Title: Generalized Geometric Structures (Part 2)

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Abstract:

H odd degree closed form

curvature of a $(n-3)$-gerbe

close

ature of a $(n-3)$-gerbe

ize ?

st(

$$f^* H^\bullet = dF^\bullet$$

odd

even.

$$d + H^{\circ} \wedge \; : C^{\infty}(\Lambda^{ev} T^{*})$$

$$T^{*}) \rightleftarrows C^{\infty}(\Lambda^{0,d}T^{*})$$

$$d_H = d + H\wedge \quad : C^\infty(\Lambda^{ev} T^*) \rightleftarrows$$

$$\Rightarrow H^{\bullet}_{d_H}(N) \quad \text{twisted coh.}$$

$$\rho \in \Omega^{\bullet}(N) \qquad \text{st}$$

$\Lambda^{ev} T^*) \rightleftarrows C^\infty(\Lambda^{od} T^*)$

ed coh.

$)$ st $d_H \rho = 0$.

$\Omega^{\cdot}(N)$ st c

$$d\left(e^{F^{\cdot}} f^{*} \rho\right) = dF^{\cdot}$$

$+$

$$\text{st} \quad d_H \rho = 0.$$

$$) = dF \wedge e^F f^* \rho$$

$$+ e^F f^* \left(-H|_M \wedge \rho \right)$$

$$- dF \wedge \rho$$

$$d\left(e^{F} f^{*} \rho\right) = \quad dF$$

$$+$$

$$e^{F} f^{*} : H_{H}^{\cdot}(N) \longrightarrow H^{\cdot}(M)$$

$$H^{\bullet}(M) \longrightarrow H_H^{\bullet + \mathrm{codim}(M)}(N$$

$$d_H : \begin{array}{ccc} S & \longrightarrow & S \\ \| & & \| \\ C^\infty(\Lambda^\bullet T^*) & \longrightarrow & C^\infty(\Lambda^\bullet T^*) \end{array}$$

$$C^\infty(\Lambda^\cdot T^*) \longrightarrow C^\infty(\Lambda^\cdot T^*)$$

generates a bracket operation on

$CL(T \oplus T^*)$

$$[A, B]_H \cdot \rho = [[A, d_H], B] \cdot \rho$$

$\mathbb{H}^{\cdot} \rightarrow d_H : S \rightarrow S$

$C^\infty(\Lambda T^*) \rightarrow C^\infty(\Lambda T^*)$

$\Rightarrow$ derived bracket construction generates a bracket operation on $CL(T \oplus T^*)$

$$[A, B]_H \cdot \rho = [[A, d_H], B] \cdot \rho$$

if $A, B \in T \oplus \Lambda^{\cdot} T^*$, then $[A,B]_H$ is a section of $T \oplus \Lambda^{\cdot} T^*$.

$$H^{\bullet} \to d_H : S \to S$$

$$\begin{array}{ccc} S & \to & S \\ \cup & & \| \\ C^\infty(\Lambda^\bullet T^*) & \to & C^\infty(\Lambda^\bullet T^*) \end{array}$$

derived bracket construction

$\Rightarrow$ generates a bracket operation on

$$CL(T \oplus T^*) \cong \Lambda^\bullet(T \oplus T^*)$$

$$[A, B]_H \cdot \rho = [[A, d_H], B] \cdot \rho$$

$A, B \in$

is a section of

$$[X + \xi , Y + \eta]_H =$$

T $\quad \Lambda^\cdot T^*$ $\quad \Lambda^\cdot T^*$

$\oplus \Lambda^{\cdot} T^*$, then $[A, B]_H$

$T \oplus \Lambda^{\cdot} T^*$..

$[X, Y] + L_X \eta - L_Y \xi - \frac{1}{2} d(i_X \eta - i_Y \xi)$

$\oplus \Lambda^{\cdot} T^{*}$, then $[A, B]_H$

$T \oplus \Lambda^{\cdot} T^{*}$ ..

$X, Y] + L_X \eta - L_Y \xi - \frac{1}{2} d(i_X \eta - i_Y \xi)$

$+ i_X i_Y H$

$$T \oplus T^{*} \longleftrightarrow H^{(5)}$$

Enlarged symmetry gr

$$\mathcal{D}iff_H \subset Sym([\,,\,]_H)$$

$$e^b[A,B]_H = [e^bA, e^bB]_{H+db}$$

group

if $db = 0$

then e^{b} is

a symmetry.

