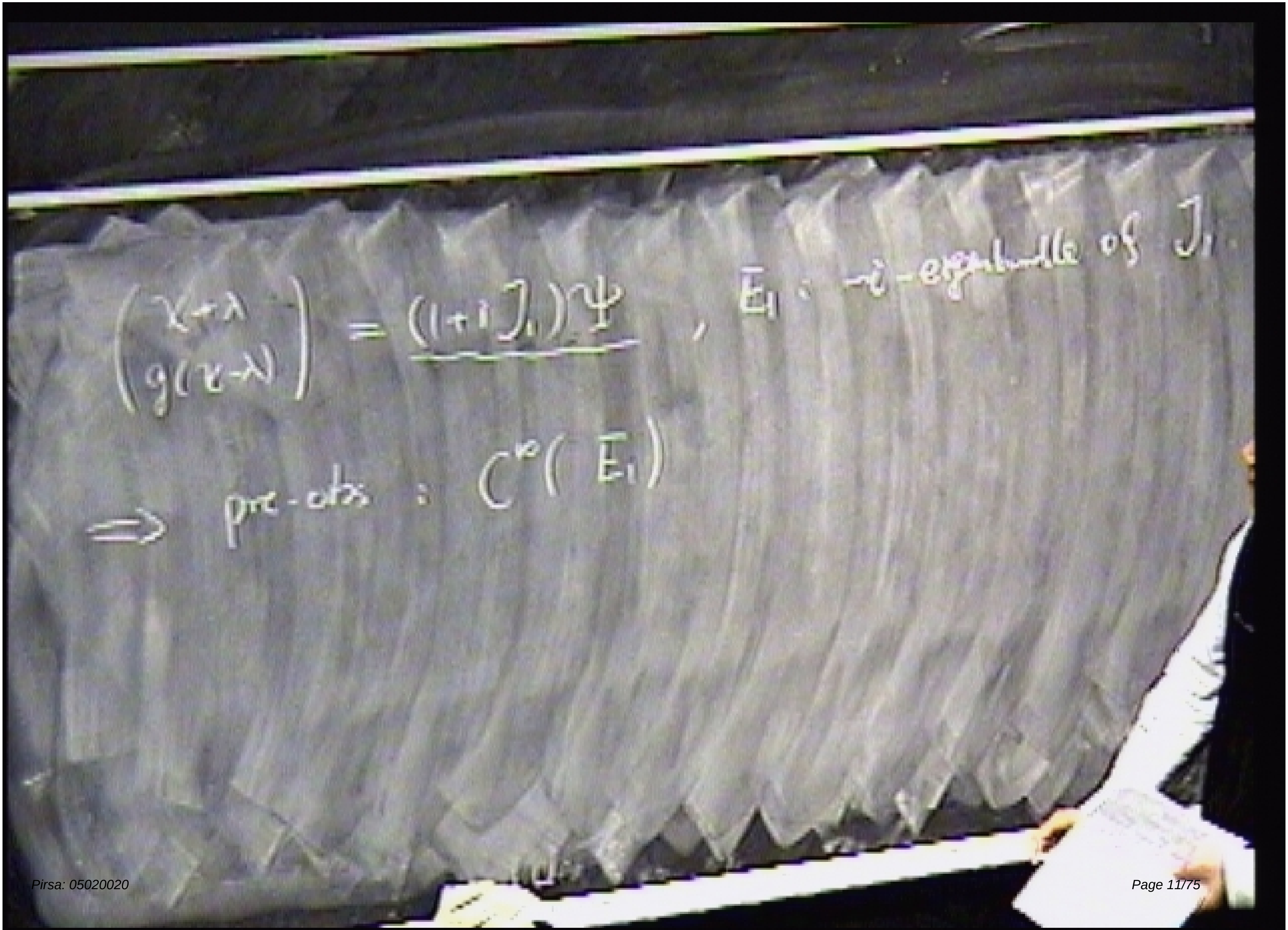
$$\psi = \frac{1}{\sqrt{2}}(\psi_+ + i\psi_-) \qquad \rho = \frac{1}{\sqrt{2}}\, g(\psi_+ - i\psi_-)$$

$$\cap \qquad\qquad \cap$$

$$\Gamma(TM) \qquad \Gamma(TM^*)$$

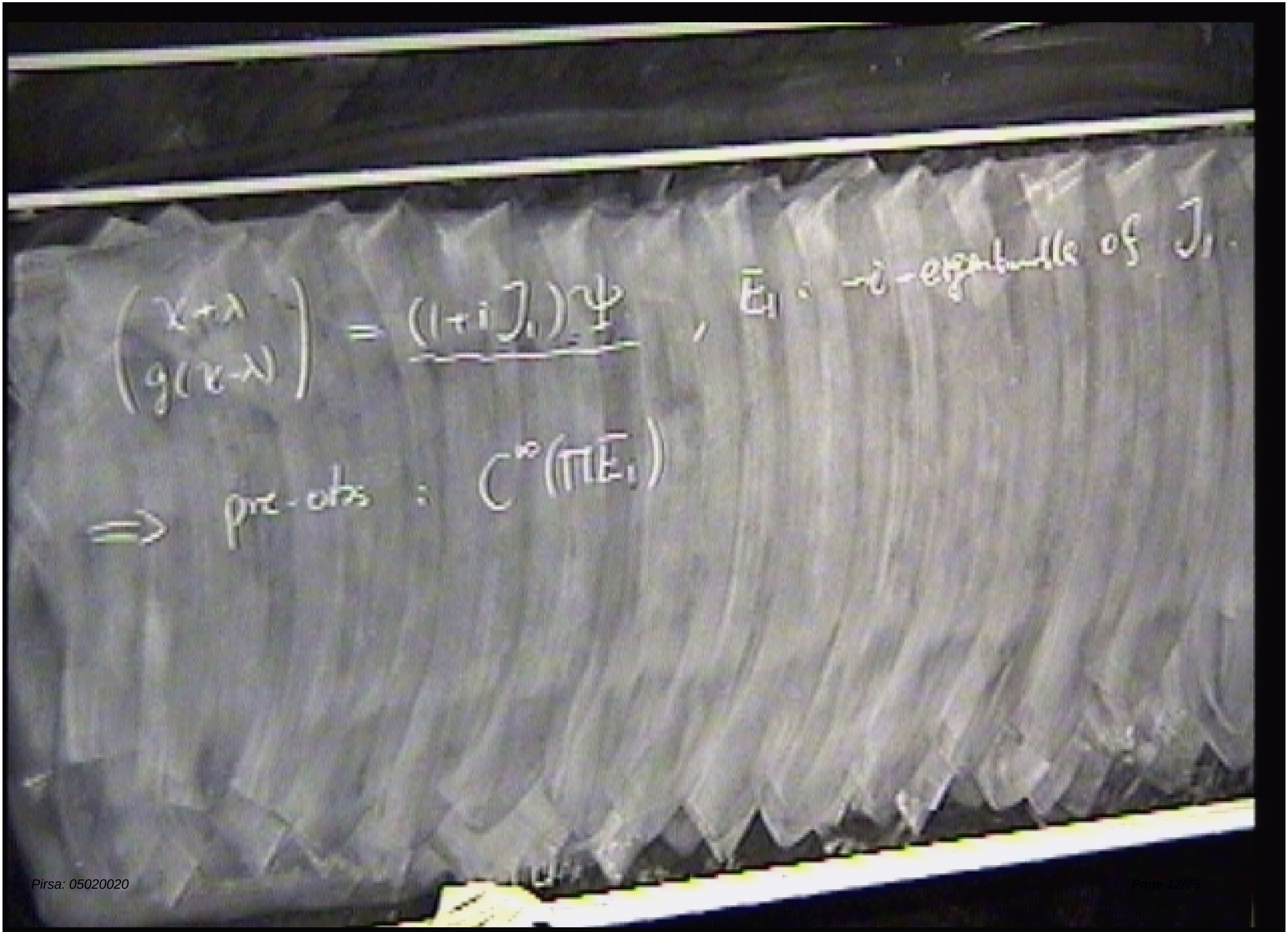
$$\underline{\Psi} = \begin{pmatrix} \psi \\ \rho \end{pmatrix} \in \Gamma(TM \oplus TM^*)$$

$$\begin{pmatrix} \chi + \lambda \\ g(\chi - \lambda) \end{pmatrix} = (1 + i J_1)\Psi$$



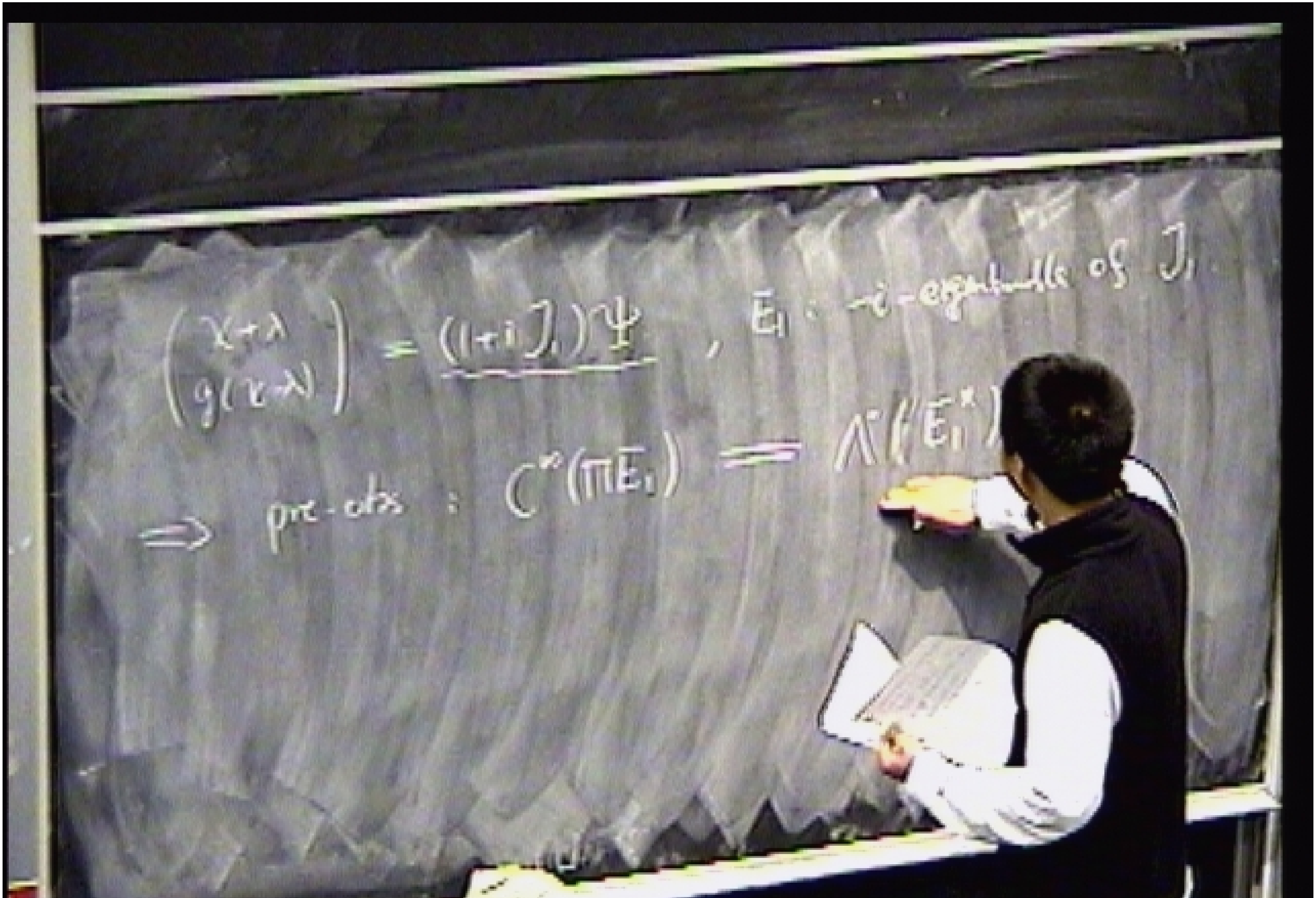
$$\begin{pmatrix} \xi + \lambda \\ g(\xi - \lambda) \end{pmatrix} = \underline{(1 + i J_1)\Psi} \quad , \quad \bar{E}_1 : -i\text{-eigenbundle of } J_1$$

$\Longrightarrow$ pre-obs : $C^\infty(E_1)$



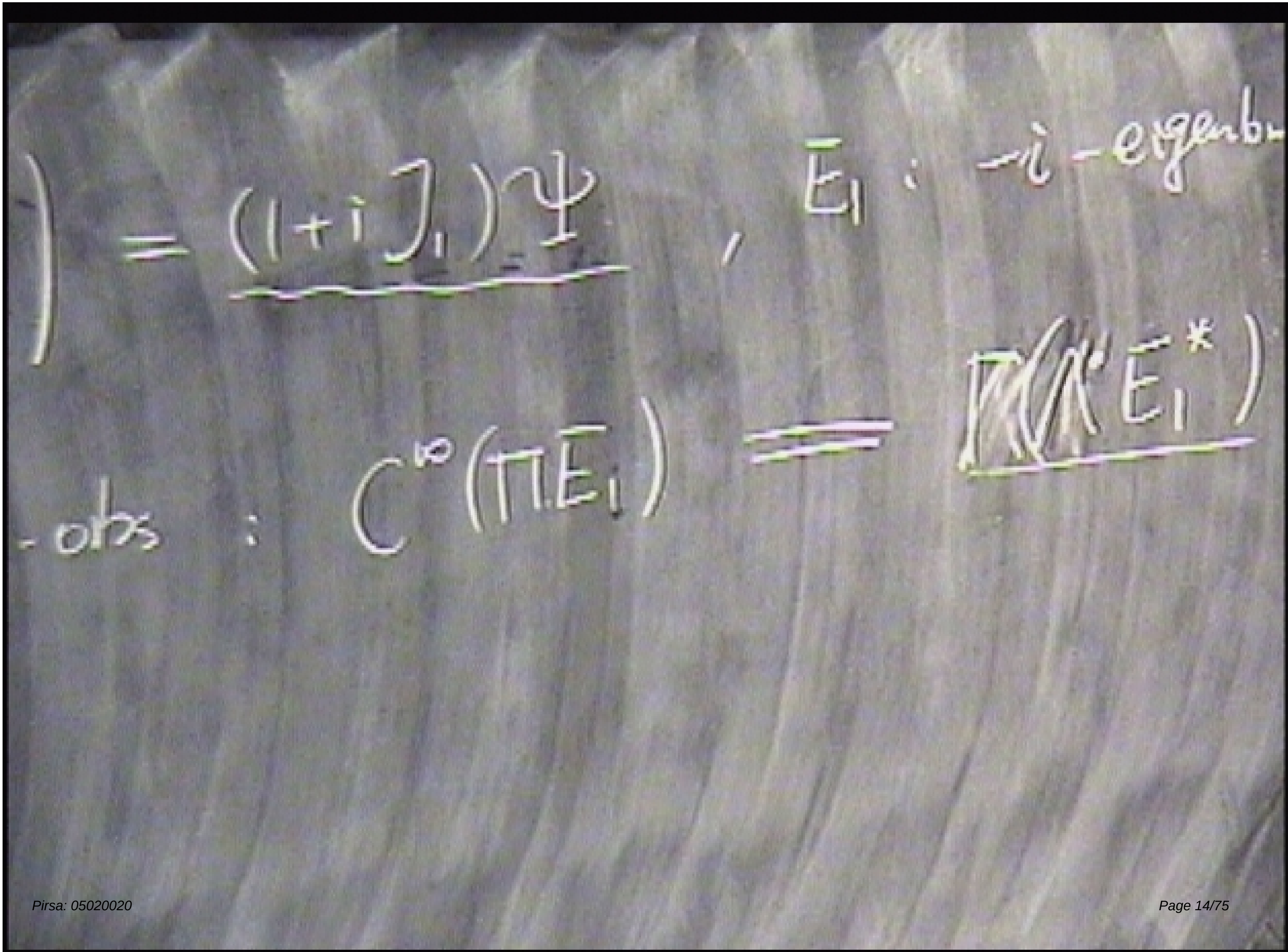
$$\begin{pmatrix} \chi + \lambda \\ g(\chi - \lambda) \end{pmatrix} = \underline{(1 + i J_1)\Psi} \quad , \quad \bar{E}_1 : -i\text{-eigenbundle of } J_1 .$$

$\Longrightarrow$ pre-obs : $C^\infty(\Pi \bar{E}_1)$



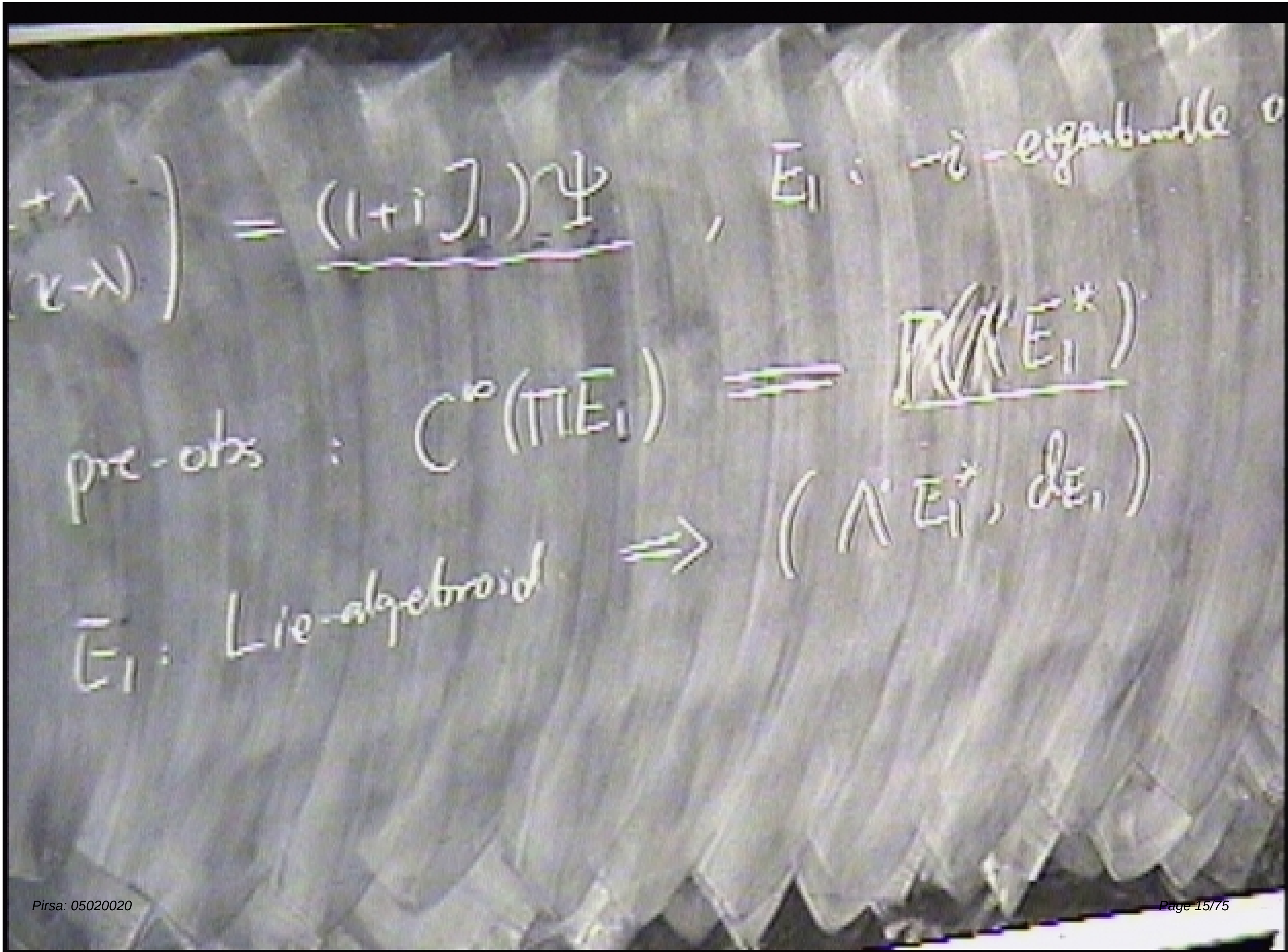
$$\begin{pmatrix} \chi+\lambda \\ g(\chi-\lambda) \end{pmatrix} = \underline{\underline{(1+iJ_1)\Psi}}, \quad E_1 : -i\text{-eigenbundle of } J_1$$

$$\Longrightarrow \text{pre-obs} : \; C^\infty(\Pi E_1) \;=\!=\; \Lambda^\bullet(E_1^*)$$



$$) = \underline{(1 + i J_1)\Psi} \quad , \quad E_1 : -i\text{-eigenb}$$

$$\text{-obs} : \quad C^{\infty}(\Pi E_1) = \underline{\Gamma(\Lambda^{\bullet} E_1^{*})}$$

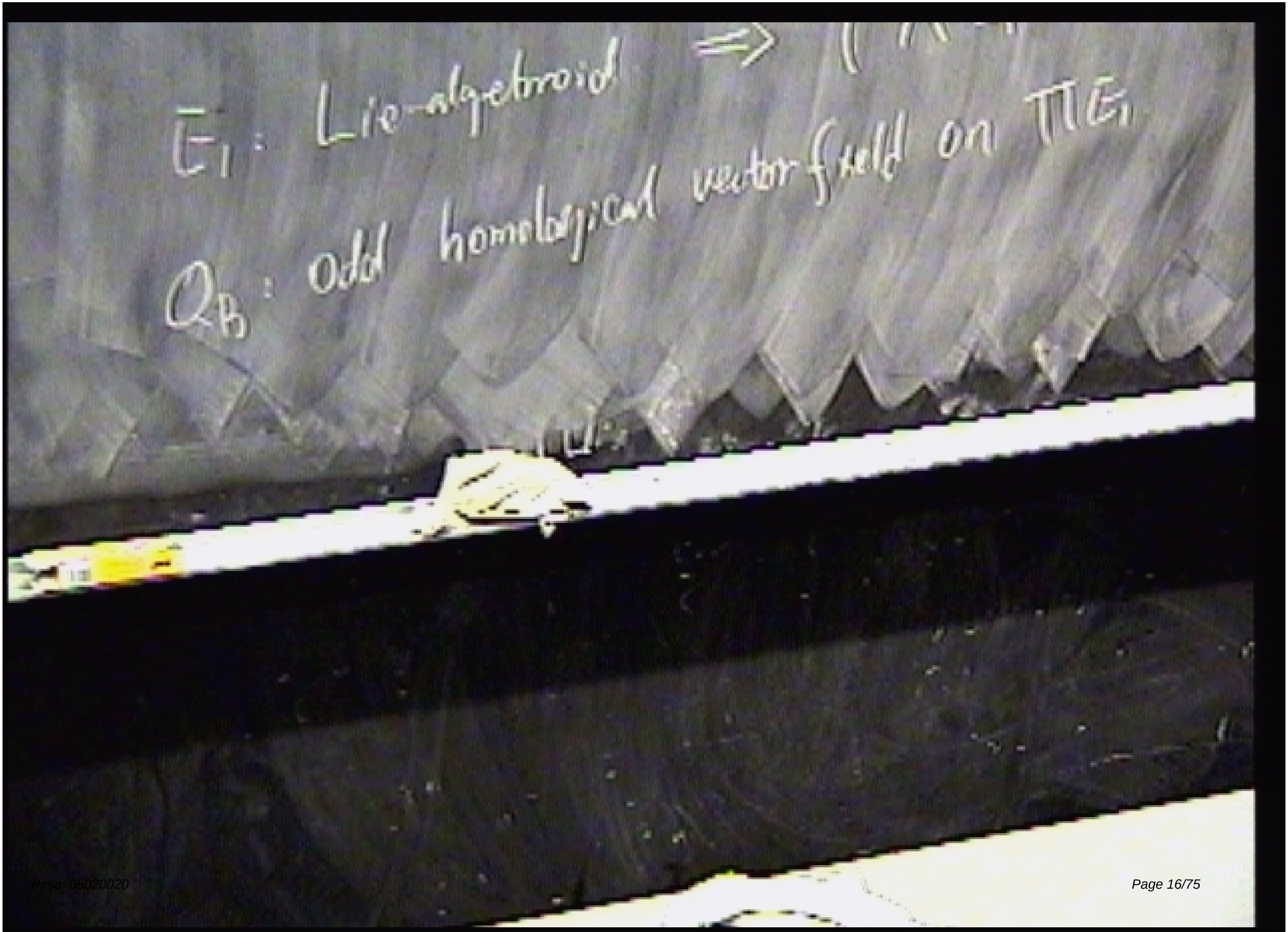


$\left(\begin{matrix} \;+\lambda \\ (\chi-\lambda) \end{matrix} \right) = \underline{(1+i\,J_1)\Psi}$, $\overline{E}_1$: $-i$ - eigenbundle o

pre-obs : $C^\infty(\Pi \overline{E}_1) = \underline{\Gamma(\Lambda^\bullet E_1^*)}$

