Title: Generalized Geometric Structures (Part 1)

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Abstract:

E real $2n$-dim^l v. space, $\langle\,,\rangle$ signature (n,n)

$V \oplus V^*$

$X \quad \xi$

$$\langle X+\xi, Y+\eta\rangle = \frac{1}{2}\left(\xi(Y)+\eta(X)\right).$$

Signature (n,n)

$$Y+\eta\rangle = \frac{1}{2}\left(\xi(Y)+\eta(X)\right).$$

$$A \subset E$$

- A – definite $\pm$
- $A^\perp \cap A = \{0\}$.

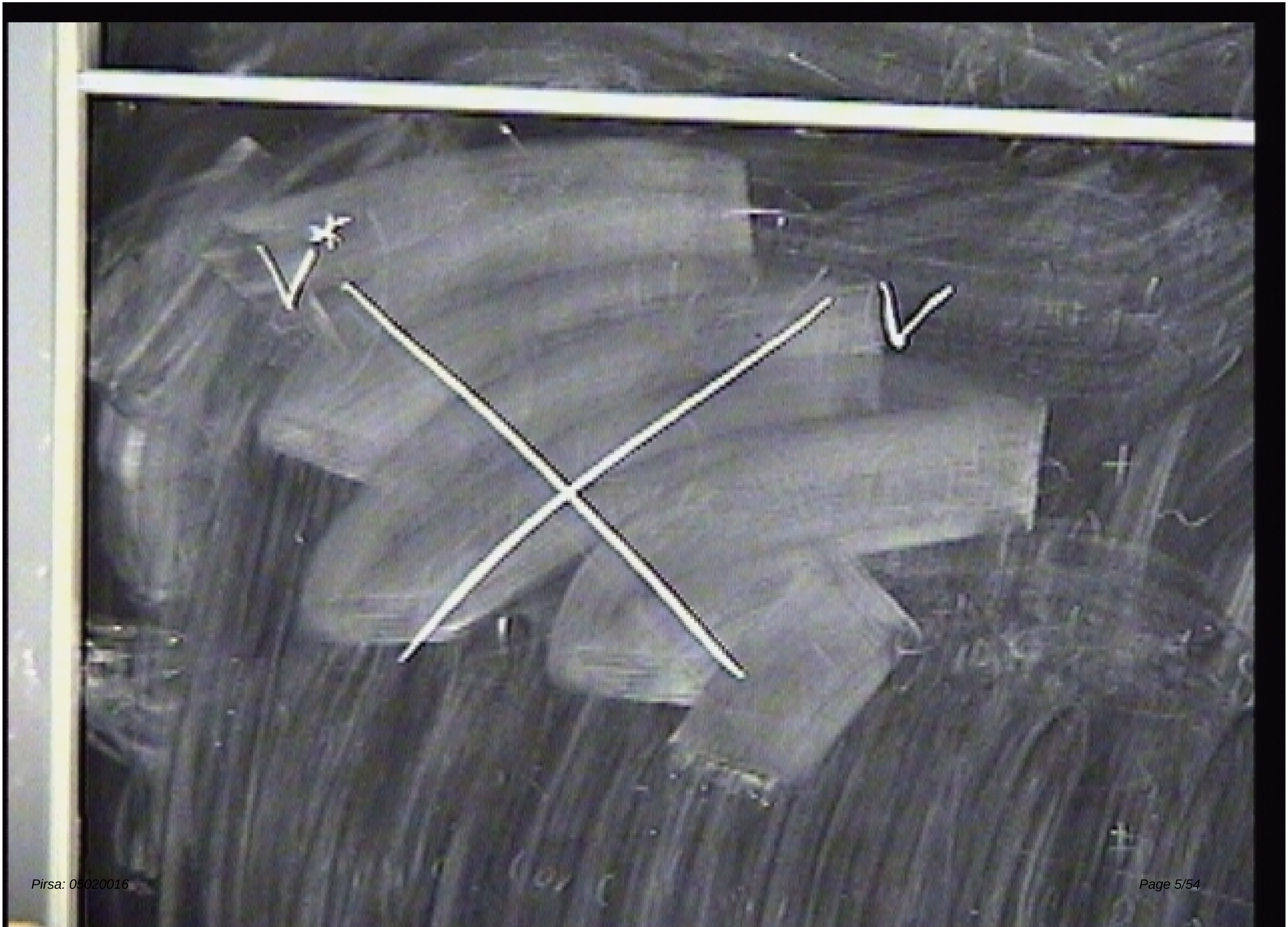
- A definite $\begin{cases} + \\ - \end{cases}$

$A^\perp \cap A = \{0\}$.

- $A \subset A^\perp \Rightarrow A$ isotropic
- $A \supset A^\perp \Rightarrow$ coisotropic.

