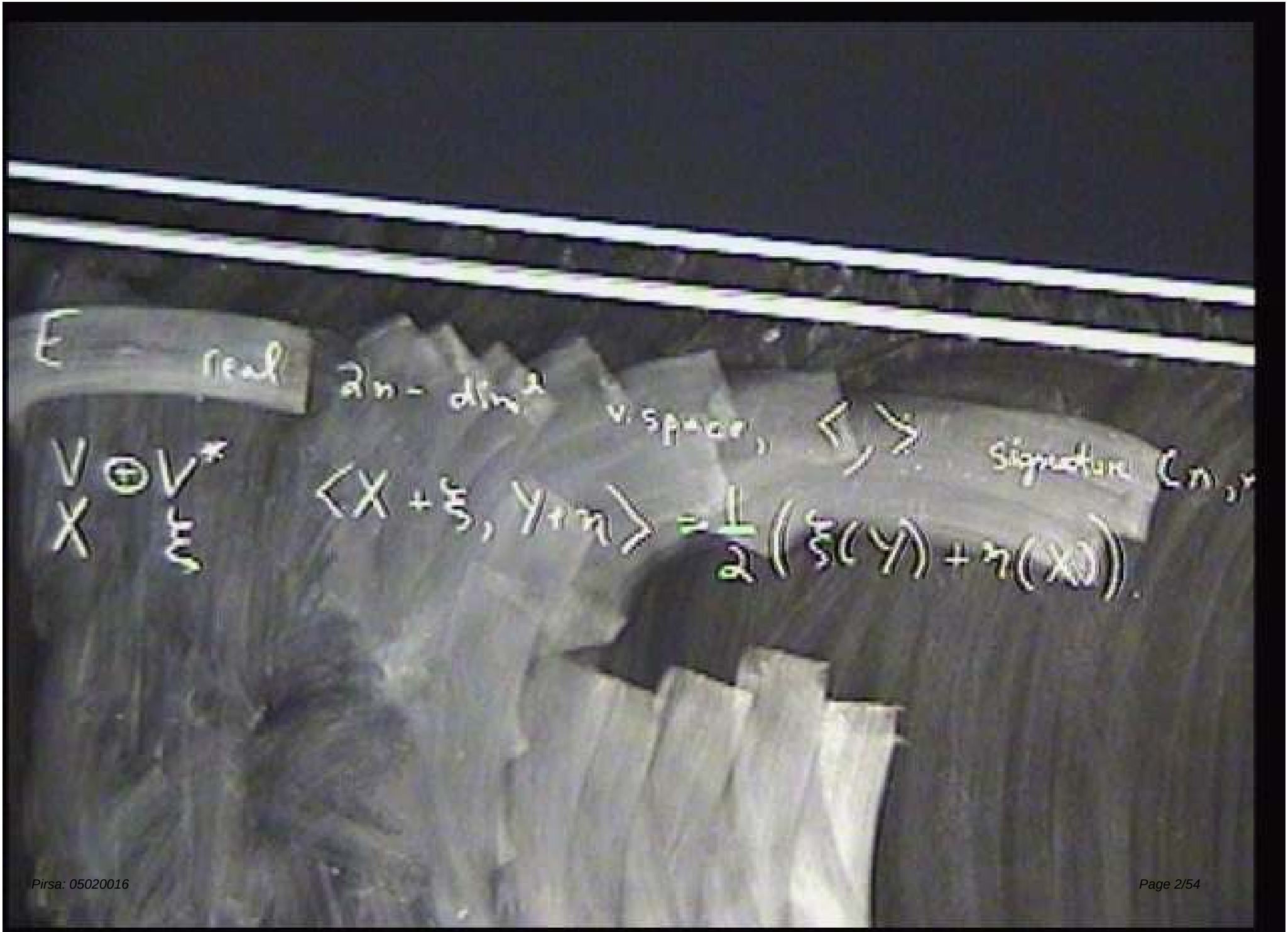
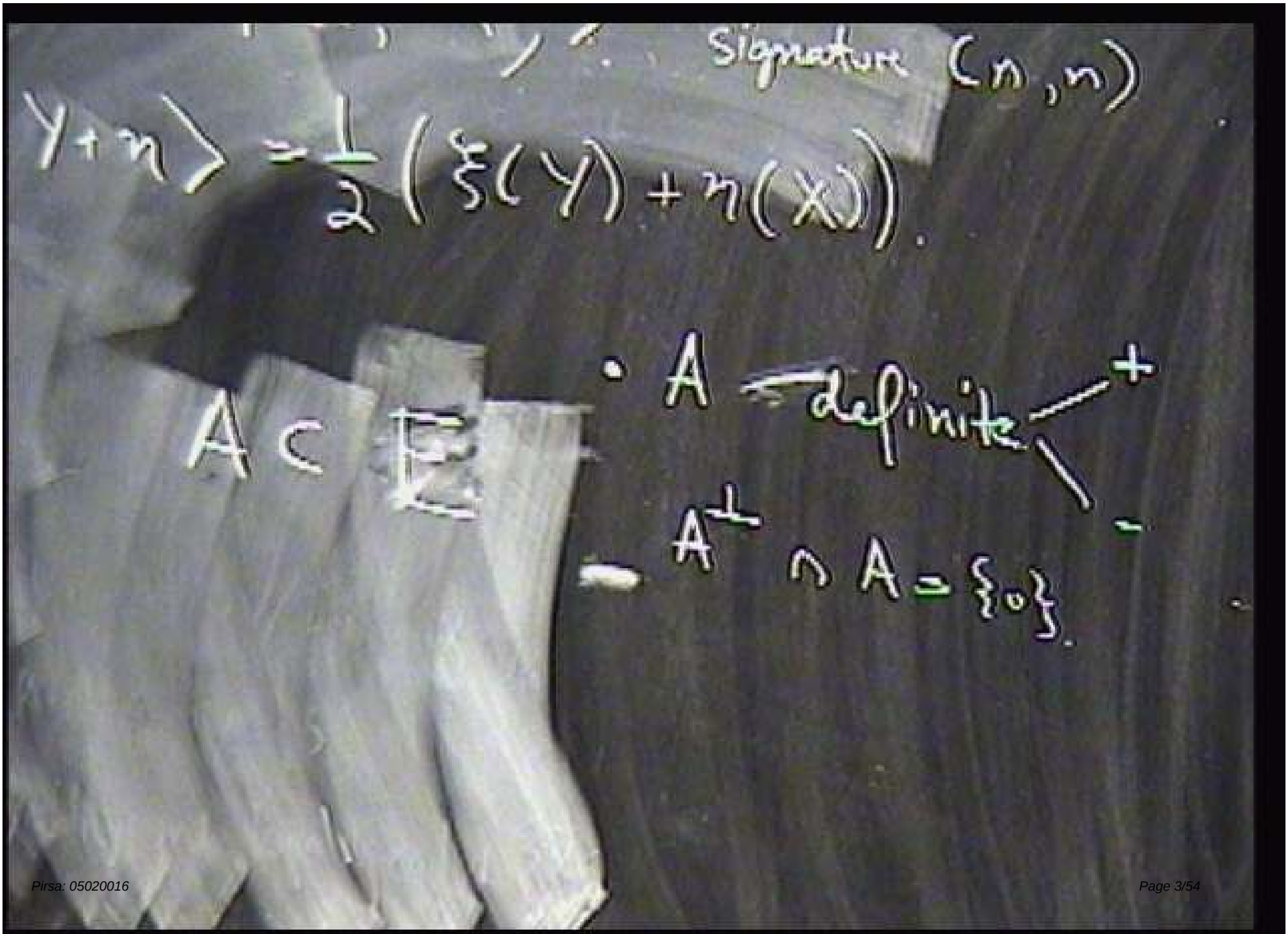
Title: Generalized Geometric Structures (Part 1)

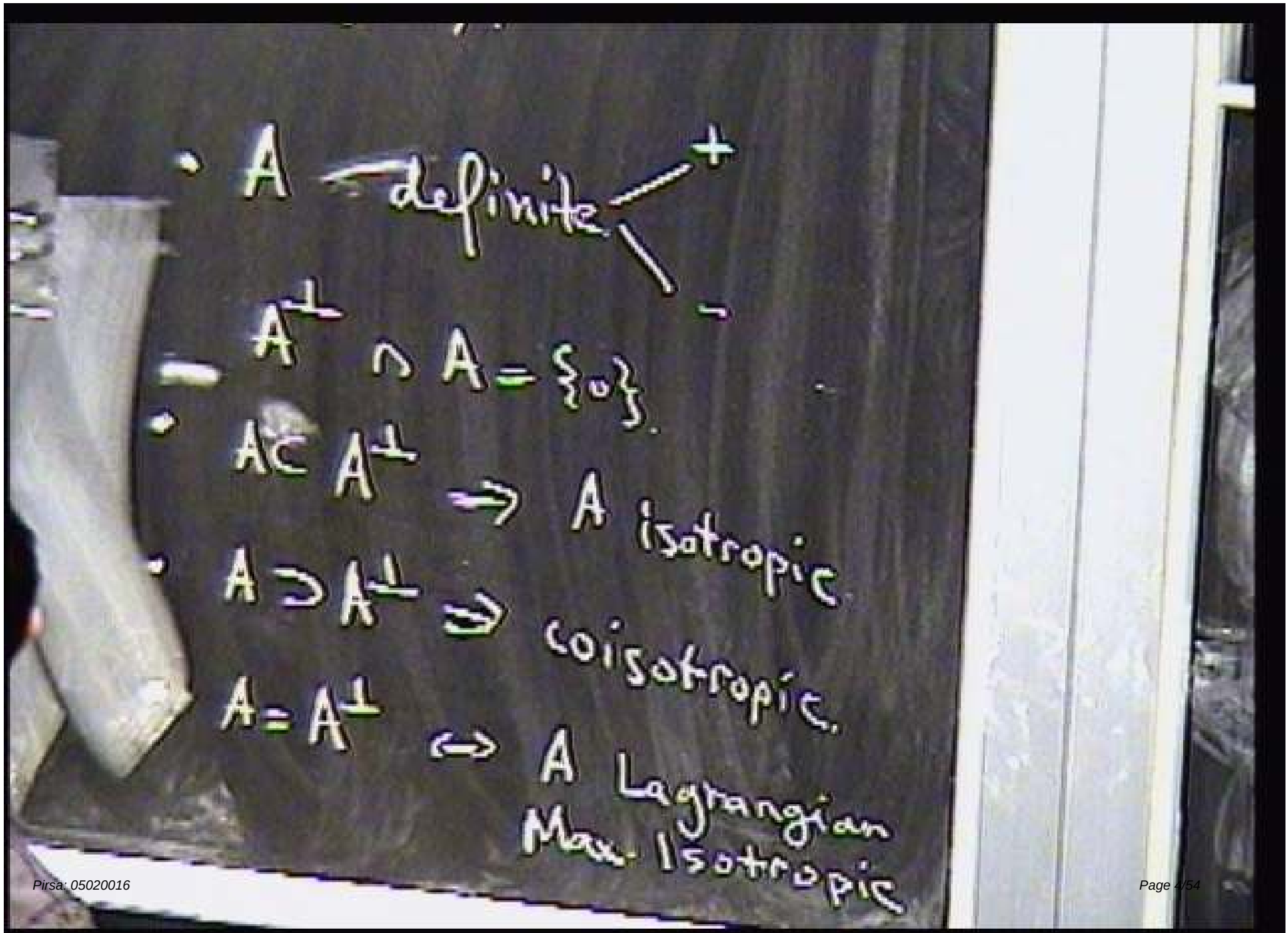
Date: Feb 15, 2005 11:00 AM

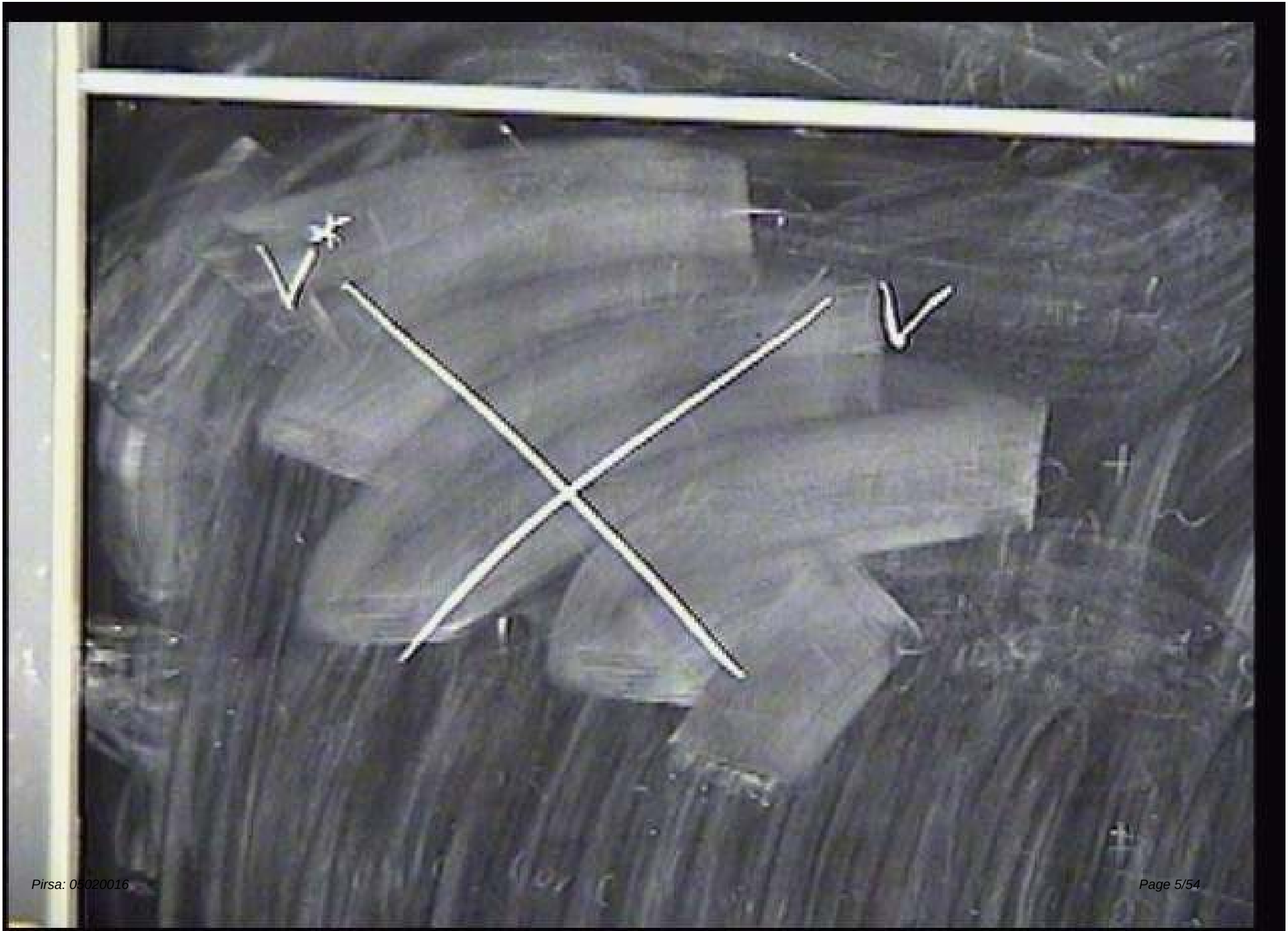
URL: http://pirsa.org/05020016

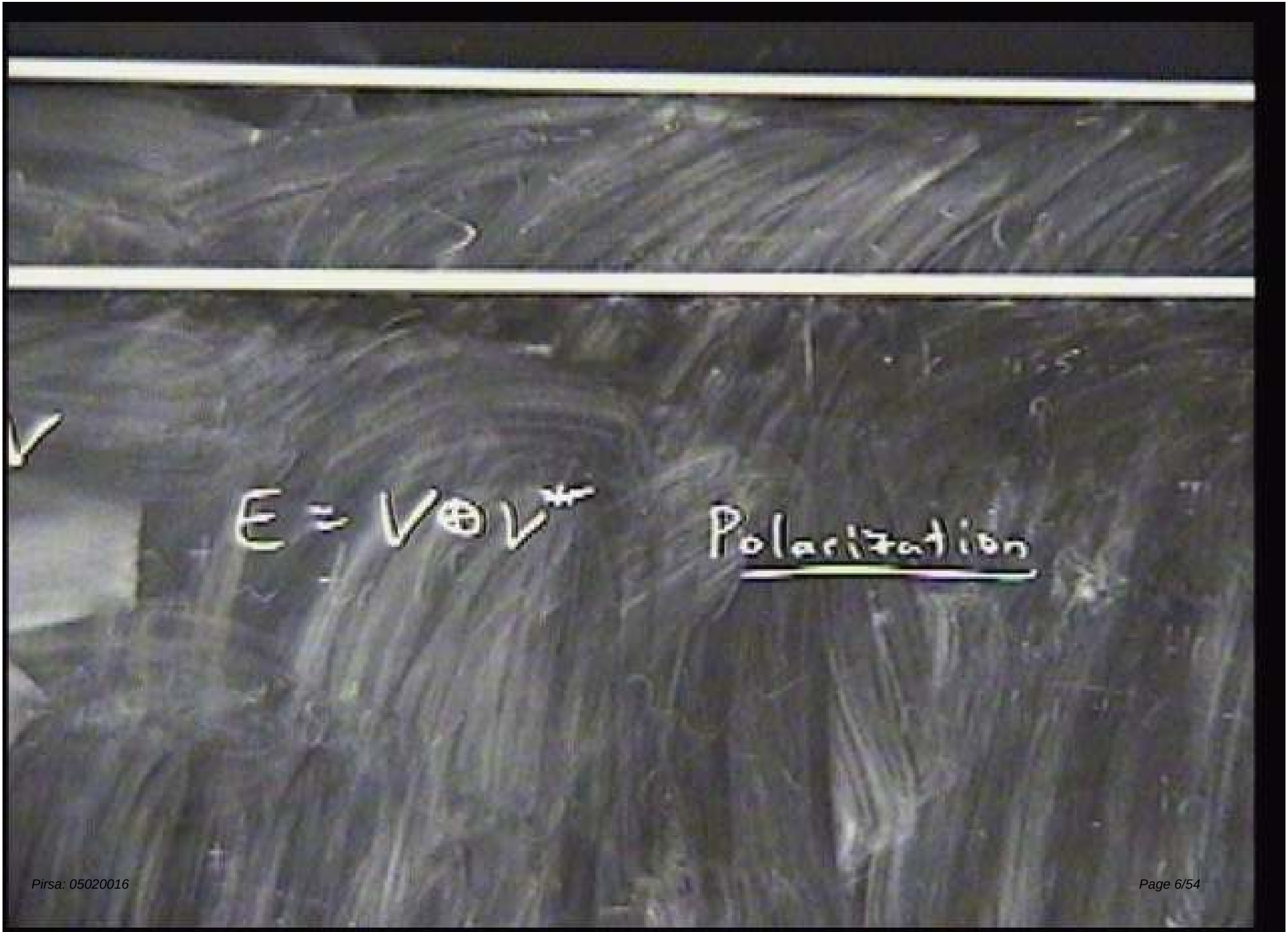
Abstract:

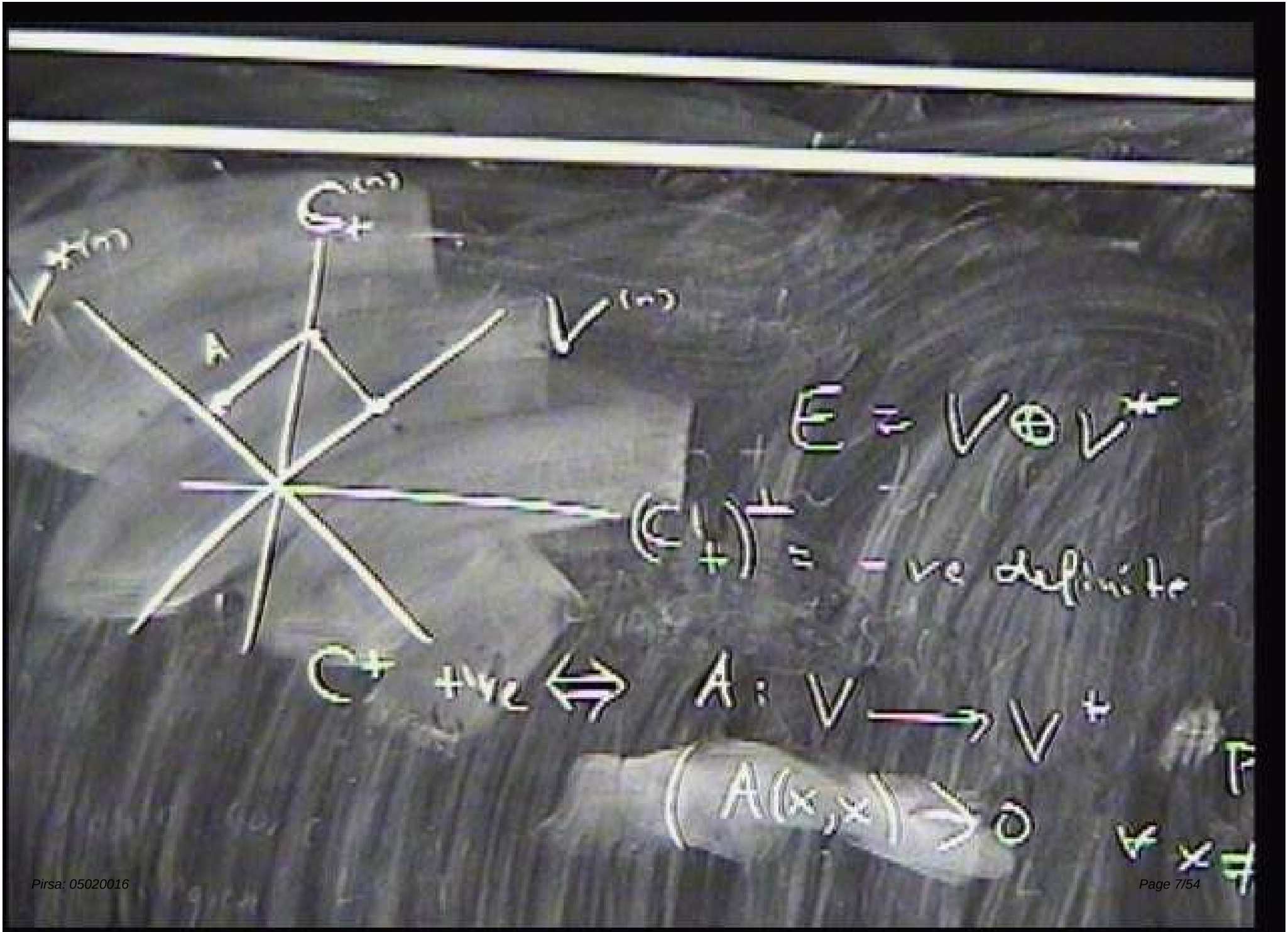

E
real
$2n$ - $\dim_{\mathbb{R}}$
v. space, $\langle , \rangle$
Signature (n, n)
$V \oplus V^*$
$X \quad \xi$
$\langle X + \xi, Y + \eta \rangle = \frac{1}{2}(\xi(Y) + \eta(X))$

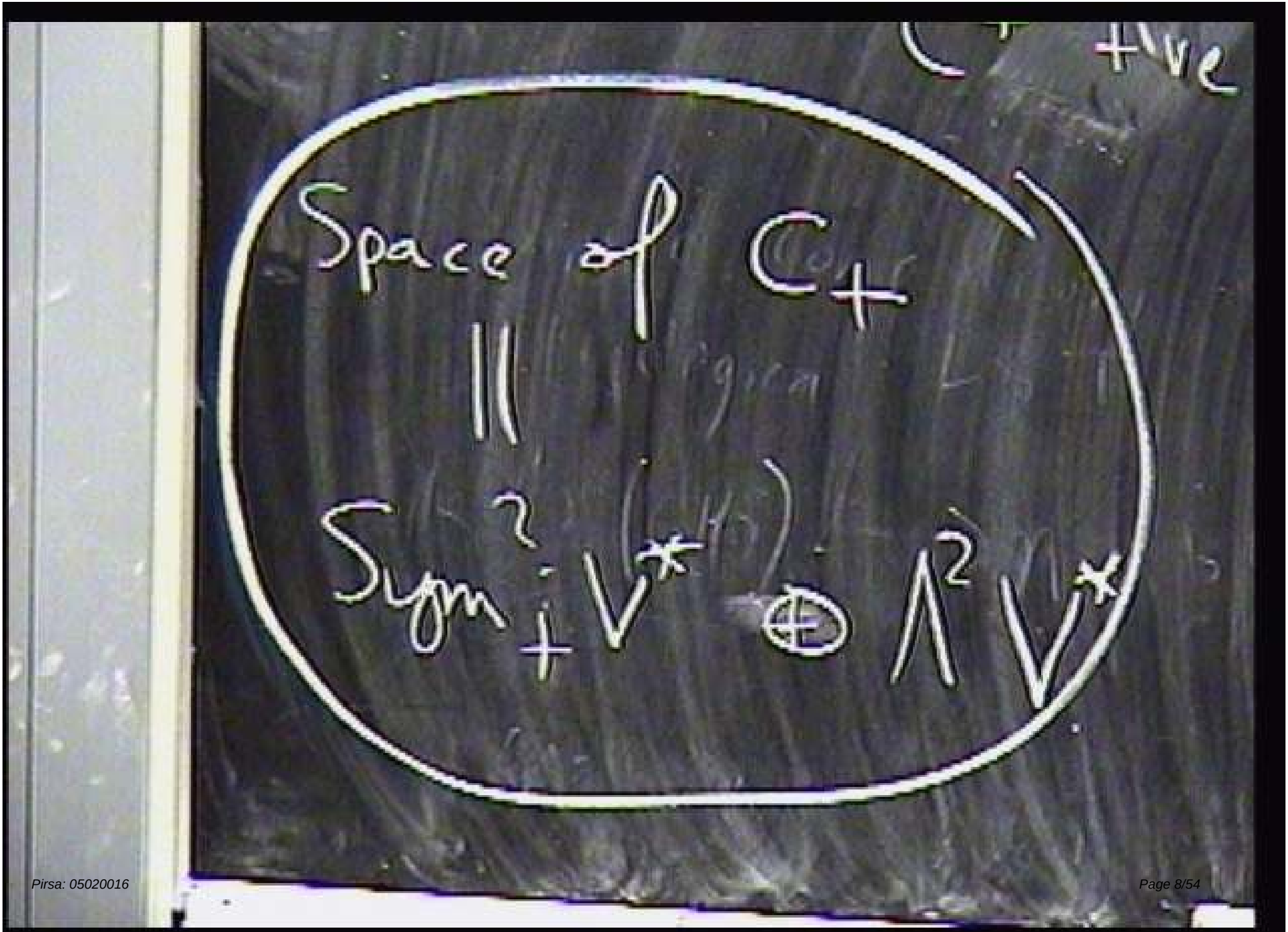

Signature (n, n)
Y + η⟩ = 1/2 (ξ(Y) + η(X)).
A ⊂ E
• A — definite
+
−
A⊥ ∩ A = {0}.


A definite + -
$A^\perp \cap A = \{0\}$
$A \subset A^\perp \Rightarrow$ A isotropic
$A \supset A^\perp \Rightarrow$ coisotropic
$A = A^\perp \Leftrightarrow$ A Lagrangian
Max. Isotropic


V*
V


V
E ~ V ⊗ V*
Polarization


$C_+^{(n)}$
$V^{+(n)}$
$V^{(n)}$
$E = V \oplus V^*$
$(C_+^{(n)})^\perp$ = -ve definite
C_+ +ve $\Leftrightarrow$ $A : V \longrightarrow V^+$
$A(x, x) > 0$
$\forall x \neq$


+ve
Space of C_+
$\| \cong$
$\mathrm{Sym}^2 V^* \oplus \Lambda^2 V^*$

$(C^1_+)^{\perp}$ = - ve definite.

$\Leftrightarrow$ $A : V \longrightarrow V^*$ Pos def.

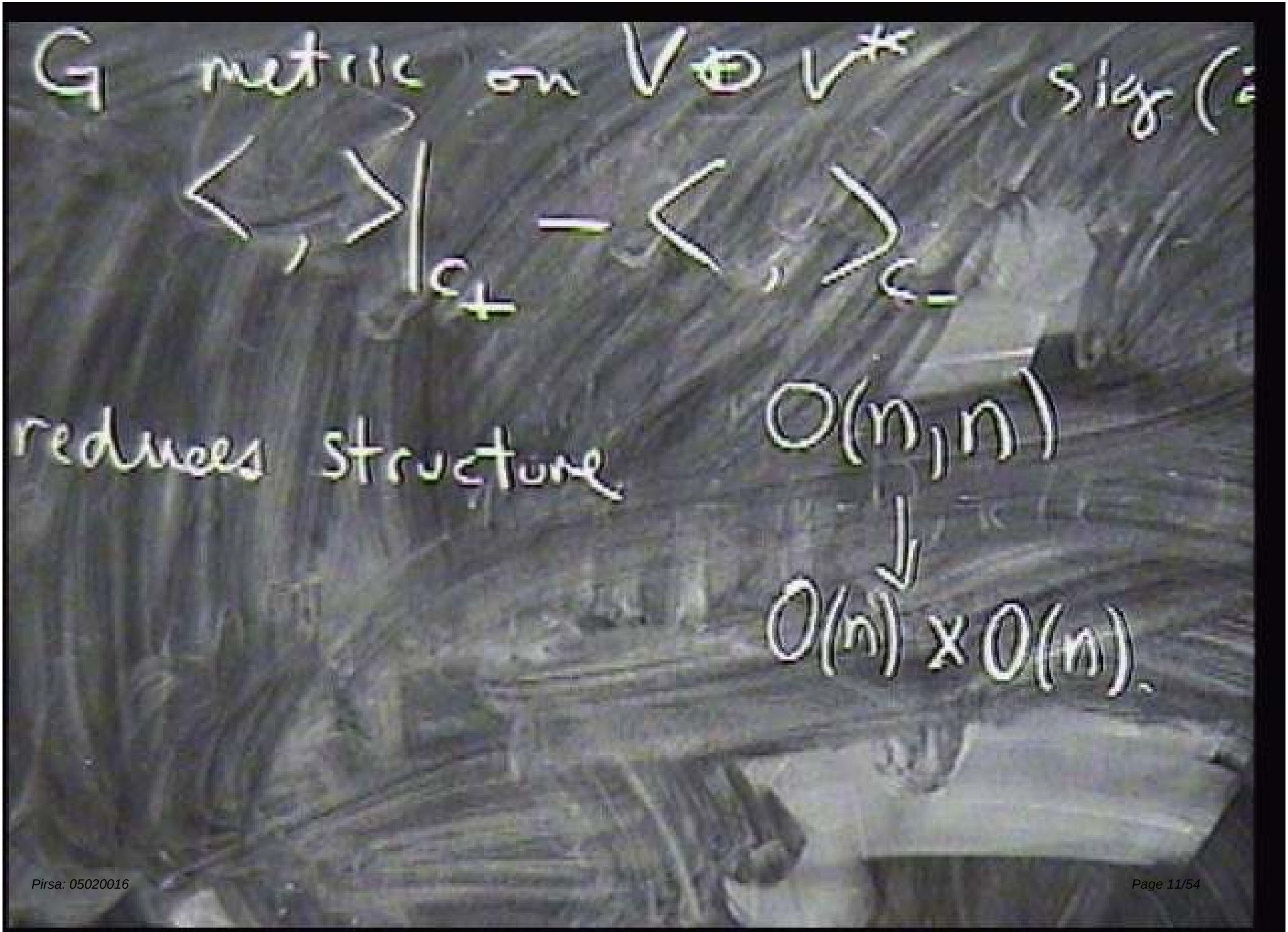
$(A(x,x) > 0 \quad \forall x \neq 0)$

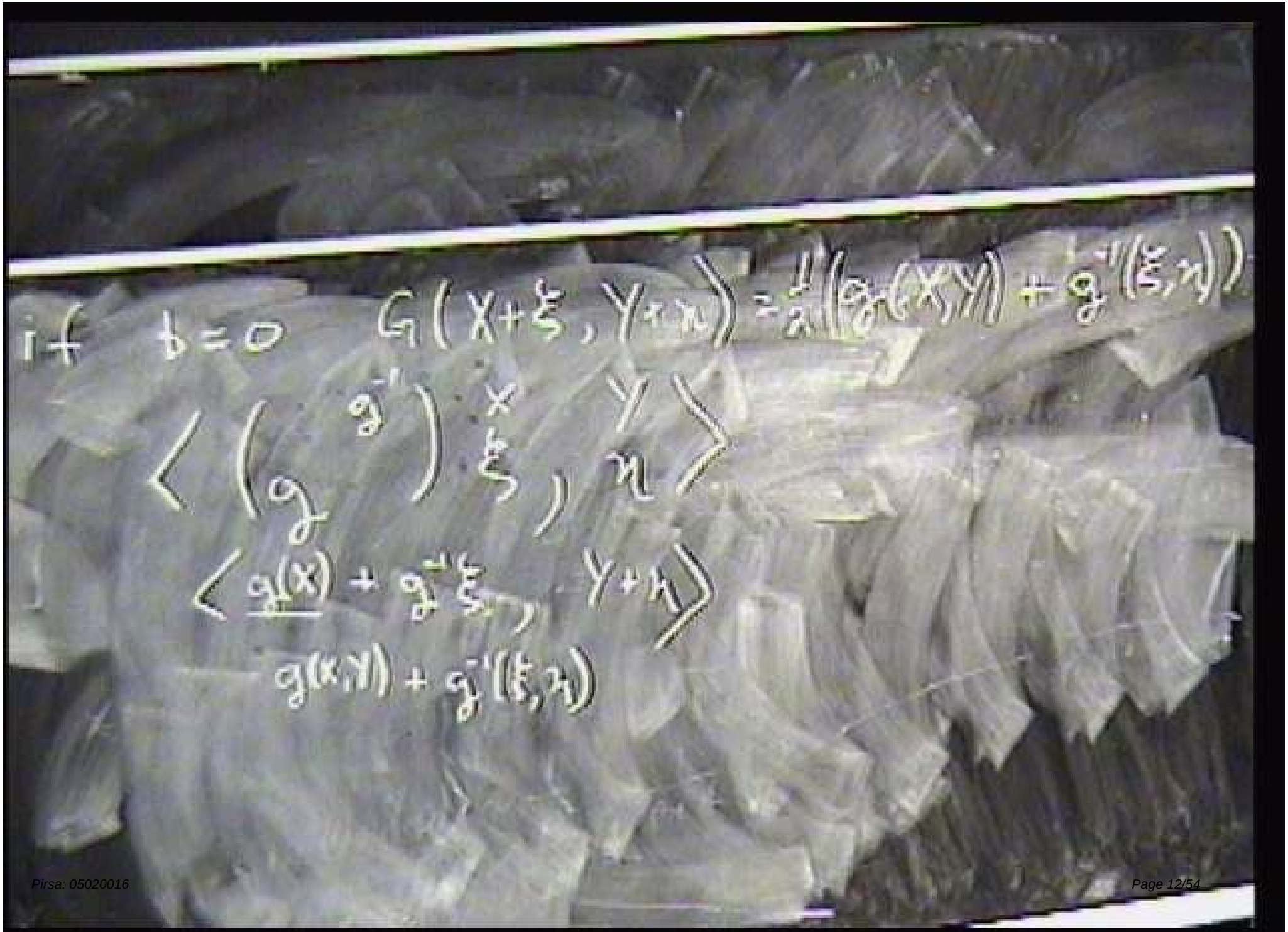
$A \in V^* \otimes V^*$

$A = g + b \in \mathrm{Sym}^2_+ V^* \oplus \Lambda^2 V^*$

G metric on $V \oplus V^*$ $\quad$ sig $(2n, 0)$

$$\langle , \rangle|_{C_+} - \langle , \rangle_{C_-}$$


G metric on V ⊕ V*
sig (
⟨ , ⟩|c+ − ⟨ , ⟩c−
reduces structure
O(n, n)
O(n) × O(n)


if b = 0
$G_1(X+\xi, Y+\eta) = \frac{1}{2\pi}\left(g_0(X,Y) + g^{-1}(\xi,\eta)\right)$
$\left\langle \left(g^{-1}\right) X, Y \right\rangle$
$\left\langle \xi, \eta \right\rangle$
$\left\langle g(X) + g^{-1}\xi, Y+\eta \right\rangle$
$g(X,Y) + g^{-1}(\xi,\eta)$

D] Action of $\Lambda^2 V^*$ on $V \oplus V^*$

$V \oplus V^*$ $\quad O(n,n)$ $\quad \mathfrak{so}(V \oplus V^*) = \Lambda^2(V \oplus V^*$

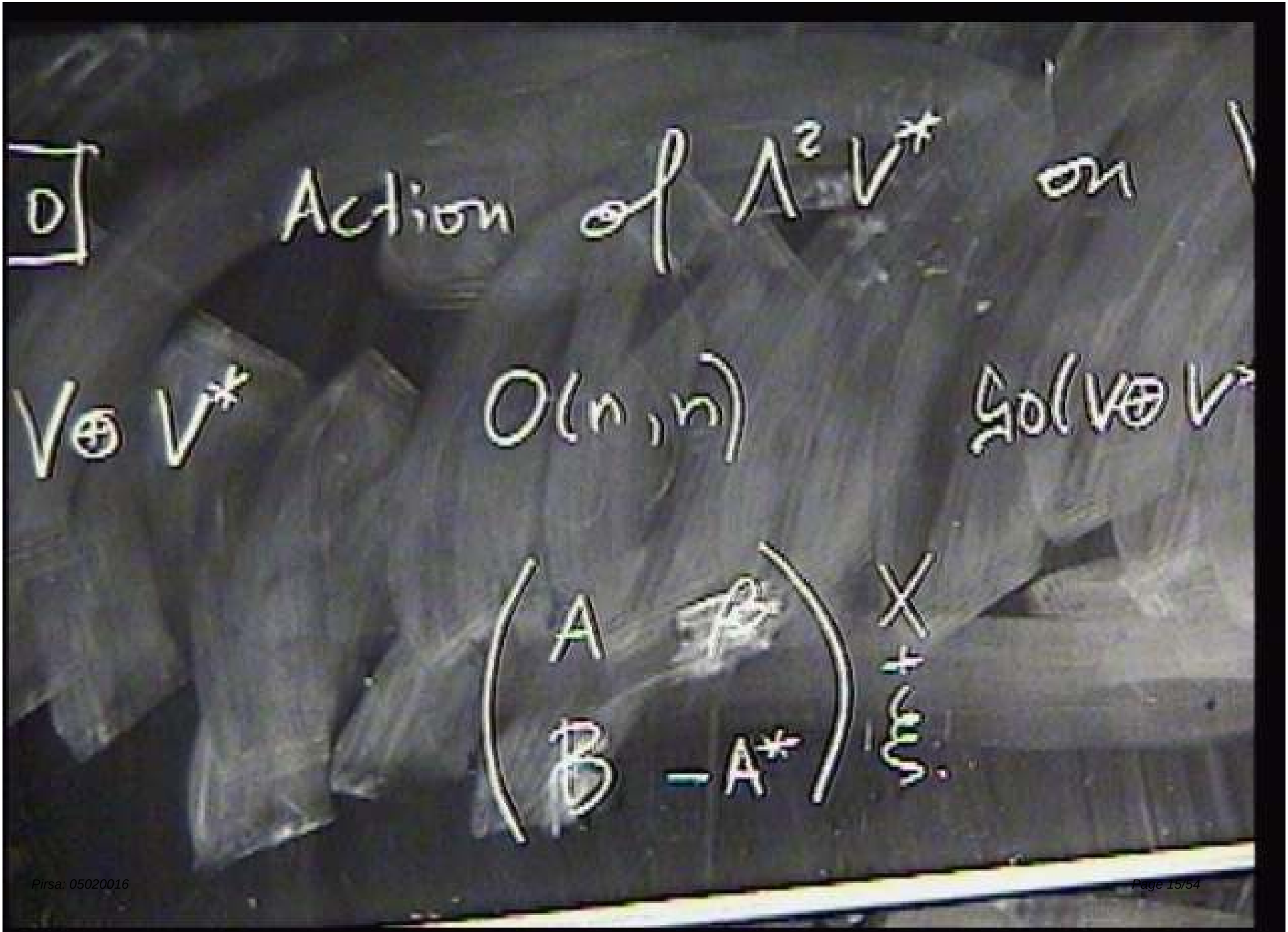
$\cup$

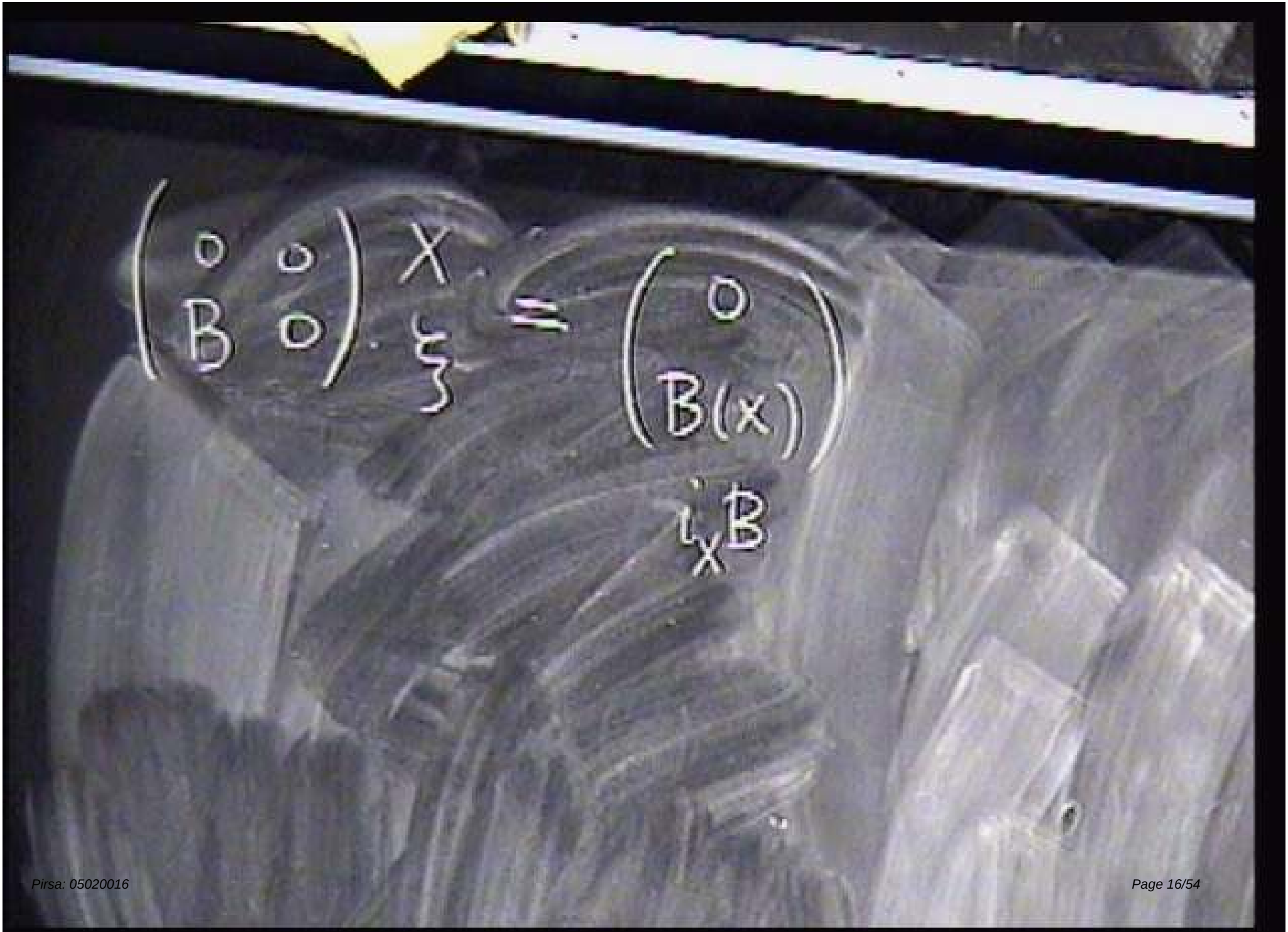
$\Lambda^2 V \oplus \Lambda^2 V^* \oplus$