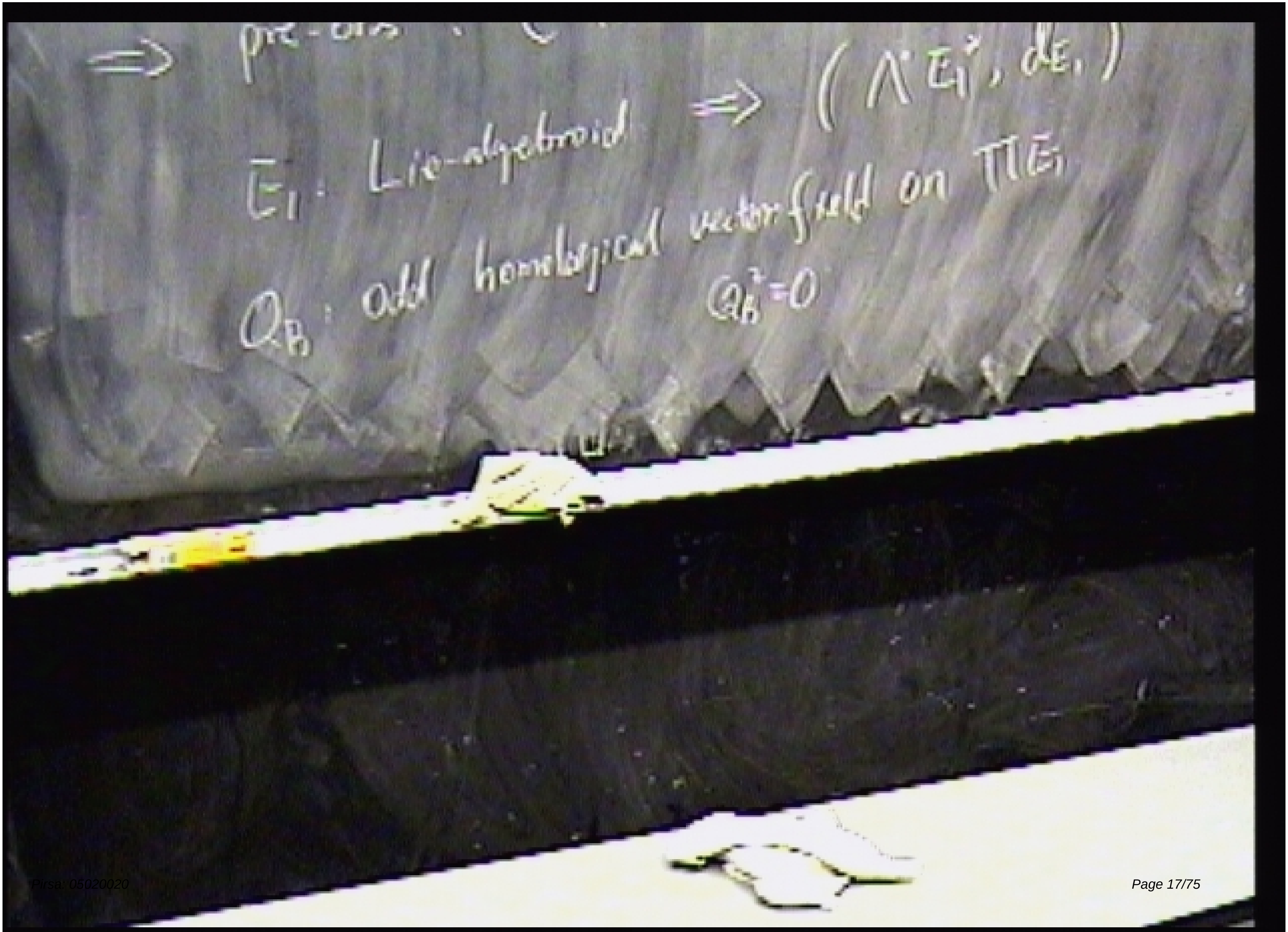
$(\Lambda^\bullet E_1^*, d_{E_1})$

$\overline{E}_1$: Lie-algebroid $\Longrightarrow$



E_1 : Lie-algebroid $\Longrightarrow$ (

Q_B : odd homological vector field on ΠE_1

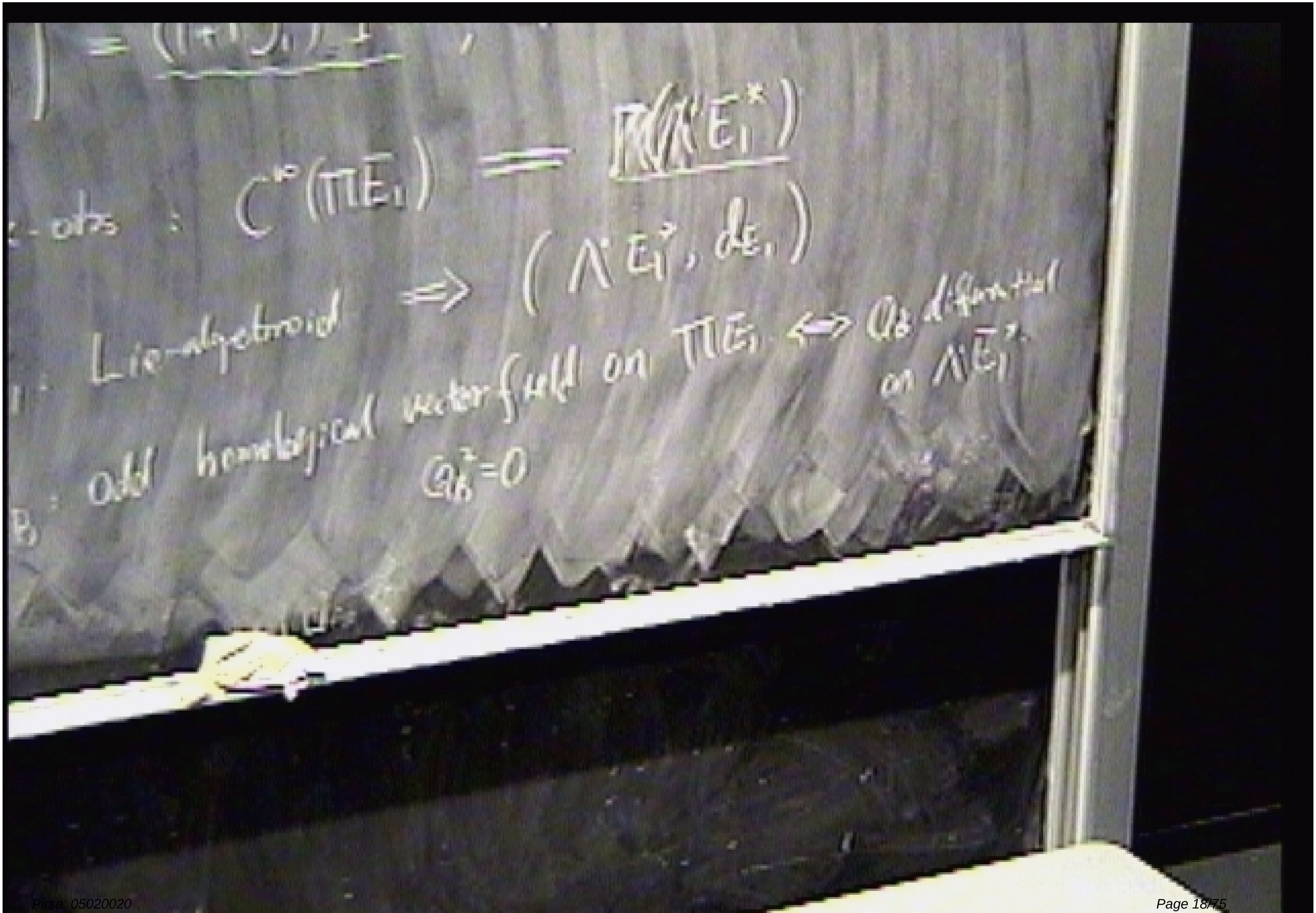


$\Longrightarrow$ pre-

E_1 : Lie-algebroid $\Longrightarrow$ $(\Lambda^\bullet E_1^*, d_{E_1})$

Q_B : odd homological vector field on ΠE_1

$Q_B^2 = 0$

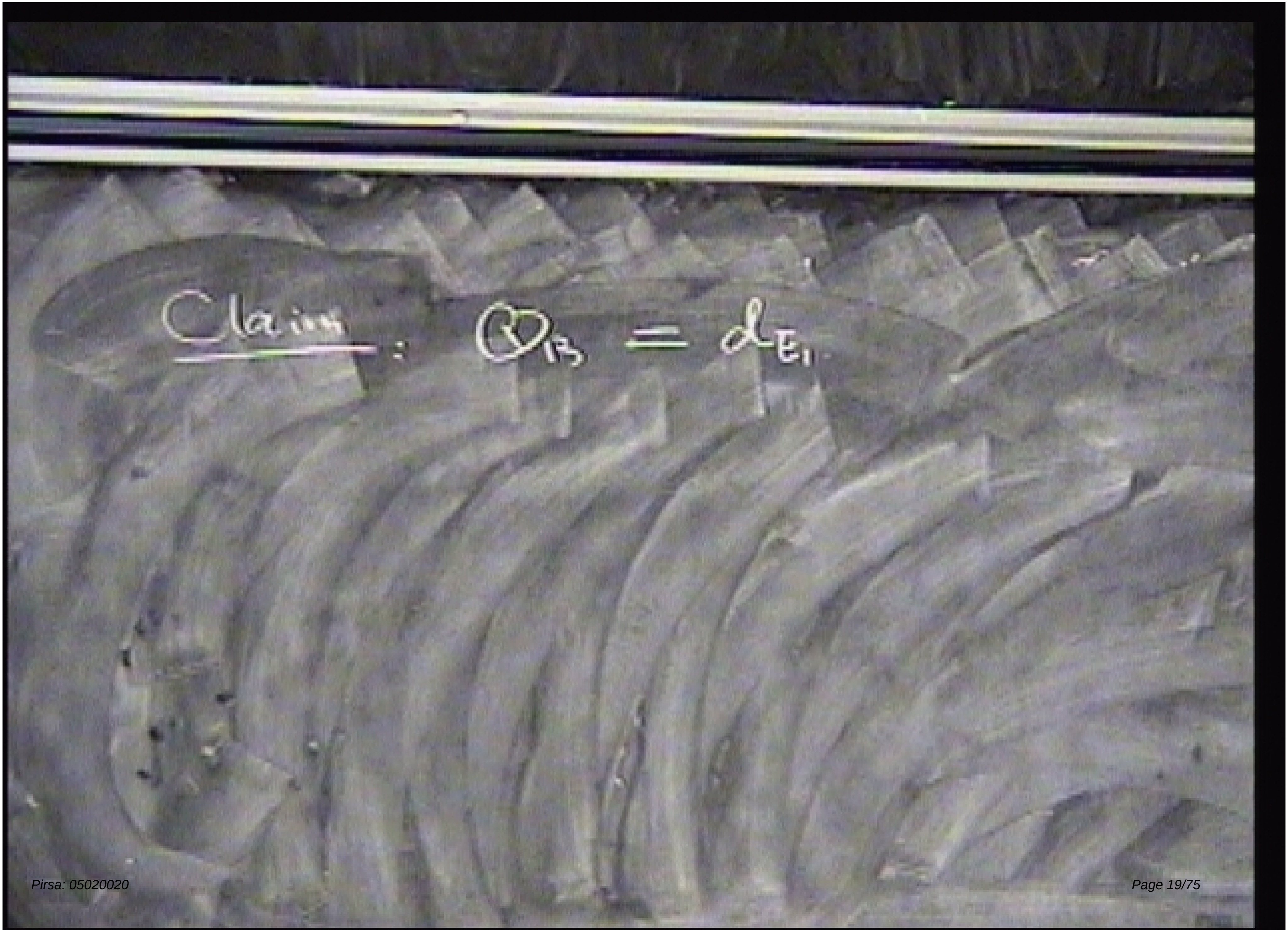


-alts : $C^\infty(\Pi E_1) = \Gamma(\Lambda^\bullet E_1^*)$

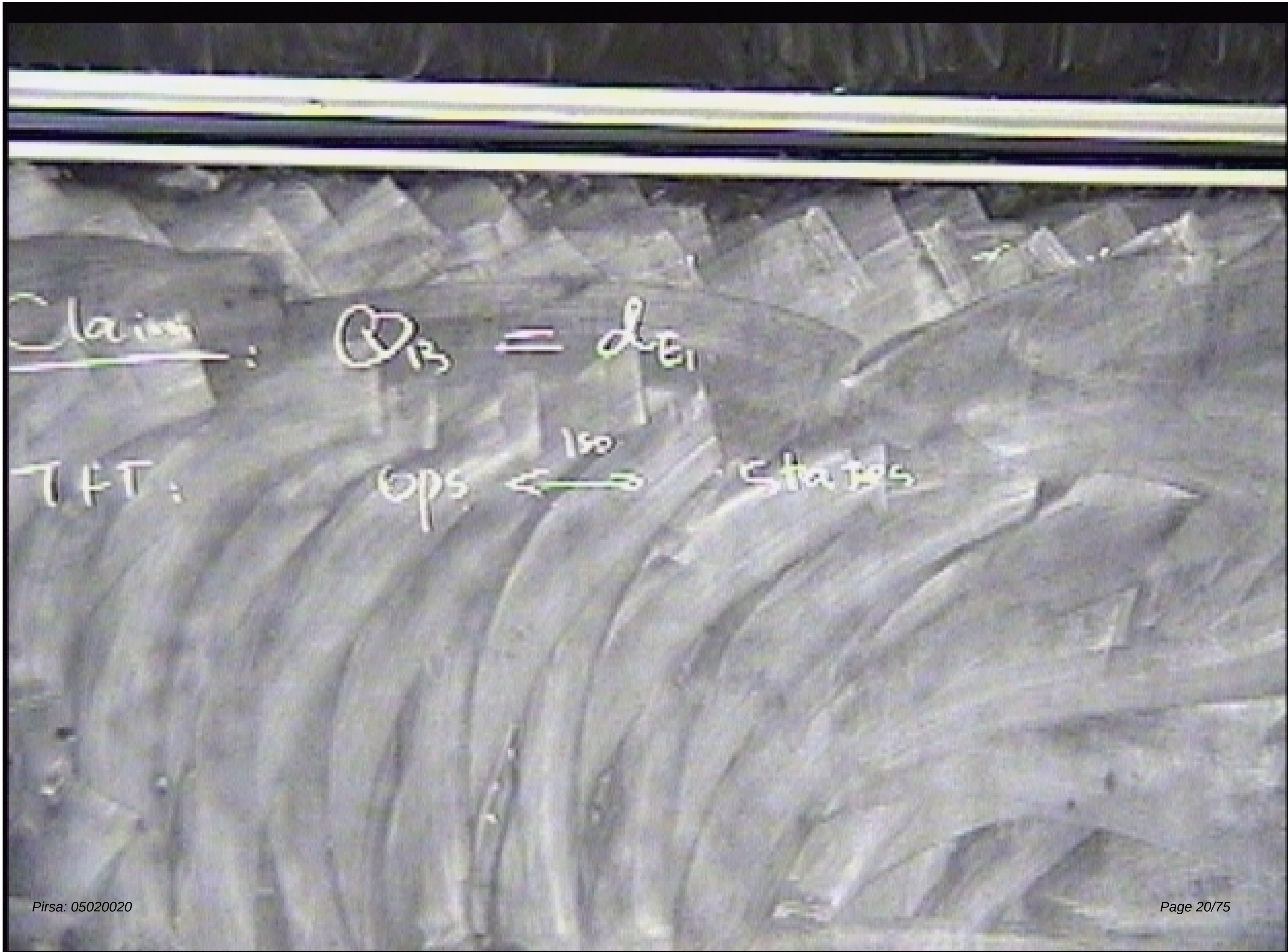
Lie-algebroid $\Longrightarrow (\Lambda^\bullet E_1^*, d_{E_1})$

Q_B : odd homological vector field on $\Pi E_1 \Longleftrightarrow Q_B$ differential on $\Lambda^\bullet E_1^*$

$Q_B^2=0$

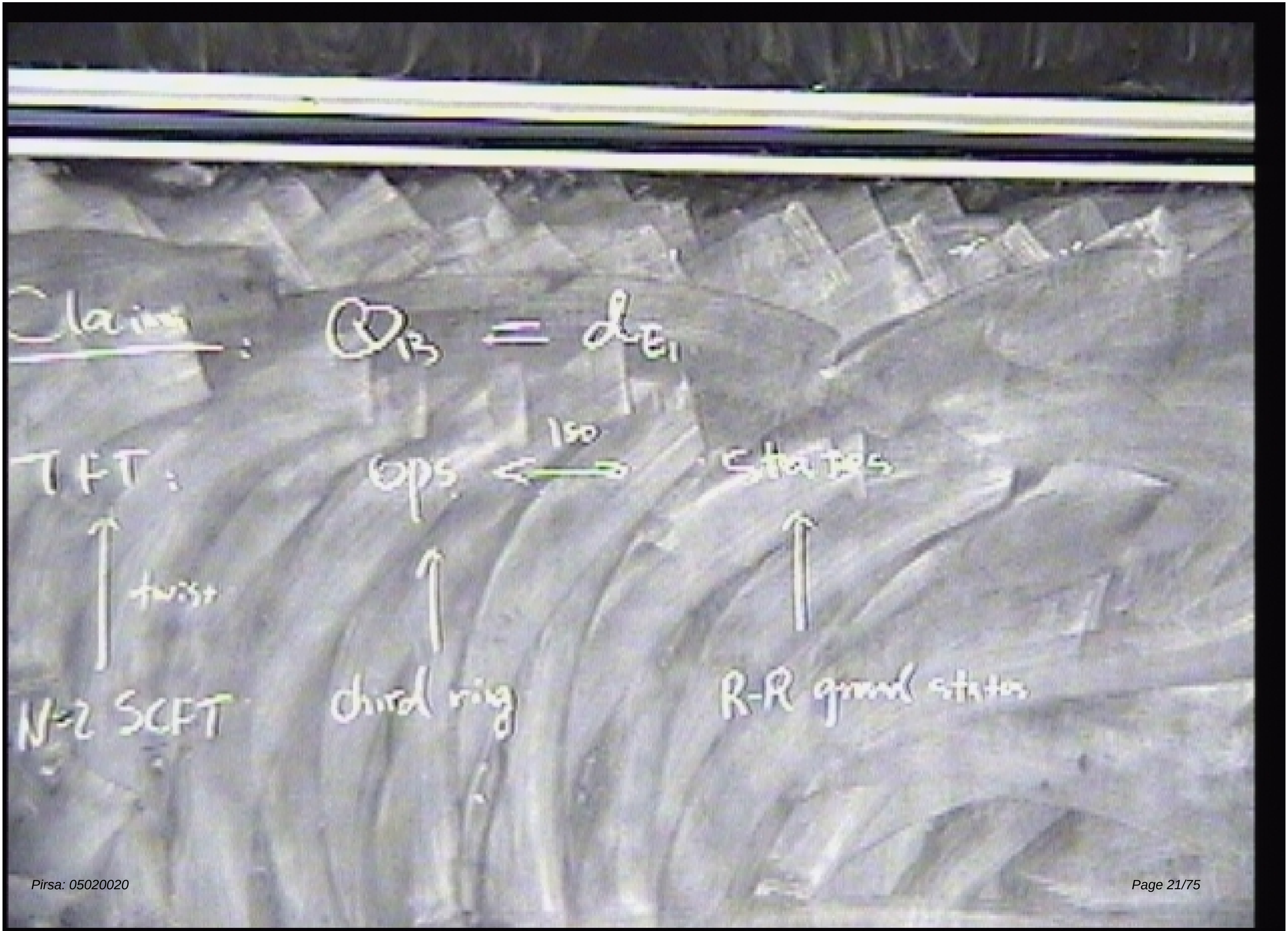


Claim: $\mathcal{D}_{13} = d_{E_1}$



Claim: $Q_B = d_{E_1}$

TFT: ops $\xleftrightarrow{\text{iso}}$ states



Claim: $Q_B = d_{E_1}$

$$\begin{tikzcd} \text{TFT:} & \text{Ops} \arrow[r, leftrightarrow, "\text{iso}"] & \text{States} \\ N=2 \text{ SCFT} \arrow[u, "\text{twist}"'] & \text{chiral ring} \arrow[u] & \text{R-R ground states} \arrow[u] \end{tikzcd}$$

Claim: $\mathcal{O}_B = \mathcal{L}_{E_1}$

$$\begin{tikzcd}
\text{TFT:} & \text{Ops} \arrow[r, leftrightarrow, "\text{iso}"] & \text{States} \\
\mathcal{N}=2\ \text{SCFT} \arrow[u, "\text{twist}"'] & \text{chiral ring} \arrow[u] \arrow[r, shift left] & \text{R-R ground states} \arrow[u] \arrow[l, shift left]
\end{tikzcd}$$

Claim: $Q_B = d_{E_1}$

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\text{TFT:} & \text{Ops} \arrow[rr, leftrightarrow, "\text{iso}"] & & \text{States} \\
N=2 \text{ SCFT} \arrow[u, "\text{twist}"'] & \text{chiral ring} \arrow[u] \arrow[rr, shift left, "\text{spec flow}"] & & \text{R-R ground states} \arrow[u] \arrow[ll, shift left]
\end{tikzcd}$$