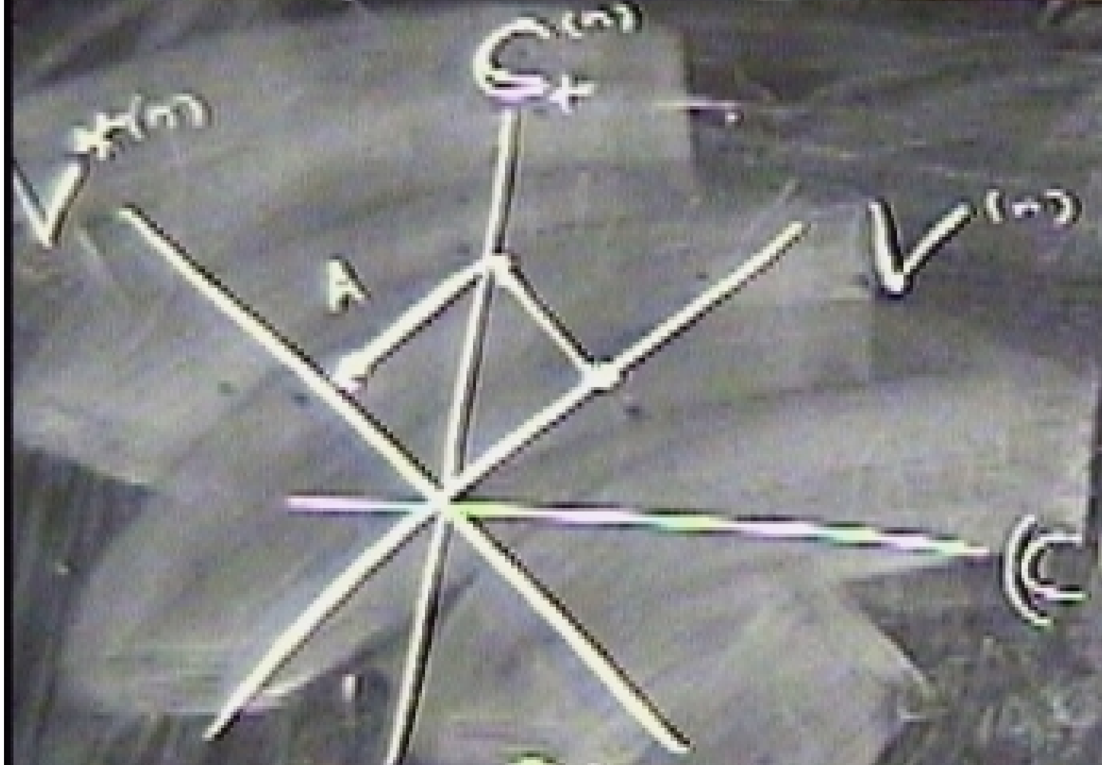
$A = A^\perp \Leftrightarrow A$ Lagrangian

Max. Isotropic


V^*
V

$E = V \otimes V^*$

Polarization



$$E = V \oplus V^*$$

$$(C_+^{\perp})^{\perp} = -\text{ve definite}$$

$$C^+ \ +\text{ve} \Leftrightarrow A : V \longrightarrow V^*$$

$$\left(A(x,x) \right) > 0 \quad \forall\, x \neq$$

C_+ +ve

Space of C_+

$$\| $$

$$\mathrm{Sym}^2_+ V^* \oplus \Lambda^2 V^*$$

$(C^1_+)^\perp$ = $-$ve definite

$\Leftrightarrow$ $A : V \longrightarrow V^*$ pos def

$(A(x,x) > 0 \quad \forall x \neq 0)$

$A \in V^* \otimes V^*$

$\boxed{A = g + b} \in \mathrm{Sym}^2_+ V^* \oplus \Lambda^2 V^*$

G metric on $V \oplus V^*$ sig$(2n, 0)$

$$\langle \,,\, \rangle|_{C_+} - \langle \,,\, \rangle_{C_-}$$

G metric on $V \oplus V^*$ sig(

$$\langle\, , \rangle\big|_{C_+} - \langle\, , \rangle_{C_-}$$

reduces structure

$$O(n,n) \downarrow O(n) \times O(n).$$

if $b=0$ $\quad G(X+\xi, Y+\eta) = \frac{1}{2}\left(g(X,Y) + g^{-1}(\xi,\eta)\right)$

$$\left\langle \begin{pmatrix} & g^{-1} \\ g & \end{pmatrix} \begin{matrix} X \\ \xi \end{matrix}, \begin{matrix} Y \\ \eta \end{matrix} \right\rangle$$

$$\left\langle \underline{g(X)} + g^{-1}\xi, \; Y+\eta \right\rangle$$

$$g(X,Y) + g^{-1}(\xi,\eta)$$

$\boxed{D}$ Action of $\Lambda^2 V^*$ on $V \oplus V^*$

$V \oplus V^*$ $\quad O(n,n) \quad$ $\mathfrak{so}(V \oplus V^*) = \Lambda^2 (V \oplus V^*$