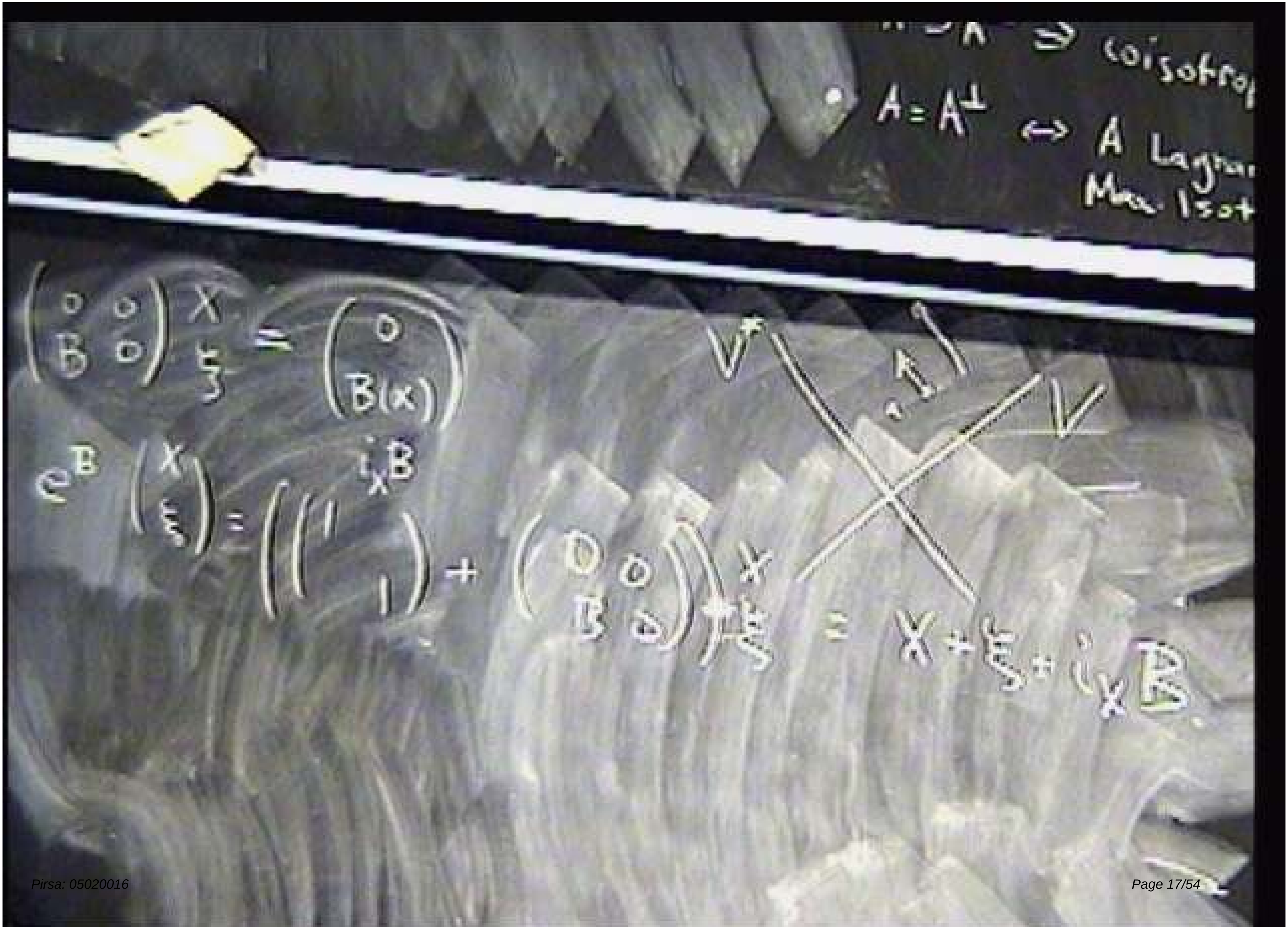
on $V \oplus V^*$

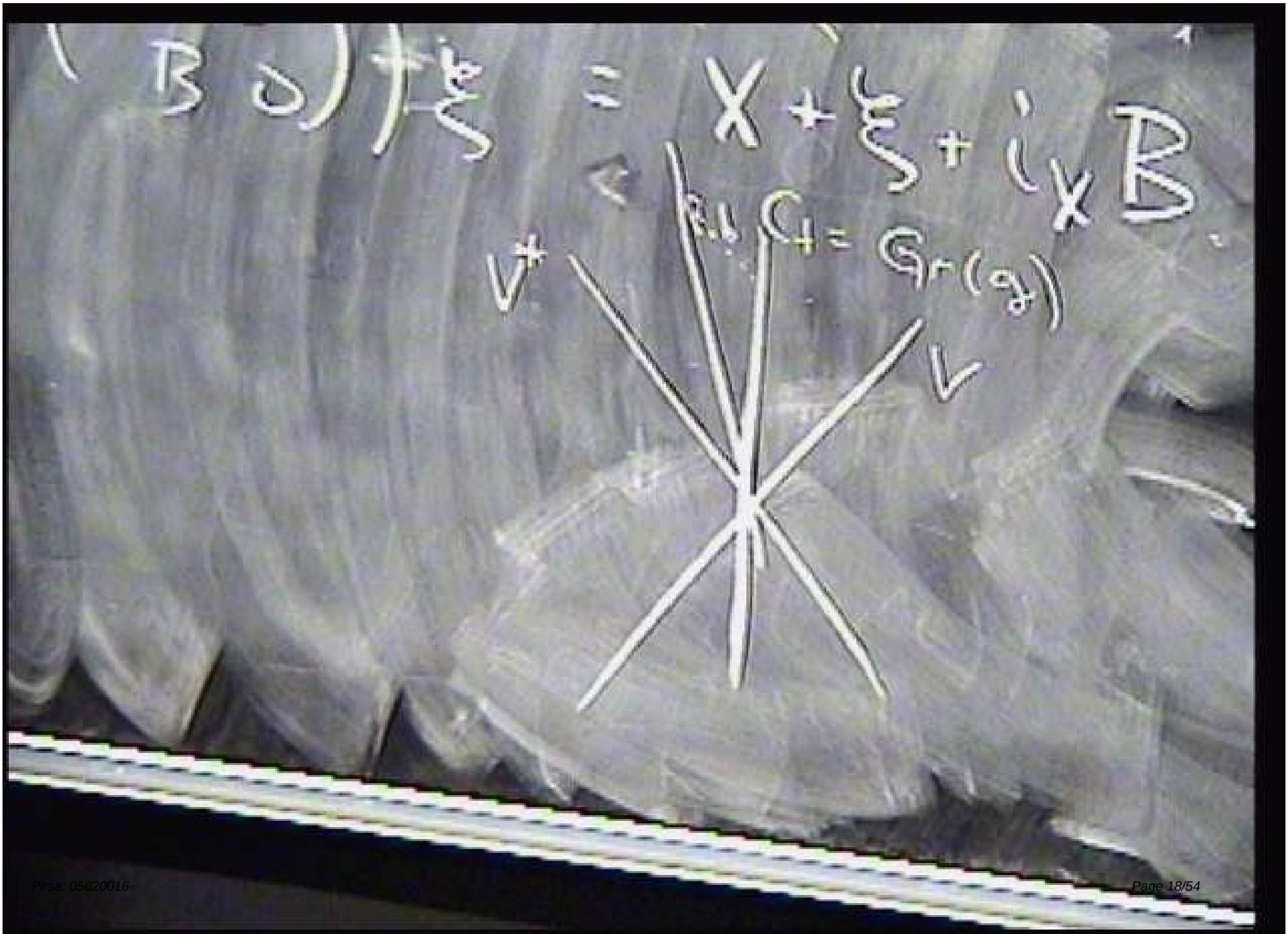
$$\mathfrak{so}(V \oplus V^*) = \Lambda^2(V \oplus V^*)$$

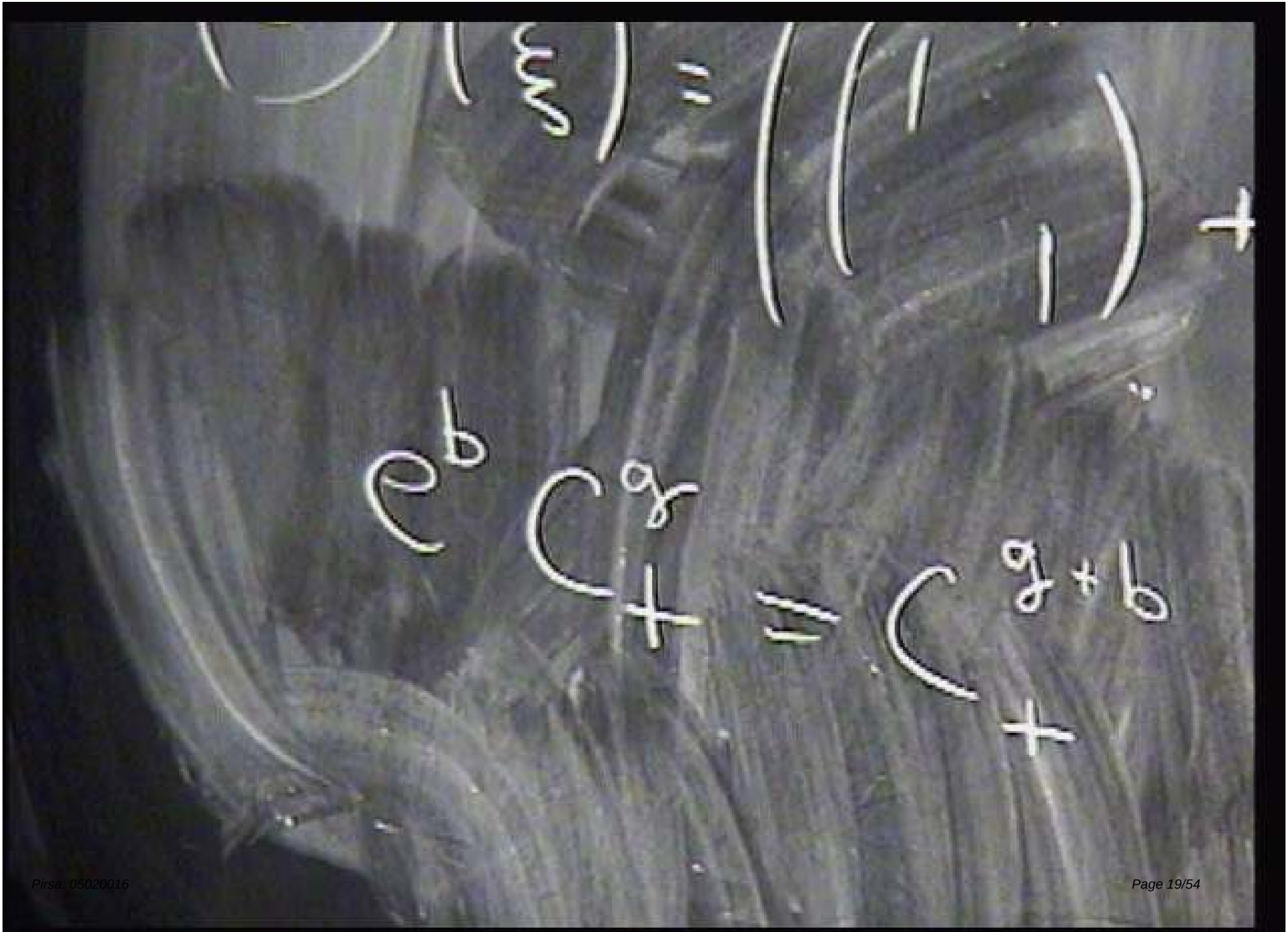
$$= \Lambda^2 V \oplus \Lambda^2 V^* \oplus \mathrm{End}\, V$$

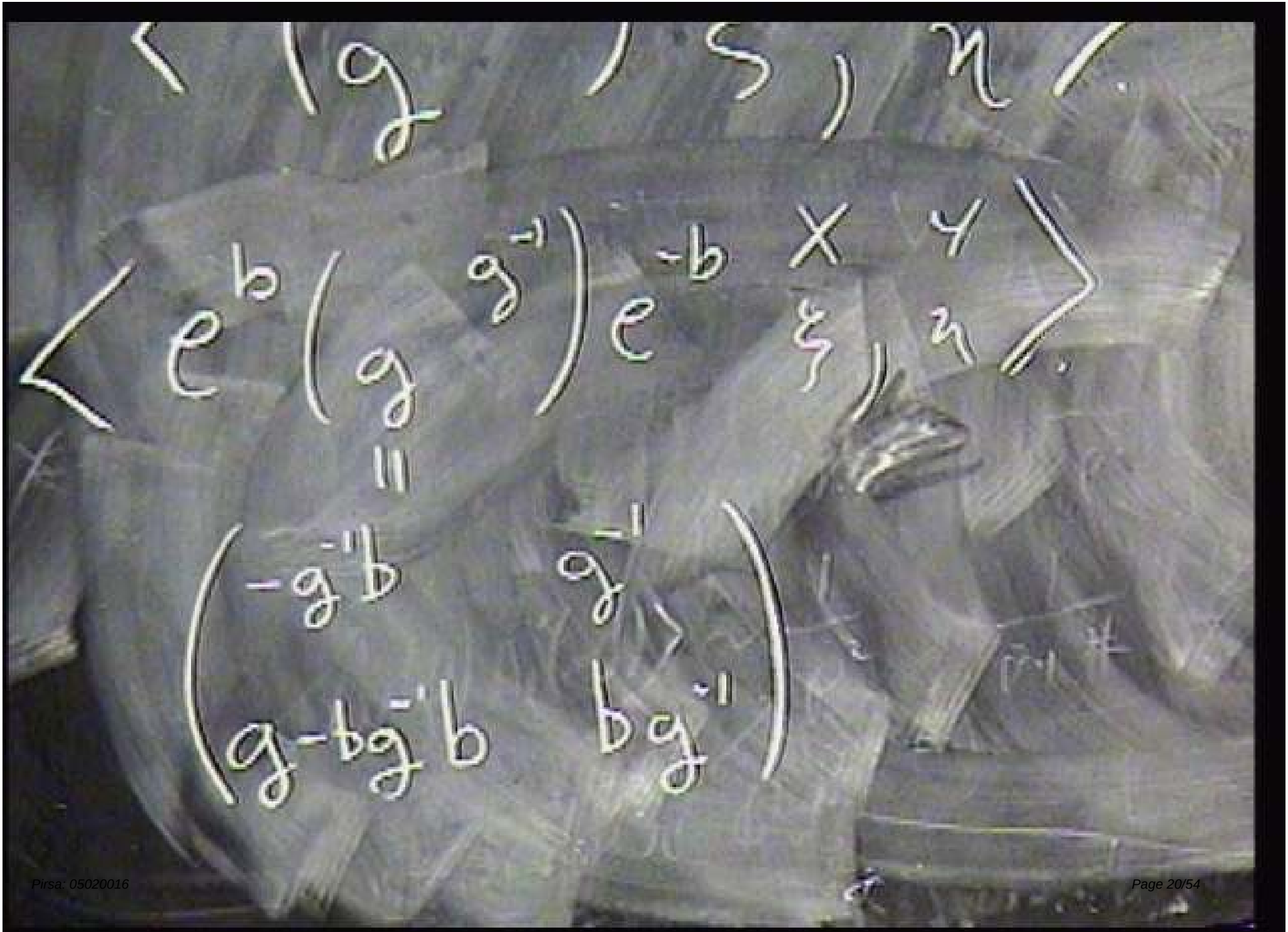

D Action of $\Lambda^2 V^*$ on
$V \oplus V^*$
$O(n,n)$
$\mathfrak{so}(V \oplus V^*$
$\begin{pmatrix} A & \beta \\ B & -A^* \end{pmatrix} \begin{matrix} X \\ + \\ \xi. \end{matrix}$


$\begin{pmatrix} 0 & 0 \\ B & 0 \end{pmatrix} \begin{matrix} X \\ \xi \end{matrix} = \begin{pmatrix} 0 \\ B(x) \end{pmatrix}$
$i_x B$


coisotrop
A = A⊥ ⇔ A Lagra
Max. Isot
e^B (X ξ) = ((1 1) + (0 0 B 0)) (X ξ) = X + ξ + i_X B
(0 0 B 0) (X ξ) = (0 B(X))
V*
V