Claim: $\mathcal{O}_B = d_{E_1}$

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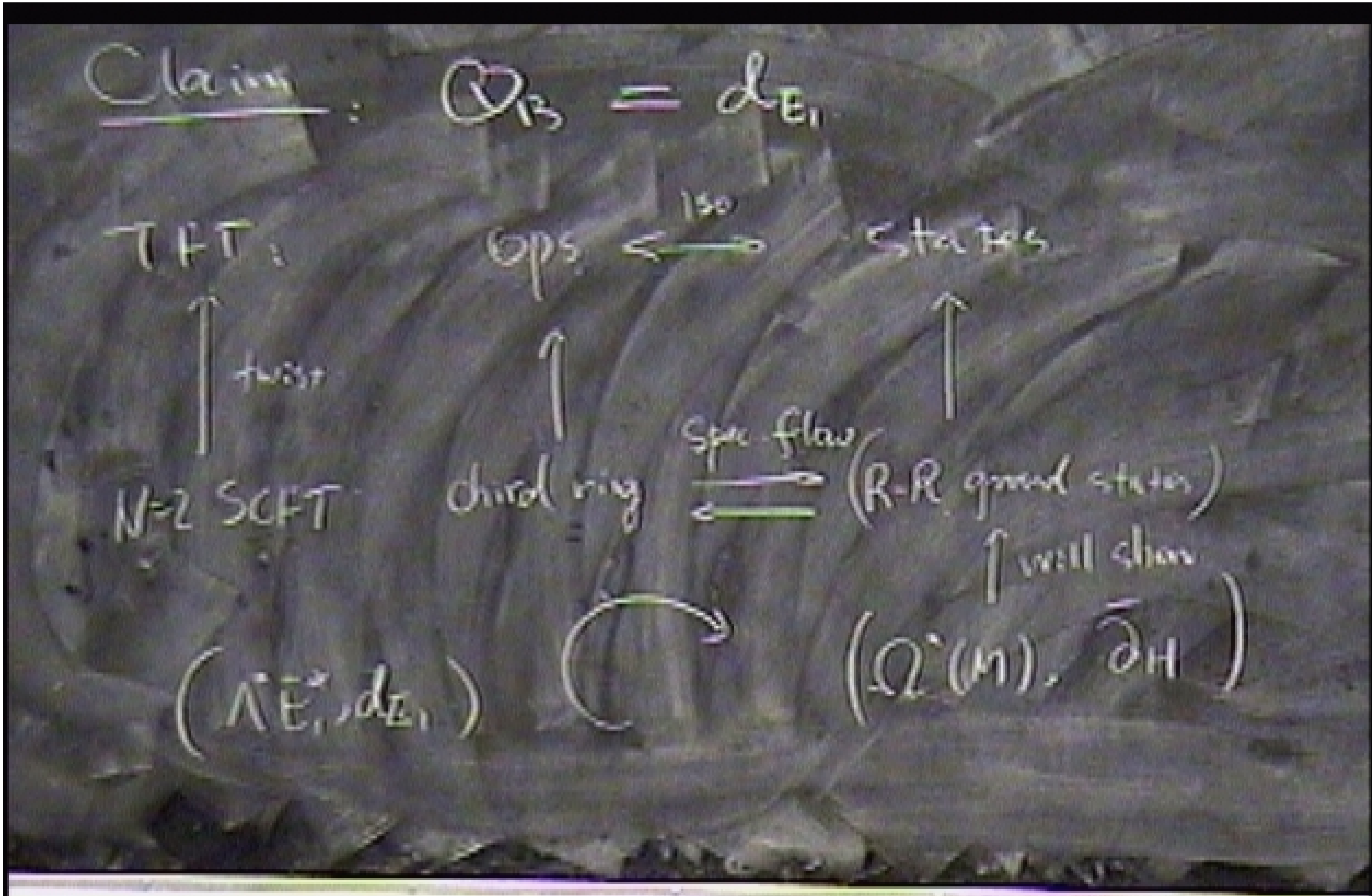
 & & (\Omega^{\bullet}(M), \bar{\partial}_H) \arrow[u, "\text{will show}"']
\end{tikzcd}$$



Claim: $Q_B = d_{E_1}$

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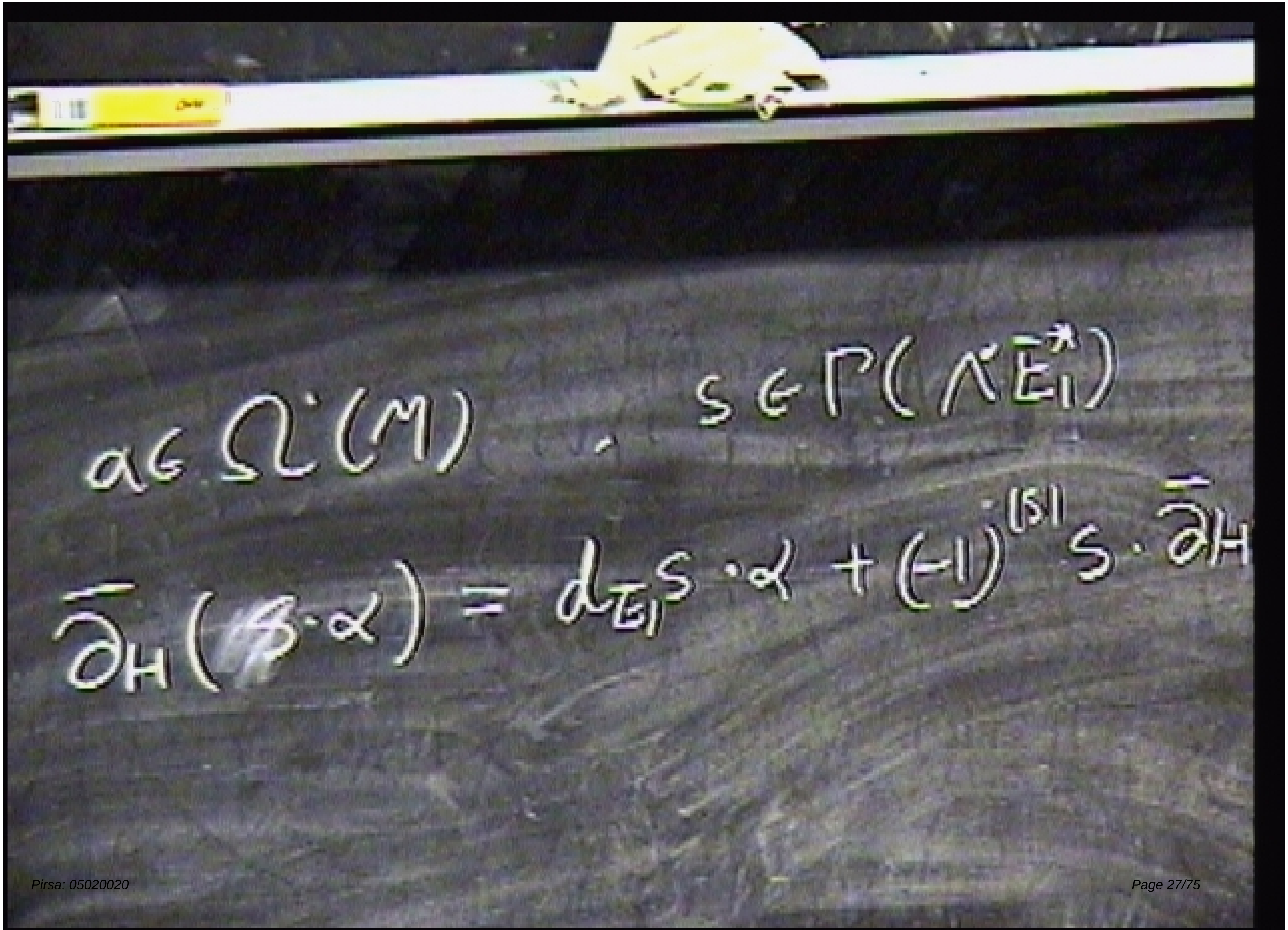
(\Lambda^\bullet E_1^*, d_{E_1}) & (\arrow[r, bend left] & (\Omega^\bullet(M), \bar{\partial}_H) \arrow[u, "\text{will show}"]
\end{tikzcd}$$

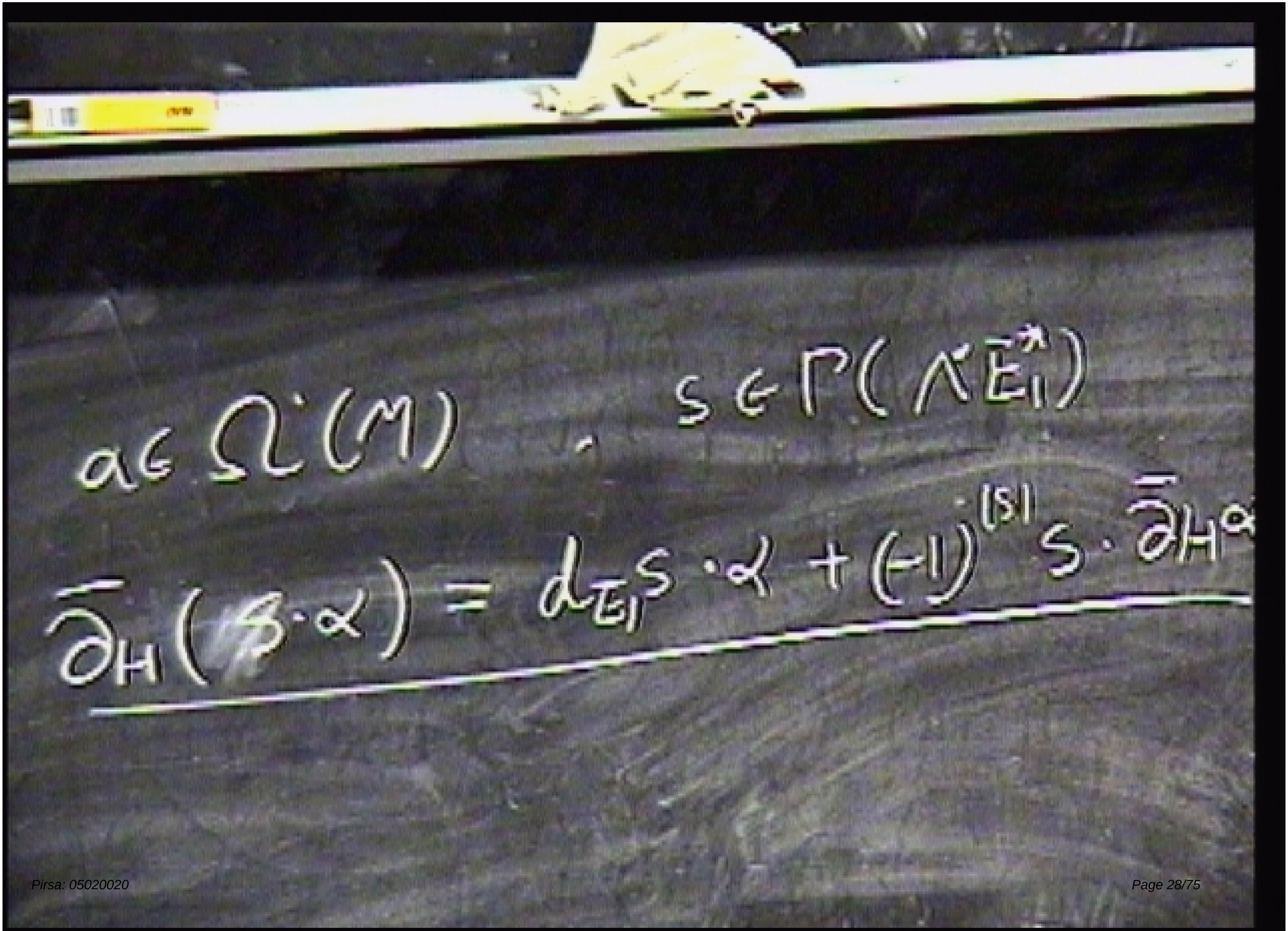
Claim: $Q_B = d_{E_1}$

$$\begin{tikzcd}
\text{TFT}: & \text{Ops} \arrow[r, leftrightarrow, "\text{iso}"] & \text{States} \\
N=2\ \text{SCFT} \arrow[u, "\text{twist}"'] & \text{chiral ring} \arrow[u] \arrow[r, shift left, "\text{spec. flow}"] & (\text{R-R ground states}) \arrow[u] \arrow[l, shift left] \\
(\Lambda^\cdot E_1^*, d_{E_1}) \arrow[rr, bend right, "\text{DG-module}"'] & & (\Omega^\cdot(M), \bar{\partial}_H) \arrow[u, "\text{will show}"']
\end{tikzcd}$$



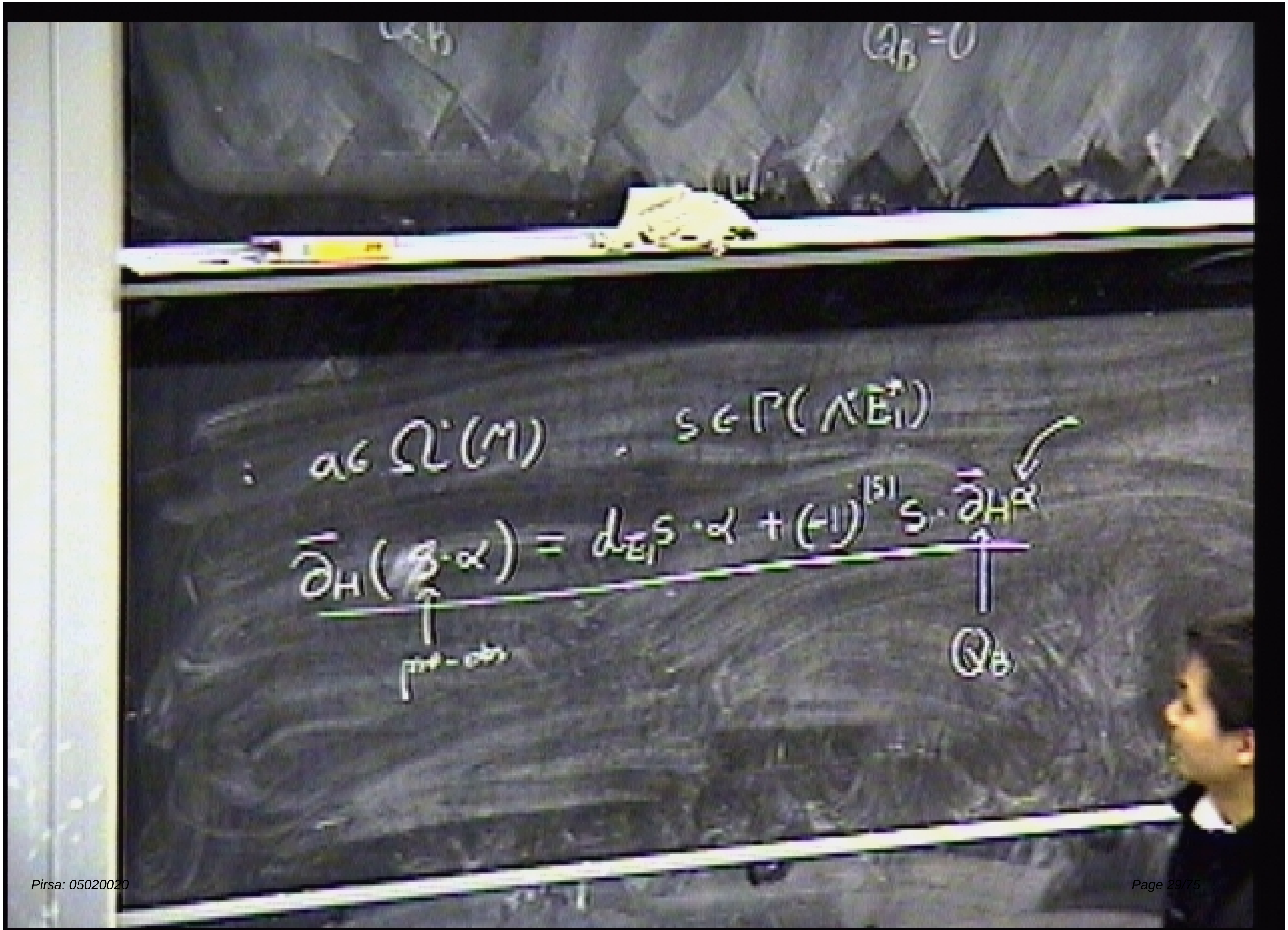
$\alpha \in \Omega^{\cdot}(M)$ $\quad$ $s \in \Gamma(\Lambda^{\cdot} E_1^{*})$

$$\bar{\partial}_H(s \cdot \alpha) = d_{E_1} s \cdot \alpha + (-1)^{|s|} s \cdot \bar{\partial}_H$$



$a \in \Omega^{\cdot}(M)$. $s \in \Gamma(\wedge^{\cdot} E_1^{*})$

$$\underline{\bar{\partial}_H(s \cdot \alpha) = d_{E_1} s \cdot \alpha + (-1)^{|s|} s \cdot \bar{\partial}_H \alpha}$$



Q_B $\quad$ $Q_B = 0$

: $\alpha \in \Omega^{\cdot}(M)$. $s \in \Gamma(\wedge^{\cdot} E_1^{*})$

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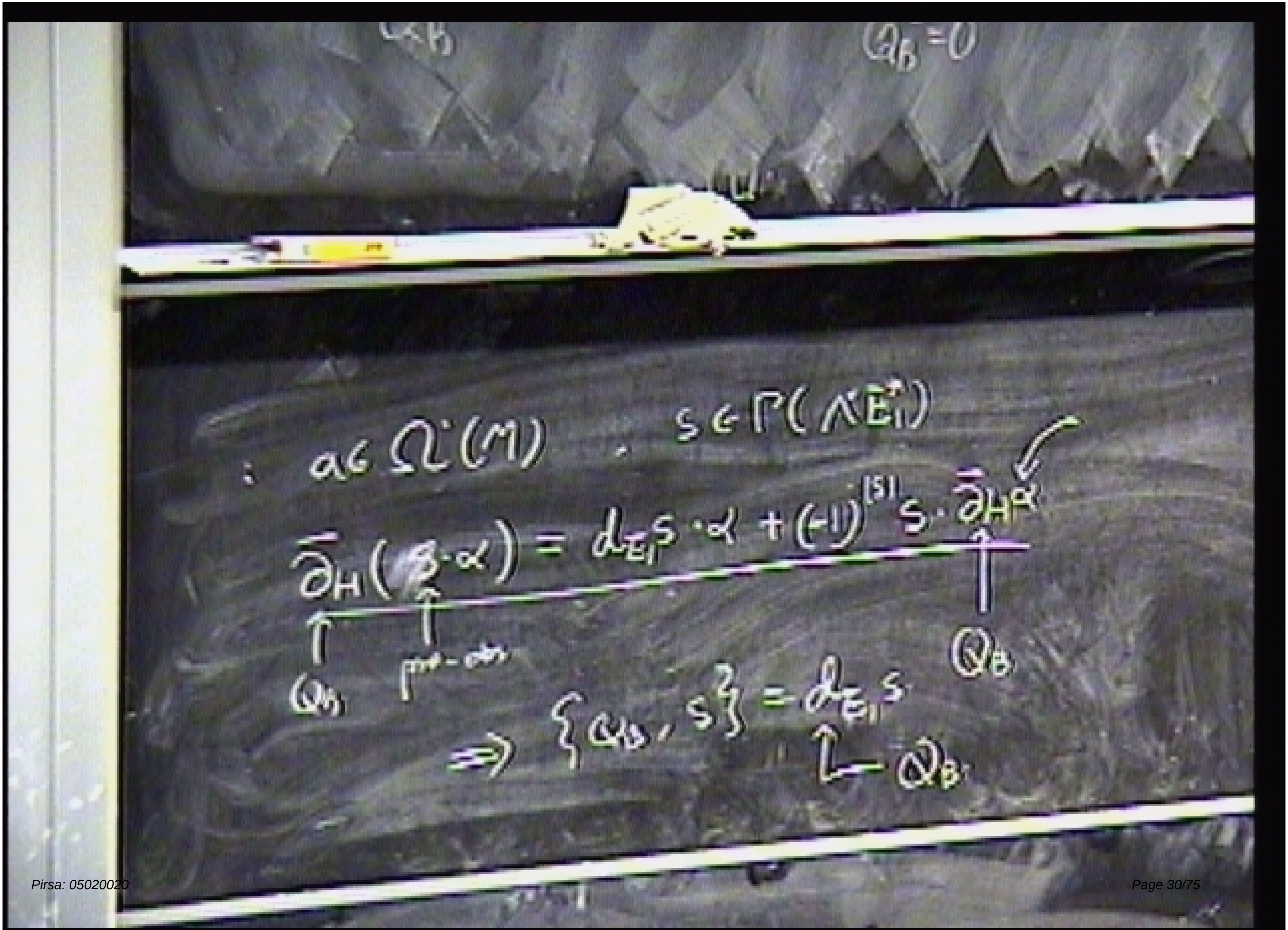
[illegible]

Q_B



Q_B $\qquad$ $Q_B = 0$

$: \quad \alpha \in \Omega^{\cdot}(M) \quad . \quad s \in \Gamma(\Lambda^{\cdot} E_1^{*})$

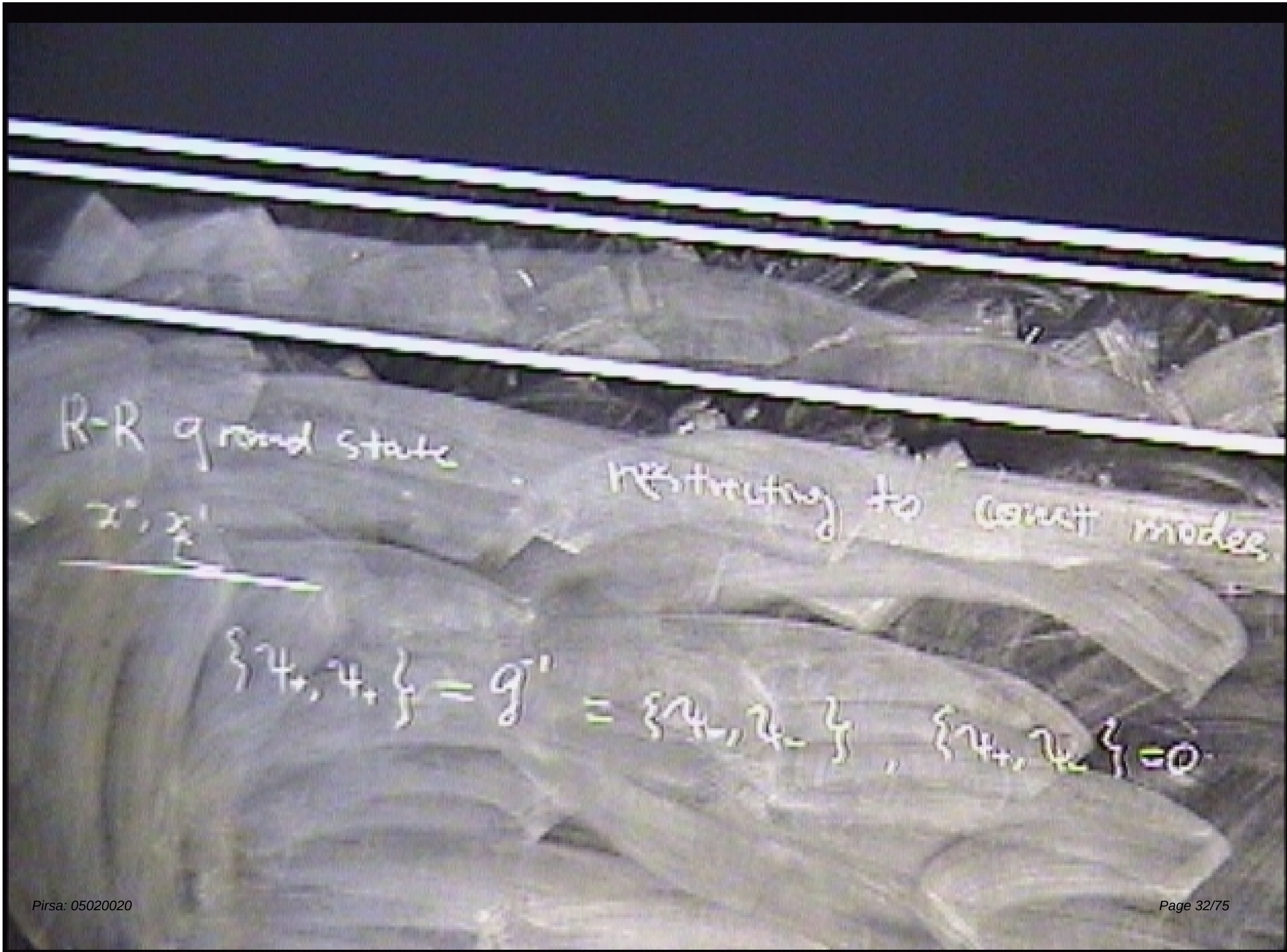
$$\underset{Q_H}{\underline{\bar{\partial}_H}}\big(\underset{\text{mult-ops}}{s}\cdot\alpha\big) = d_{E_1} s \cdot \alpha + (-1)^{|s|}\, s \cdot \underset{Q_B}{\bar{\partial}_H \alpha}$$

$$\Rightarrow \{Q_B, s\} = \underset{Q_B}{d_{E_1}} s$$

R-R ground state

x^0, ξ^1

restricting to const modes.