$\cup$

$\Lambda^2 V \oplus \Lambda^2 V^* \oplus$

on $V \oplus V^*$

$$\mathfrak{so}(V \oplus V^*) = \Lambda^2 (V \oplus V^*)$$

$$\cup$$

$$\Lambda^2 V \oplus \Lambda^2 V^* \oplus \mathrm{End}\, V$$

$\boxed{D}$ Action of $\Lambda^2 V^*$ on V

$V \oplus V^*$ $\quad O(n,n) \quad$ $\mathfrak{so}(V \oplus V^*)$

$$\begin{pmatrix} A & \beta \\ B & -A^* \end{pmatrix} \begin{matrix} X \\ + \\ \xi. \end{matrix}$$

$$\begin{pmatrix} 0 & 0 \\ B & 0 \end{pmatrix} \begin{matrix} X \\ \xi \end{matrix} = \begin{pmatrix} 0 \\ B(X) \end{pmatrix}$$

$$i_X B$$

$\ni$ coisotrop...

$A = A^{\perp} \iff A$ Lagran...
Max. Isot...

$$\begin{pmatrix} 0 & 0 \\ B & 0 \end{pmatrix} \begin{pmatrix} X \\ \xi \end{pmatrix} = \begin{pmatrix} 0 \\ B(x) \end{pmatrix}$$

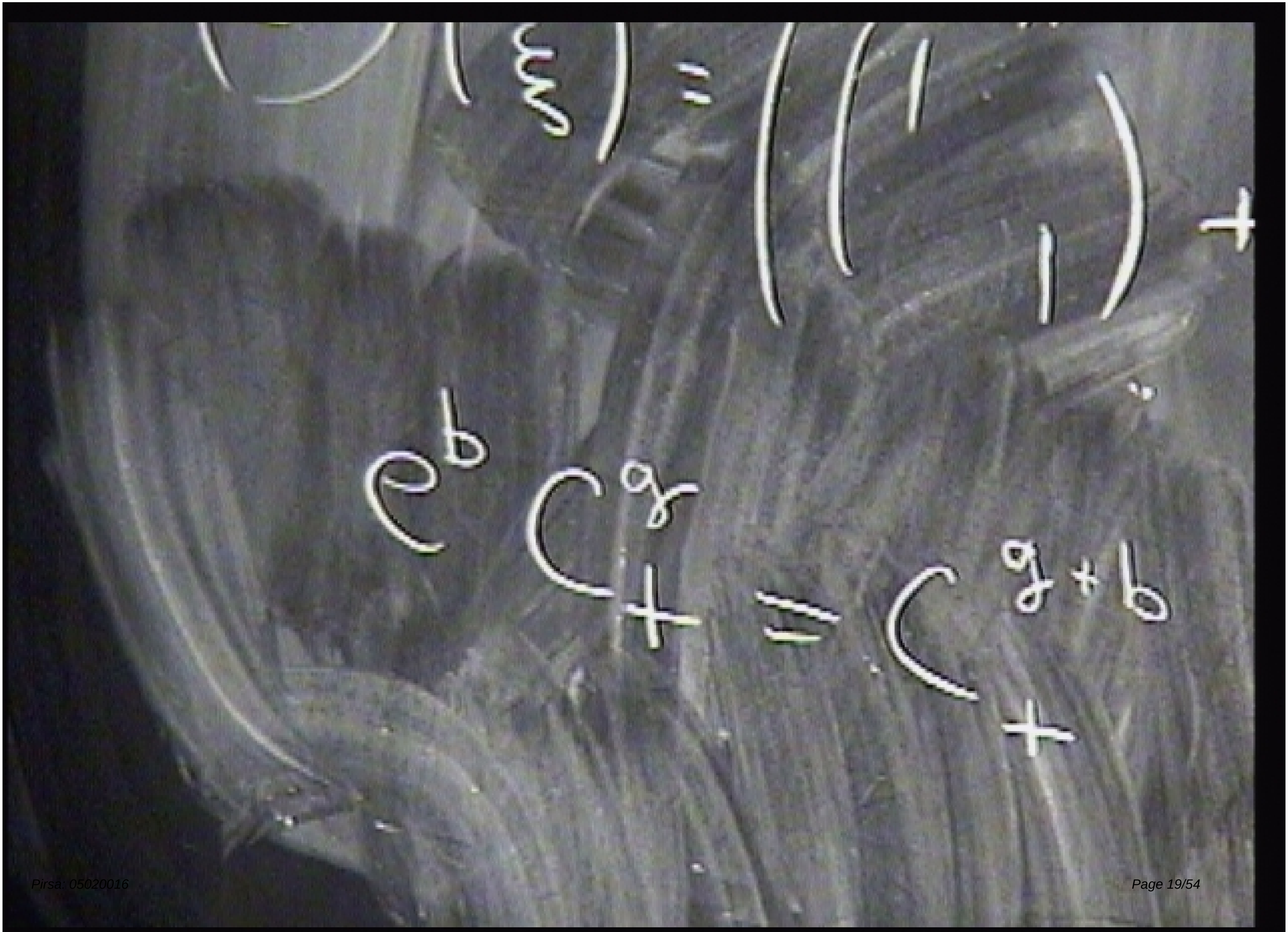
$$e^B \begin{pmatrix} X \\ \xi \end{pmatrix} = \left(\begin{pmatrix} 1 & \\ & 1 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ B & 0 \end{pmatrix} \right) \begin{pmatrix} X \\ \xi \end{pmatrix} = X + \xi + i_X B$$