Diff_H

T^* T

e^{b} ev

$[A, B$

$$X+\xi \mapsto X+\xi+i_X b$$

$$
\begin{tikzcd}
T_1 \oplus T_1^* \arrow[r, "F"] \arrow[d] & T_2 \oplus T_2^* \arrow[d] \\
N_1 \arrow[r, "f"'] & N_2
\end{tikzcd}
$$

nmetry group: for [

ourant bracket $\rightarrow T\oplus T^*$ is <u>not</u> a Lie brack

$$\mathrm{Jac}(A,B,C) \equiv [[A,B],C] + \text{c.p.}$$

$$= d\, \mathrm{Nij}(A,B,C)$$

$$\mathrm{Nij}(A,B,C) = \frac{1}{3}\left(\langle [A,B],C\rangle + \cancel{i_z i_y i_x dH} + \text{c.p.}\right)$$

$L \subset T \oplus T^*$ subbundle

L ~~maxi~~ Lagrangian (max. isotropic)

2. $C^\infty(L)$ closed under Courant bracket

ourant & Weinstein

f we find $L \subset T \oplus T^*$ subbundle

st

1. L ~~...~~ Lagrangian (max. isotropic)

2. $C^\infty(L)$ closed under Courant bracket

hen $N_{ij}|_L = 0 \Rightarrow [\,,\,]$ is a Lie bracket on $C^\infty(L)$

if we find $L \subset$

$\boxed{\text{Dirac Structure}} \begin{cases} 1.\ L = \\ 2.\ C^a(L) \end{cases}$

then $N_{ij}\big|_L = 0 \Rightarrow [$

T closed if $H = 0$

$$[X, Y]_H = [X, Y] + i_X i_Y H$$

a

T

$$[X, Y]_H = [X, Y] +$$

$$\hat{T}$$

$$e^b T = \{ X + i_X b \} = Gr(b)$$

this is Dirac $\Leftrightarrow$ $db = H$.

$$[X,Y]_H = [X,Y] + i_X i_Y H$$

$$\underset{T}{\underset{\cap}{X}}$$

$$e^b T = \{X + i_X b\} = Gr(b)$$

this is Dirac $\Leftrightarrow$ $db = H$

$H=0$ then closed 2-forms are Dirac

2. T^*

Dirac.

$$[\xi, \eta] = 0$$

$] = 0$

$\beta \in C(\tilde{\Lambda}^2 T)$

$\beta_1 \in \Gamma O(T \oplus T^*)$

irac.

$\xi, \eta] = 0$

$\beta^* H \in \Lambda^3 T$

$H \in \Lambda^3 T^*$

$\beta \in C(\tilde{\Lambda}^2 T)$

$\beta \in \mathfrak{so}(T \oplus$

$\beta : T^* \to T$

$\Gamma^\beta T^*$

Dirac $\Leftrightarrow$ $[\beta, \beta] = \beta^* H$

$H = 0$

$$\left(C^\infty(L), [\ ,\ ]\right) \text{ Lie algebr}$$

$$C^\infty(\Lambda^k L^*) \xrightarrow{d_L} C^\infty(\Lambda^{k+1} L^*$$

$(C^\infty(L), [\ ,\ ])$ Lie algebroid

$C^\infty(\Lambda^k L^*) \xrightarrow{d_L} C^\infty(\Lambda^{k+1} L^*)$

$d_L^2 = 0$ diff complex.

$H^\bullet_{d_L}(L)$ Lie algebroid cohomol

$C^\infty(\Lambda^\bullet T^*) \circlearrowleft d$ — Poincaré lem — exact as complexes of sheaves

$C^\infty(\Lambda^{0,\bullet} T^*) \circlearrowleft \bar{\partial}$ — Dolbeault lem.