$\xi = X + \xi + i_X B$
$C_t = Gr(g)$
V^*
V



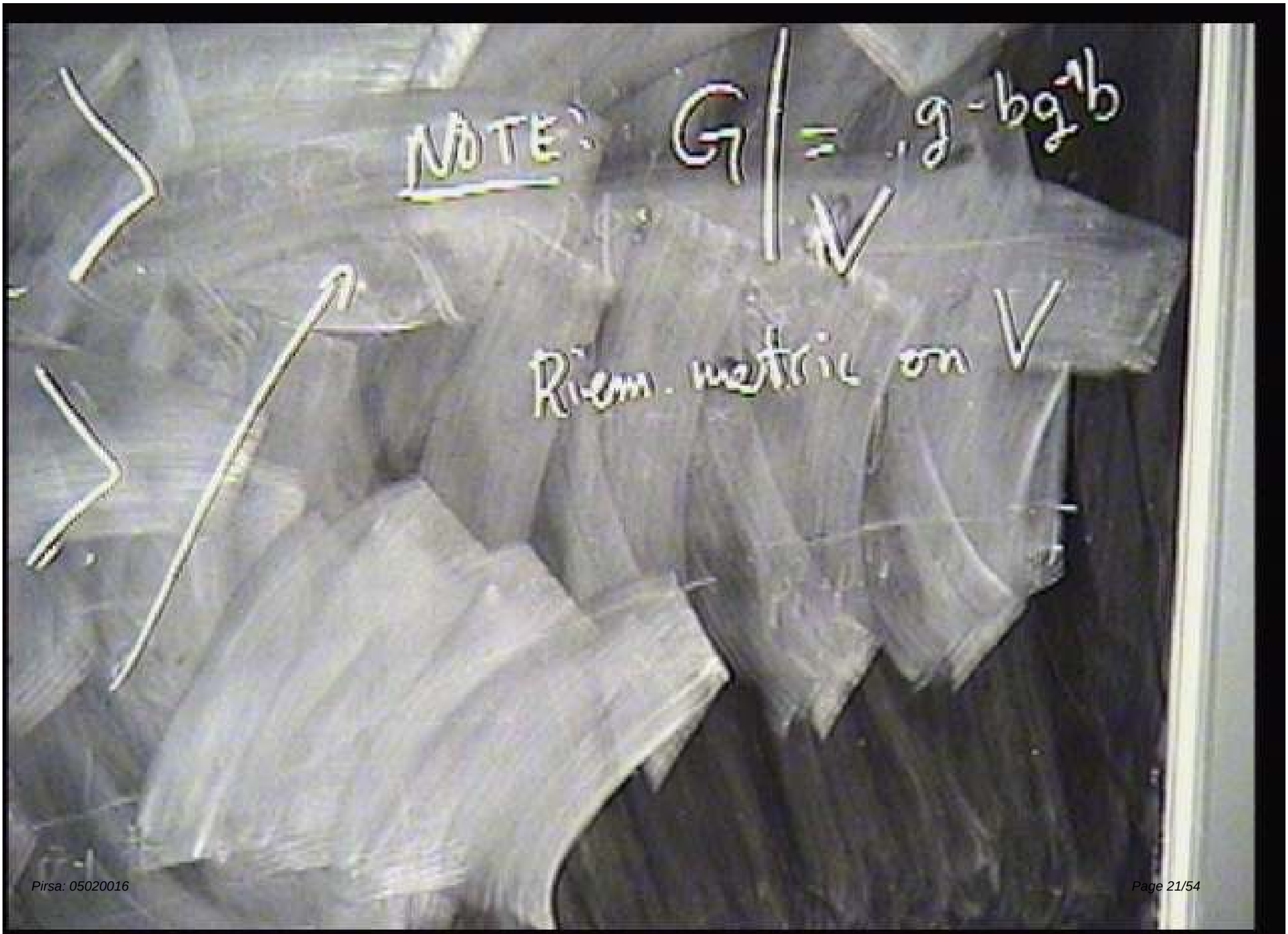


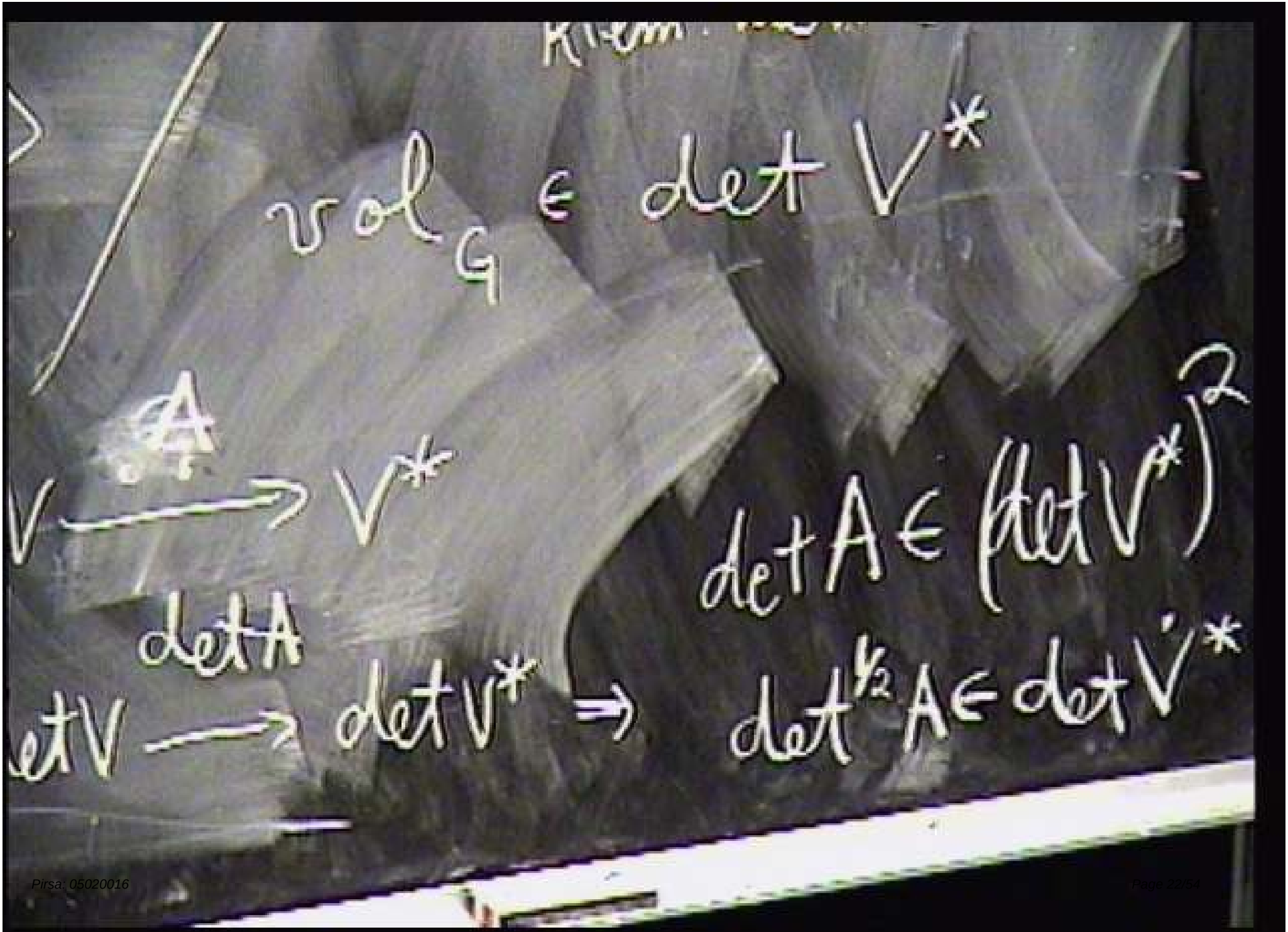
$$\langle (g \quad) \xi, \eta \rangle.$$

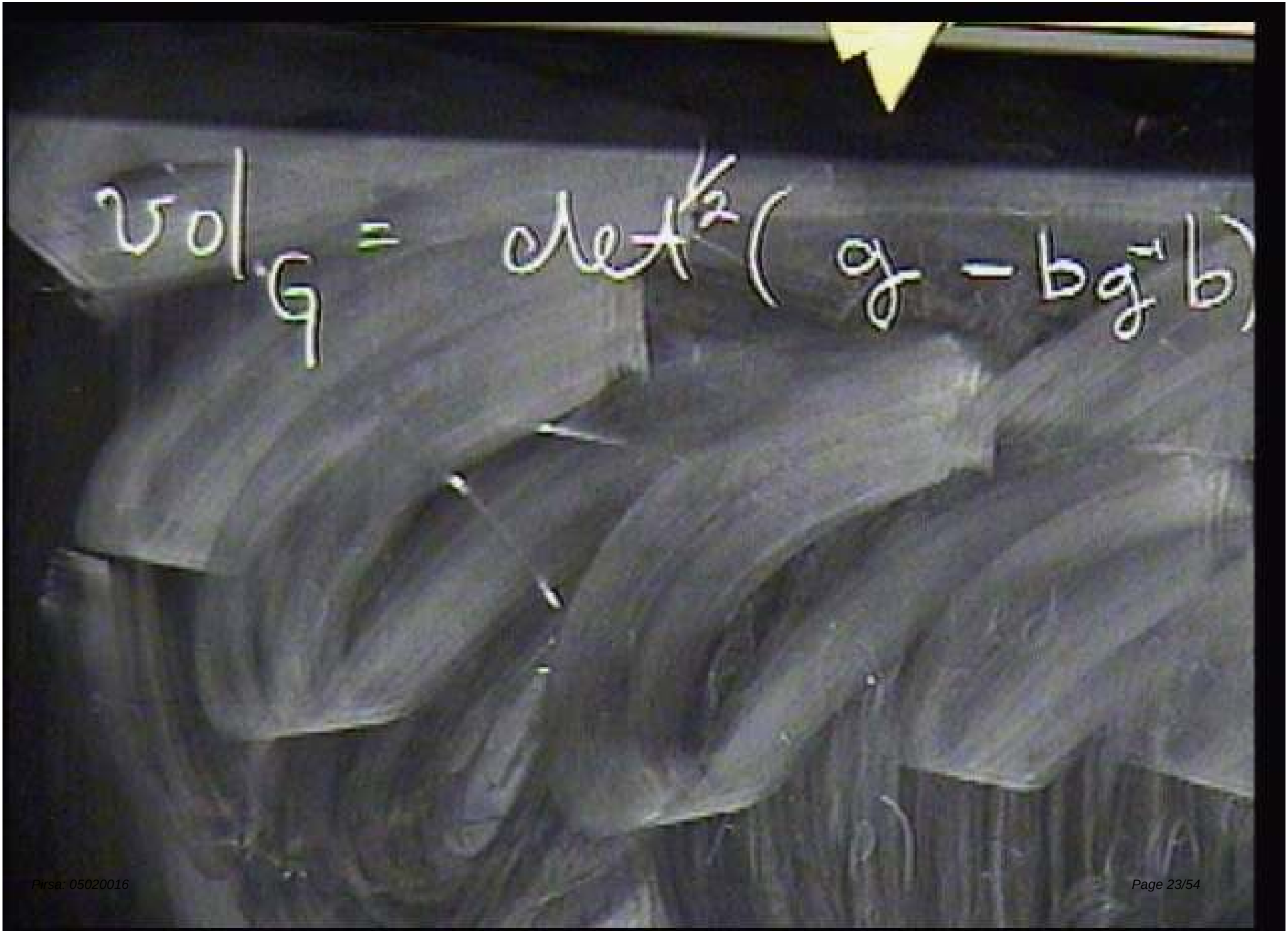
$$\left\langle e^{b} \begin{pmatrix} & g^{-1} \\ g & \end{pmatrix} e^{-b} \begin{matrix} X & Y \\ \xi, & \eta \end{matrix} \right\rangle.$$

$$\parallel$$

$$\begin{pmatrix} -g^{-1}b & g^{-1} \\ g - bg^{-1}b & bg^{-1} \end{pmatrix}$$


NOTE:
$G|_V = g - b g^{-1} b$
Riem. metric on V

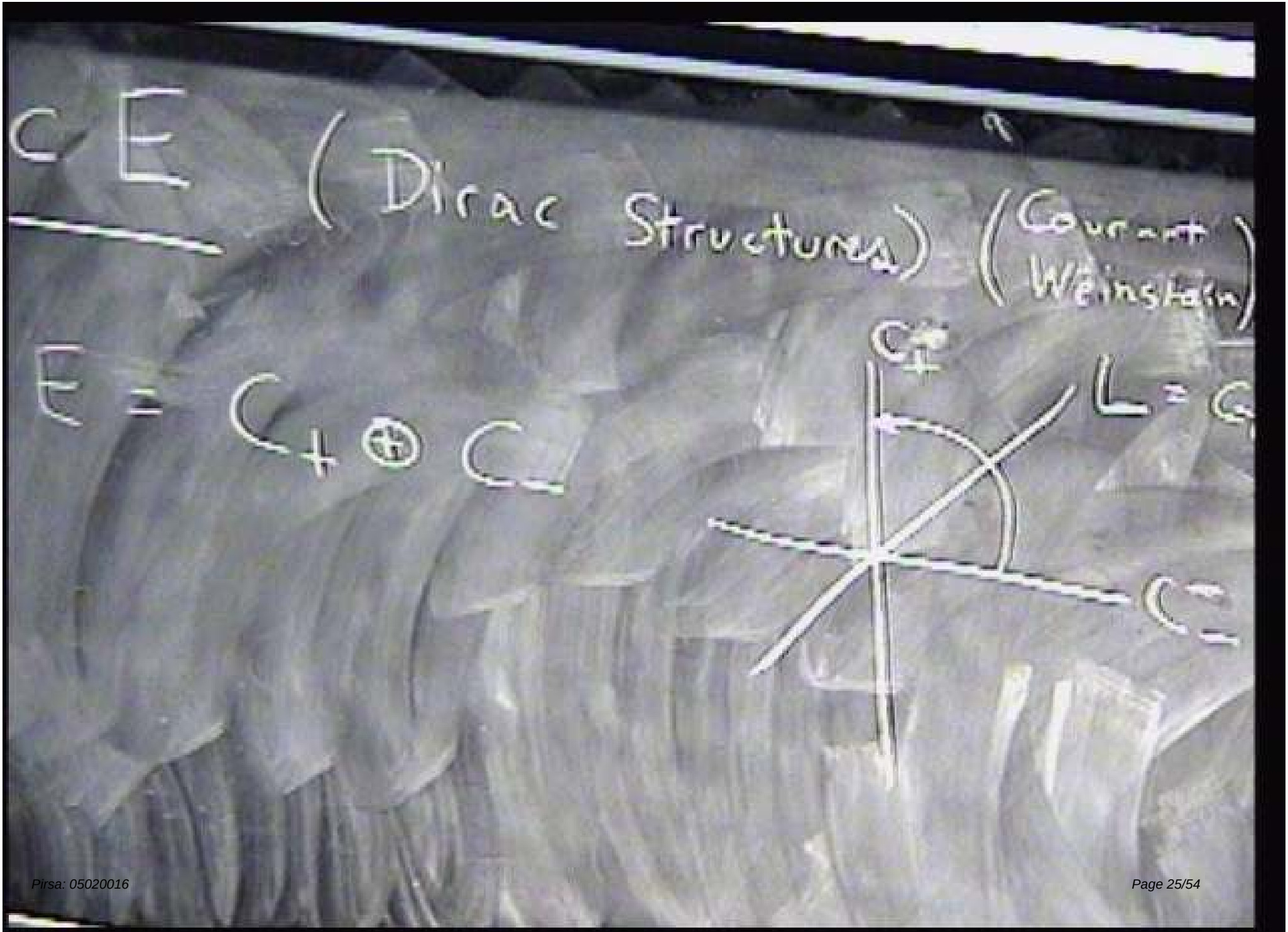

vol_G ∈ det V*
A
V → V*
det A
det V → det V*
det A ∈ (det V*)^2
det^½ A ∈ det V*

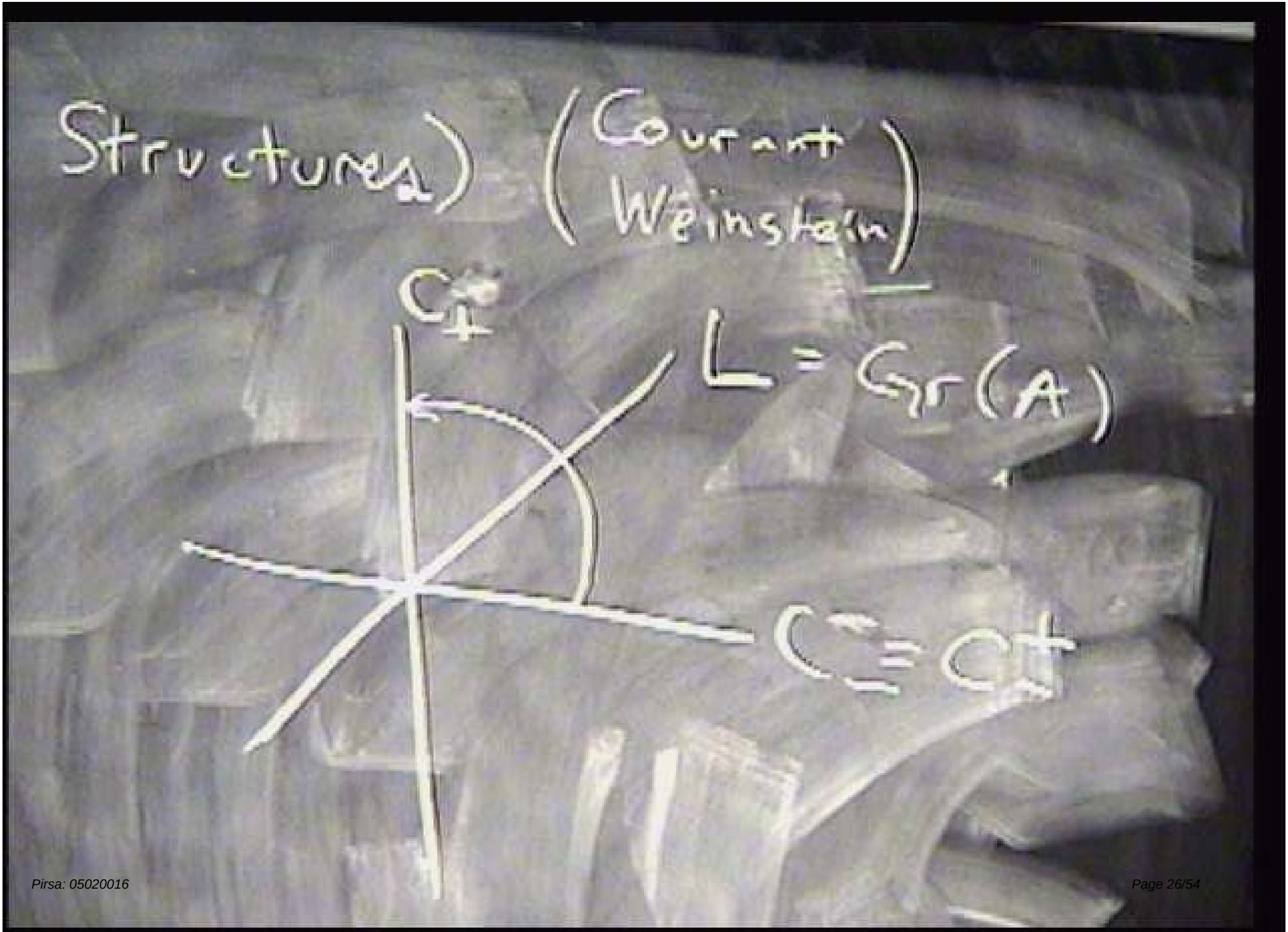


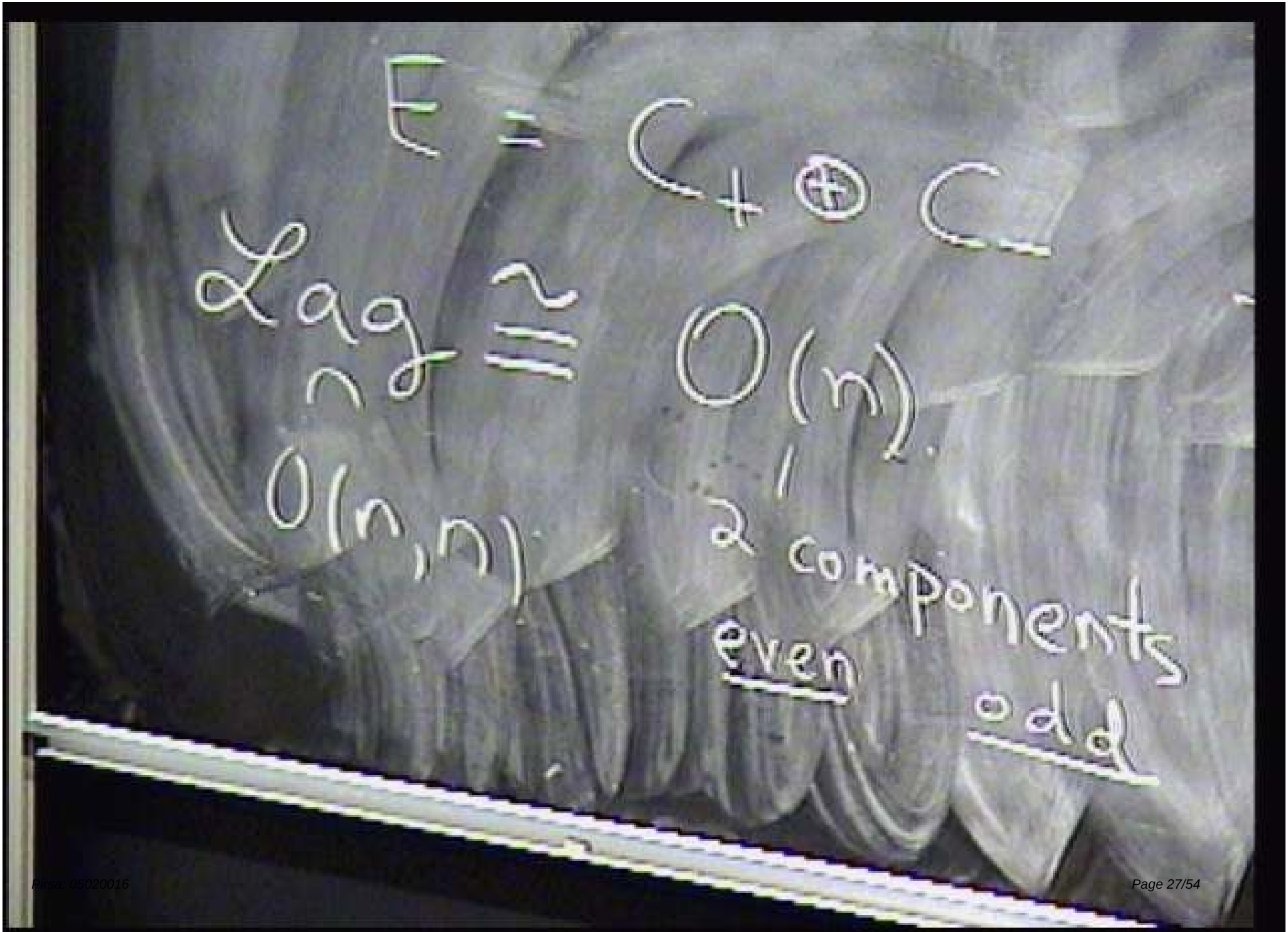
$$\mathrm{vol}_G = \det{}^{1/2}\left(g - b g^{-1} b\right)$$