$i_X B$

V^* V

$\begin{pmatrix} & \\ B & 0 \end{pmatrix} \xi = X + \xi + i_X B$

e.g. $C_+ = Gr(g)$


$e^b \, \mathbb{C}^g_+ = \mathbb{C}^{g+b}_+$

$$\left\langle \begin{pmatrix} & \\ g & \end{pmatrix} \xi , \eta \right\rangle .$$

$$\left\langle e^{b} \begin{pmatrix} & g^{-1} \\ g & \end{pmatrix} e^{-b} \begin{matrix} X \\ \xi \end{matrix} , \begin{matrix} Y \\ \eta \end{matrix} \right\rangle .$$

$$\begin{matrix} \| \end{matrix}$$

$$\begin{pmatrix} -g^{-1}b & g^{-1} \\ g - b g^{-1} b & b g^{-1} \end{pmatrix}$$

NOTE: $G\big|_V = g - b g^{-1} b$

Riem. metric on V

Riem.

$$\mathrm{vol}_G \in \det V^*$$

$$V \xrightarrow{A} V^*$$

$$\det V \xrightarrow{\det A} \det V^* \Rightarrow \det A \in (\det V^*)^2$$

$$\det^{1/2} A \in \det V^*$$

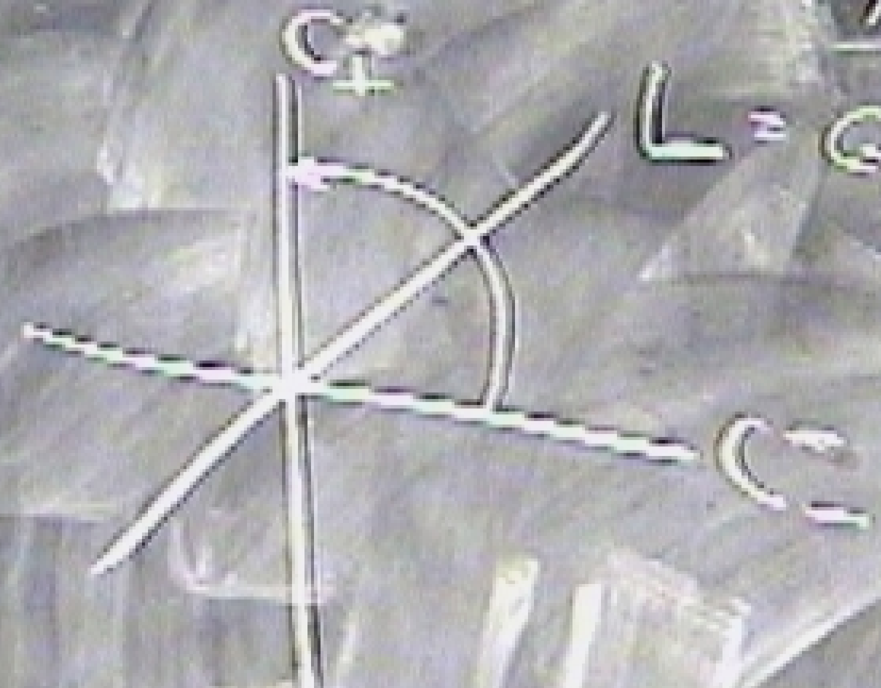
$$\mathrm{vol}_{G} = \det{}^{1/2}\left(g - b g^{-1} b\right)$$