$C^\infty(L^\bullet) \circlearrowleft d_L$ <u>not</u> exact as cx of shea[ves]

$\mathcal{H}^\bullet(L)$ cohomology sheaves of $C^\infty(\Lambda^\bullet L^*)$

$$E_2^{p,q} = H^p(N, \mathcal{H}^\bullet(L))$$

$$\Downarrow \quad \longleftarrow \text{Hypercoh. s.s.}$$

$$H^\bullet_{d_L}(L).$$

$\mathbb{J} : T\oplus T^* \circlearrowleft \quad \mathbb{J}^2 = -1$

$T\oplus T^*$

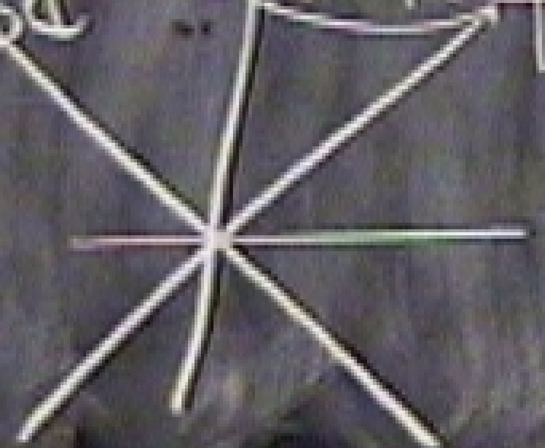
$O(2n,2n)$

$\downarrow$

$(M \text{ even}) \quad U(n,n)$

$\mathbb{J}$ orthog in $\langle\, , \rangle$

$\mathbb{J}$ <u>integ</u> $\quad L = +i$ eigenbundle <u>closed</u> <u>under Cour</u>



$\ell_1, \ell_2 \in L$

$$\langle \ell_1, \ell_2\rangle = -\langle i\ell_1, i\ell_2\rangle$$
$$= -\langle \mathbb{J}\ell_1, \mathbb{J}\ell_2\rangle$$
$$= -\langle \ell_1, \ell_2\rangle$$
$$= 0$$

$$\boxed{\text{G Cx Str}}$$

$$\updownarrow$$

$$\boxed{\begin{array}{l}\text{Cx. Dirac Structure} \\ L \text{ st } L \cap \bar{L} = \{0\}\end{array}}$$

ex:

$$\mathcal{J}_{\mathfrak{g}} = \begin{pmatrix} -J & \\ & J^* \end{pmatrix}$$

integ $\Rightarrow$ $\Downarrow$ integ (cx) and $\mathbb{J}_\omega = \begin{pmatrix} & -\omega^{-1} \\ \omega & \end{pmatrix}$.

$\downarrow$

$d\omega = 0$

ex: $\mathcal{J}_J = \begin{pmatrix} -J & \\ & J^* \end{pmatrix}$ and $\mathcal{J}_\omega = \begin{pmatrix} & -\omega^{-1} \\ \omega & \end{pmatrix}$

Integ $\Rightarrow$ integ cx

$C^\infty(\Lambda^\bullet L^*)$ is complex of sheaves

$\Downarrow$

$d_\omega = 0$

$\left(\quad \mathcal{J}^* \right)$ and $\mathbb{J}_w = \left({}_w \right.$

$\downarrow$ $\qquad\qquad\qquad\qquad\qquad\qquad \downarrow$

Integ $\Rightarrow$ integ cr $\qquad\qquad dw = 0$

$C^\infty(\Lambda^\cdot L^*)$ is elliptic complex of sheaves.

governs deformation theory of $\mathbb{J}$.

cx mfld

$$L = T_{0,1} \oplus T^*_{1,0}$$

$$C^\infty(M,\mathbb{C}) \qquad C^\infty(T^*_{0,1}) \oplus C^\infty(T_{1,0}) \qquad C^\infty(\Lambda^2 T^*_{0,1}) \oplus C^\infty(T_{1,0} \otimes T^*_{0,1}) \oplus C^\infty(\Lambda^2 T_{1,0})$$

