

Title: An informal update on little string theories

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Abstract:

An informal update on "little string theories"

Based on ... , OA, Vasiliev, Kutasov, hep-th/0404016

- 1) Review of LSTs
- 2) Review of DSLSTs
- 3) An apparent paradox
- 4) LSZ in LST
- 5) Other plots in LST correlators and UV/IR mixing
- 6) Application I: Fixing operator mixings
- 7) Application II: Holographic duality and topological LSTs

LSTs

Decoupled theories on NS5 / geometric
singularity

LSTs

Decoupled theories on NS5 / geometric singularities
Simplest case - k flat NS5 in type IIB
 G $n=1(0,1)$ 6d

LSTs

Decoupled theories on NS5 / geometric singularities

Simplest case -

$$g_s^2 \alpha'^4 \sim G_N \rightarrow 0$$

g_m fixed

k flat NS5

in type IIB

$$n = 1(1,1)$$

6d

LSTs

Decoupled theories on NS5 / geometric singularities

Simplest case - k flat NS5 in type IB

$g_s^2 \propto 1/r$ $G_N \rightarrow 0$ g_m fixed

$n = k(1,1)$ 6d
 $\downarrow$ low energies
 $U(k)$ sym

LSTs

Decoupled theories on NS5 / geometric singularities

Simplest case -

$$g_s^2 \alpha'^4 \sim G_4 \rightarrow 0$$

g_m fixed

k flat NS5 in type IB

$$g_m = 1(1/1) \quad G_4$$

↓ low energies

$U(k)$

sym

$$g_m = \alpha'$$

on NS5

geometric
singularities

k flat NS5 in type IIB
 g_{YM} fixed

$n = 1(1,1)$ 6d

↓ low energies
sym

$U(k)$

$$g_{YM}^2 \sim \alpha'$$

$$(D5 \quad g_{YM}^2 \sim g_s \alpha')$$

LSTs

Decoupled theories on

Simplest case - k

$$g_s^2 \propto 1/\Lambda^4 \xrightarrow{\Lambda \rightarrow 0} G_N \rightarrow 0$$

G_N fixed

Usual decoupling = $\Lambda' \rightarrow 0$

LSTs

Decoupled theories on

Simplest case - K

$$g_s^2 \propto 1/\Lambda \sim G_N \rightarrow 0$$

G_M fixed

Usual decoupling $\equiv \Lambda' \rightarrow 0$

Decoupled theories on
 Simplest case - k
 $g_s^2 \propto 1/\alpha' \sim G_N \rightarrow 0$ g_{YM} fixed

Usual decoupling = $\alpha' \rightarrow 0$
 For NS5 = α' fixed, $g_s \rightarrow 0$

LSTs

Decoupled theories on NS5 / geometric singularities

Simplest case - k flat NS5 in type IB

$$g_s^2 \alpha'^4 \rightarrow G_4 \rightarrow 0 \quad \text{grn field}$$

Usual decoupling: $\alpha' \rightarrow 0$

For NS5 = $\boxed{1, g_s \rightarrow 0}$

$\downarrow$ low energies
 $U(1)$ sym
 $g_m \sim \alpha'$
 $(D5 \quad g_m^2 = 2\pi\alpha')$

LSTs

Decoupled theories on NS5 / geometric singularities

Simplest case - k flat NS5 in type IB

$$g_s^2 \alpha'^4 \rightarrow G_4 \rightarrow 0 \quad g_m \text{ fixed}$$

Usual decoupling: $x' \rightarrow \sigma$

For NS5 = $x' \text{ fixed, } g_s \rightarrow 0$

$\downarrow$ low energies
 $U(1)$ sym
 $g_m \sim g_s'$
 $(D5 \quad g_m^2 = g_s'^2)$

Direct definitions:

DLCQ

Acconstruction

hobgraphic

Direct definitions:

DLCQ

deconstruction

✓ holographic

Background of K NS5-branes:

$$ds^2_{10} = dx_\mu dx^\mu + \left(1 + \frac{K\alpha'}{r^2}\right) dx^i dx^i \quad \mu=$$

intions: DLCQ
 deconstruction
 ✓ holographic

of K NS5-branes:

$$k_m dx^m + \left(1 + \frac{k \alpha'}{r^2}\right) A x^i dx^i$$

$$\mu_i = 0, -1, 5$$

$$i = 6, 7, 8$$

Direct definitions:

DLCQ

deconstruction

✓ holographic

Background of k NS5-branes:

$$\begin{cases} ds^2_{10} = dx_{\mu} dx^{\mu} + \left(1 + \frac{k\alpha'}{r^2}\right) dx^i dx^i & \mu = 0, \dots, 9 \\ e^{2\tilde{\Phi}} = g_s^2 \left(1 + \frac{k\alpha'}{r^2}\right) \\ H_{ijk} = -\epsilon_{ijkl} \partial^l \tilde{\Phi} \end{cases}$$

Direct definitions:

DLCQ

1 construction

✓ holographic

Background of K NS5-branes:

$$\begin{cases} ds^2_{10} = dx_{\mu} dx^{\mu} + \left(1 + \frac{K \alpha'^2}{r^2}\right) dx^i dx^i & \mu = 0, \dots, 5 \\ e^{2\tilde{\Phi}} = g_s^2 \left(1 + \frac{K \alpha'^2}{r^2}\right) & g_s \rightarrow 0 \\ H_{ijk} = -\epsilon_{ijkl} \partial^l \tilde{\Phi} \end{cases}$$

Direct definitions:

DLCQ

deconstruction

✓ holographic

Background of k NS5-branes:

$$\begin{cases} ds^2_{10} = dx_\mu dx^\mu + \left(1 + \frac{k\alpha'}{r^2}\right) dx^i dx^i & \mu = 0, 1, 2, 3 \\ e^{2\tilde{\Phi}} = g_s^2 \left(1 + \frac{k\alpha'}{r^2}\right) & g_s \rightarrow 0 \\ H_{ijk} = -\epsilon_{ijk} \partial^l \tilde{\Phi} & r \rightarrow 0 \end{cases}$$

Holog. dual of LSTs

$$ds^2 = dx_{\mu} dx^{\mu}$$

Holog. dual of $LS T_5$

$$ds^2 = dx_\mu dx^\mu + K^2 d\Omega_3^2 + d\phi^2$$

Holog. dual of LSTs

$$ds^2 = dx_\mu dx^\mu + \kappa \alpha' d\Omega_3^2 + d\phi^2$$

$$\bar{\Phi} = -\frac{Q}{2}\phi$$

$$Q = \frac{2}{\sqrt{\kappa \alpha'}}$$

$$H_{ijk} \rightarrow \dots$$

Holog. dual of LST

$$ds^2 = \frac{dx_\mu dx^\mu}{R^2} + R^2 d\Omega_3^2 + d\phi^2$$

$$\bar{\phi} = -\frac{Q}{2\pi} \phi$$

$$Q = \frac{2\pi}{\sqrt{\kappa_2}}$$

$$\mathbb{R}^{5,1} \times \mathbb{R}_\phi \times$$

$$H_{ijk} \rightarrow \dots$$



eg. dual of LST:

$$ds^2 = \underbrace{dx_\mu dx^\mu}_{\text{spatial}} + \kappa \alpha' d\tau^2 + \alpha \phi^2$$

$$\bar{\Phi} = -\frac{Q}{2}\phi$$

$$Q = \frac{2}{\sqrt{\kappa \alpha'}}$$

$$\left. \begin{array}{l} ds^2 \\ \bar{\Phi} \\ Q \end{array} \right\} \mathbb{R}^{5,1} \times \mathbb{R}_\phi \times \text{SU}(2)_\chi$$

$$H_{ijk} \rightarrow \dots$$

Hijk

Standard features:

Hijk = - - -

Standard features:
Fields in bulk

$$\Phi = -\frac{Q}{r} \phi$$

$$Q \approx \sqrt{\frac{2}{K_9}}$$

$$\mathbb{R} \times (K_9 \times \mathbb{R})$$

hijk

Standard features:

Fields in bulk $\leftrightarrow$ off-shell operators in

$$\bar{\Phi} = -\frac{1}{\sqrt{2}} \phi$$

$$Q \equiv \frac{1}{\sqrt{2\kappa\alpha'}} \frac{\partial}{\partial \phi}$$

$$\left. \begin{array}{l} R^{5,1} \times R_9 \end{array} \right\}$$

$$H_{ijk} = \dots$$

Standard features:

- 1) Fields in bulk $\longleftrightarrow$ Off-shell operators in LST
- non / non-pert.

$$\bar{\Phi} = -\frac{Q}{2}\phi$$

$$Q \equiv \sqrt{\kappa \alpha'}$$

$$\left. \begin{array}{l} R^{5,1} \times R_9 \end{array} \right\}$$

$$H_{ijk} = \dots$$

Standard features:

1) Field in bulk $\leftrightarrow$ Off-shell operators in LST

non- $e^{-\alpha p^2}$ non- $e^{-\alpha p^2}$ $\leftrightarrow$ Correlators in LST

2) Chiral fields in bulk $\Leftrightarrow$ Chiral ops in LST
" Chiral ops in 4d SYM

2) Chiral fields in bulk $\leftrightarrow$ Chiral q's in LST
 $\parallel$
 Chiral q's in Cd SYM
 $A_\mu, \psi, X^1, X^2, X^3, X^4$

bulk $\Leftrightarrow$ Chiral ops in LST
 ||
 Chiral ops in 6d SYM
 $A_M, \psi, X^{\mu}, X^{\mu\dagger}, X^{\mu}, X^{\mu\dagger}$
 $\text{tr}(X^{i_1} X^{i_2} \dots X^{i_n})$

2) Chiral fields in bulk $\leftrightarrow$ Chiral ops in LST

||
Chiral ops in 6d SYM

$A_n, \psi, \chi^a, \tilde{\chi}^a, \chi^a, \tilde{\chi}^a$

$\text{tr}(\chi^a \chi^a \dots \chi^a)$

Precise agreement

2) Chiral fields in bulk $\leftrightarrow$ Chiral ops in LST

Chiral ops in 6d SYM
 $A_\mu, \psi, X^1, X^2, X^3, X^4$

Trace agreement $\rightarrow$

$\underline{\underline{\theta_2}} \leftrightarrow \underline{\underline{\theta_1}} = \text{tr}(X^1 X^2 \dots X^k)$
 $n=2, \dots, k$

2) Chiral fields in bulk $\leftrightarrow$ Chiral ops in LST

Chiral ops in 4d SYM
 $A_\mu, \psi, \chi^1, \chi^2, \chi^3, \chi^4$

Trace agreement $\rightarrow$

$\hat{Q}_n$ $\leftrightarrow$ $\theta_n = \text{tr}(\chi^1 \chi^2 \dots \chi^n)$
 $n = 0, \dots, 4$

Stringy - features

1) T-duality

$$R \leftrightarrow \frac{\alpha'}{R}$$

1) gauge theory

Stringy features

1) T-duality

$$R \leftrightarrow \alpha'$$

Non-local
UV Ringov.

Stringy - features

1) T-duality

$$R \leftrightarrow \frac{\alpha'}{R}$$

Non-local
Int. Stringy.

2)

Stringy - features

1) T-duality

$$R \leftrightarrow \alpha'$$

Non-local
Int String.

2)

Stringy features

1) T-duality

$$R \longleftrightarrow \frac{\alpha'}{R}$$

Non-local
Int. stringy.

2) High-energy density

Stringy features

1) T-duality

$$R \leftrightarrow \frac{\alpha'}{R}$$

Non-local
Int. string.

2) High-energy density :

$$\rho(\epsilon) \propto e^{\beta_H \epsilon}$$

$$\beta_H = \frac{1}{2\pi\alpha' \sqrt{g}}$$

Stringy features

1) T-duality

$$R \leftrightarrow \frac{\alpha'}{R}$$

Non-local
Int. string.

2) High-energy density :

$$\rho(E) \propto E^2$$

like in free string theory

$$R_H \sim \alpha' \sqrt{E}$$

1) T-duality

$$R \leftrightarrow \frac{\alpha'}{R}$$

Non-local
Int. string v.

2) High-energy density:

$$\rho(\epsilon) \propto e^{\beta_H \epsilon}$$

like in free string th

$$\beta_H = \alpha' \sqrt{K_2}$$

sharp feature =

1) T-duality

$$R \leftrightarrow \frac{\alpha'}{R}$$

Non-local
Int. string v.