R-R ground state

x^0, x_0^1

Restricting to const modes.

$\{\psi_+, \psi_+\} = g' = \{\psi_-, \psi_-\}$, $\{\psi_+, \psi_-\} = 0$

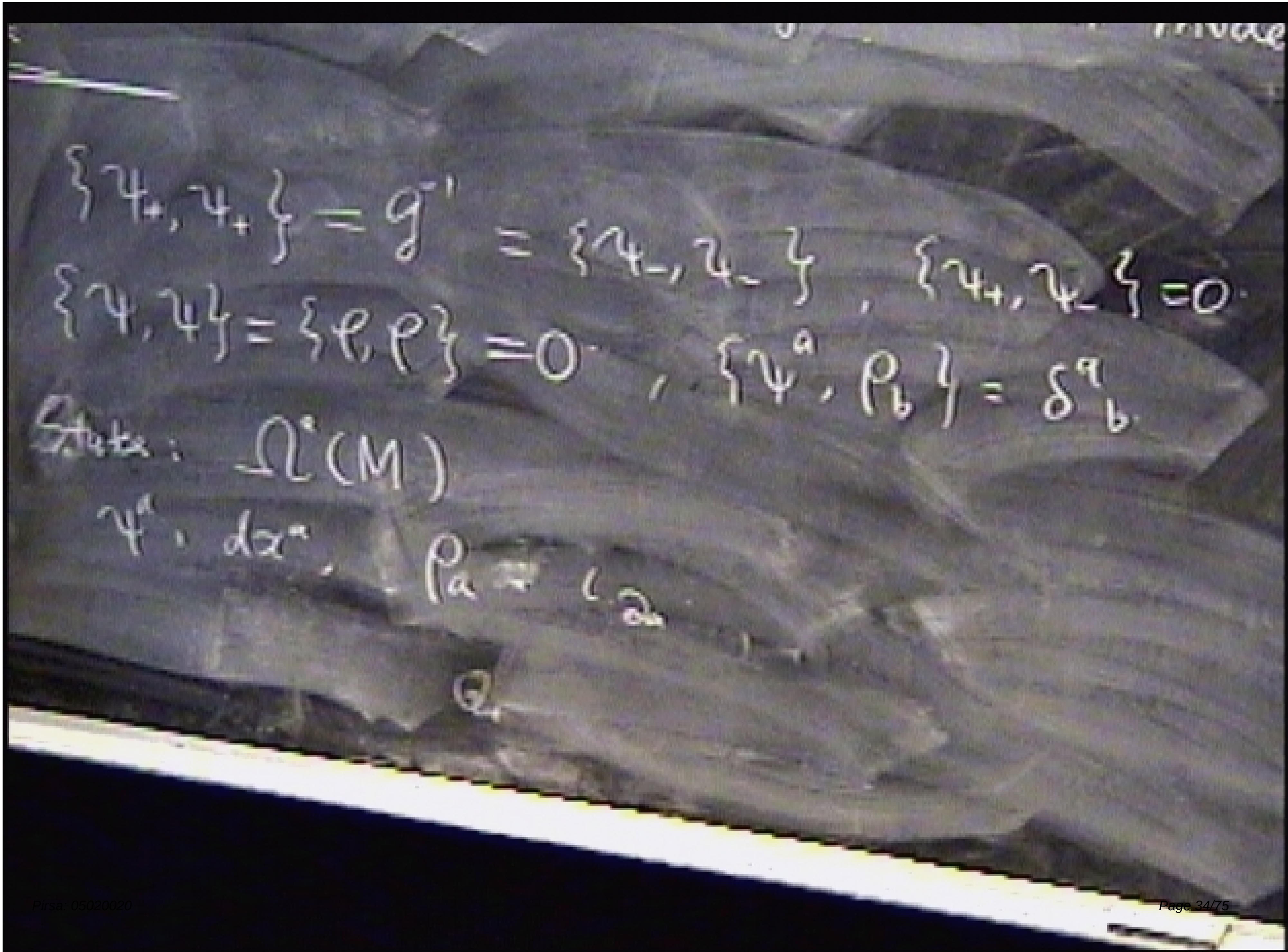
R-R ground state : Restricting to const modes.

x^i, ξ^i

$$\{\psi_+, \psi_+\} = g^{-1} = \{\psi_-, \psi_-\} \quad , \quad \{\psi_+, \psi_-\} = 0$$

$$\{\psi, \psi\} = \{\rho, \rho\} = 0 \quad , \quad \{\psi^a, \rho_b\} = \delta^a_b$$

State :



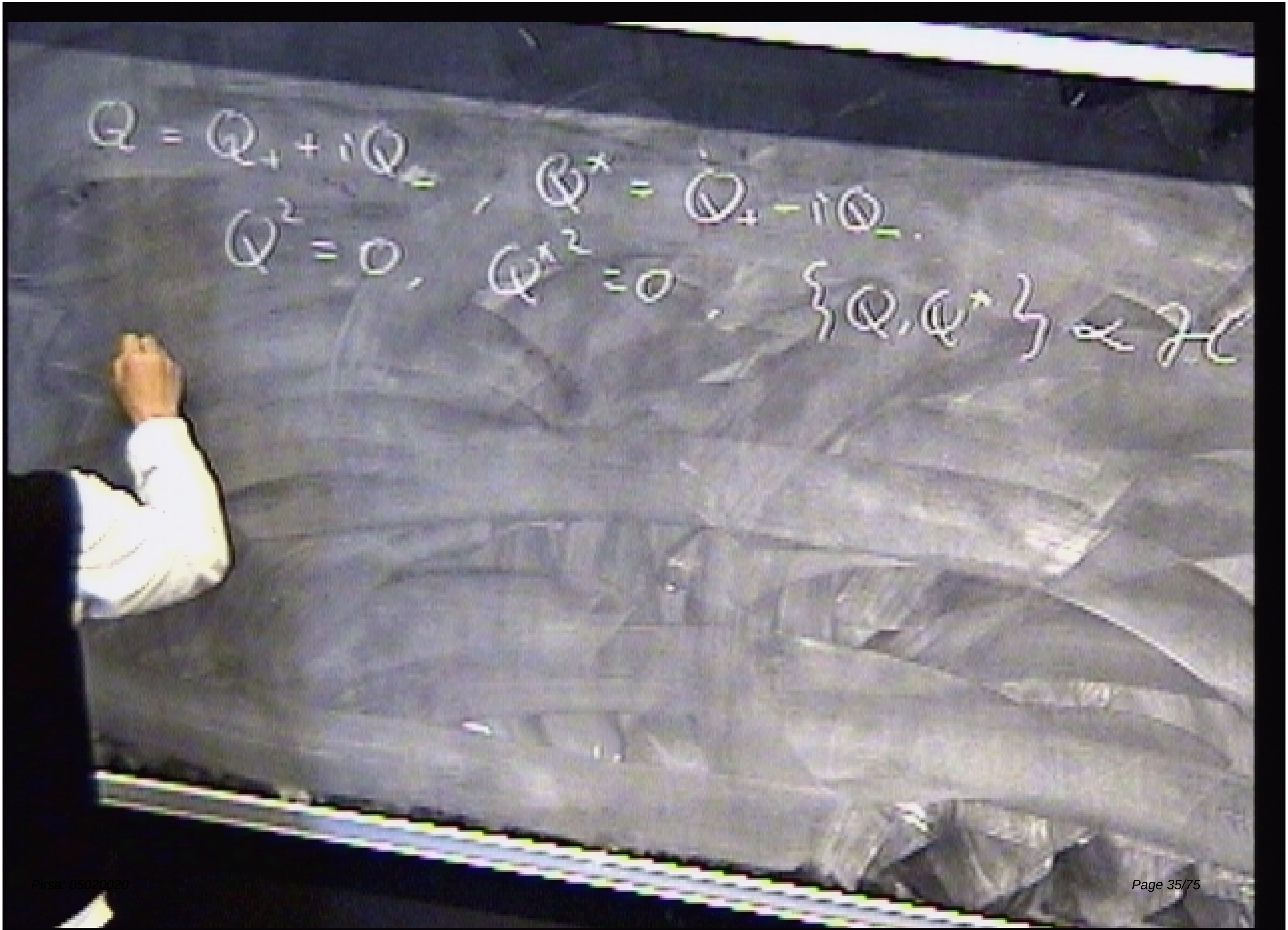
$$\{\psi_+, \psi_+\} = g^{-1} = \{\psi_-, \psi_-\}\ , \quad \{\psi_+, \psi_-\} = 0$$

$$\{\psi, \psi\} = \{\rho, \rho\} = 0\ , \quad \{\psi^a, \rho_b\} = \delta^a_b$$

States: $\Omega^{\bullet}(M)$

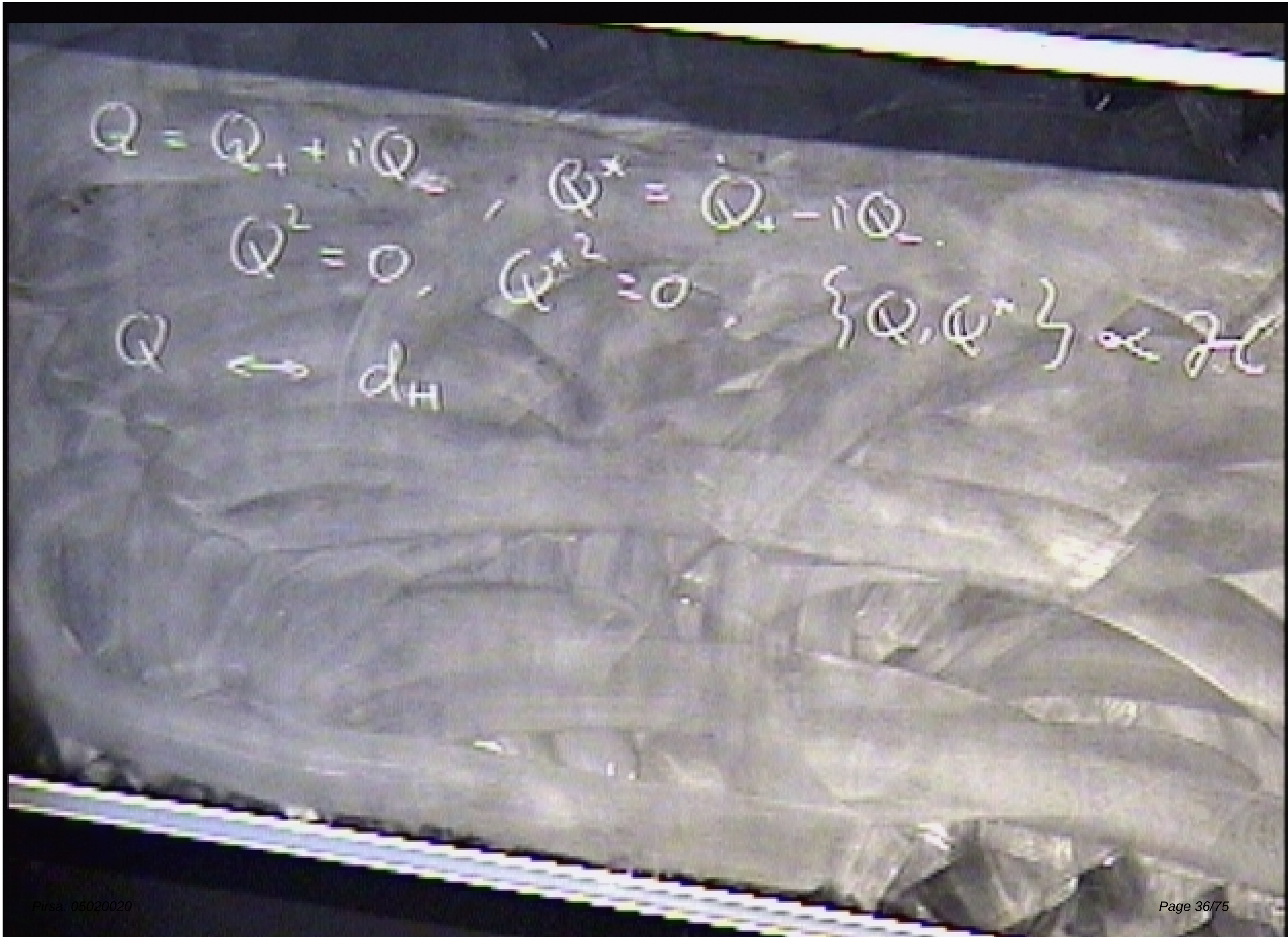
$\psi^a : dx^a\ , \quad \rho_a = \iota_{\partial_a}$

Q



$$Q = Q_+ + iQ_- \,, \quad Q^* = Q_+ - iQ_- .$$

$$Q^2 = 0 \,, \quad Q^{*2} = 0 \,. \quad \{Q, Q^*\} \propto \mathcal{H}$$



$$Q = Q_+ + iQ_- \,, \quad Q^* = Q_+ - iQ_- .$$

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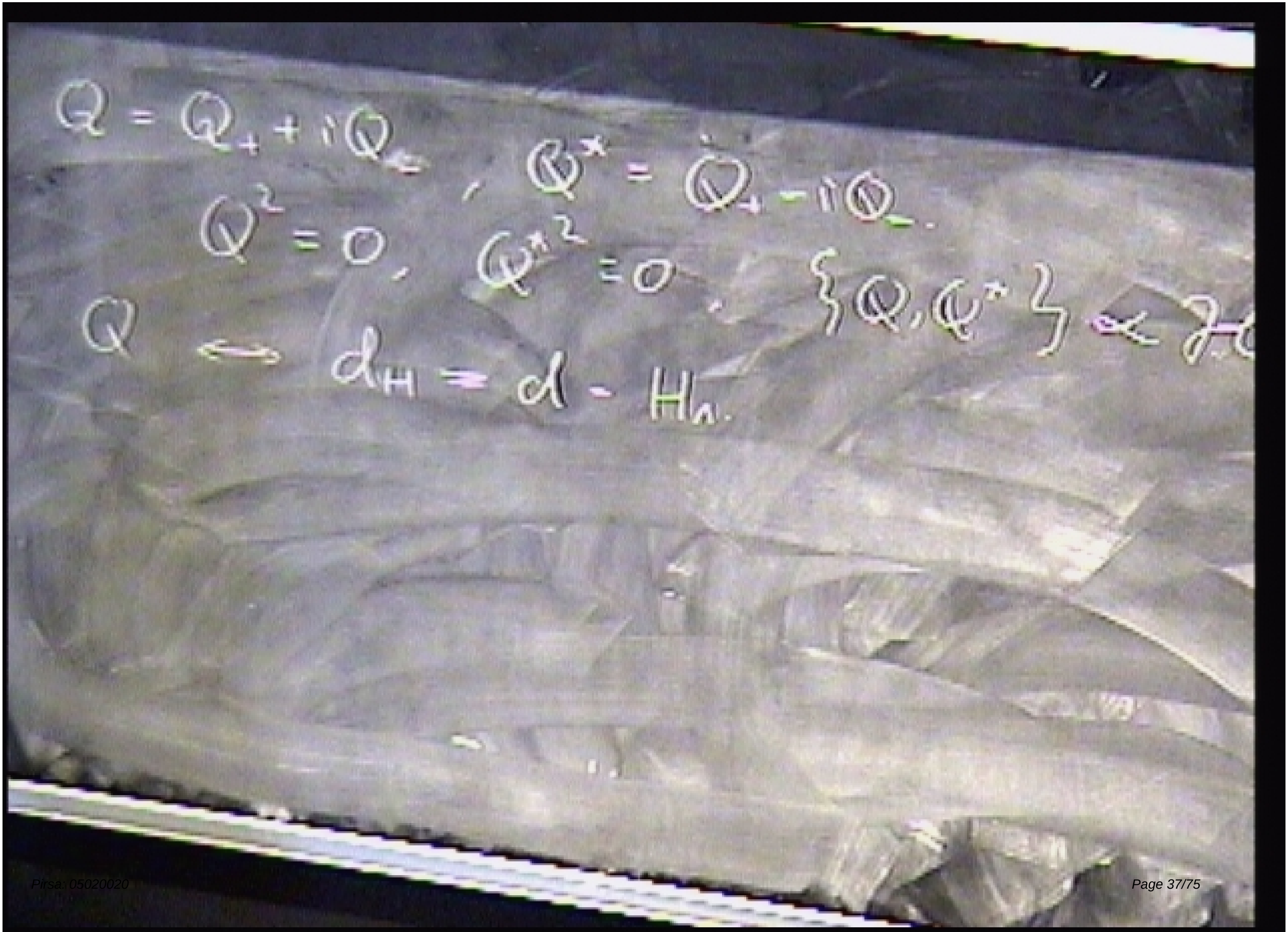
$$Q \longleftrightarrow d_H$$



$$Q = Q_+ + iQ_- \,, \quad Q^* = Q_+ - iQ_- .$$

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$$Q \Longleftrightarrow d_H = d - H\wedge .$$

$Q \quad = 0$, $\{Q, Q^*$

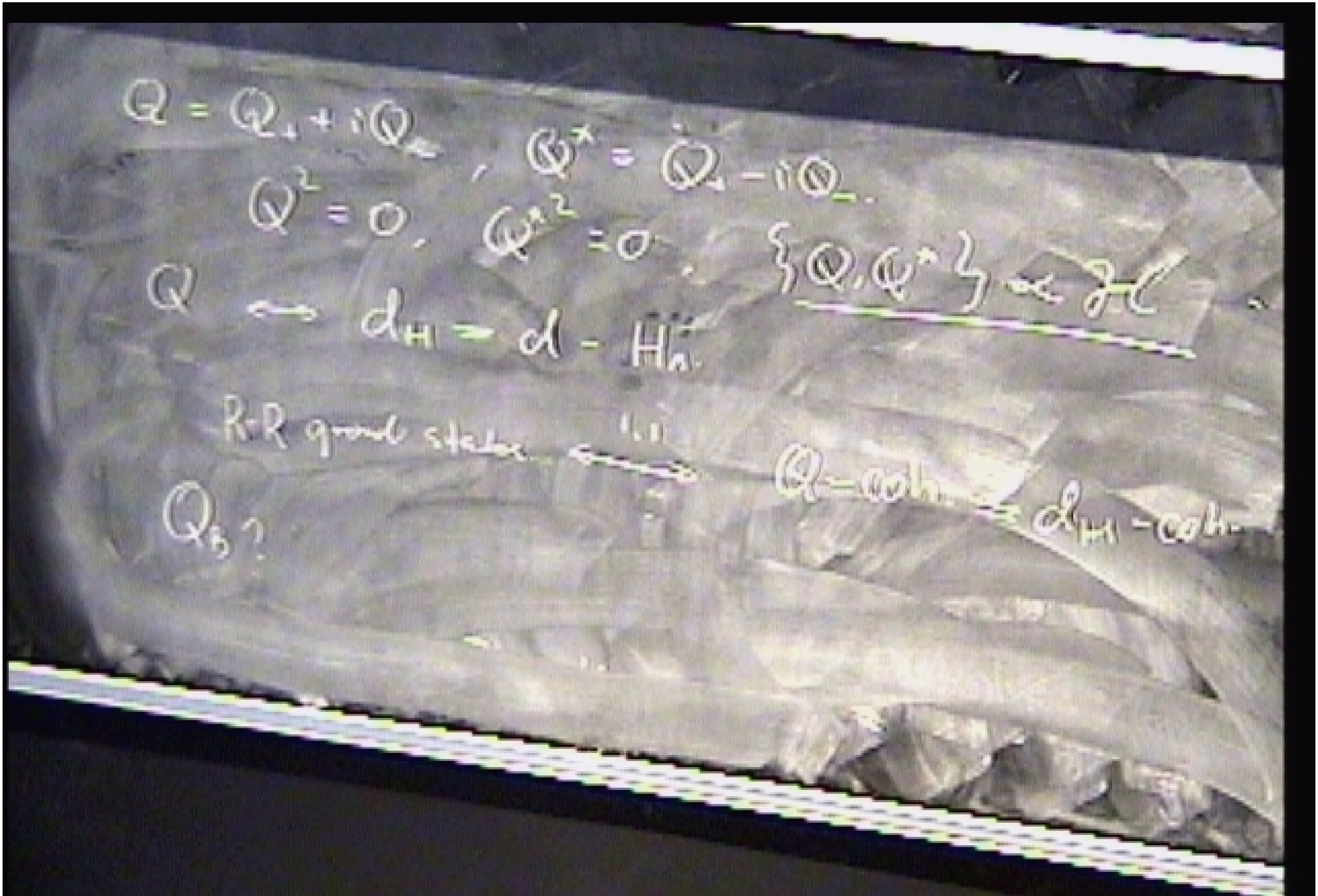
$\Longleftrightarrow \quad d_H = d - H\wedge .$

R-R ground states $\xleftrightarrow{1:1}$ Q-coh.

$$\{Q, Q^*\} \propto \mathcal{H}$$

1.

$$\Rightarrow \quad Q\text{-coh.} = d_H\text{-coh.}$$



$Q = Q_+ + iQ_-$, $Q^* = Q_+ - iQ_-$.

$Q^2 = 0$, $Q^{*2} = 0$. $\underline{\{Q, Q^*\} \propto \mathcal{H}}$.

$Q \longleftrightarrow d_H = d - H\wedge$

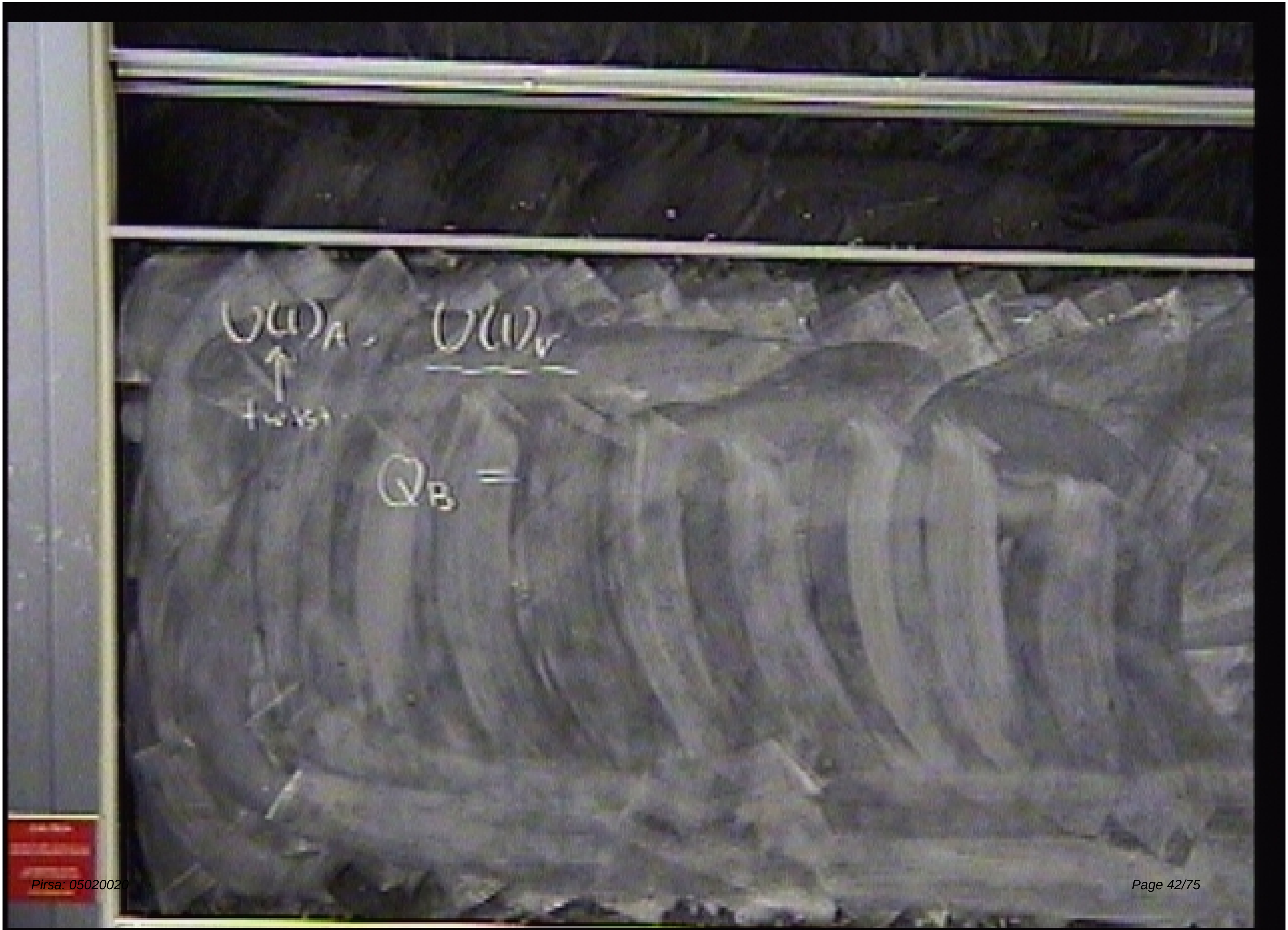
R-R ground states $\overset{1:1}{\longleftrightarrow}$ Q-coh. $= d_H$-coh.