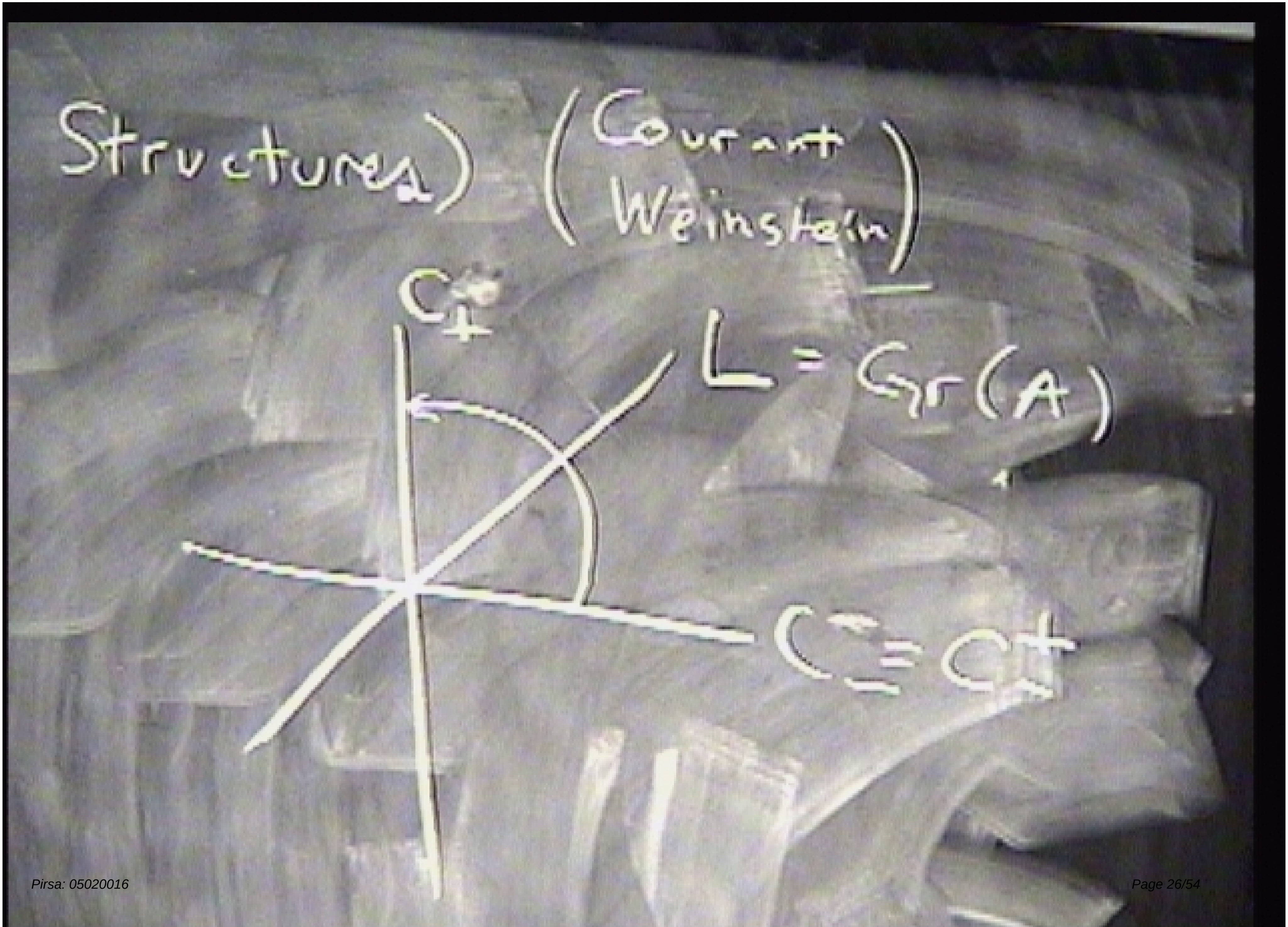
$$\ldots - b g^{-1} b) = (\det g)(1 - g^{-1} b g^{-1} b \ldots$$

$$\mathrm{vol}_G = \frac{\det(g+b)}{\det^{1/2} g}.$$

$C \subset E$ (Dirac Structures) (Courant Weinstein)