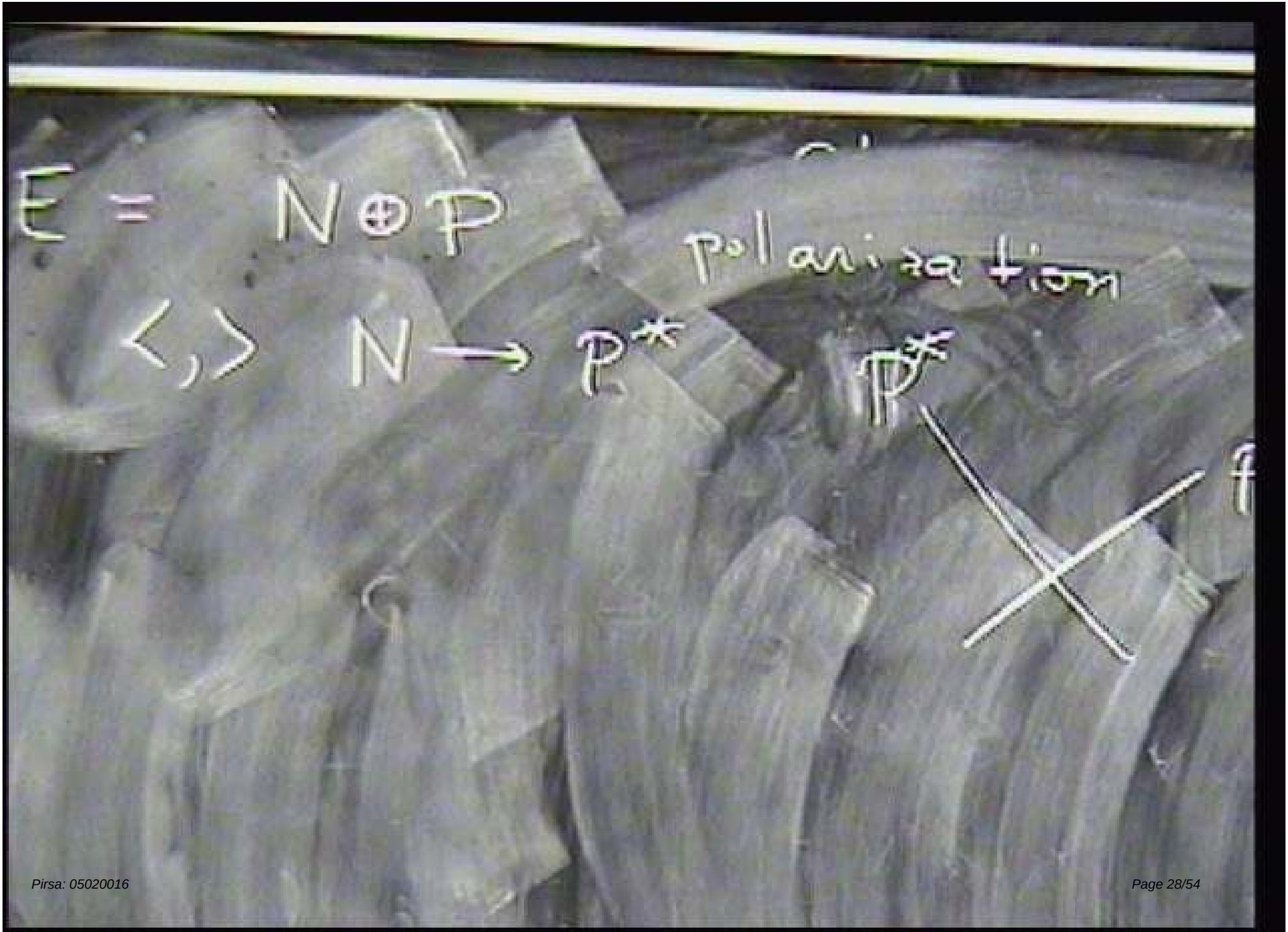
$$- - b g^{-1} b) = (\det g \det(1 - g^{-1} b g^{-1}$$

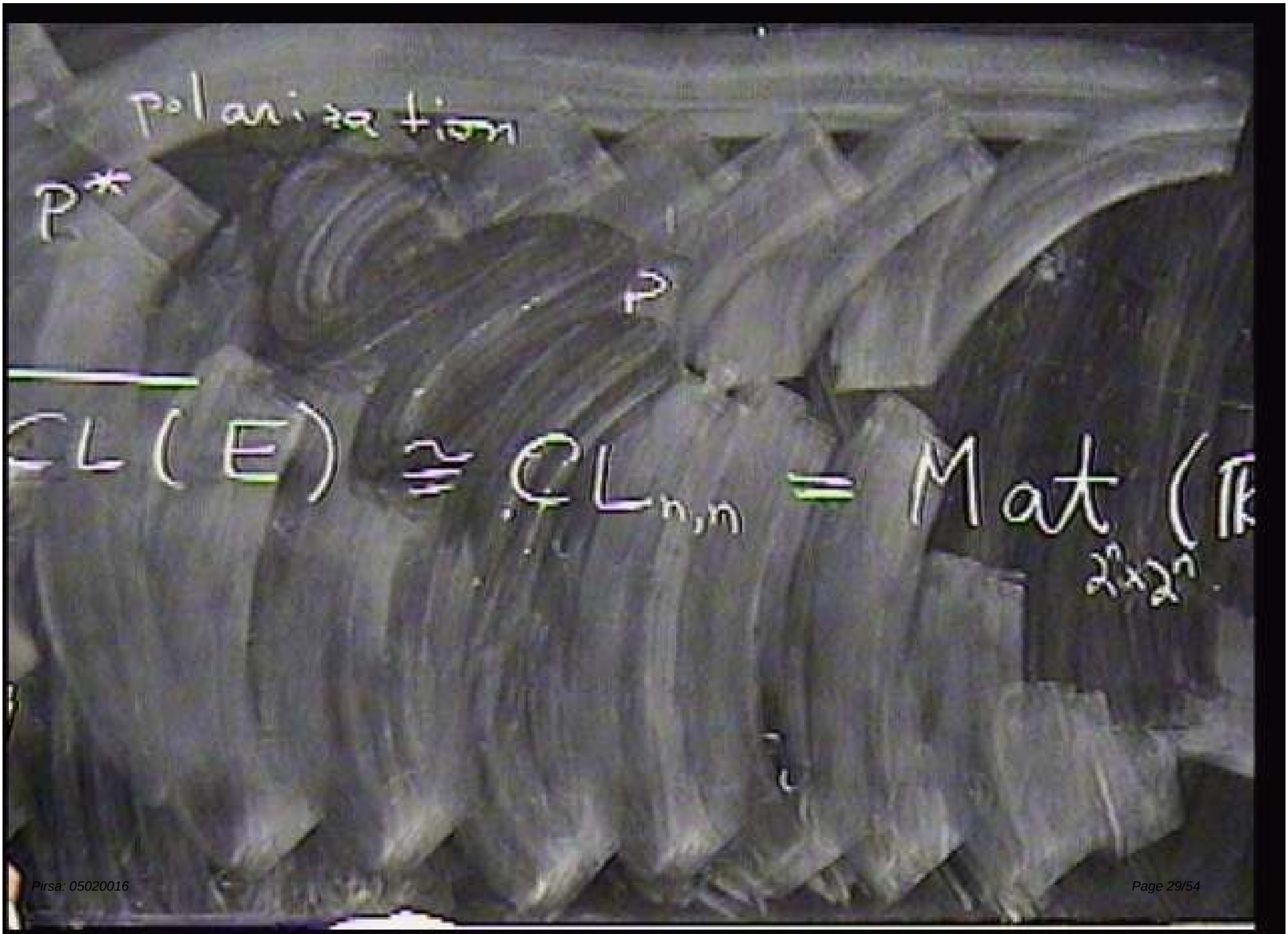
$$\mathrm{Vol}_G = \frac{\det(g+b)}{d\mu_A^{1/2}\, g}.$$