$$\Lambda^0 L^* \qquad \Lambda^1 L^*$$

mfld

$$L = T_{0,1} \oplus T^*_{1,0}$$

$$C^\infty(M,\mathbb{C}) \xrightarrow{\bar\partial} \begin{matrix} C^\infty(T^*_{0,1}) \\ \oplus \\ C^\infty(T_{1,0}) \end{matrix} \xrightarrow{\bar\partial} \begin{matrix} C^\infty(\Lambda^2 T^*_{0,1}) \\ \oplus \\ C^\infty(T_{1,0} \otimes T^*_{0,1}) \\ \oplus \\ C^\infty(\Lambda^2 T_{1,0}) \end{matrix} \xrightarrow{\bar\partial}$$

$$\Lambda^0 L^* \qquad \Lambda^1 L^* \qquad \Lambda^2 L^*$$

$\Lambda^0 L^*$

$\Lambda^1 L^*$

$$H^0(L) = \Lambda^0 \mathcal{T}$$

$$\mathbb{J}_\omega = \begin{pmatrix} & -\omega^{-1} \\ \omega & \end{pmatrix}$$

$$L_\omega = Gr(i\omega)$$

$$\mathbb{J}_\omega = \begin{pmatrix} & -\omega^{-1} \\ \omega & \end{pmatrix}$$

$$L_\omega = \mathrm{Gr}(i\omega) \quad T^*$$

$$\Lambda^i L_\omega^* \cong \Lambda^i T^* \otimes \mathbb{C}$$

$$d) = \begin{cases} 0 & \text{for all } k > 0 \\ \mathbb{C} & \text{for } k = 0. \end{cases}$$

$$\mathbb{J}_\omega = \begin{pmatrix} & -\omega^{-1} \\ \omega & \end{pmatrix}$$

$$L_\omega = \mathrm{Gr}(i\omega) \subset T \otimes \mathbb{C}$$

$$\Lambda^{\cdot} L_\omega^* \cong \Lambda^{\cdot} T^* \otimes \mathbb{C}$$

$$H^k(\Lambda^{\cdot} T^* \otimes \mathbb{C}, d) = \begin{cases} 0 & \text{for all } k > 0 \\ \mathbb{C} & \text{for } k = 0. \end{cases}$$

L Lag subbundle $\subset (T \oplus T^*) \otimes \mathbb{C}$

$\updownarrow$

Pure spinor lines $U \subset \Lambda^* T^*$

$L = \mathrm{Ann}(U)$

Canonical Line bundle of $\mathbb{J}$

ub bundle $\subset (T \oplus T^*) \otimes \mathbb{C}$

line $U \subset \Lambda^{\cdot} T^* \otimes \mathbb{C}$

$L = \mathrm{Ann}(U)$ "

Line bundle of $\mathbb{J}$

$\Rightarrow \exists\, L$ — Lag. subbundle $\subset (T\oplus T^*)\otimes\mathbb{C}$

~~Pure~~ spinor line $U \subset \Lambda^\bullet T^* \otimes \mathbb{C}$

$L = \mathrm{Ann}(U)$ "

Canonical Line bundle of $\mathbb{J}$

$\rho = c\, e^{B+i\omega}\, \theta_1 \wedge \dots \wedge \theta_k \longrightarrow k$ called the <u>type</u> of $\mathbb{J}$ at

$k \in \{0, \dots, n\}$

L — Max subbundle $\subset (T \oplus T^*) \otimes \mathbb{C}$

Pure spinor lines $U \subset \Lambda^\bullet T^* \otimes \mathbb{C}$

$L = \mathrm{Ann}(U)$ "

Canonical Line bundle of $\mathbb{J}$

$\rho = c\, e^{B + i\omega}\, \theta_1 \wedge \ldots \wedge \theta_k \longrightarrow k$ called the **type** of $\mathbb{J}$ at x.

$k \in \{0, \ldots, n\}$

$L n L = 0 \Leftrightarrow \omega^{n-k} \wedge$

cx mfld

$\rho =$

$$\omega^{n-k} \wedge \Omega \wedge \bar{\Omega} \neq 0 \qquad e^{B}\Omega.$$

$$\rho = f \det T^{*}_{0,1} \Rightarrow \Omega^{n,0} \wedge \bar{\Omega} \neq$$

$$\rho = e^{i\omega} \qquad \omega^{n} \neq 0$$

$$\xrightarrow{B} e^{B+i\omega}$$

$$\omega^{n-k} \wedge \Omega \wedge \bar{\Omega} \neq 0 \qquad e^{B}\Omega.$$

$$\rho = f \det T^{*}_{0,1} \Rightarrow \Omega^{n,0} \wedge \bar{\Omega} \neq 0.$$

$$\rho = e^{i\omega} \qquad \omega^{n} \neq 0$$

$$\xrightarrow{B} e^{B+i\omega}$$

$\mathcal{U}$ $\quad \Gamma\mathcal{U}$ $\quad \Lambda^2\Gamma\mathcal{U}$ $\quad \cdots$ $\quad \det\Gamma\cdot\mathcal{U}$