$E = C_+ \oplus C_-$




Structures) (Courant Weinstein)
C_+
$L = Gr(A)$
$C_- = C_+^\perp$

$$E = C_+ \oplus C_-$$

$$\mathcal{L}ag \cong O(n)$$

$$\cap$$

$$O(n,n)$$

2 components

even odd

$E = N \oplus P$

polarization

$\langle , \rangle \quad N \to P^*$

P^*

f

polarization

P^*

P

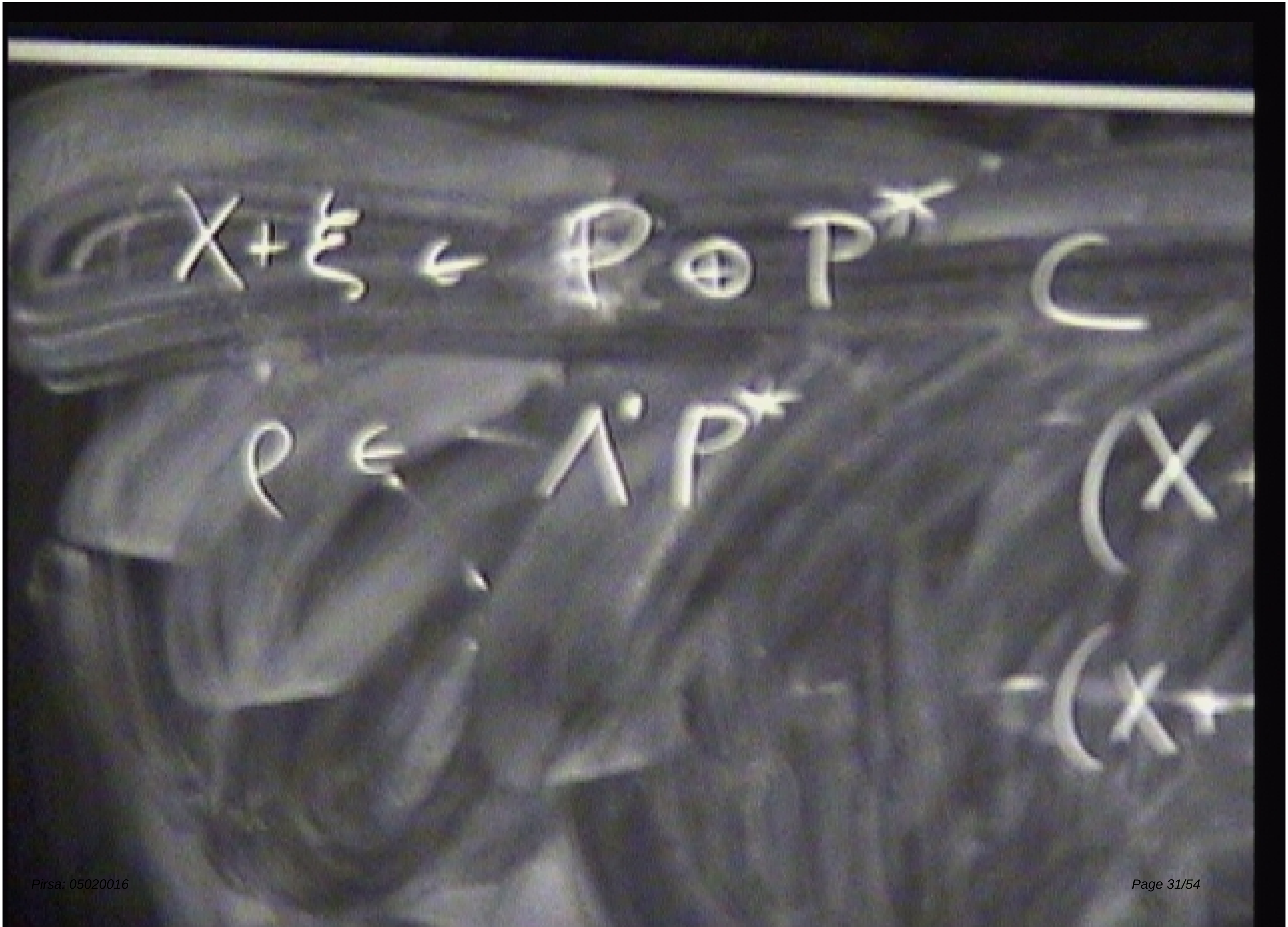
$$Cl(E) \cong Cl_{n,n} = \mathrm{Mat}_{2^n \times 2^n}(\mathbb{R}$$