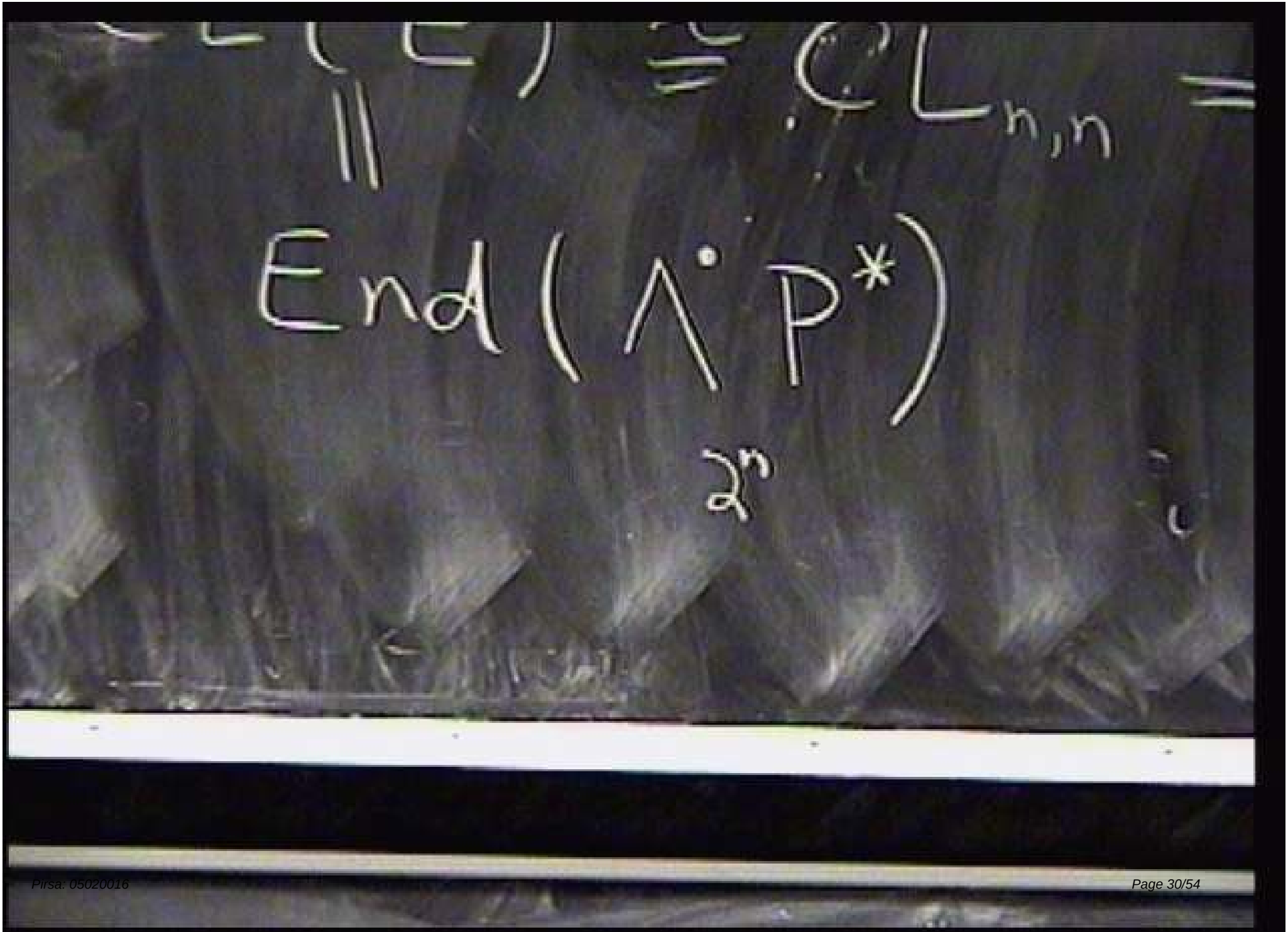

C E
(Dirac Structures)
(Covariant Weinstein)
E = C+ ⊕ C-
C+
L = C
C-

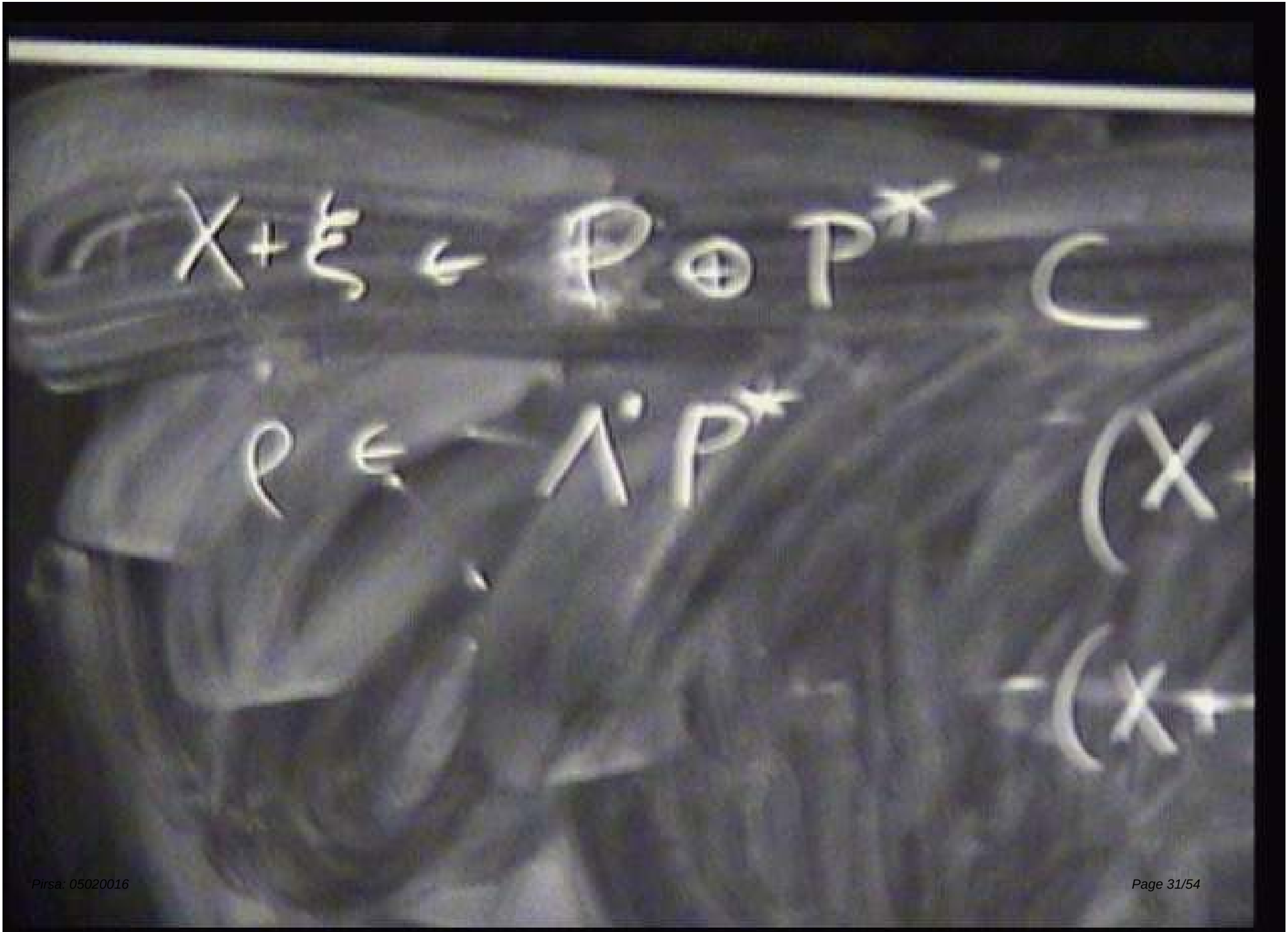

Structures
Courant
Weinstein
C_t^*
$L = Gr(A)$
$C = C_t$

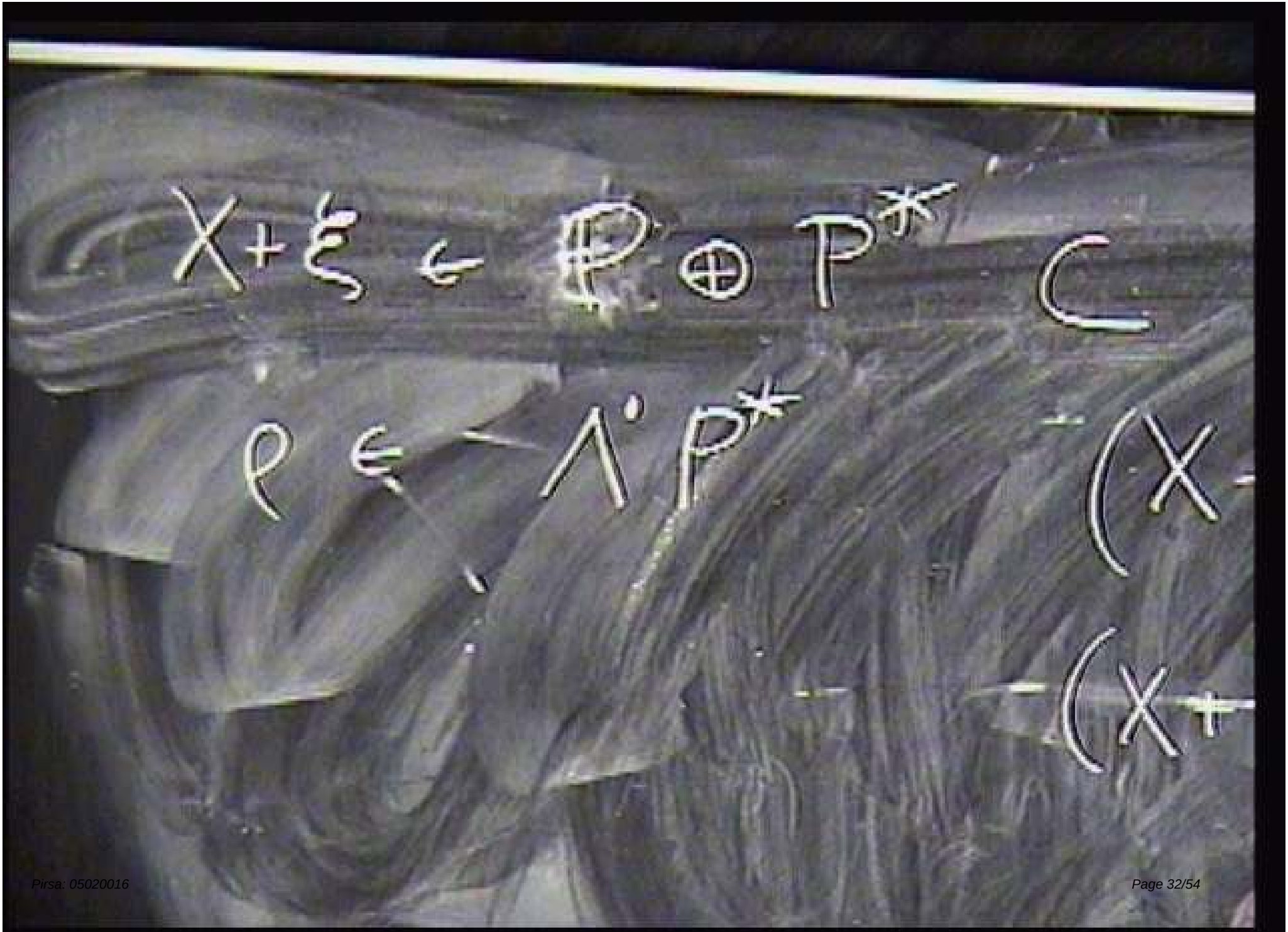

E = C+ ⊕ C-
Lag ≅ O(n)
O(n,n)
2 components
even
odd

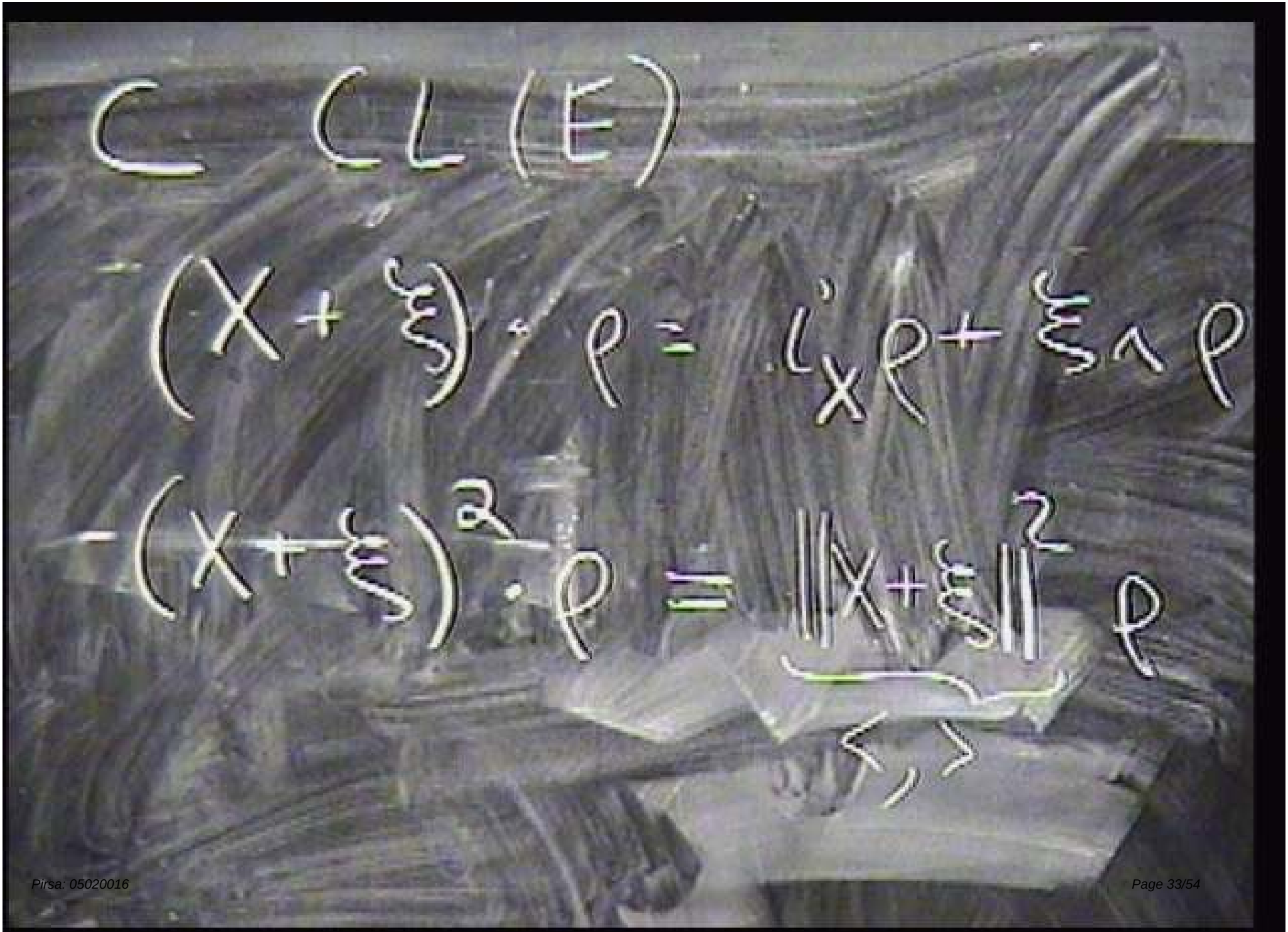

E = N ⊕ P
Polarization
⟨,⟩ N → P*
P*

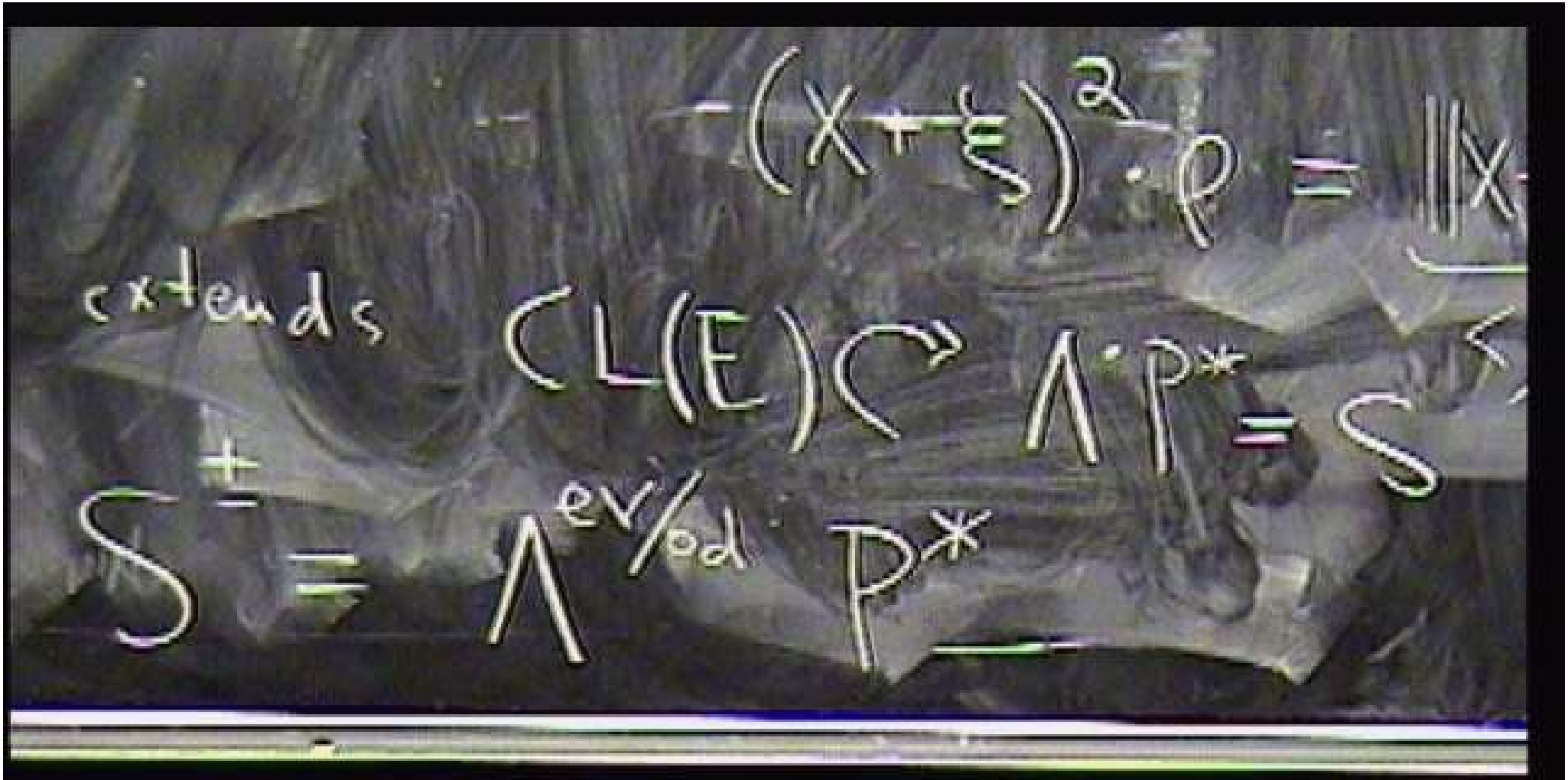

polarization
P^*
$CL(E) \cong CL_{n,n} = Mat_{2^n \times 2^n}(\mathbb{R}$


$\mathbb{C}L(\mathbb{C}) \cong \mathbb{C}L_{n,n} =$
$End(\Lambda^\bullet P^*)$
2^n


X+ξ ∈ P ⊕ P*
C
ρ ∈ Λ P*
(X+
-(X+




C CL(E)
$-(X+\xi)\cdot\rho = i_X\rho + \xi\wedge\rho$
$-(X+\xi)^2\cdot\rho = \|X+\xi\|^2\rho$
$\langle\ ,\ \rangle$



$$-(X+\xi)^2 \cdot \rho = \|X$$

extends $CL(E) \circlearrowright \Lambda \cdot P^* = S$

$$S^{\pm} = \Lambda^{ev/od} P^*$$

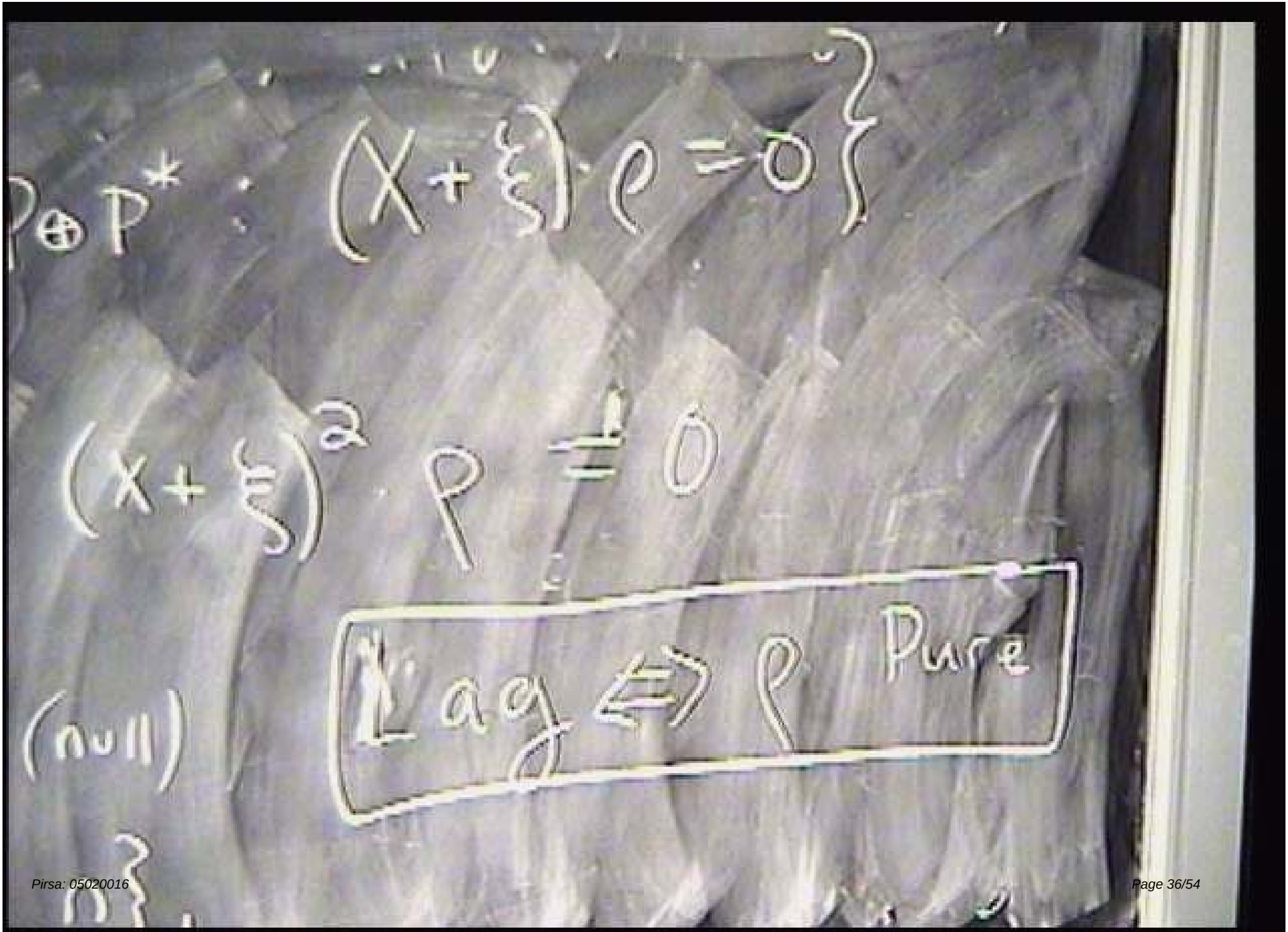
of $\rho \in \Lambda^{\cdot} P^*$

$$\mathrm{Ann}(\rho) = \{ X + \xi \in P \oplus P^* : \}$$

$$\forall\, X+\xi \quad \| X+\xi \|^2_{\rho} = ((X+\xi$$

$\Rightarrow$ $\mathrm{Ann}(\rho)$ Isotropic (null)

dimension $\in \{0, 1, \ldots, n\}$.


$P \oplus P^*$: $(X+\xi)\cdot\rho = 0$
$(X+\xi)^2 \cdot \rho = 0$
(null)
$L_{a_g} \Leftrightarrow \rho$ Pure

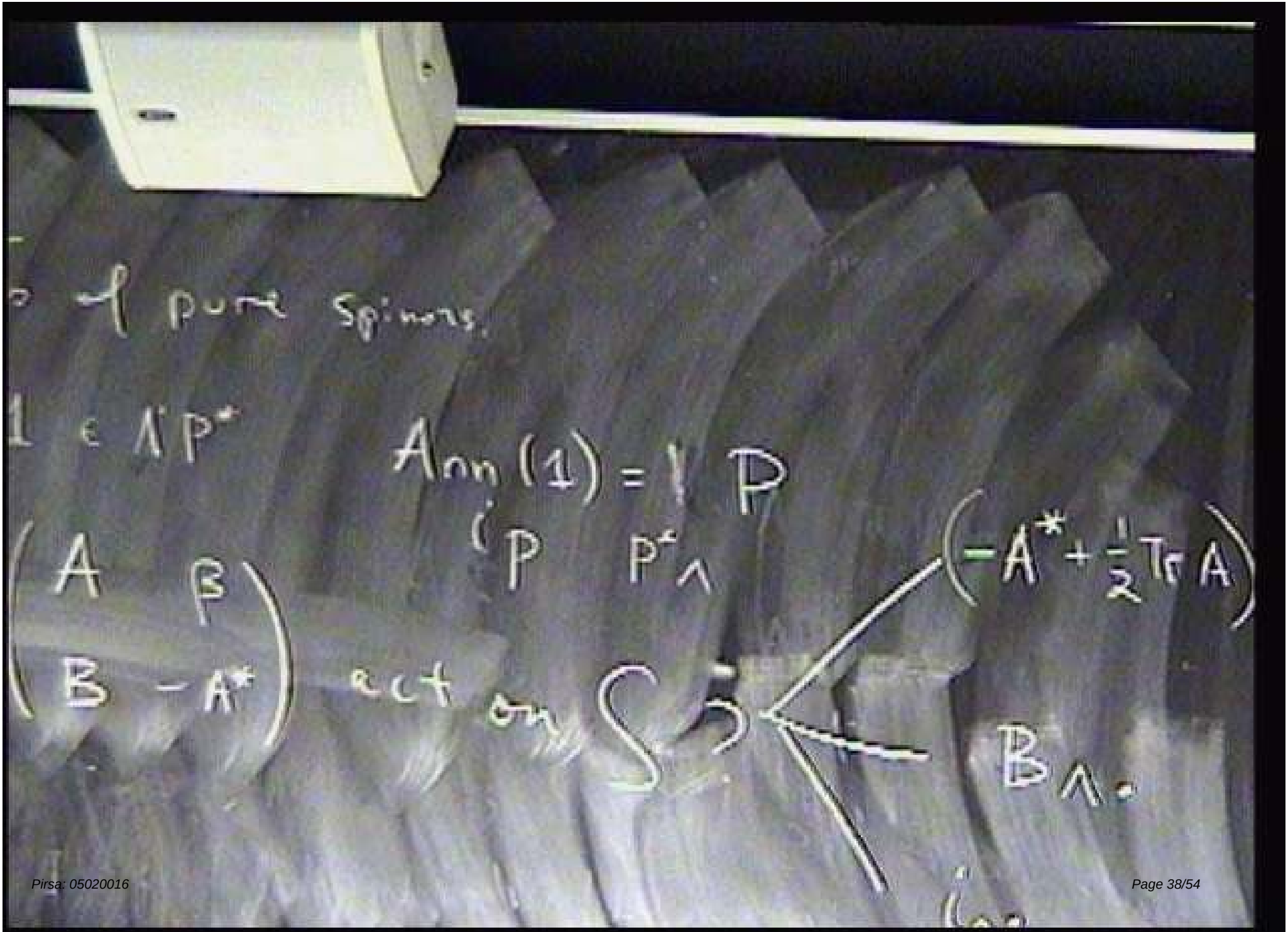
$E = P \oplus P^*$

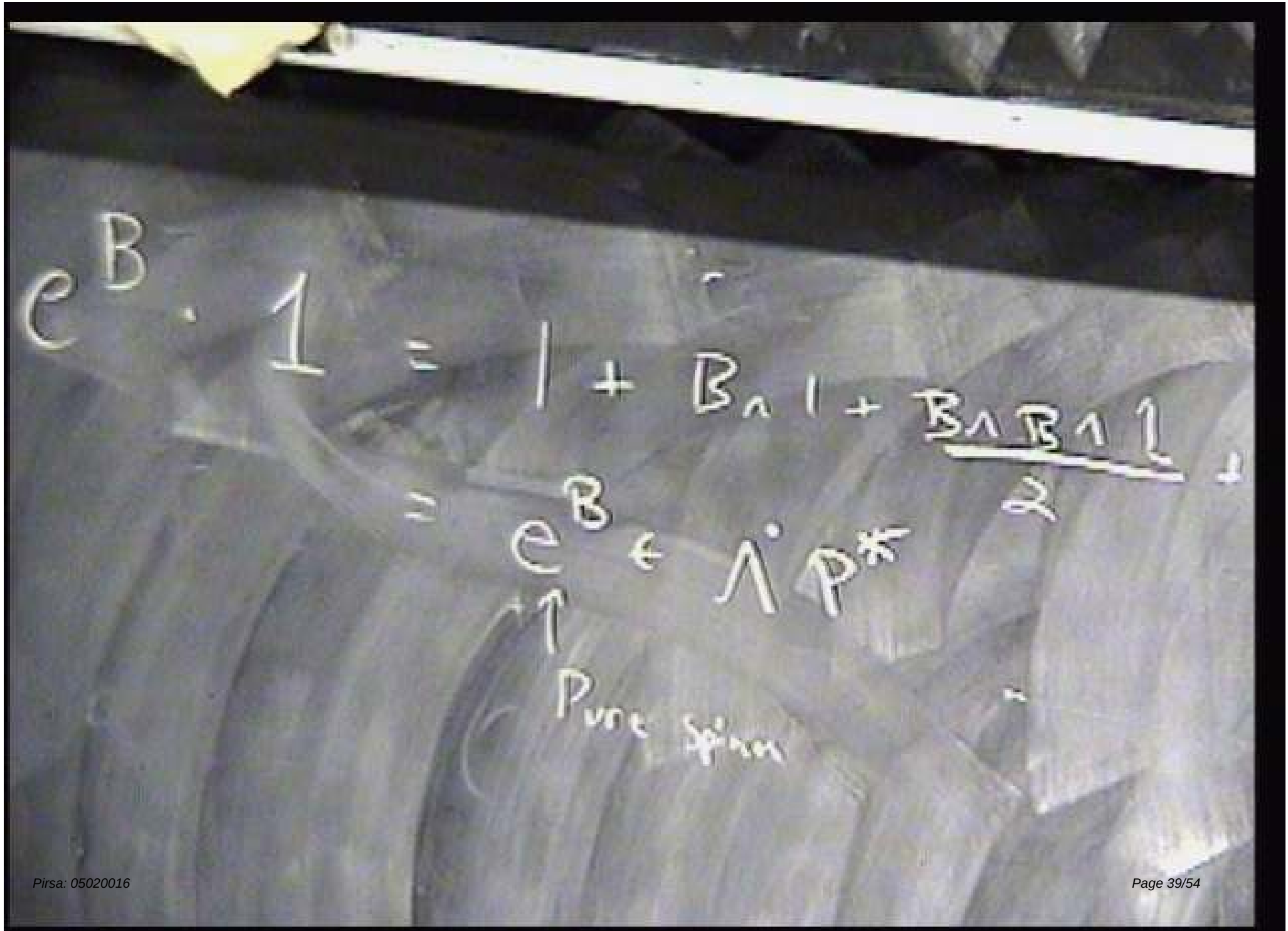
Examples of pure spinors.