Q_B ?

$U(1)_A \,, \quad \underline{U(1)_V}$

$\uparrow$

twist.

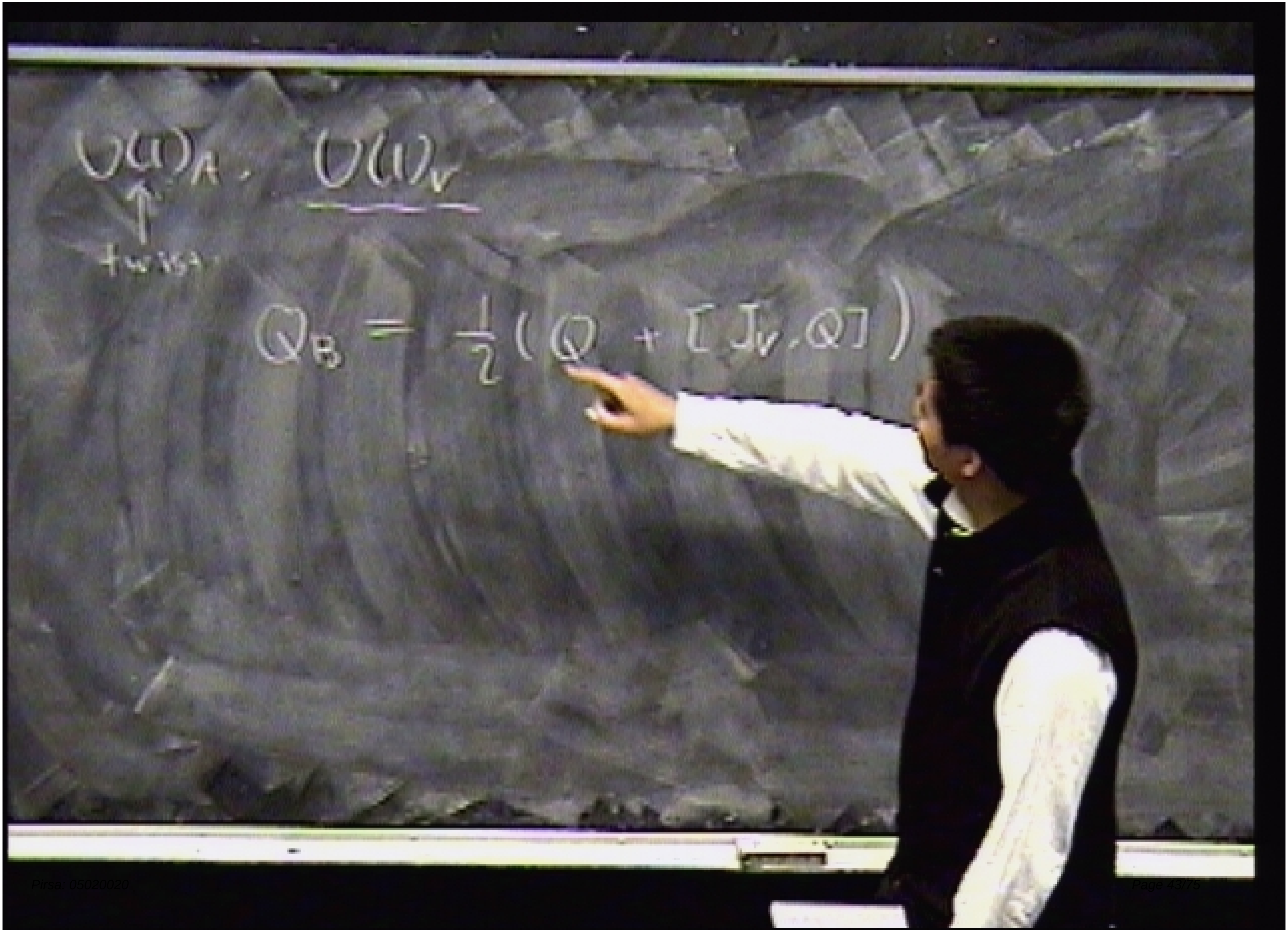


$U(1)_A$, $\underline{U(1)_V}$

$\uparrow$

twist

$Q_B =$



$U(1)_A$, $\underline{U(1)_V}$

$\uparrow$

twist.

$$Q_B = \frac{1}{2}\left(Q + [J_V, Q] \right)$$

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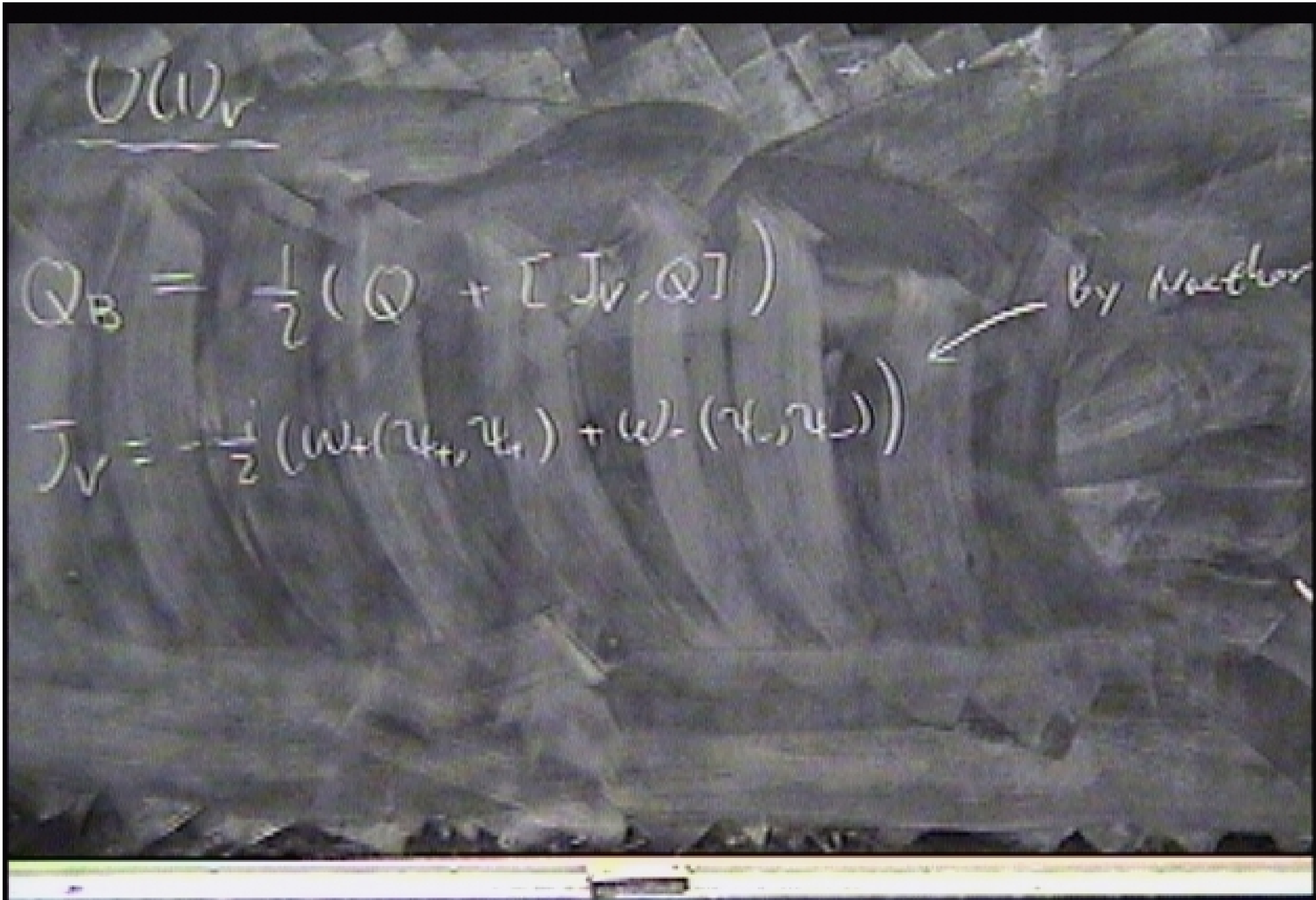
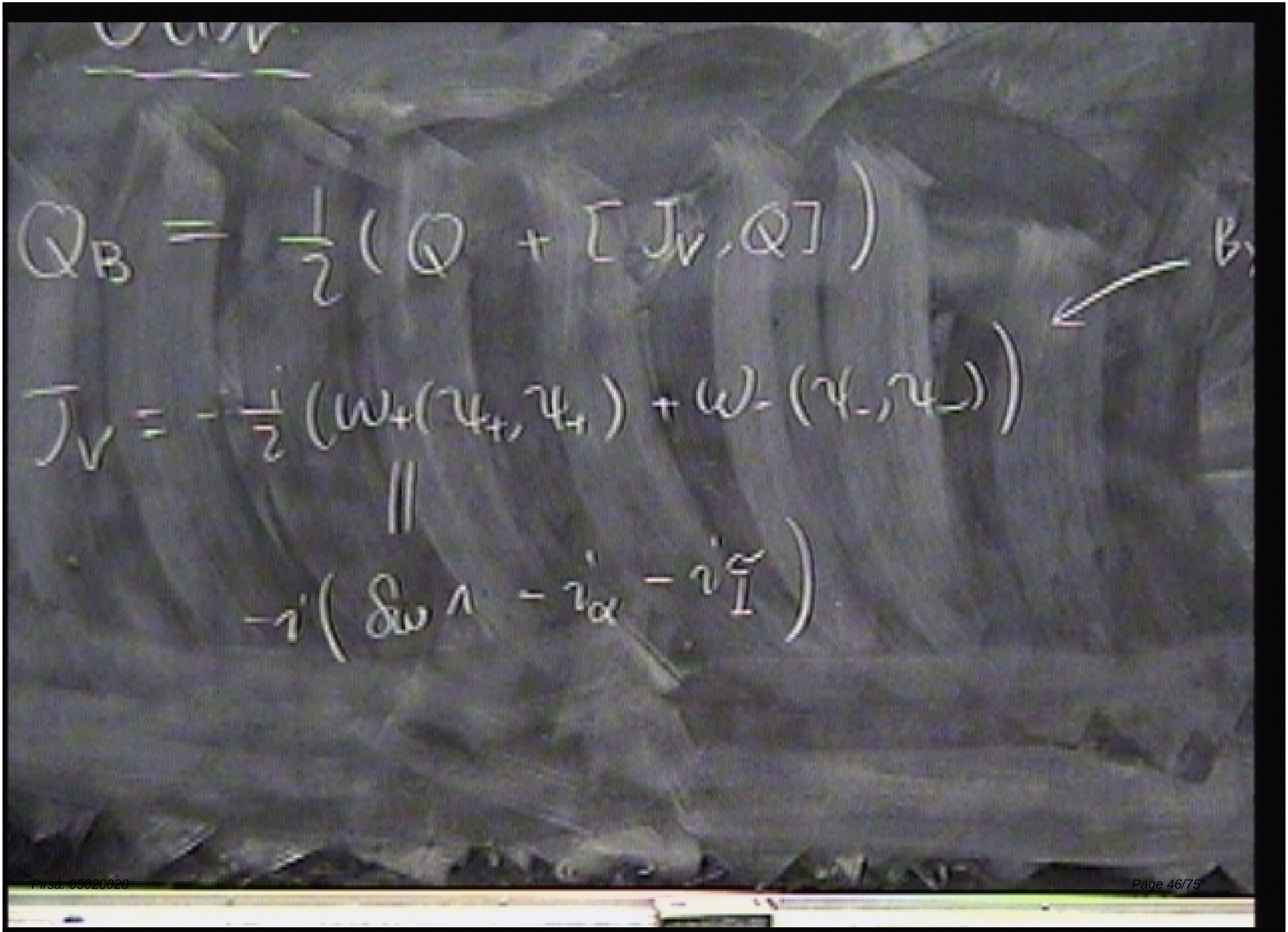


$\underline{U(1)_V}$

$$Q_B = \frac{1}{2}\left(Q + [J_V, Q]\right)$$

by Noether

$$J_V = -\frac{i}{2}\left(\omega_+(\psi_+,\psi_+) + \omega_-(\psi_-,\psi_-)\right)$$



$O(n)_V$

$$Q_B = \frac{1}{2}\left(Q + [J_V, Q] \right)$$

B,

$$J_V = -\frac{i}{2}\left(\omega_+(\psi_+,\psi_+) + \omega_-(\psi_-,\psi_-) \right)$$

$$\|$$

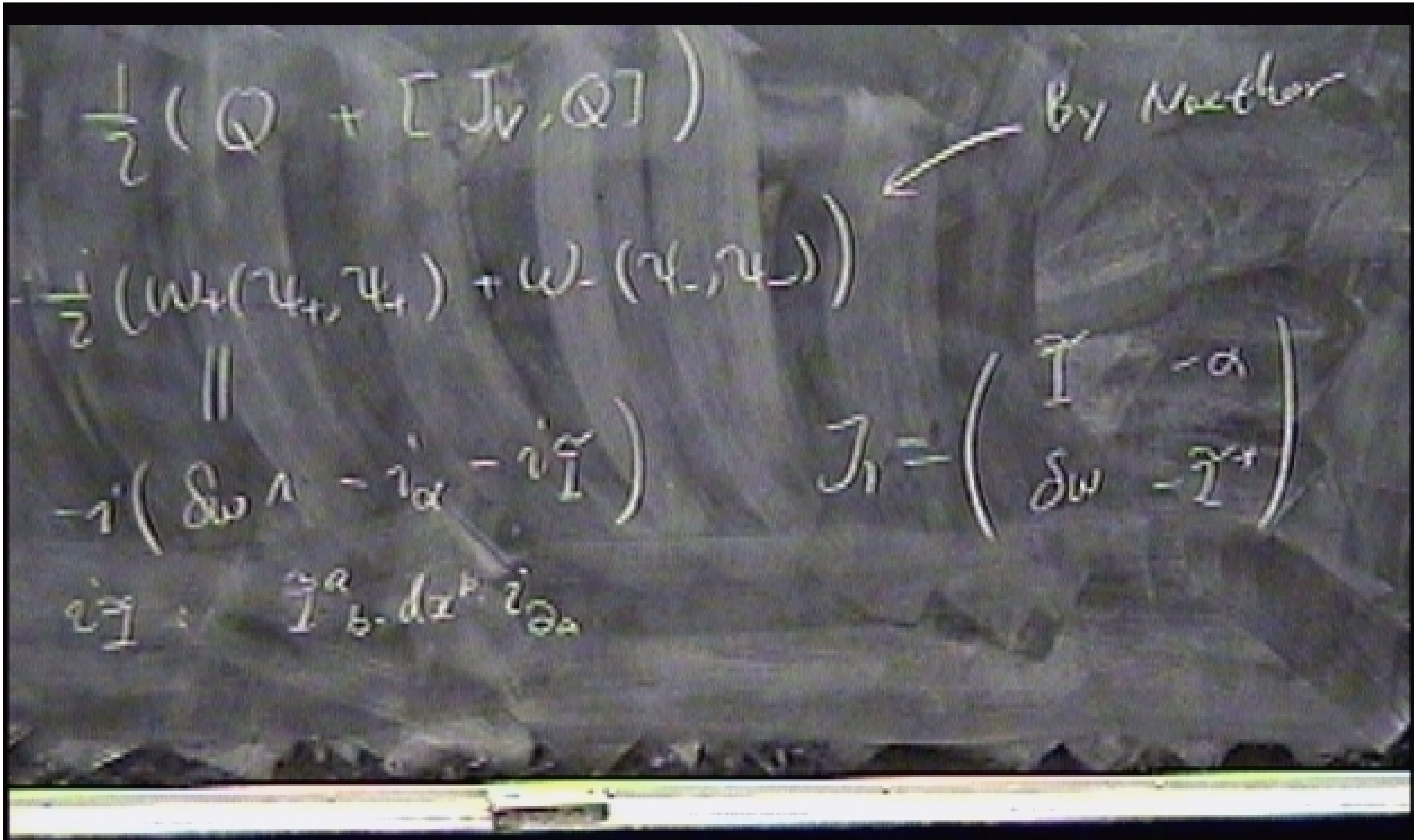
$$-i\left(\delta\omega \wedge - i_\alpha - i\tilde{I} \right)$$

$\mathcal{J}_V, Q])$ by

$+\omega_-(\psi_-,\psi_-))$

$-i\tilde{I})$

$$\mathcal{J}_1 = \begin{pmatrix} \tilde{I} & -\alpha \\ \delta\omega & -\tilde{I}^* \end{pmatrix}$$



$$\frac{1}{2}\left(Q + [\mathcal{J}_v, Q]\right)$$

By Noether

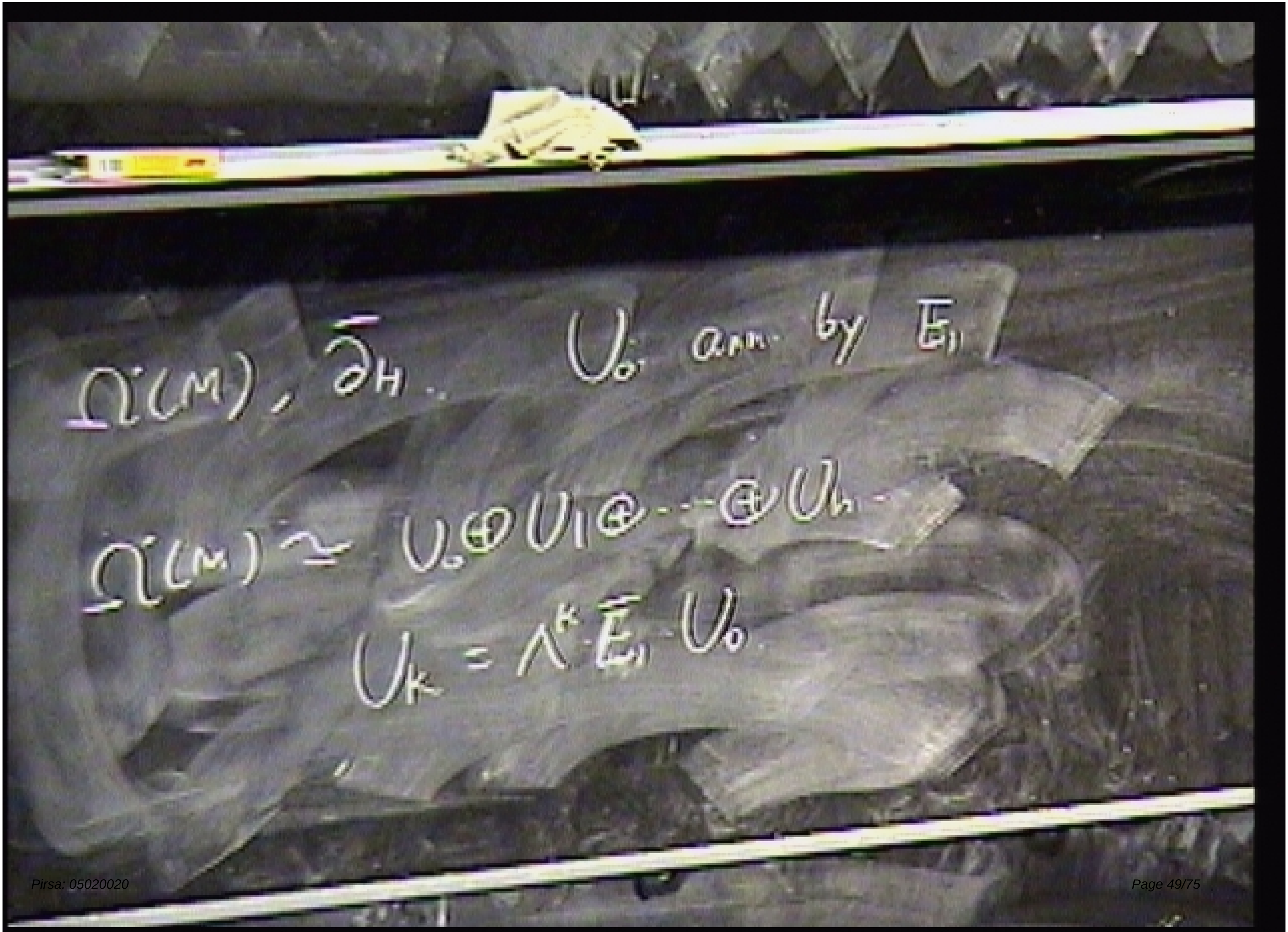
$$\frac{i}{2}\left(\omega_+(\psi_+,\psi_+) + \omega_-(\psi_-,\psi_-)\right)$$

$$\|$$

$$-i\left(\delta\omega\wedge - i_\alpha - i\tilde{I}\right)$$

$$\mathcal{J}_1 = \begin{pmatrix} \tilde{I} & -\alpha \\ \delta\omega & -\tilde{I}^{*} \end{pmatrix}$$