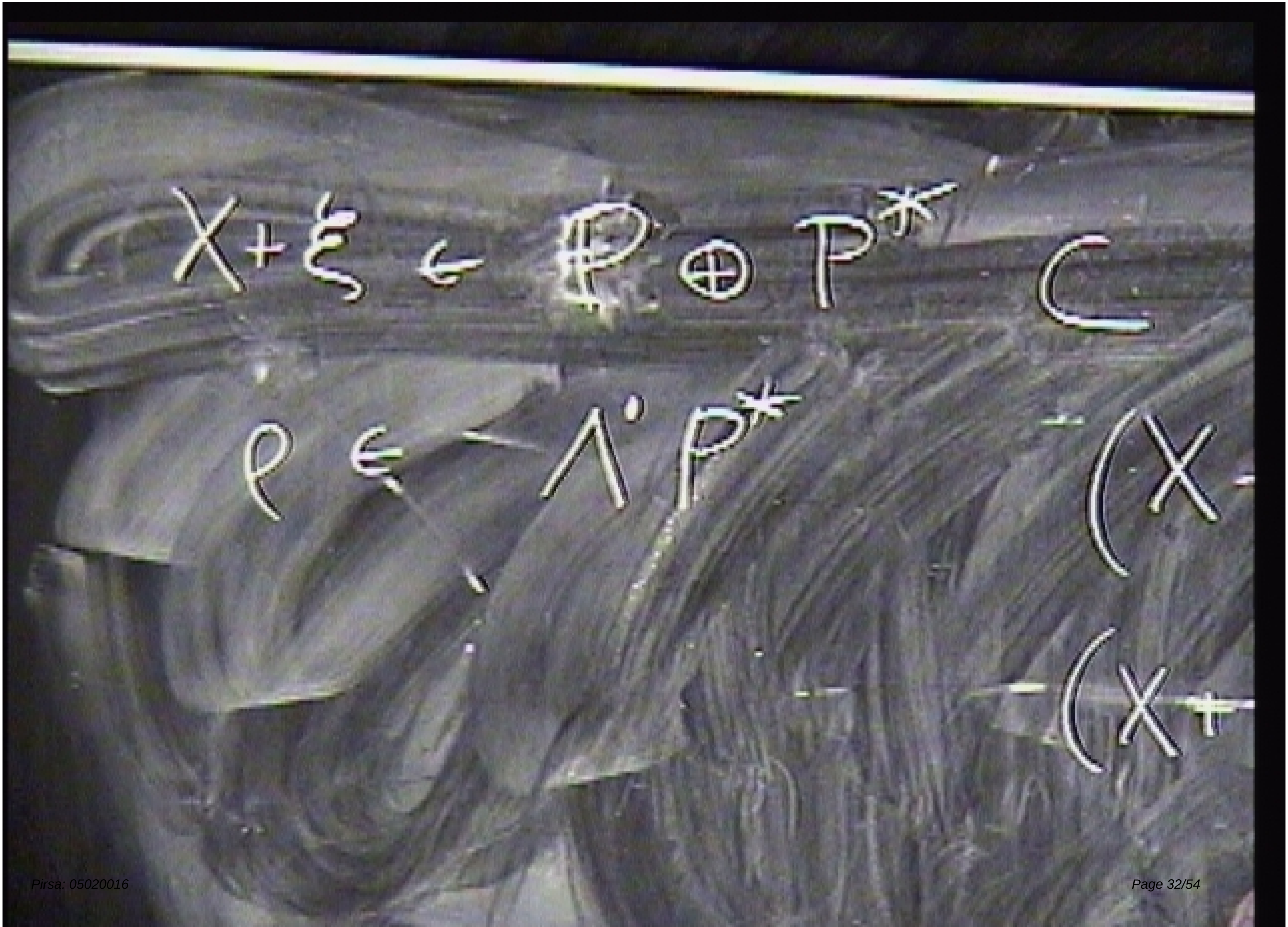
$$Cl(E) \cong \mathbb{C}L_{n,n} =$$

$$\|$$

$$\operatorname{End}(\Lambda^{\bullet} P^{*})$$

$$2^n$$


$X+\xi \in P \oplus P^* \subset$
$\rho \in \Lambda^{\bullet} P^*$
$(X+$
$-(X+$


$X+\xi \in P \oplus P^* \subset$
$\rho \in \Lambda^\bullet P^*$
$(X$
$(X+$

$$C \; C\ell(E)$$

$$(X+\xi)\cdot\rho = i_X\rho + \xi\wedge\rho$$

$$(X+\xi)^2\cdot\rho = \underbrace{\|X+\xi\|^2}_{\langle\,,\,\rangle}\,\rho$$

$$-(x+\xi)^2 \cdot \rho = \|x$$

extends $Cl(E) \curvearrowright \Lambda^{\cdot} P^* = S$

$$S^{\pm} = \Lambda^{ev/od} P^*$$

of $\rho \in \Lambda^{\cdot} P^*$

$$\mathrm{Ann}(\rho) := \{ X+\xi \in P \oplus P^* :$$

$$\forall X+\xi \quad \|X+\xi\|^2_\rho = (X+\xi$$

$\Rightarrow \mathrm{Ann}(\rho)$ $\underline{\text{Isotropic}}$ (null)

dimension $\in \{0, 1, \ldots, n\}$

$$P \oplus P^* : (X+\xi)\rho = 0\}$$

$$(X+\xi)^2 \rho = 0$$

(null)

$$\boxed{\text{Lag} \Leftrightarrow \rho \text{ Pure}}$$

$E = P \oplus P^*$

examples of pure spinors.

- $1 \in \Lambda^0 P^*$

$\mathrm{Ann}(1) = P$

$i_P \qquad P^* \wedge$

-

of pure spinors.

$1 \in \Lambda^{\cdot} P^{*}$ $\quad \mathrm{Ann}(1) = P$

$\begin{pmatrix} A & \beta \\ B & -A^{*} \end{pmatrix}$ act on S

$i_P \quad P^{*}\wedge$

$\left(-A^{*} + \frac{1}{2}\mathrm{Tr}\,A\right)$

$B\wedge .$

$$e^{B}\cdot 1 = 1 + B\wedge 1 + \frac{B\wedge B\wedge 1}{2} + \dots$$

$$= e^{B} \in \Lambda^{\bullet} P^{*}$$

Pure spinor

$$= e^{B} \in \Lambda^{\bullet} P^{*}$$

$\uparrow$ Pure spinor

• $\boxed{\Omega = \theta_1 \wedge \theta_2 \wedge \cdots \wedge \theta_k}$ decomposable k-f