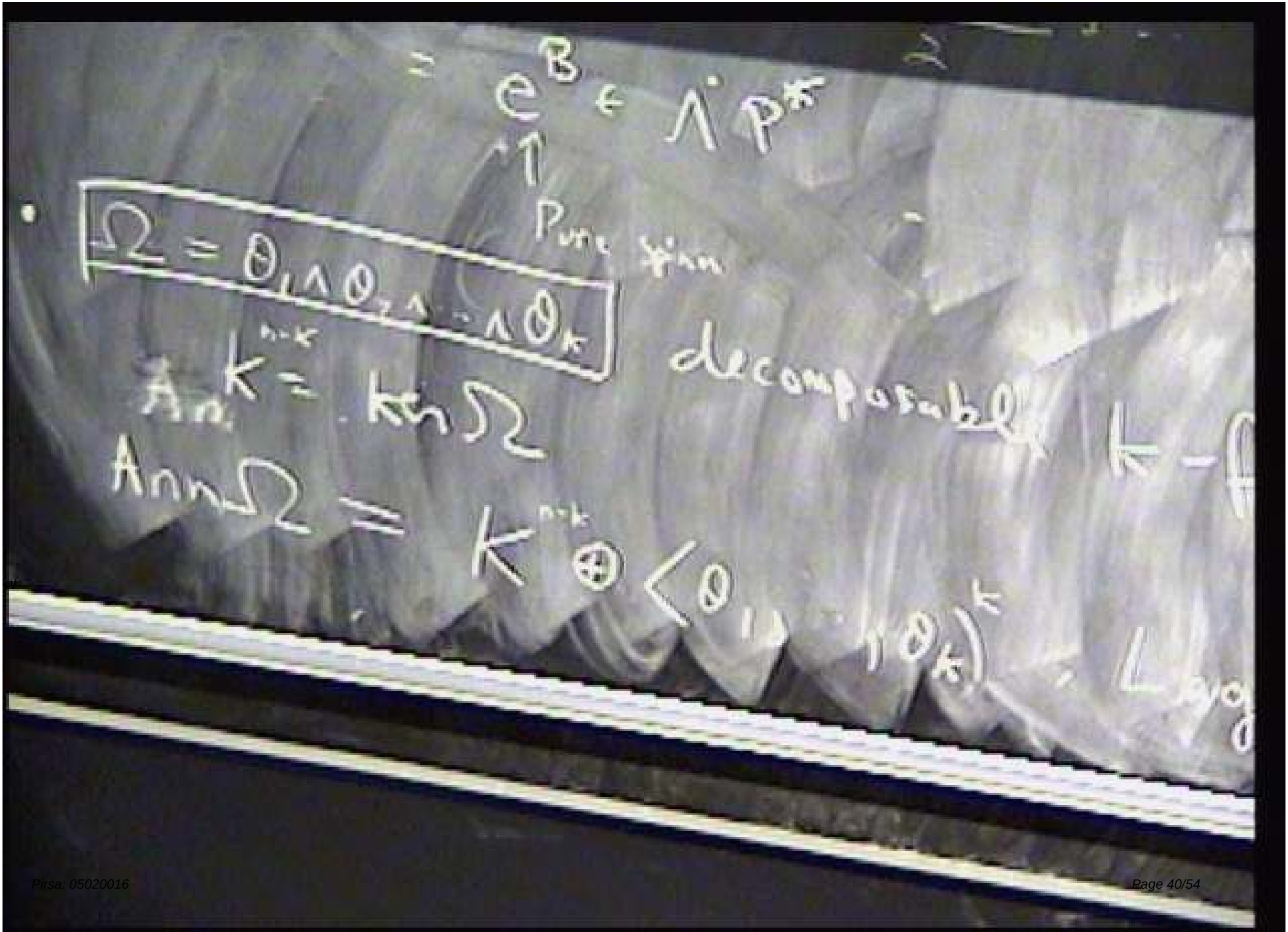
- $1 \in \Lambda^{\cdot} P^*$

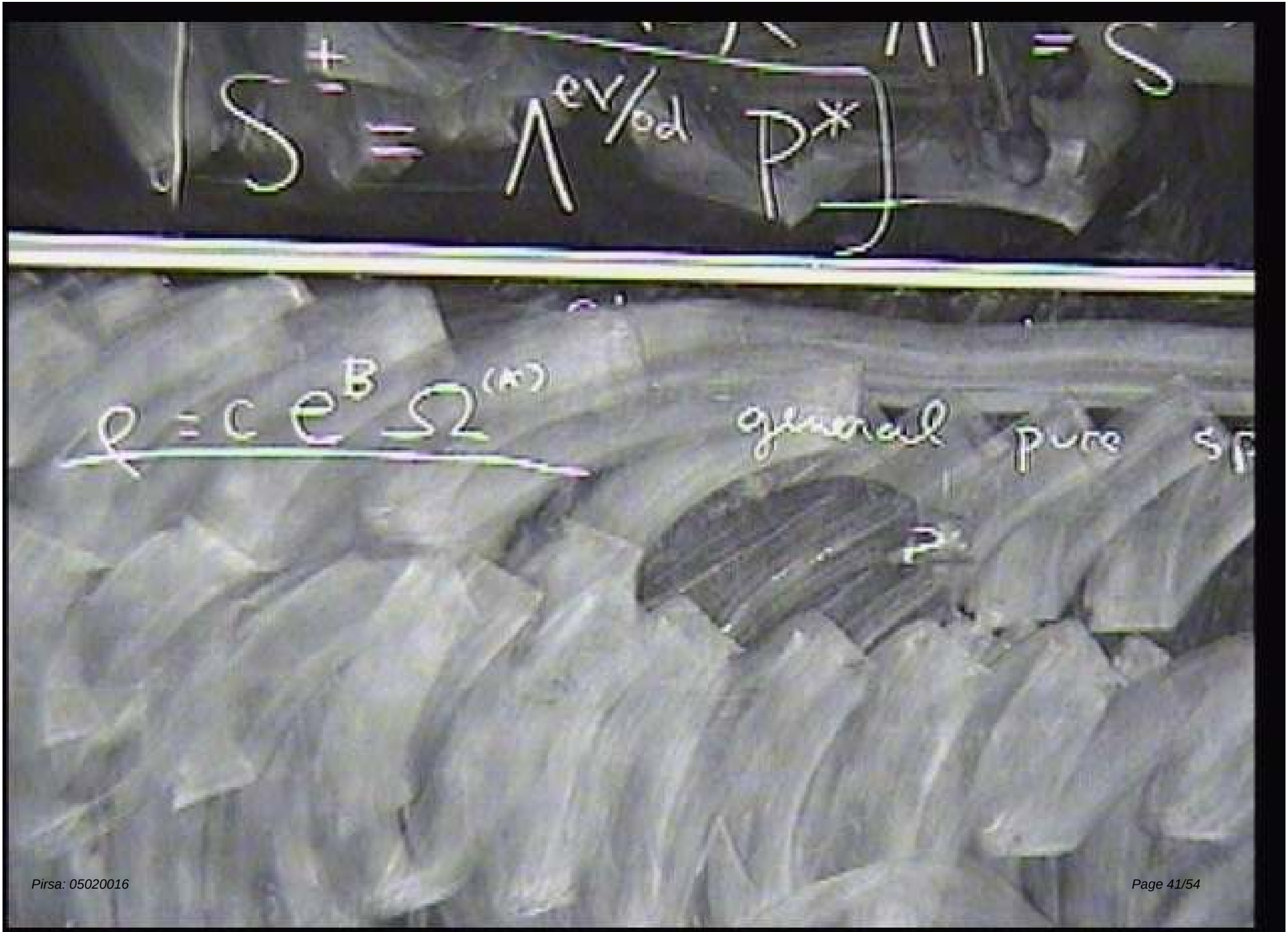
$\mathrm{Ann}(1) = \mathbb{P}$

$i_P \quad P^* \wedge$


of pure spinors.
1 ∈ Λ^i P*
Ann(1) = P
iP P*Λ
(A β
B −A*) act on
(−A* + ½ Tr A)
BΛ.


$e^B \cdot 1 = 1 + B \wedge 1 + \frac{B \wedge B \wedge 1}{2} + \ldots$
$= e^B \in \wedge^{\bullet} V^*$
Pure Spinor


$\Omega = \theta_1 \wedge \theta_2 \wedge \cdots \wedge \theta_k$
pure spinor
decomposable

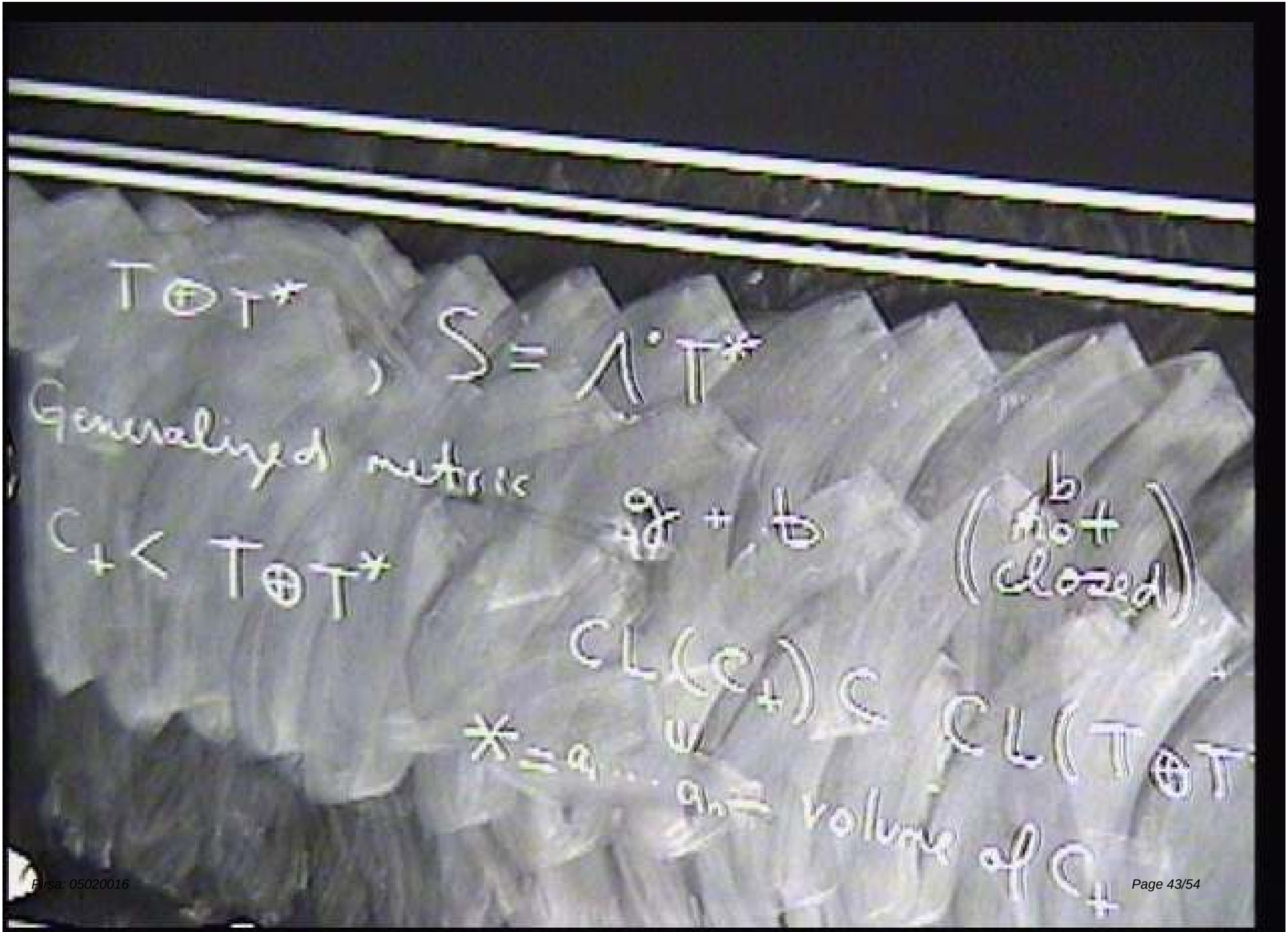

$S^{\pm} = \Lambda^{ev/od} P^{*}$
$\rho = c\, e^{B}\, \Omega^{(*)}$
general pure sp

$$\boxed{S = \Lambda^{ev/od} P^*}$$

$$\underline{\rho = c\, e^{B}\, \Omega^{(*)}}$$ general pure spinor

$$\mathrm{Ann}\,\rho = \{ X + \xi \;:\; X \in \mathrm{Ker}(\Omega)^{\perp},\ \xi\big|_{\ker\Omega} = i_X B$$

$$\{\mathrm{Lag}\} \xleftrightarrow{1:1} \left\{\begin{matrix}\text{pure}\\ \text{spinor}\\ \text{lines}\end{matrix}\right\}$$



$T \oplus T^*$, $S = \Lambda^\bullet T^*$

Generalized metric

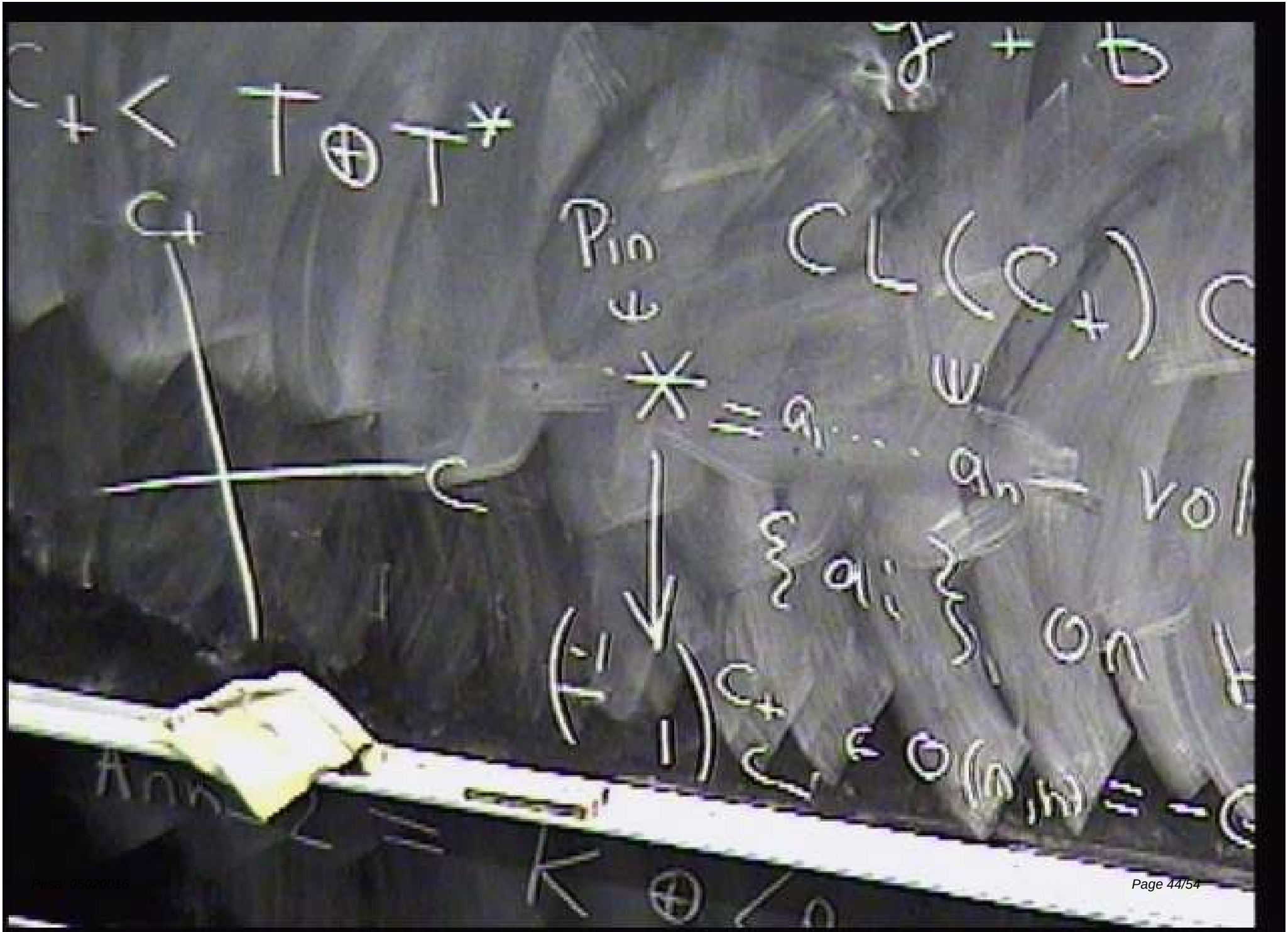
$C_+ < T \oplus T^*$

$g + b$ $\left(\begin{matrix} b \\ \text{not} \\ \text{closed} \end{matrix}\right)$