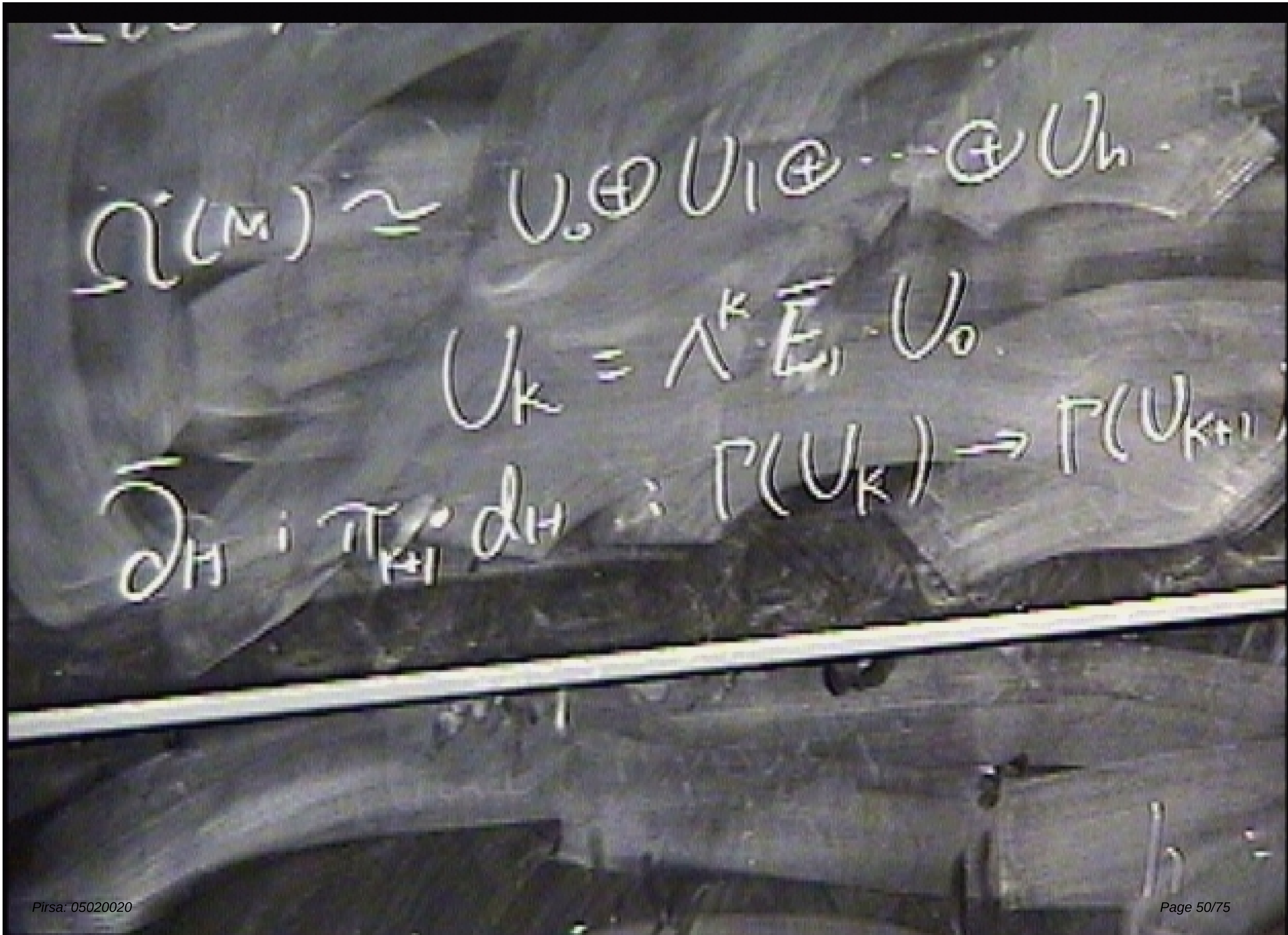
$$i_{\tilde{I}} : \quad \tilde{I}^a{}_b \, dx^b \, i_{\partial_a}$$



$\Omega^{\cdot}(M)$, $\bar{\partial}_H$ $\quad U_0$ ann. by $\bar{E}_1$

$\Omega^{\cdot}(M) \simeq U_0 \oplus U_1 \oplus \cdots \oplus U_n$

$U_k = \wedge^k \bar{E}_1 \cdot U_0$



$$\Omega^i(M) \simeq V_0 \oplus V_1 \oplus \cdots \oplus V_n$$

$$V_k = \Lambda^k \bar{E}_1 V_0$$

$$\bar{\partial}_H := \pi_{k+1} d_H : \Gamma(V_k) \to \Gamma(V_{k+1})$$

$h :$

$\Omega^{\cdot}(M)$, $\bar{\partial}_H$ .. U_0 ann. by $\bar{E}_1$

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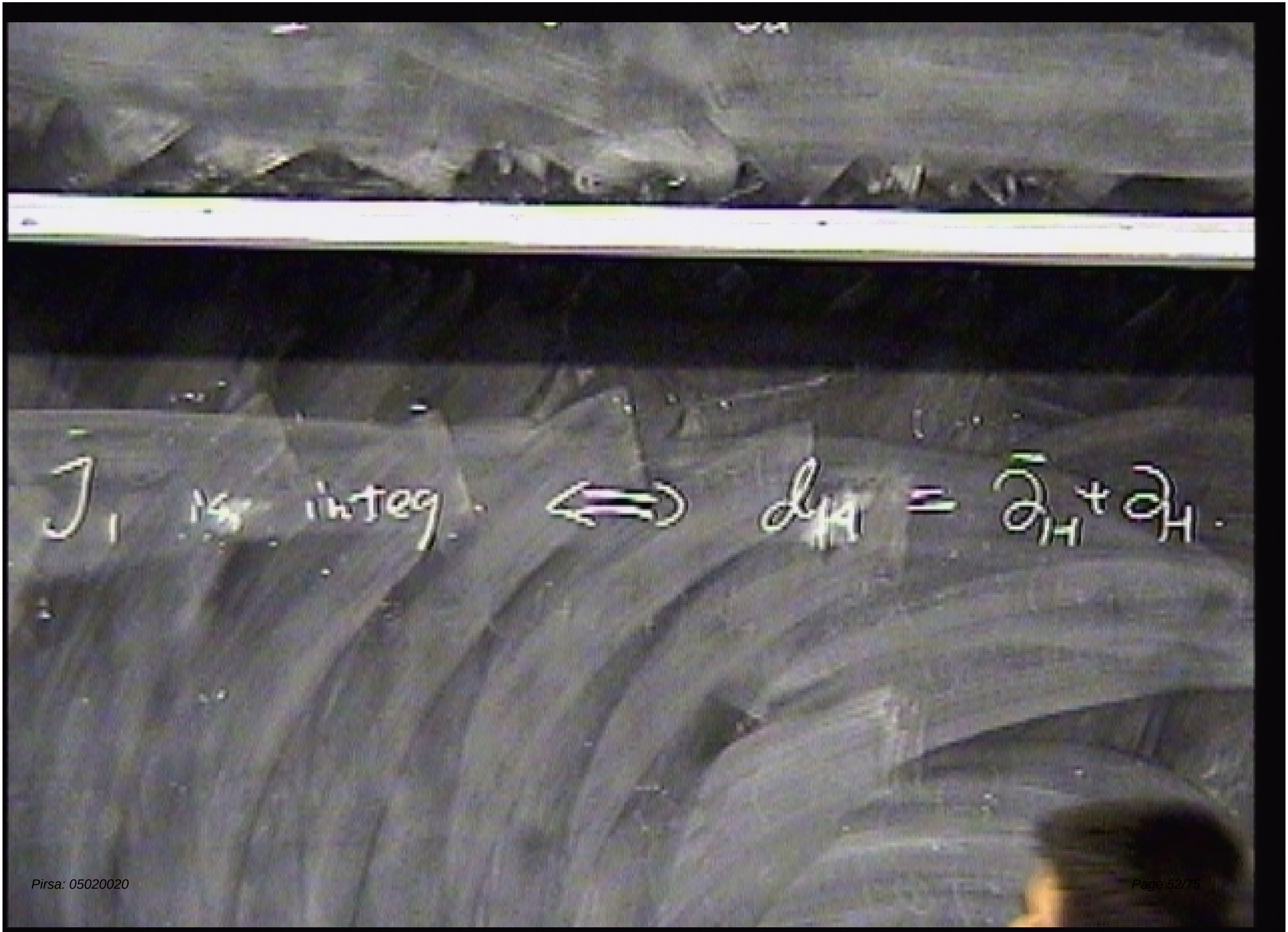
$\bar{\partial}_H : \pi_{k+1} d_H : \Gamma(U_k) \to \Gamma(U_{k+1})$, ∂_H.



J_1 is integ. $\Longleftrightarrow$ $d_H = \bar{\partial}_H + \partial_H$.

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R_{J_1} ... : grading op.

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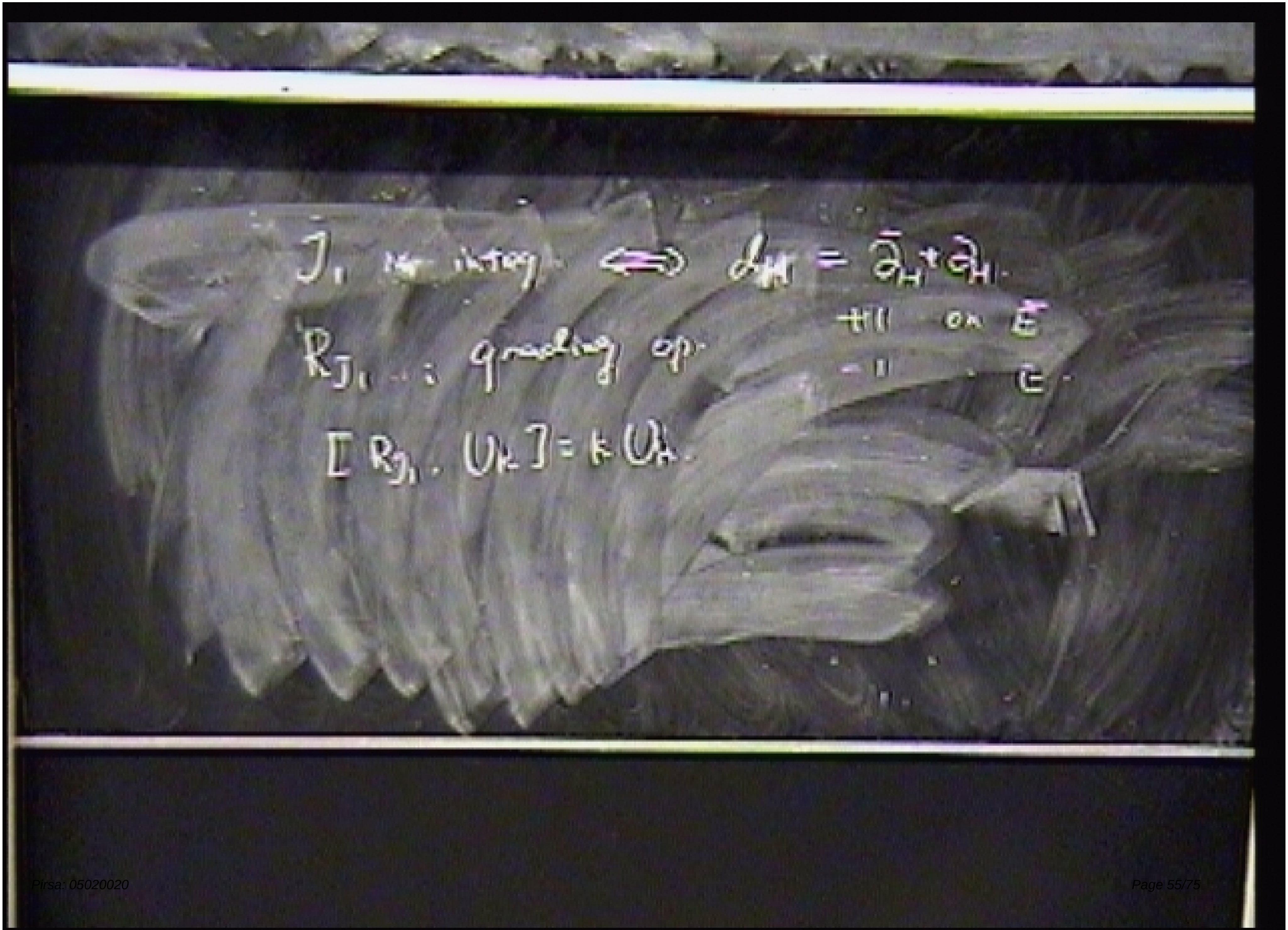
R_{J_1} : grading op.

$[R_{J_1}, U_k] = k\, U_k$.

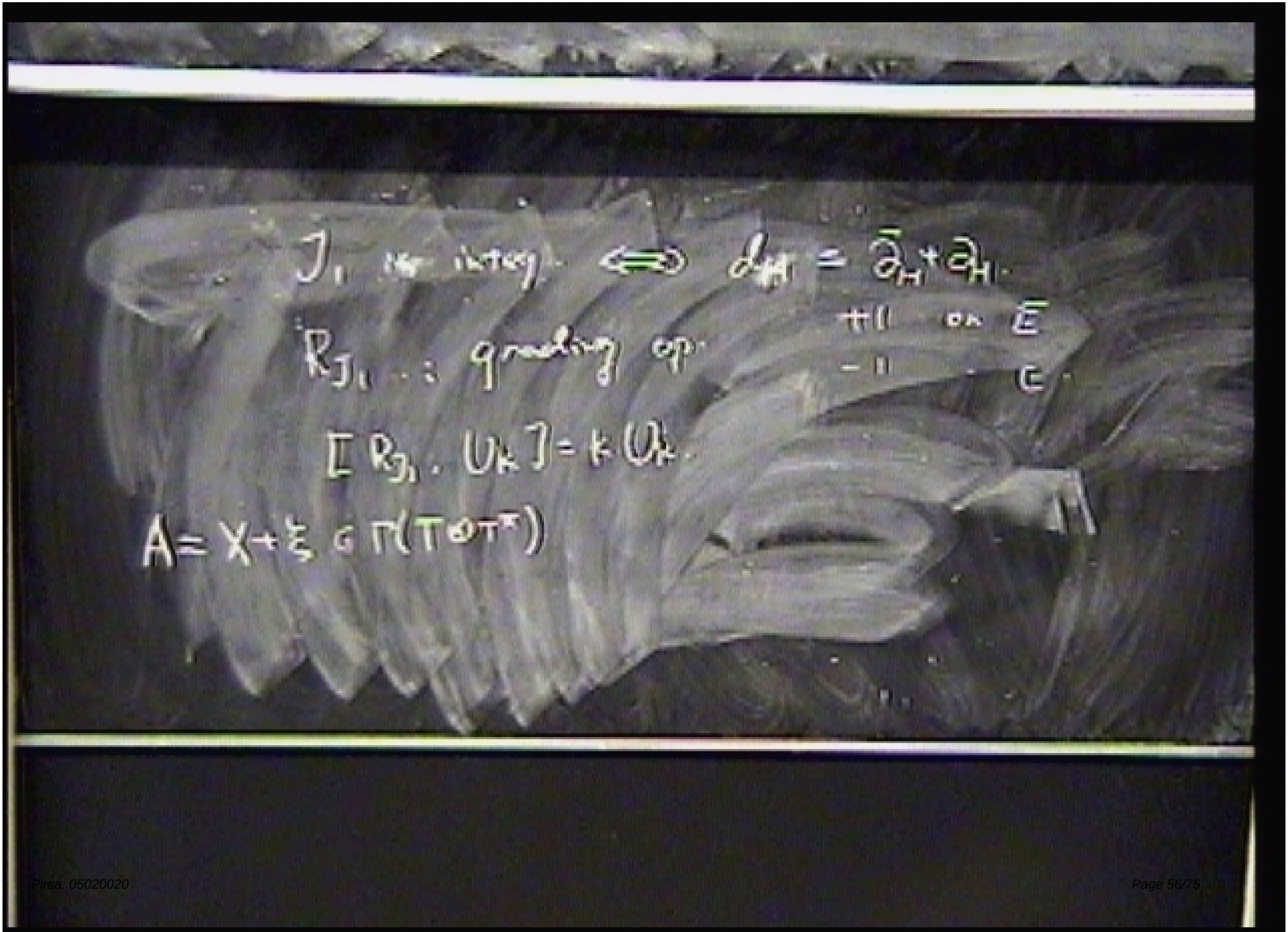


J_1 is integ. $\Longleftrightarrow$ $d_H = \bar{\partial}_H + \partial_H$.

R_{J_1} : grading op. $+1$ on $\bar{E}$

-1 on E.

$[R_{J_1}, \mathcal{U}_k] = k\,\mathcal{U}_k$.



J_1 is integ. $\Longleftrightarrow$ $d_H = \bar{\partial}_H + \partial_H$.

R_{J_1} : grading op. $+i$ on $\bar{E}$

$-i$ " E.

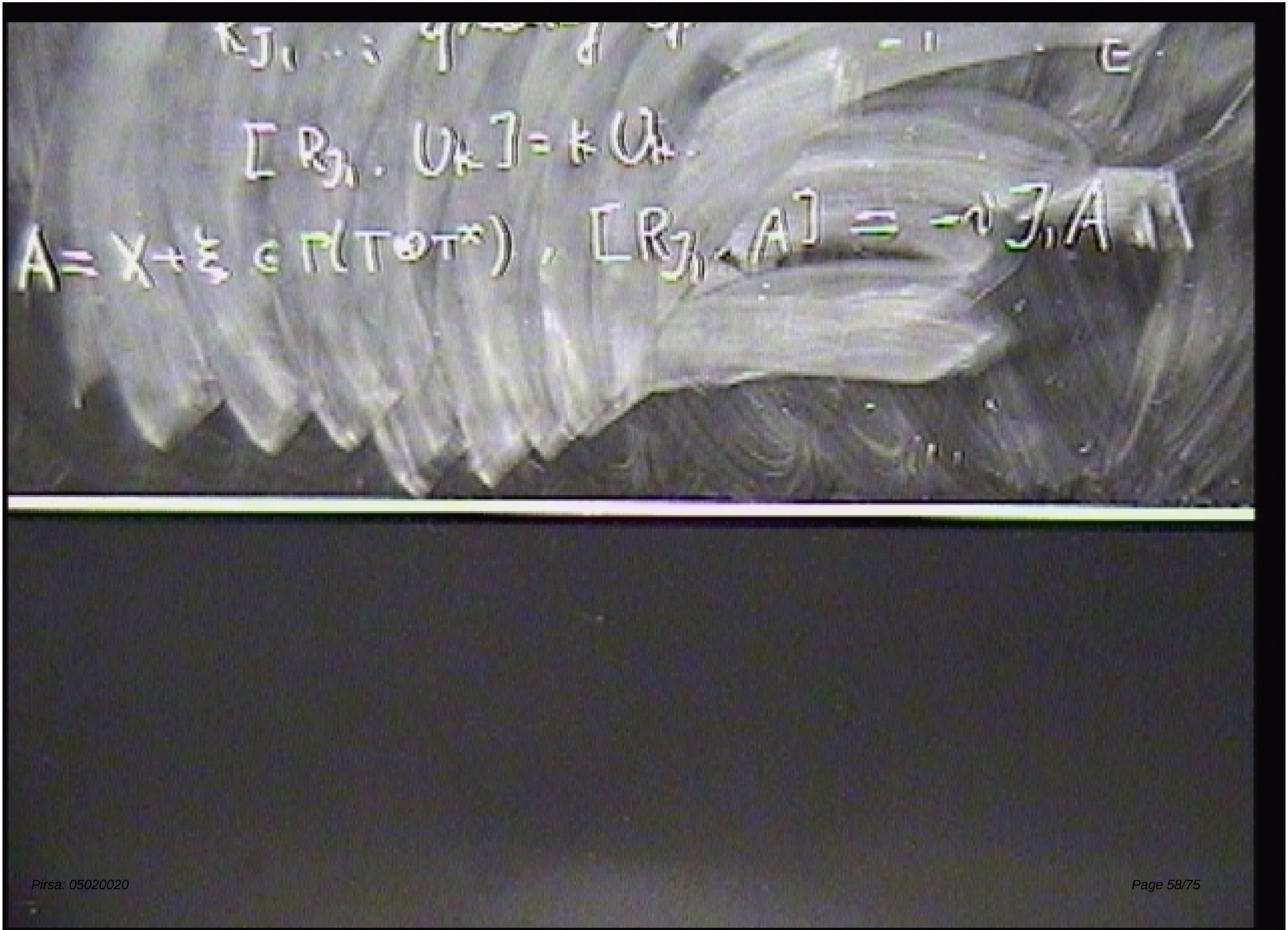
$[R_{J_1}, U_k] = k\, U_k$.

$A = X + \xi \in \Gamma(T \oplus T^*)$

$R_{J_1}, \dots ;$ [illegible]