$$K^{n-k} = \ker \Omega$$

$$\mathrm{Ann}\,\Omega = K^{n-k} \oplus \langle \theta_1, \ldots, \theta_k \rangle^{k} \;:\; \mathrm{Lag}$$

$$\boxed{S^{\pm} = \Lambda^{ev/od} P^*}$$

$$\underline{\rho = c\, e^{B}\, \Omega^{(k)}}$$

general pure sp

$$\boxed{S^{\pm} = \Lambda^{ev/od}\, P^{*}}$$

$$\underline{\rho = c\, e^{B}\, \Omega^{(k)}}$$ general pure spinor

$$\mathrm{Ann}\,\rho = \{ X+\xi : X \in \mathrm{Ker}(\Omega),\ \xi|_{\ker\Omega} = i_X B \}$$

$$\{\mathrm{Lag}\} \xleftrightarrow{1:1} \left\{\begin{matrix}\text{pure}\\ \text{spinor}\\ \text{lines}\end{matrix}\right\}$$

$T \oplus T^*$, $S = \Lambda^{\bullet} T^*$

Generalized metric

$C_+ < T \oplus T^*$ $\quad g + b$ (b not closed)

$$CL(C_+) \subset CL(T \oplus T^*)$$

$$* = a_1 \ldots a_n = \text{volume of } C_+$$

$C_+ < T \oplus T^*$

$g + b$

Pin

$\Downarrow$

$CL(C_+) \subset$

$\Downarrow$

$* = a_1 \cdots\cdots a_n = \mathrm{vol}$

C_+

C_-

$\xi\, a_i\, \xi_i$ on

$\begin{pmatrix} -1 & \\ & 1 \end{pmatrix}_{C_-}^{C_+} \in O(n,n) = -$

$\mathrm{Ann}\, L = K \oplus L$

$$* \, \sigma(*) = 1$$

$$\sigma(*) = *^{-1}$$

$$*^2 = (-1)^{\frac{n(n-1)}{2}}$$

$$\sigma(*) = a_n \cdots a_1$$

$$* \, \sigma(x) = \ldots$$

$$\sigma(*) = \ldots$$

$$*^2 = (-1 \ldots$$

$$\langle \varphi_n, \psi_n \rangle = \left[\varphi \wedge \sigma(\psi) \right]_n$$

$\Lambda^n p^*$

$$\langle \varphi_n, \psi_n \rangle = [\varphi \wedge \sigma(\psi)]_n \qquad \hat{\Lambda}^{\cdot} p^*$$

$$*^2 = (-1)^{\frac{n(n-1)}{2}}$$

$$(-1)^{\frac{k(k-1)}{2}}$$

$$\langle e^B, e^B \rangle = e^B \wedge e^{-B} = 1 = \langle e^{-B} e^B, 1 \rangle$$

Spin

$$\langle \varphi, \psi \rangle = (-1)^{\frac{n(n-1)}{2}} \langle \psi, \varphi \rangle$$

$$\langle \varphi, \sigma(*) \psi \rangle =$$ Born - Infeld

$$\langle \varphi, \psi \rangle = (-1)^{\frac{n(n+1)}{2}} \langle \psi, \hat{\varphi} \rangle$$

$$\langle \varphi, \underset{\text{Generalised metric}}{\sigma(x)} \psi \rangle$$ Born-Infeld pairi

$$\langle \varphi, \sigma(x) \psi \rangle \quad \text{symmetric} \quad = G(\varphi, \psi)\, vol($$

$\langle \varphi, \sigma(x)\psi \rangle$: Born-Infeld pairing
Generalized metric

$\langle \varphi, \sigma(x)\psi \rangle$ symmetric $= \underbrace{G(\varphi,\psi)}_{\text{pos. def.}} vol_G$

if $b=0$ then

$\langle \varphi, \sigma(x)\psi \rangle = (\varphi \wedge * \psi) = g(\varphi,\psi)\, vol_g$

$d_H = d + H\wedge$

H closed 3-form

$d_H^2 = 0$

$H^{\cdot}_{d_H}$ twisted cohomology

$d_H^* = * \, d_H *^{-1}$

H closed

$d_H^2 = 0$

$H^\cdot_{d_H}$ twisted coho

$d_H^* = \pm * d_H *^{-1}$

$\Delta_{d_H} = (d_H + d_H^*)^2$

elliptic

every twisted coh class has ! Δ_{d_H}-harmonic represent.

$$\begin{tikzcd}
(M, g, b, H) \arrow[r, "e^{b'}"] \arrow[d, Rightarrow] & (M, g, b+b', H-db') \arrow[d] \\
\Delta_{d_H}\text{-harm elts} \arrow[r] & \Delta_{d_{H-db'}}
\end{tikzcd}$$