$CL(C_+) \subset CL(T \oplus T^*$

$\cup$

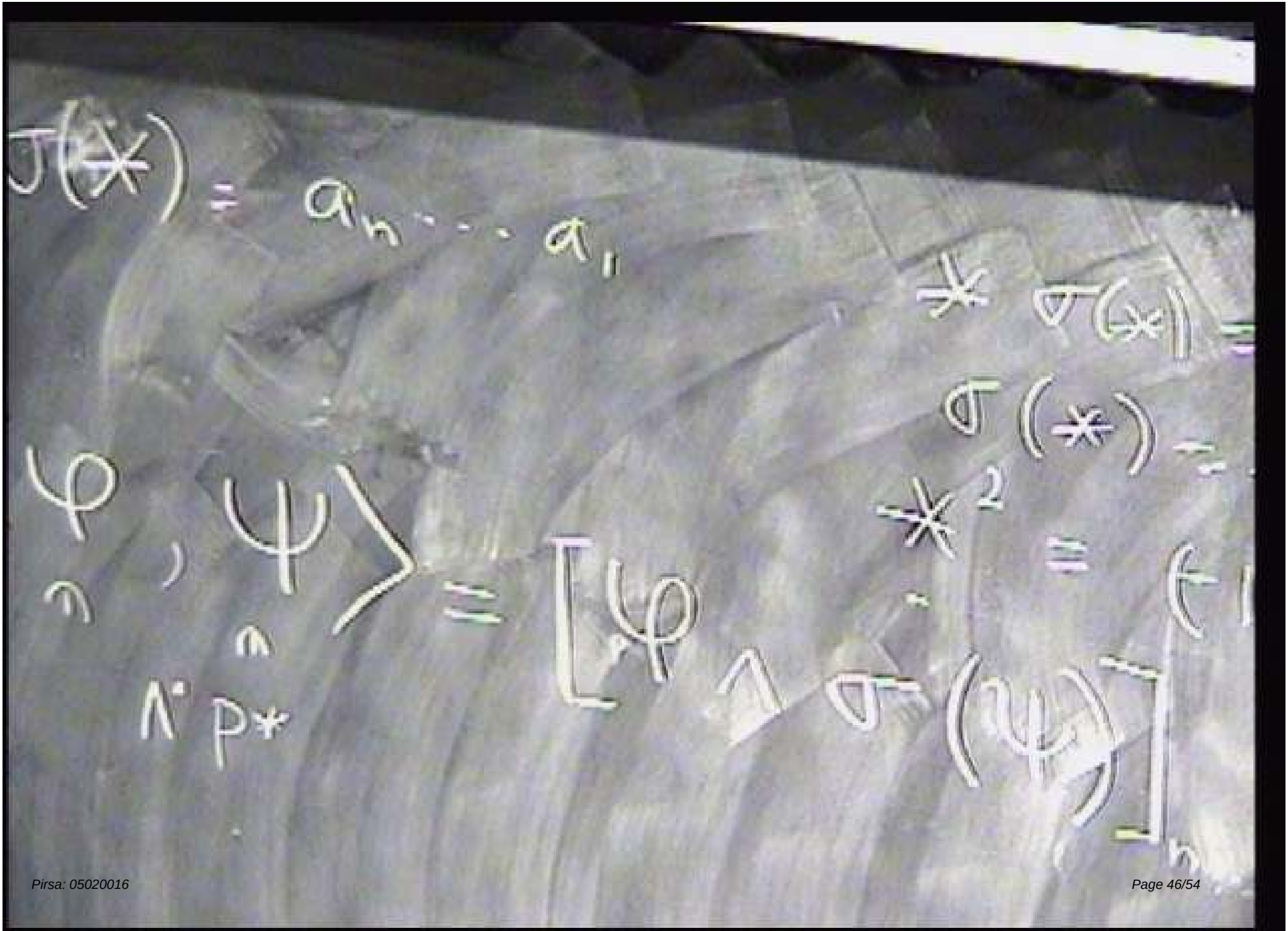
$* = a_1 \ldots a_n =$ volume of C_+

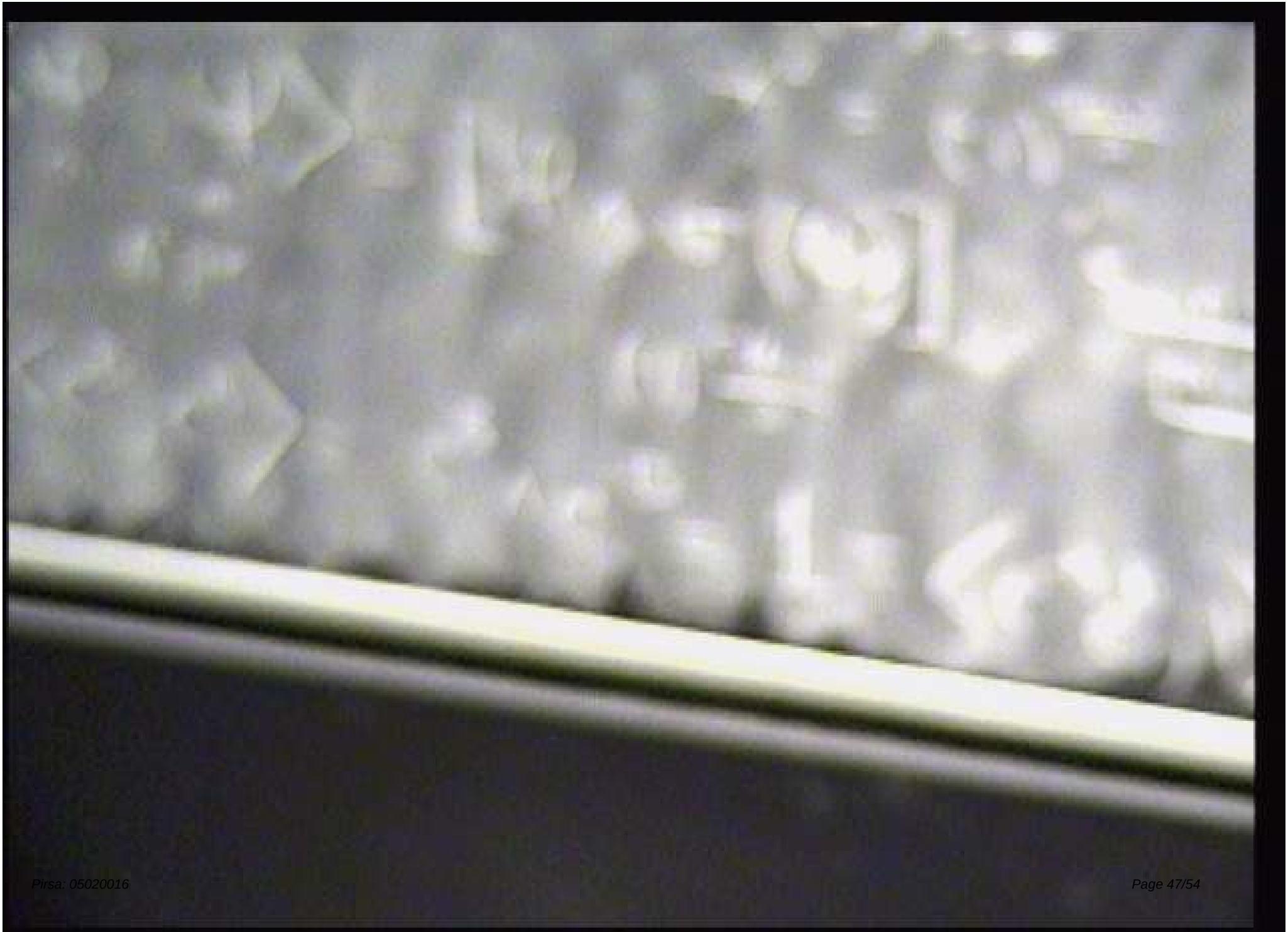


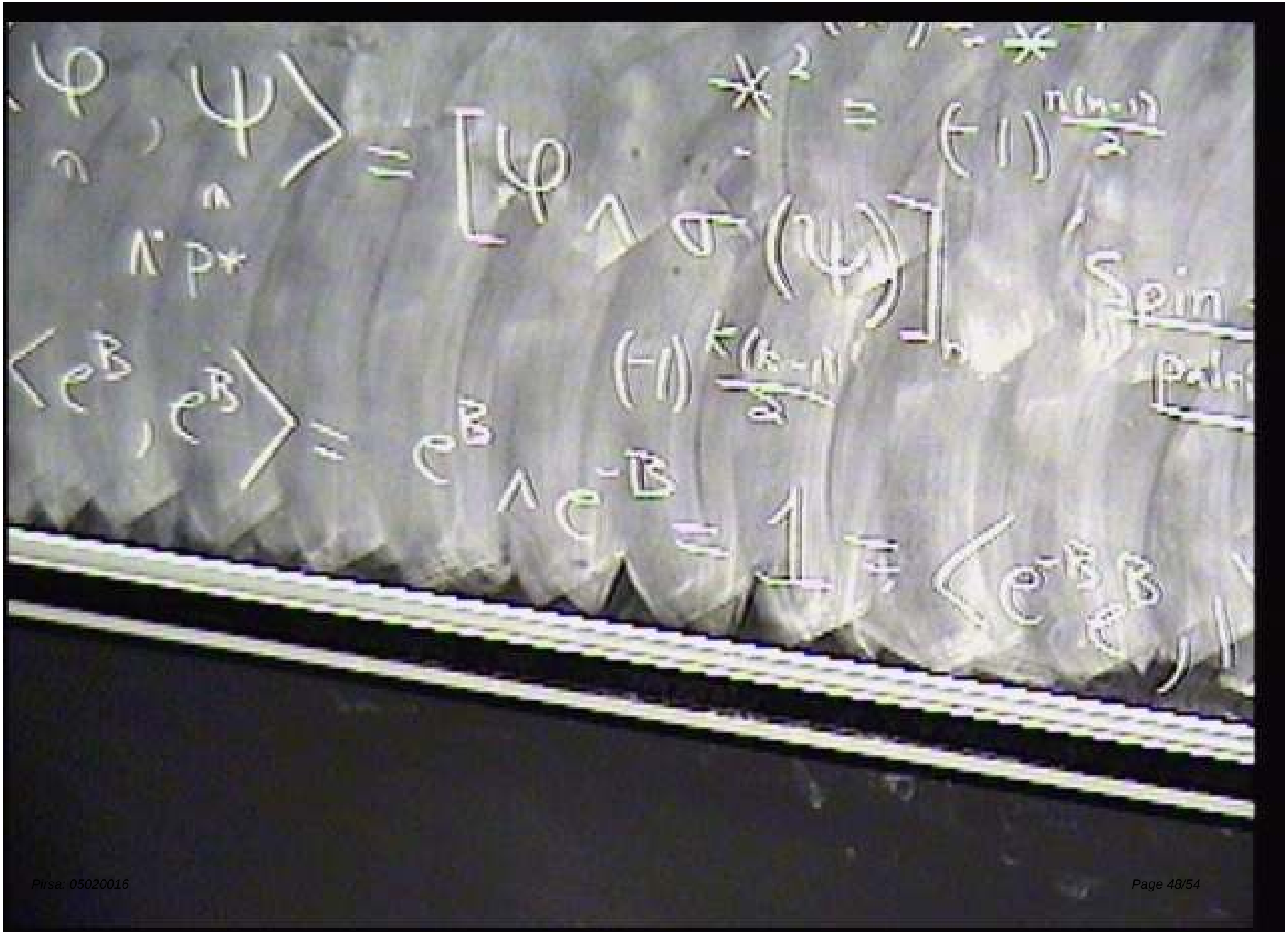
$$* \quad \sigma(*1) = 1$$

$$\sigma(*) = *^{-1}$$

$$*^2 = (-1)^{\frac{n(n-1)}{2}}$$







$$\langle \varphi, \psi \rangle = (-1)^{\frac{n(n-1)}{2}} \langle \psi, \varphi \rangle$$

$$\langle \varphi, \sigma(x)\, \psi \rangle$$

Bonn-Infeld

$$\langle \varphi, \psi \rangle = (-1)^{\frac{n(n+1)}{2}} \langle \psi, \varphi \rangle$$

$$\langle \varphi, \sigma(*)\psi \rangle$$ Born-Infeld pairin

Generalized metric

$$\langle \varphi, \sigma(*)\psi \rangle$$ symmetric $= G(\varphi, \psi)\, \mathrm{vol}$

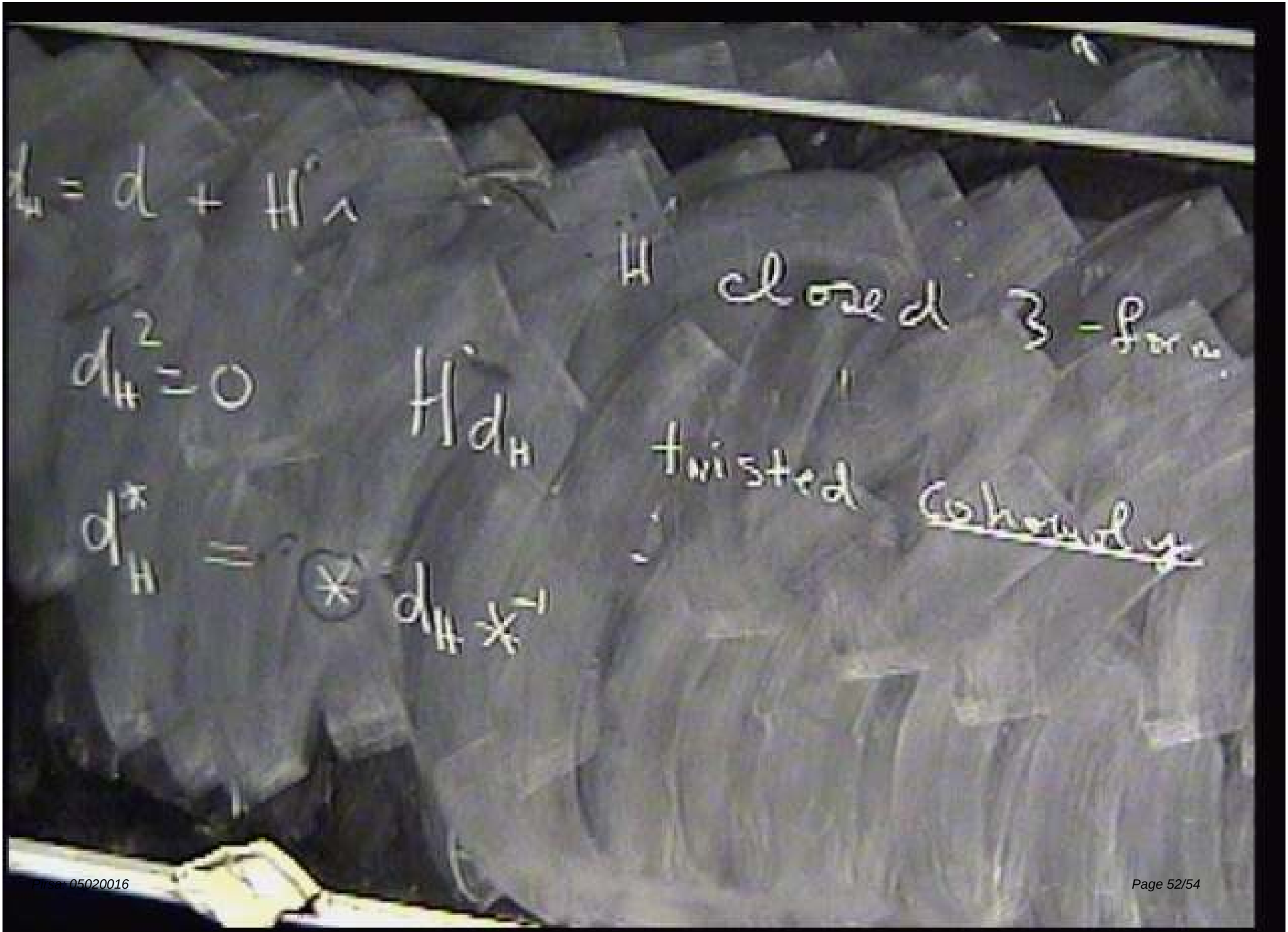
$\langle \varphi, \sigma(*)\psi \rangle$ Born-Infeld pairing

Generalized metric

$\langle \varphi, \sigma(*)\psi \rangle$ symmetric $= \underbrace{G(\varphi,\psi)}_{\text{pos def}} \mathrm{vol}_g$

If $b=0$ then

$\langle \varphi, \sigma(*)\psi \rangle = (\varphi \wedge \star \psi) = g(\varphi,\psi)\, \mathrm{vol}_g$



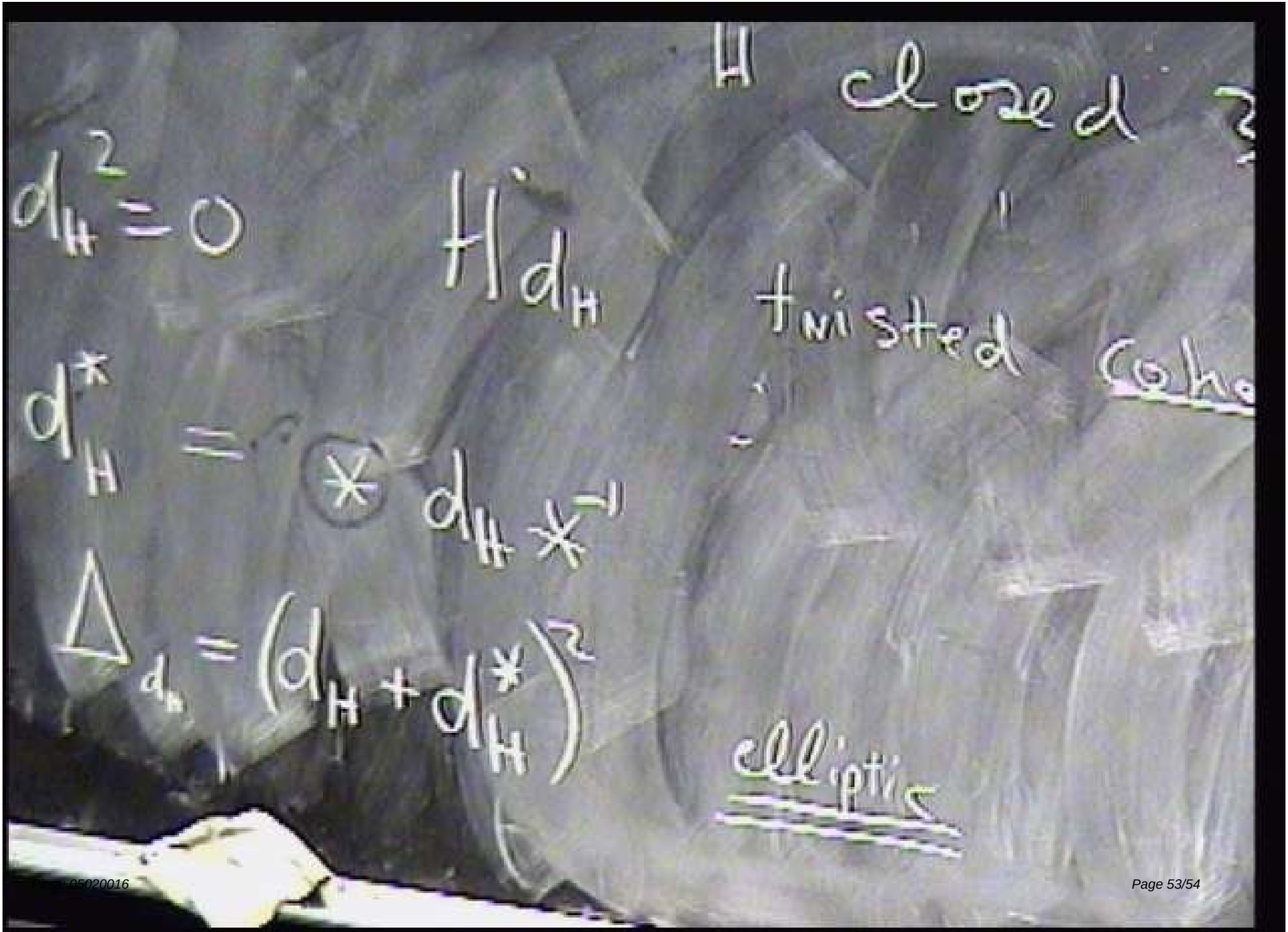
$d_H = d + H \wedge$

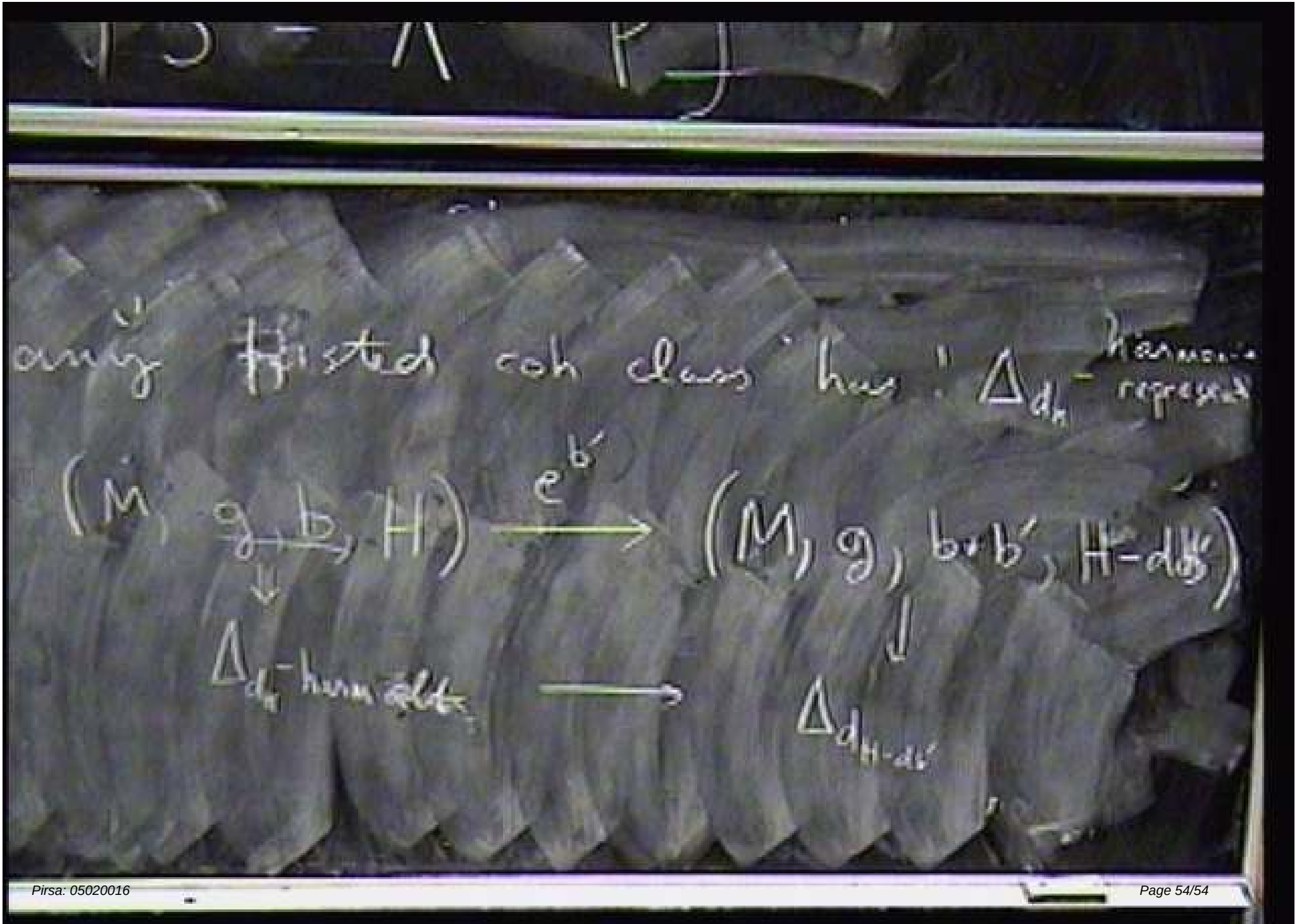
H closed 3-form

$d_H^2 = 0$

$H^\cdot_{d_H}$ twisted cohomology

$d_H^* = * \, d_H \, *^{-1}$


H closed ?
$d_H^2 = 0$
H_{d_H}
twisted coho
$d_H^* = * \, d_H \, *^{-1}$
$\Delta_{d_H} = (d_H + d_H^*)^2$
elliptic


$(M, g, b, H) \xrightarrow{e^{b'}} (M, g, b+b', H-db')$
Δ_{d_H}
$\Delta_{d_{H-db'}}$