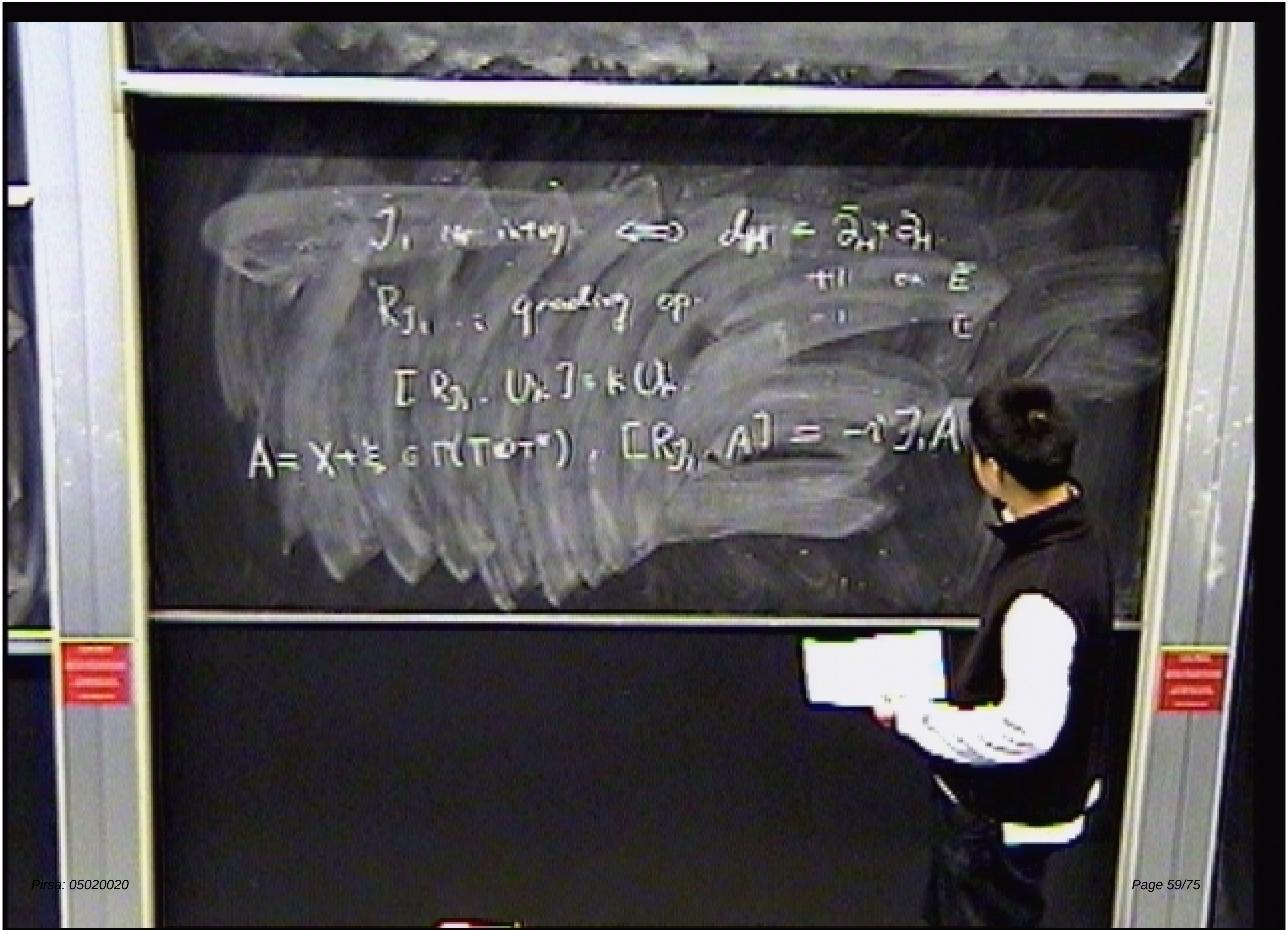
$$[R_{J_1}, U_k] = k\,U_k$$

$$A = X + \xi \in \Gamma(T \oplus T^*) , \quad [R_{J_1}, A]$$



$$[R_J, U_k] = k\,U_k.$$

$$A = X + \xi \in \Gamma(T \oplus T^*), \quad [R_J, A] = -i\,J A$$



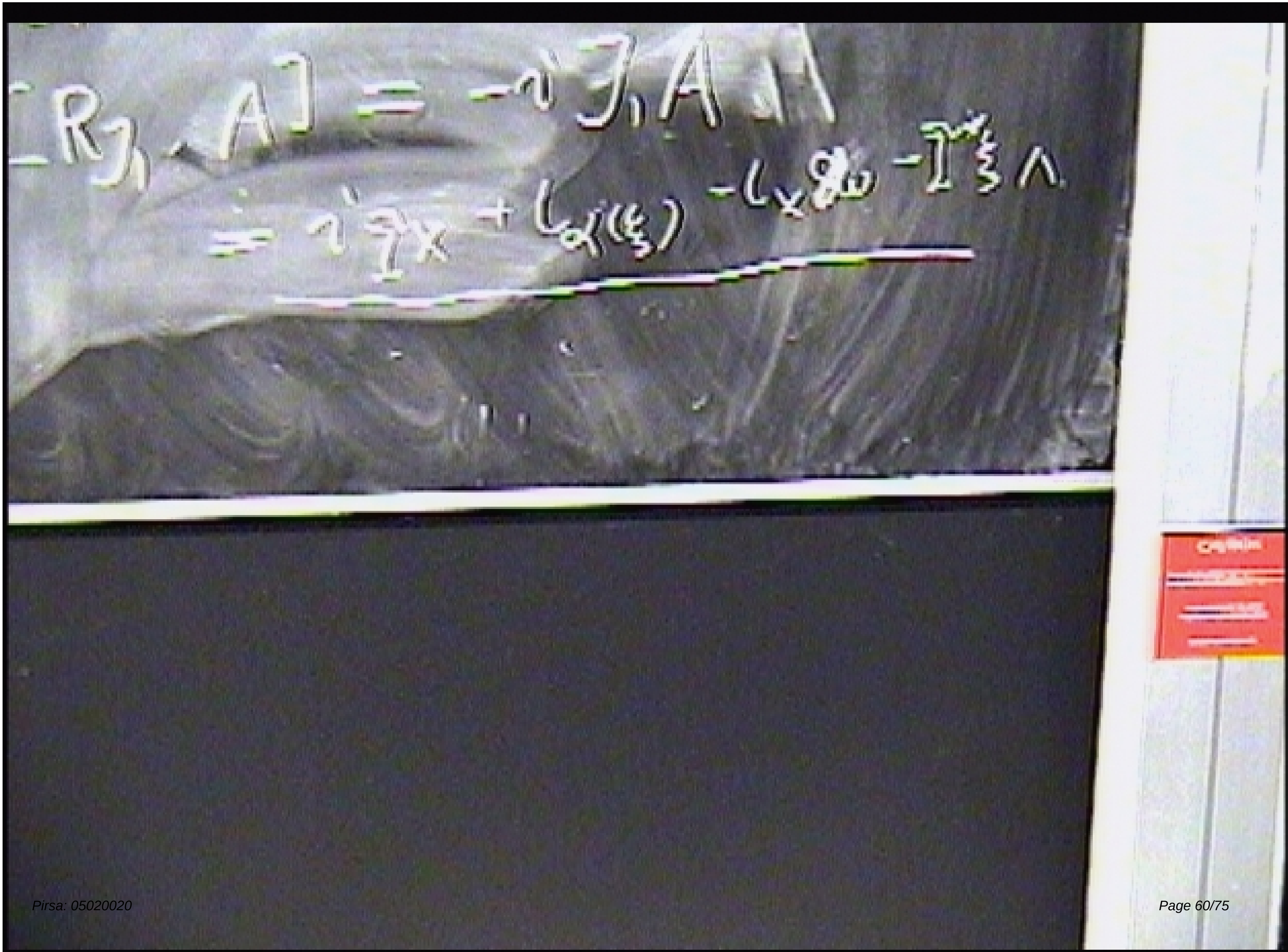
J_1 integrable $\Longleftrightarrow$ $d_H = \bar{\partial}_H + \partial_H$

R_{J_1} : grading op. $+i$ on $\bar{E}$

$-i$ — E

$[R_{J_1}, U_k] = k\,U_k$

$A = X + \xi \in \Gamma(T \oplus T^*)$, $[R_{J_1}, A] = -i\,J_1 A$



$$[R_\xi, A] = -i\, \mathcal{I}_\xi A$$

$$= i\, \mathcal{I}_\xi x + \mathcal{L}_{\alpha(\xi)} - \mathcal{L}_x g_\omega - \mathcal{I}^{\omega}_{\xi} \Lambda$$

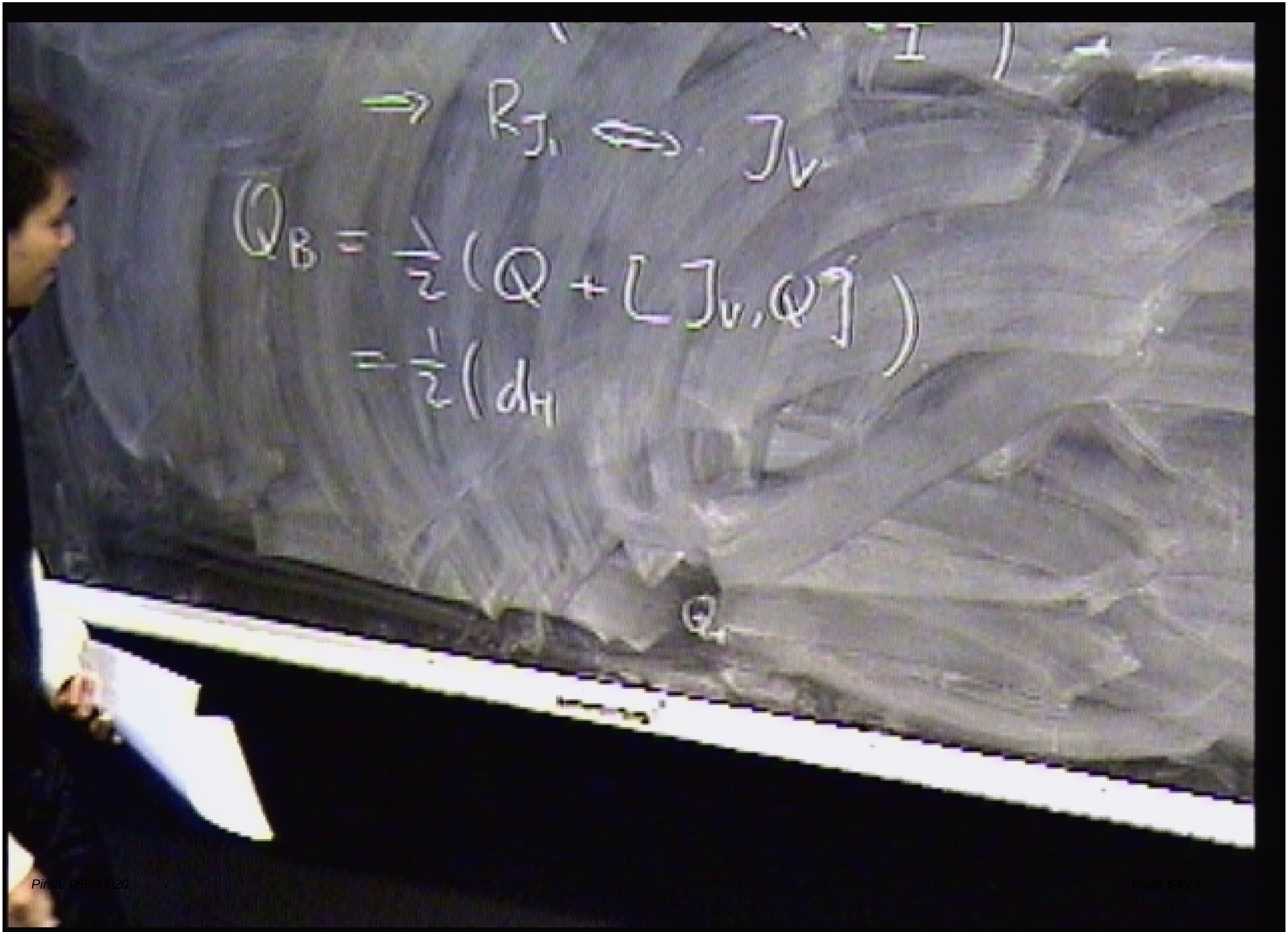
$$A] = -i\, J, A$$

$$= i\, \mathcal{I}_x + L_{\alpha(\xi)} - L_x\, \delta\omega - \mathcal{I}^*_\xi\, \Lambda$$

$$R_{J_1} = -i\Big(\delta\omega_{\Lambda} - (\omega - \omega_{\tilde{I}} \quad \Big)$$

$$R_{J_1} = -i\left(\delta_{Lor} - C_{L} - C_{I}\right)$$

$$\Rightarrow \quad R_{J_1} \Leftrightarrow J_V$$

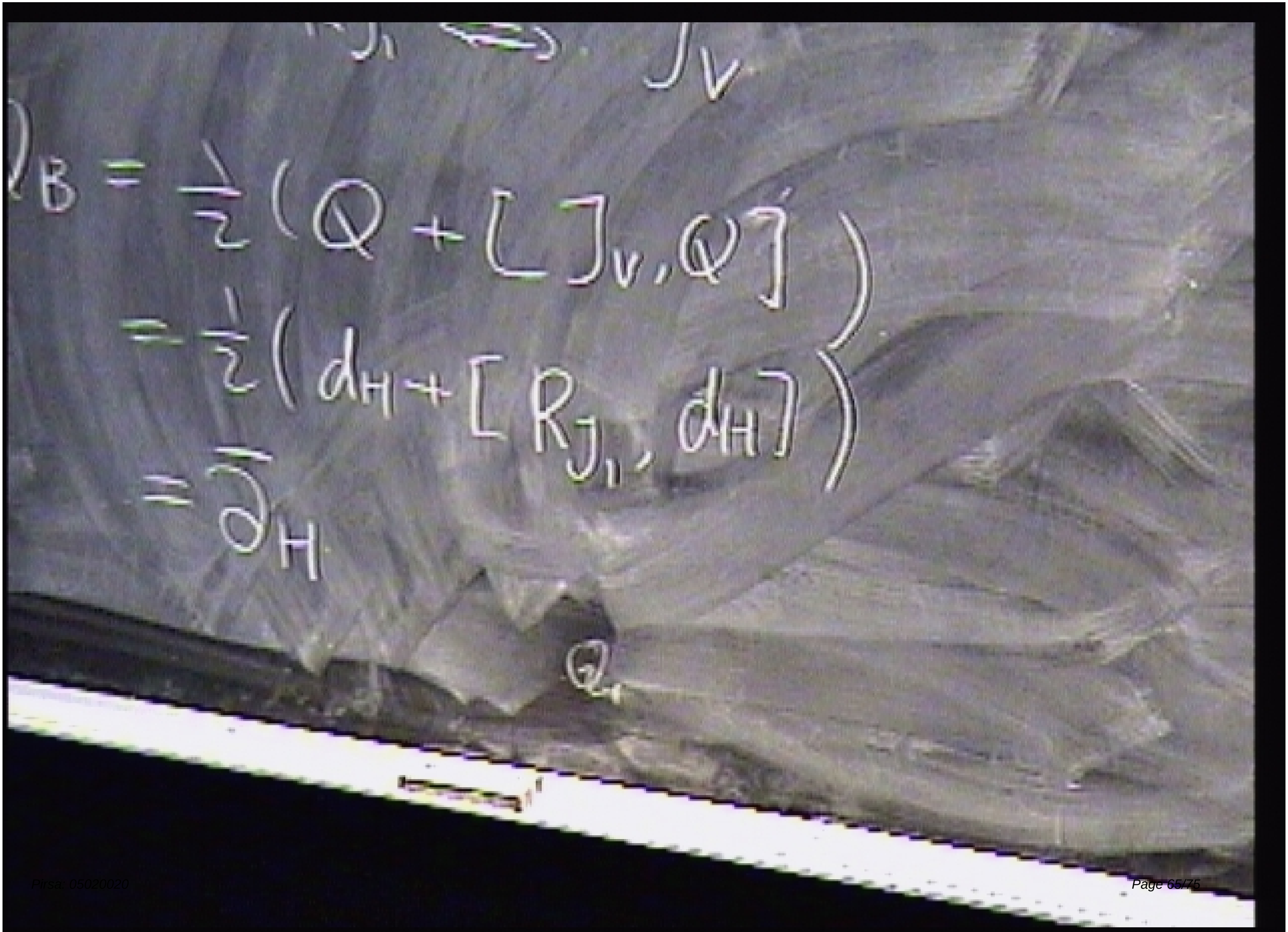


$$\Longrightarrow R_{J_1} \Longleftrightarrow J_V$$

$$Q_B = \frac{1}{2}\left(Q + [J_V, Q]\right)$$

$$= \frac{1}{2}\left(d_H\right.$$

Q



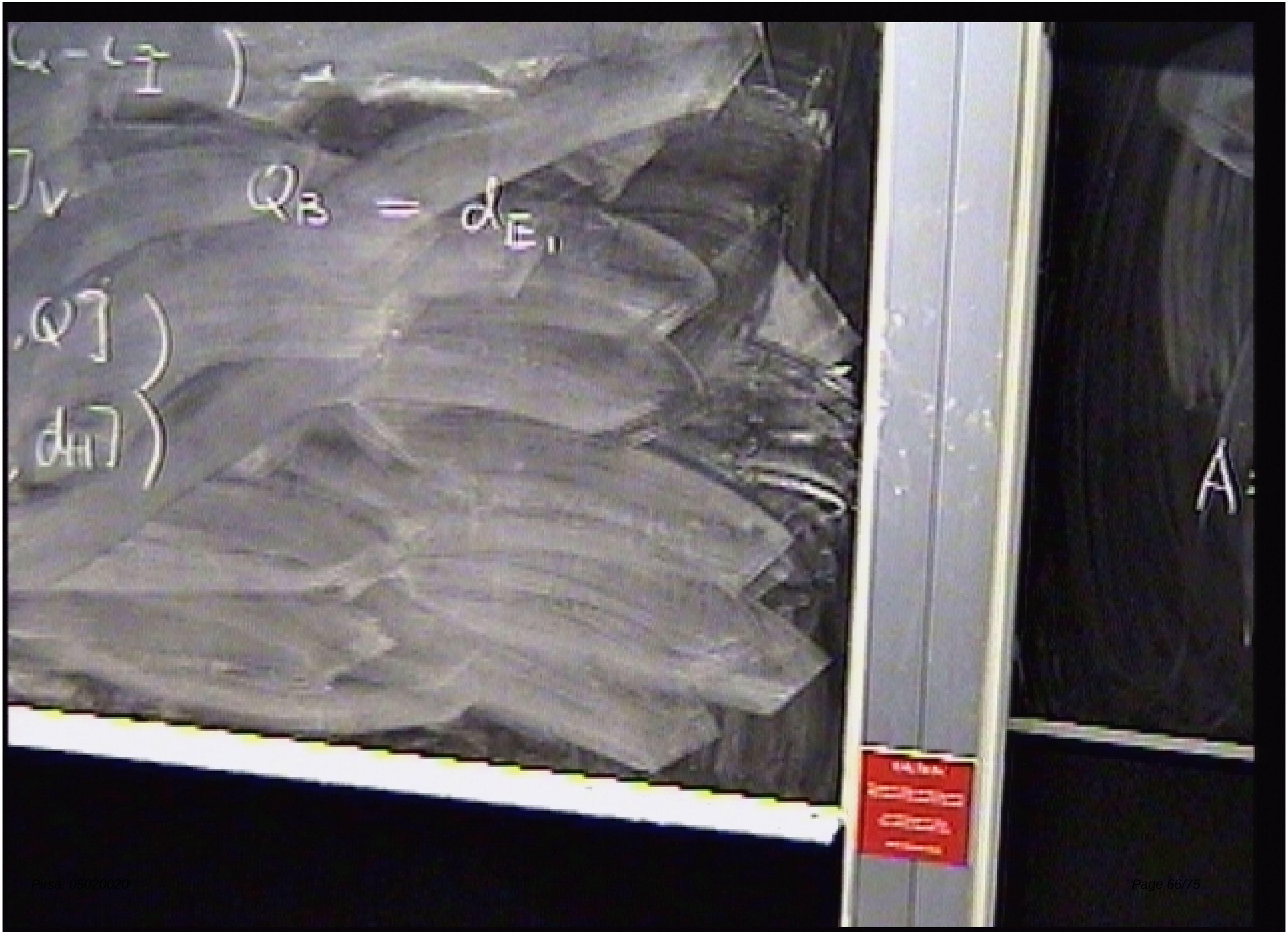
$J_1 \Longleftrightarrow J_V$

$$Q_B = \frac{1}{2}\left(Q + [J_V, Q]\right)$$

$$= \frac{1}{2}\left(d_H + [R_{J_1}, d_H]\right)$$

$$= \bar{\partial}_H$$

Q



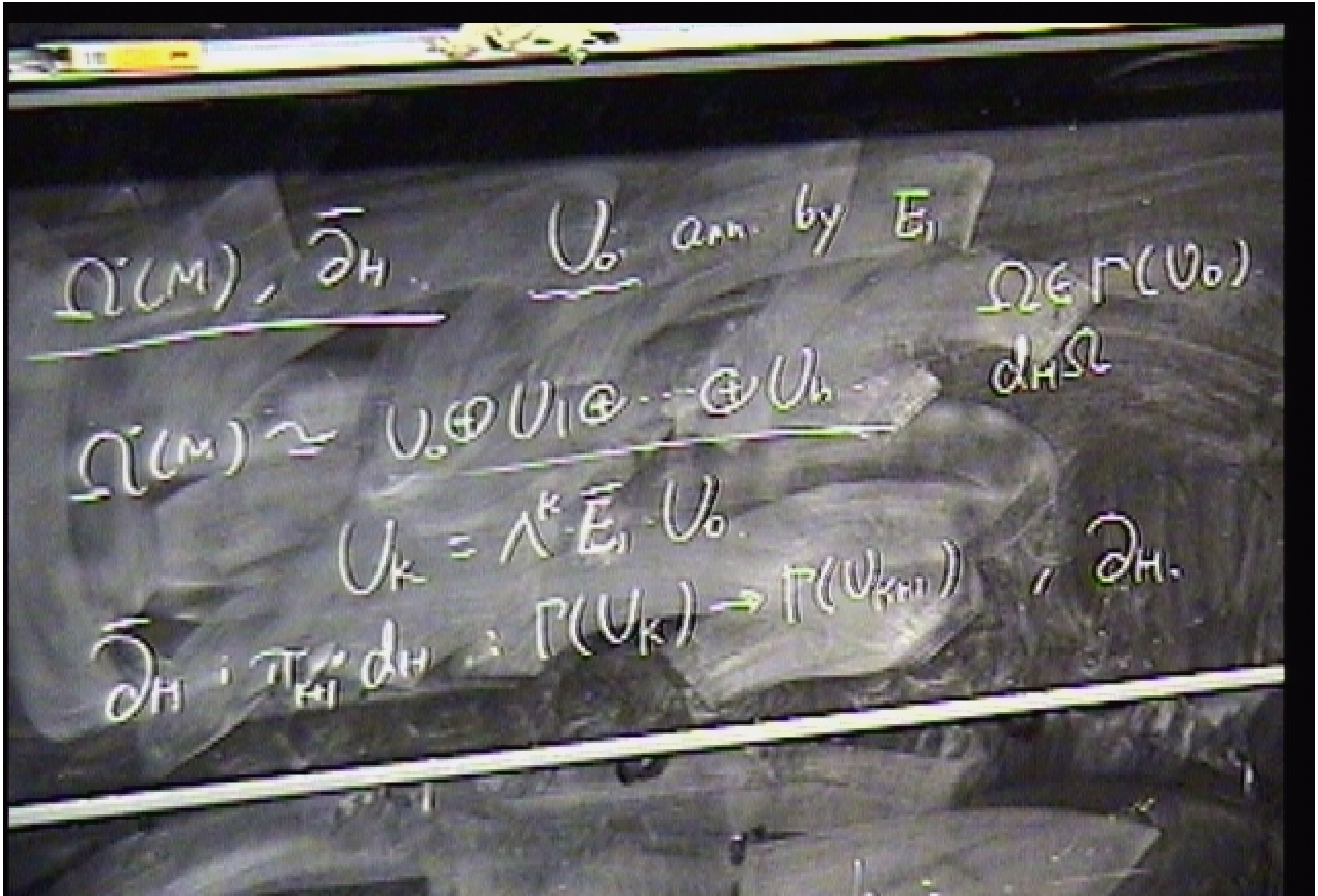
$(Q - \zeta_1 \ )$

$]_V \qquad Q_B = d_{E_1}$

$, Q] \ )$

$, d_H] \ )$

A:



$\underline{\Omega^{\cdot}(M)\,,\ \bar{\partial}_H}$. $\underline{V_0}$ ann. by E_1

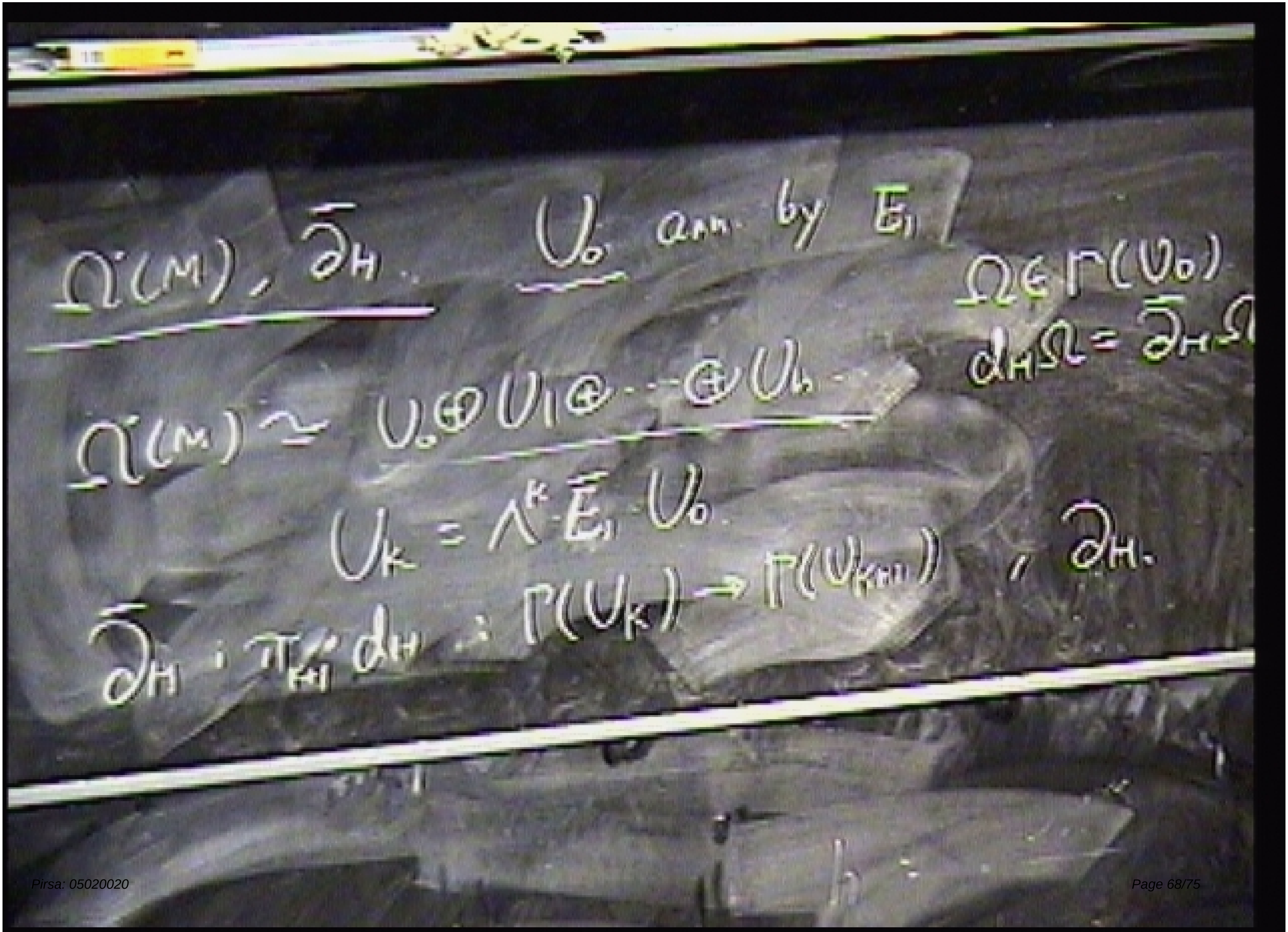
$\Omega \in \Gamma(V_0)$

$d_H \Omega$

$\Omega^{\cdot}(M) \simeq V_0 \oplus V_1 \oplus \cdots \oplus V_n$

$V_k = \Lambda^k \bar{E}_1 \cdot V_0$

$\bar{\partial}_H : \pi_{k+1} d_H : \Gamma(V_k) \to \Gamma(V_{k+1})$, ∂_H.