1 $\cdots$ dual $\quad\cdots$ dual

$$\Lambda^\bullet T^* \otimes \mathbb{C} = \mathcal{U} \oplus \Gamma\mathcal{U} \oplus \cdots \Lambda^k\Gamma\mathcal{U} \cdots \oplus \det\Gamma\,\mathcal{U}.$$

$$\Lambda T^* \otimes \mathbb{C} = \mathcal{U} \oplus L\mathcal{U} \oplus \cdots \Lambda^k L\mathcal{U} \;\cdots\; d$$

$\mathcal{U}$ alg determined from L

$\downarrow$ $\qquad\qquad\qquad\qquad\qquad\qquad \downarrow$

integ $\Leftrightarrow$ $d(\mathcal{U}) \subset L\mathcal{U}$ $\qquad$ integ!


$\Omega^{n,0}$
$\bar{\partial}$
∂
$\Omega^{n,1}$
$\oplus$
$\Omega^{n-1,0}$
U
$\bar{\partial}$
∂
$\bar{\partial}$
∂

CY condition

$\mathcal{U}$ canonical bdle

Suppose

$c_1\mathcal{U}=0$. Chose nonvanish section ϕ of $\mathcal{U}$

$d\phi \in \bar{L}\,\mathcal{U}$

$\Rightarrow \quad d\phi = \alpha\cdot\phi, \quad \alpha \in \bar{L} = L^*$

$\Rightarrow \quad d_L\alpha = 0 \quad \text{in } C^\infty(\Lambda^2 L^*)$

$[\alpha] \in H^1_{d_L}(L)$

$(\alpha) = 0 \iff \exists$ section $\tilde{\phi} \in U$

$\Updownarrow$

CY-condition

$c_1(K) = 0 \quad [K] \in H^1(M, \mathcal{O}) \longrightarrow H^1(M, \mathcal{O}^*)$

section $\tilde{\phi} \in \mathcal{U}$ st $d\tilde{\phi} = 0$

$$H^1(\mathcal{O}) \longrightarrow \underset{K}{H^1(M, \mathcal{O}^*)} \xrightarrow{c_1} H^2(M, \mathbb{Z})$$

$$\ldots + dz_1 \wedge dz_2 .$$

- if $z_1 = 0$ $\quad (dz_1 \wedge dz_2) \wedge (d\bar{z}_1 \wedge d\bar{z}_2) \neq 0$
- $z_1 \neq 0$

$$z_1 \left(1 + \frac{dz_1 \wedge dz_2}{z_1} \right)$$

$$= z_1 e^{\frac{dz_1 \wedge dz_2}{z_1}} \quad \text{also pure.}$$

$$d\rho = dz_1 \frac{\partial}{\partial z_2} \cdot \rho$$

$\ldots) + dz_1 \wedge dz_2.$

- if $z_1 = 0$: $(dz_1 \wedge dz_2) \wedge (d\bar{z}_1 \wedge d\bar{z}_2) \neq 0$
- $z_1 \neq 0$