2) High-energy density:

$$\rho(E) \propto e^{\beta_H E}$$

$$\beta_H = \frac{1}{2\pi\alpha' \sqrt{R\alpha'}}$$

like in free string theory

String feature:

Stringy features

1) T-duality

$$R \leftrightarrow \frac{\alpha'}{R}$$

Non-local
in string.

2) High-energy density :

$$\rho(\epsilon) \propto \frac{1}{\epsilon} \left(\frac{\epsilon}{\Lambda^2} \right)^{1/2}$$

like in free string theory

String feature :



Stringy features

1) T-duality

$$R \leftrightarrow \frac{\alpha'}{R}$$

Non-local
in string.

2) High-energy density :

$$\rho(\epsilon) \propto e^{-\beta_H \epsilon}$$

$$\beta_H = 2\pi\sqrt{\alpha'}$$

like in free string theory

String feature:



5-form nontrivial

Stringy features

1) T-duality

$$R \leftrightarrow \frac{\alpha'}{R}$$

Non-local
in string theory

2) High-energy density :

$$\rho(E) \propto e^{\beta_H E}$$

$$\beta_H = \frac{1}{2\pi\alpha' \sqrt{R}}$$

like in free string theory

String feature :



$$\delta\text{-func variable} \sim e^{(\frac{1}{2\pi\alpha'} H^2) q}$$

Stringy features

1) T-duality

$$R \leftrightarrow \frac{\alpha'}{R}$$

Non-local
in string theory

2) High-energy density :

$$\rho(E) \propto e^{\beta_H E}$$

$$\beta_H = \frac{1}{2\pi\alpha' \sqrt{g}}$$

like in free string theory

Sharp feature :



$$5\text{-form wavefunction} \sim e^{(\frac{Q}{2} H A)} \phi$$

6d continuous spectrum

$$M^2 = \lambda^2 + C$$

T-duality

$$R \leftrightarrow \frac{\alpha'}{R}$$

Non-local
Int. string

High-energy density:

$$\rho(\epsilon) \propto e^{\beta_H \epsilon}$$
$$\beta_H = \frac{1}{2\pi\alpha' \sqrt{g}}$$

like in free string theory

large feature:

$$\delta\text{-func. wavefunction} \sim e^{i(\frac{1}{2} p^2 \alpha') \phi}$$

6d continuous spectrum

$$M^2 = \lambda^2 + C$$



DSLST

11/11

NS5-branes have a moduli space

branes:

DSLST

21.00

NS5-branes have a moduli space

$$\mathbb{R}^{4k} / S_k$$

DSLST

NS5-branes have a moduli space

$$\mathbb{R}^{4k} / S_k$$

$$A = X^p + iX^q$$

$$B = X^r + iX^s$$

$$\langle A \rangle = 0$$

$$\langle B \rangle = \tau_6$$

$$\begin{pmatrix} 1 & & & \\ & e^{2\pi i / k} & & \\ & & e^{4\pi i / k} & \\ & & & \ddots \\ & & & & e^{2\pi i / k} \end{pmatrix}$$

DSLST

NS5-branes have a moduli space

$$\mathbb{R}^{4k} / S_k$$

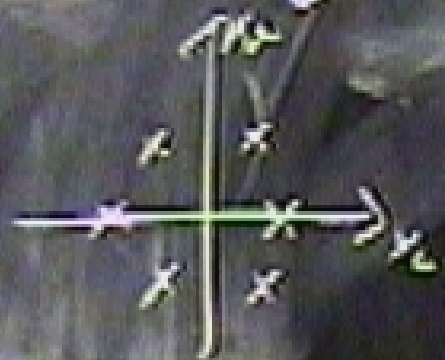
$$A = X^3 + iX^3$$

$$B = X^4 + iX^3$$

$$\langle A \rangle = 0$$

$$\langle B \rangle = \mathbb{I}_6$$

$$\begin{pmatrix} 1 & & & \\ & e^{2\pi i / k} & & \\ & & e^{4\pi i / k} & \\ & & & \ddots \\ & & & & e^{2\pi i / k} \end{pmatrix}$$