$\underline{\Omega^{\cdot}(M), \bar{\partial}_H}$ $\underline{V_0}$ ann. by E_1

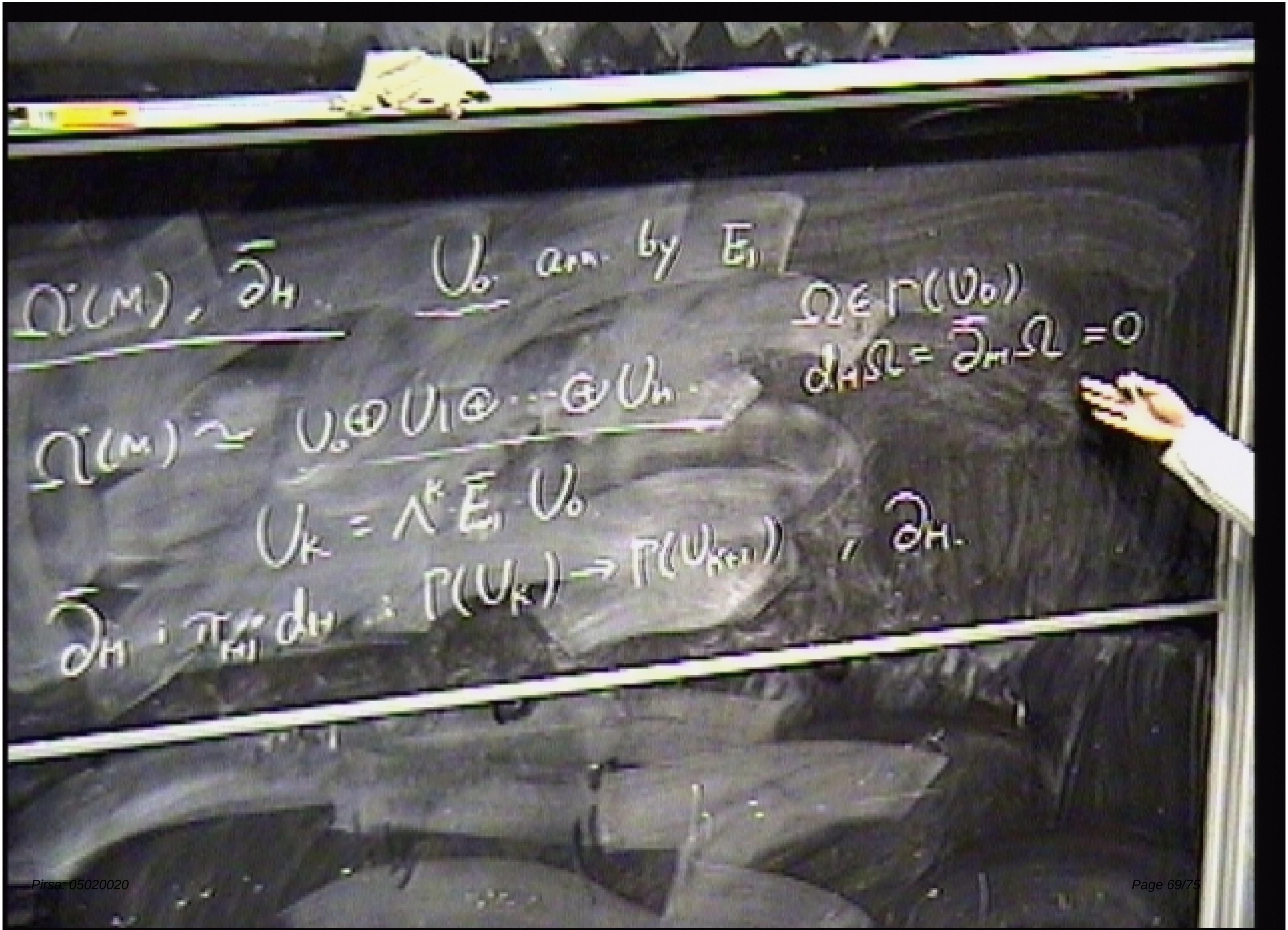
$\Omega \in \Gamma(V_0)$

$d_H \Omega = \bar{\partial}_H \Omega$

$\Omega^{\cdot}(M) \simeq \underline{V_0 \oplus V_1 \oplus \cdots \oplus V_n}$

$V_k = \Lambda^k \bar{E}_1 \cdot V_0$

$\bar{\partial}_H : \pi_{k+1} d_H : \Gamma(V_k) \to \Gamma(V_{k+1})$, ∂_H



$\underline{\Omega(M),\ \bar\partial_H}$

$\underline{V_0}$ ann. by E_1

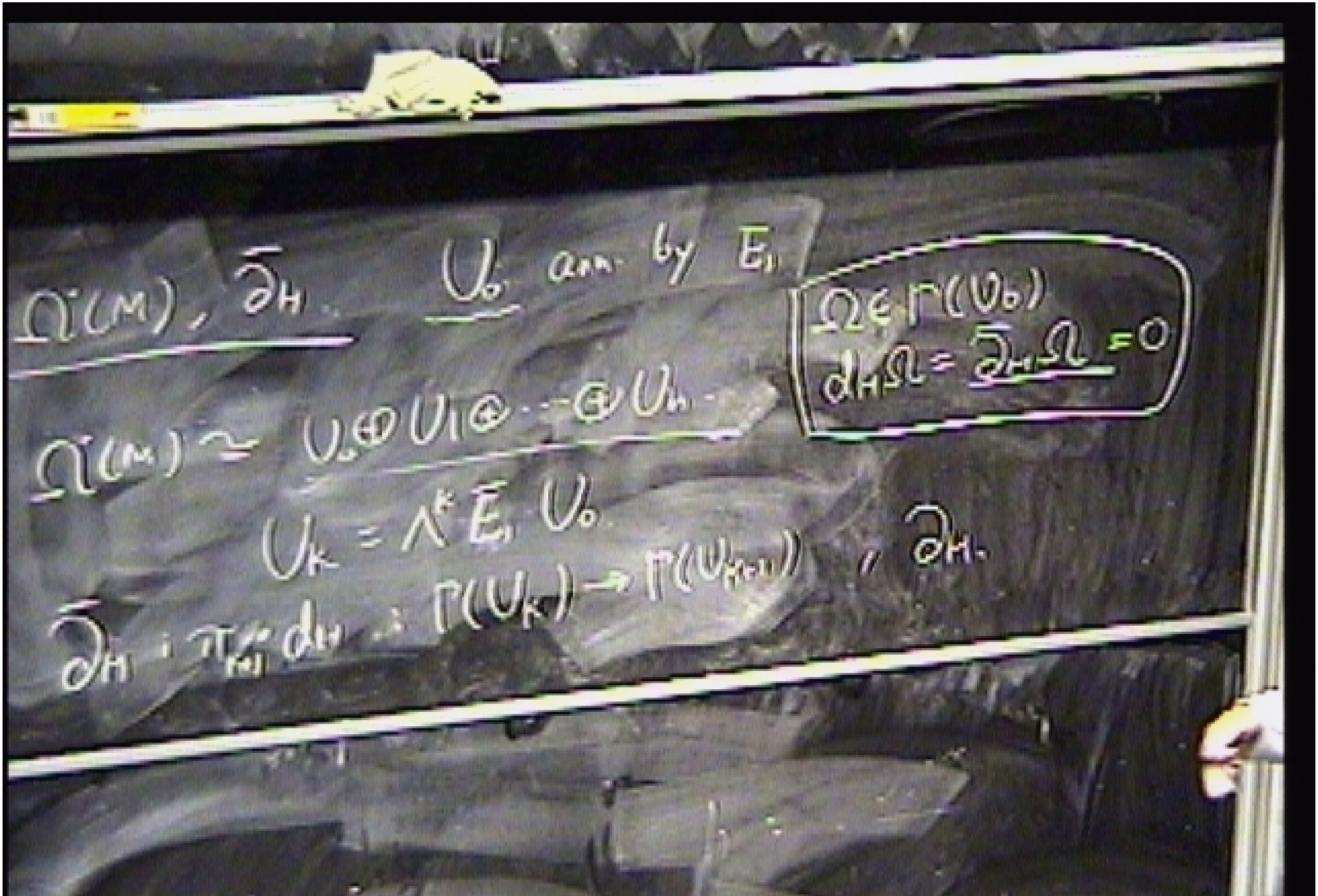
$\Omega \in \Gamma(V_0)$

$d_H \Omega = \bar\partial_H \Omega = 0$

$\underline{\Omega(M) \simeq V_0 \oplus V_1 \oplus \cdots \oplus V_n}$

$V_k = \Lambda^k \bar{E}_1 \cdot V_0$

$\bar\partial_H : \pi_{k+1} d_H : \Gamma(V_k) \to \Gamma(V_{k+1})$, ∂_H



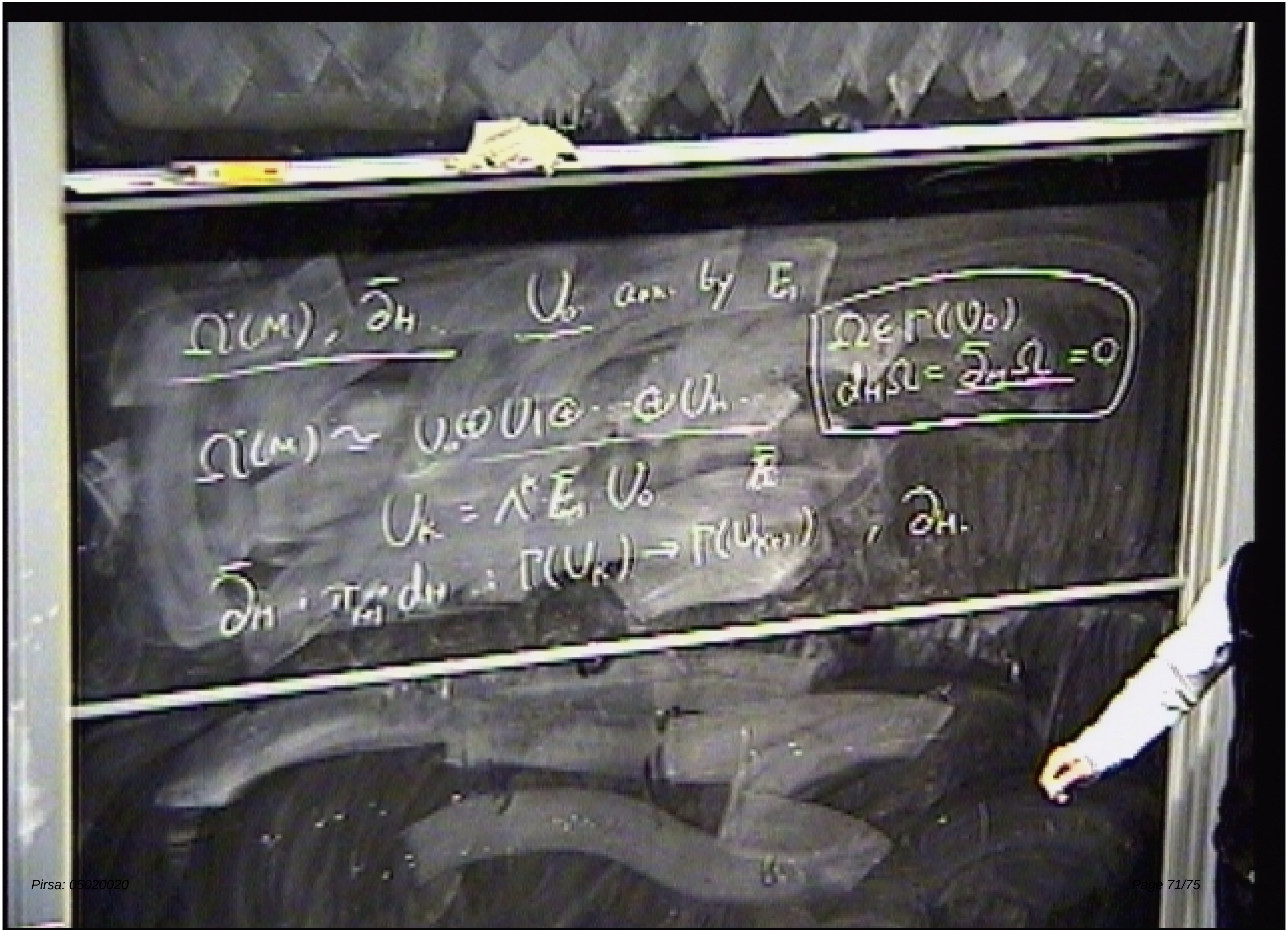
$\Omega^{\cdot}(M)$, $\bar{\partial}_H$ $\quad$ $\underline{V_0}$ ann. by E_1

$$\Omega^{\cdot}(M) \simeq V_0 \oplus V_1 \oplus \cdots \oplus V_n$$

$$V_k = \Lambda^k E_1 V_0$$

$$\bar{\partial}_H : \pi_{k+1} d_H : \Gamma(V_k) \to \Gamma(V_{k+1}) \quad , \partial_H$$

$$\boxed{\begin{array}{l} \Omega \in \Gamma(V_0) \\ d_H \Omega = \underline{\bar{\partial}_H \Omega} = 0 \end{array}}$$



$\underline{\Omega(M),\ \bar{\partial}_H}$ .. $\underline{U_0}$ ann. by E_1

$\Omega(M) \simeq \underline{U_0 \oplus U_1 \oplus \cdots \oplus U_n}$ ..

$U_k = \Lambda^k E_1 U_0$ $\bar{E}$

$\bar{\partial}_H : \pi_{k+1} d_H : \Gamma(U_k) \to \Gamma(U_{k+1})$, ∂_H.

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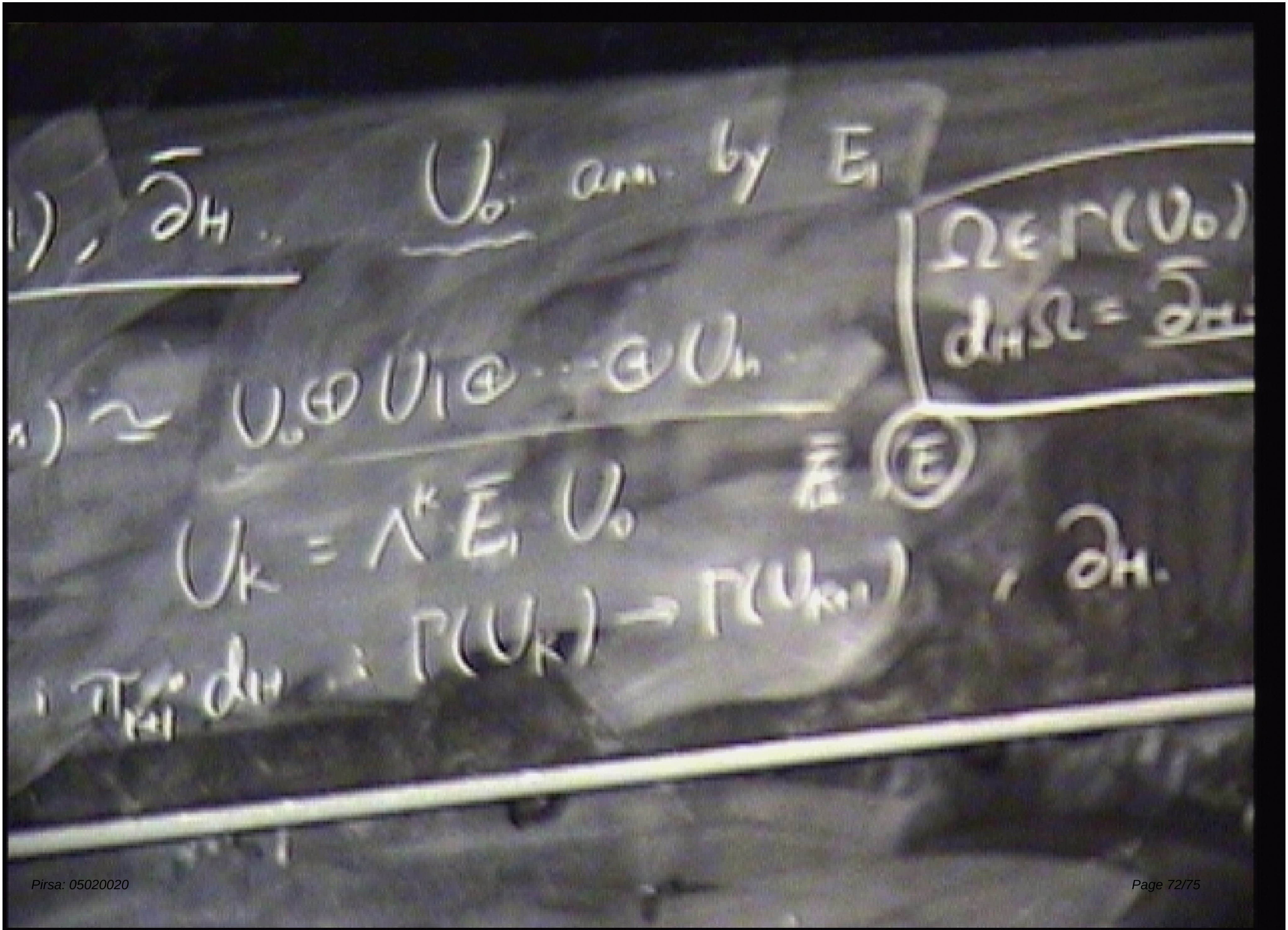


1)), $\bar{\partial}_H$ .. $\underline{V_0}$ ann. by E_1

$\Omega \in \Gamma(V_0)$
$d_H \Omega = \underline{\bar{\partial}_H}$

n) $\simeq V_0 \oplus V_1 \oplus \cdots \oplus V_n$

$\textcircled{E}$

$V_k = \Lambda^k \bar{E}_1 V_0$

: $\pi_{k+1} d_H$: $\Gamma(V_k) \to \Gamma(V_{k+1})$, ∂_H.

$U(1)_A$, $\underline{U(1)_V}$

$\uparrow$

twist

$$c_1(E_1) \overset{}{=} 0 \qquad Q_B = \frac{1}{2}\left(Q + [J_V, Q]\right)$$

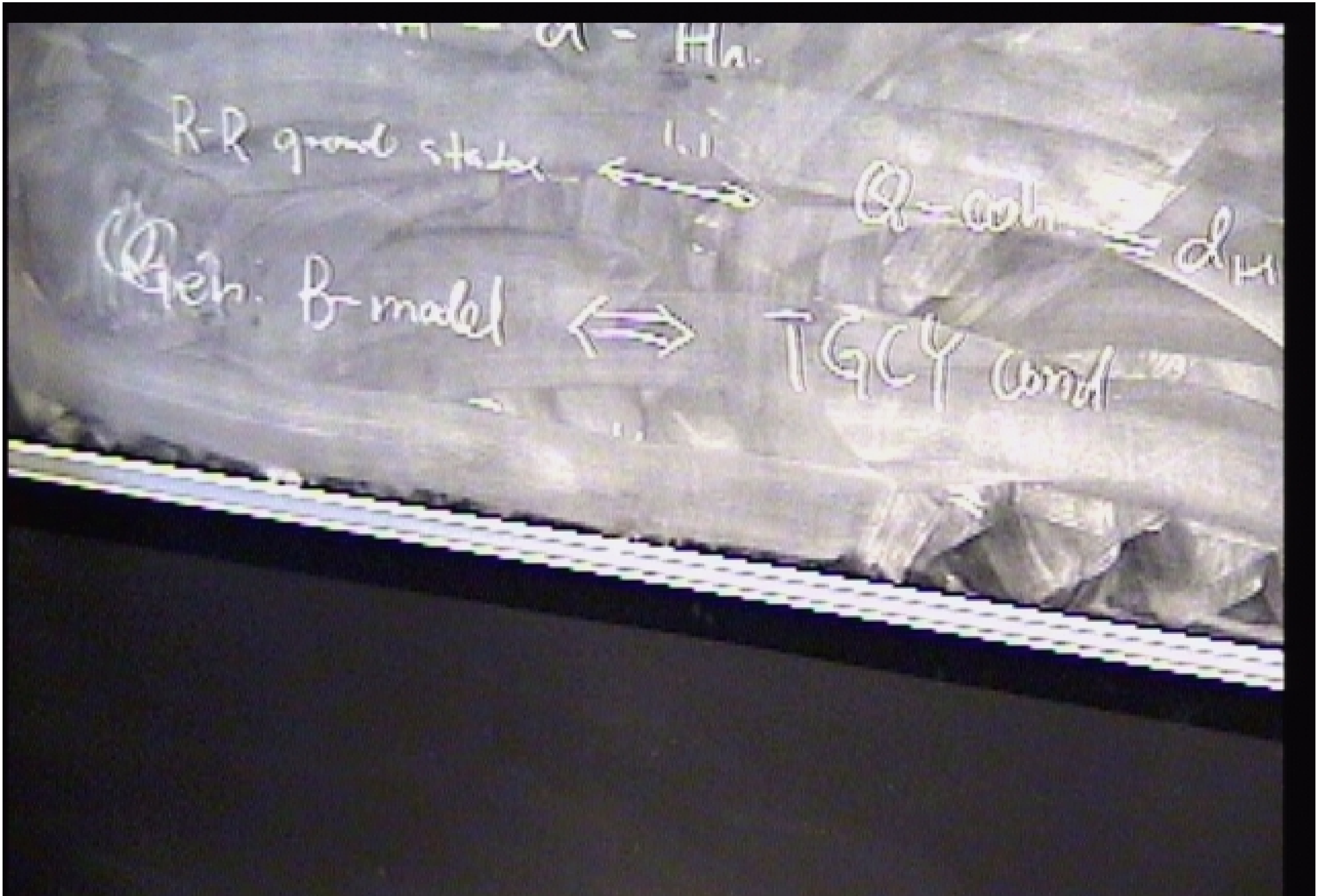
by Noether

$$J_V = -\frac{1}{2}\left(\omega_+(\nu_+,\nu_+) + \omega_-(\nu_-,\nu_-)\right)$$

$$\| $$

$$-i\left(\delta_\omega \wedge - i_\epsilon - i_{\mathcal{I}}\right) \qquad J_1 = \begin{pmatrix} \mathcal{I} & -\omega \\ \delta\omega & -\mathcal{I}^* \end{pmatrix}$$

$$i_{\mathcal{I}} = \mathcal{I}^a{}_b \, dz^b \, i_{\partial_a}$$



$= a = H_n.$

R-R ground states $\xleftrightarrow{1,1}$ Q-coh. $= d_H$

(Qeh: B-model ⟺ TGCY cond