$$z_1\left(1 + \frac{dz_1 \wedge dz_2}{z_1}\right)$$

$$= z_1 e^{\frac{dz_1 \wedge dz_2}{z_1}}$$

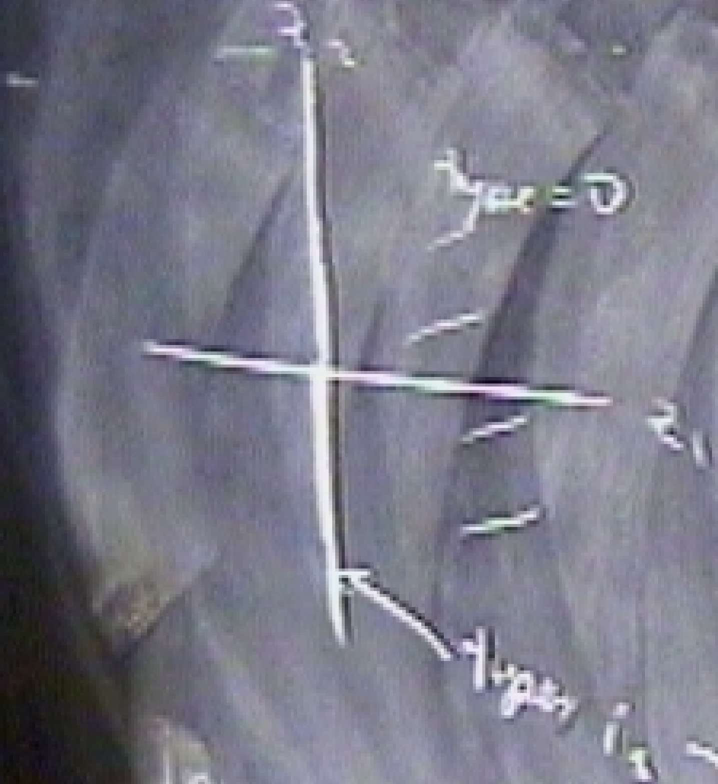
also pure.

$$d\rho = dz_1 \wedge \ldots - \frac{\partial}{\partial z_2} \cdot \rho \quad \checkmark$$

integ...

$\mathbb{C}^2$

(z_1, z_2) $\quad \rho = z_1 + dz_1 \wedge dz_2.$



- if $z_1 = 0$ $\quad (dz_1 \wedge d$
- $z_1 \neq 0$

$z_1 (1 +$

$= z_1 e^{\frac{dz_1 \wedge dz_2}{z_1}}$

In

$d\rho = dz_1 \quad \frac{\partial}{\partial z_2} \cdot \rho$

$0 \neq [-\partial_{z_2}]_0 \in H^1(M)$

$\mathbb{CP}^2$

cx structure

$$H^2_{d_L}(L) = \text{tgt space to def}^{\text{ms}}$$

$$\parallel$$

$$\underbrace{H^0(\Lambda^2 T)}_{\beta} \oplus \underbrace{H^1(T)}_{\text{usual def}^{\text{ns}} \text{ of } J} \oplus \underbrace{H^2(\mathcal{O})}_{B\text{-fields}}$$

$\beta \in C^\infty(\Lambda^{0,1} T_{\mathbb{P}})$ actual def^{n} iff $\bar\partial \beta + \frac{1}{2}\cancel{[\beta,\beta]} = 0$

iff β hol

$\beta \in C^\infty(\Lambda^2 T_{1,0})$ actual def^{m} iff $\bar\partial\beta$:

iff β hol

$\beta \in \Lambda^2 T_{1,0}$ on $\mathbb{CP}^2 = \mathcal{O}(3)$

$\beta = 0$ on a cubic

$\beta \in C^\infty(\Lambda^2 T_{1,0})$ actual def^n iff $\bar{\partial}\beta$

iff β hol

$\beta \in \Lambda^2 T_{1,0}$ on $\mathbb{CP}^2 = \mathcal{O}(3)$

$\beta = 0$ on a cubic

$e^{\beta} \mathbb{J}_J e^{-\beta}$

$\overline{L} = L^*$

L

def^{ms}

$H^2(\mathcal{O})$

B-fields

$\varepsilon : L \to L^*$

$\varepsilon \in L^* \otimes L^*$

$\varepsilon \in \Lambda^2 L^*$

Integr... $d_L \varepsilon + \frac{1}{2}[\varepsilon, \varepsilon] = 0$

$$H(X) \oplus H(Y)$$

usual defns of

$$\beta$$

$$\pi_{T\otimes\sigma} L = E$$

$$E \cap \overline{E}$$

$$\mathcal{D}_{\pi} = \{ \xi + i_{\xi}\pi \} \leftarrow T \oplus T^{*}$$

$$\Lambda^{2} T$$

$$e^{\pi} \in \Lambda^{2} T$$