DSLST

NS5-branes have a moduli space

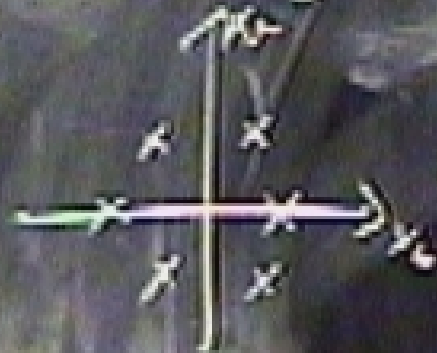
$$\mathbb{R}^{4k} / S_k$$

$$A = X^p + iX^q$$

$$B = X^r + iX^s$$

$$\langle A \rangle \rightarrow 0$$

$$\langle B \rangle = \Gamma_6$$



$$\begin{pmatrix} 1 & & & \\ & e^{2\pi i / k} & & \\ & & e^{4\pi i / k} & \\ & & & \ddots \\ & & & & e^{-2\pi i / k} \end{pmatrix}$$

gauge $SU(k) \rightarrow U(1)^{k-1}$
 global $SO(4)$

DSLST

NS5-branes have a moduli space

$$\mathbb{R}^{4k} / S_k$$

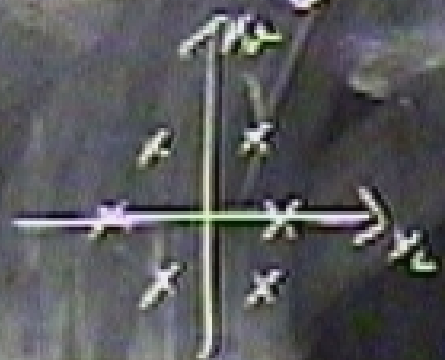
$$A = X^3 + iX^2$$

$$B = X^4 + iX^3$$

$$\langle A \rangle = 0$$

$$\langle B \rangle = \Gamma_6$$

$$\begin{pmatrix} 1 & e^{2\pi i/k} & & \\ & e^{4\pi i/k} & & \\ & & \ddots & \\ & & & e^{2\pi i/k} \end{pmatrix}$$



group $SU(k) \rightarrow U(1)^{k-1}$

global $SO(4) \rightarrow U(1) \times \mathbb{Z}_2$

DSLST

NS 5-branes have a moduli space

$$\mathbb{R}^{4k} / S_k$$

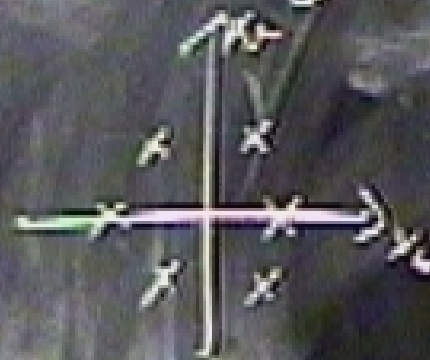
$$A = X^p + iX^q$$

$$B = X^r + iX^s$$

$$\langle A \rangle = 0$$

$$\langle B \rangle = \tau_0$$

$$\begin{pmatrix} 1 & e^{2\pi i / k} & & \\ & e^{4\pi i / k} & & \\ & & \ddots & \\ & & & e^{2\pi i (k-1) / k} \end{pmatrix}$$



gauge $SU(k) \rightarrow U(1)^{k-1}$
 global $SO(4) \rightarrow U(1) \times Z_k$

$$\frac{M_{\text{KK}}}{M_5}$$

$$\langle \psi | \hat{H} | \psi \rangle = \langle \psi | \hat{H} | \psi \rangle = 0$$

$$\langle \text{tr} b^N \rangle \text{ to } \langle \text{tr} b^N \rangle = \langle \text{tr} A^N \rangle = 0$$

Dual Backgrounds $R^{S,1} \times \left(\frac{SU(2)_L}{U(1)} \times \frac{SU(2)_C}{U(1)} \right) / \mathbb{Z}_K$

$$\langle t\phi^4 \rangle \text{ to } \langle t\phi^4 \rangle = \langle t\phi^4 \rangle = 0$$

Dual Backgrounds: $R^{5,1} \times \left(\frac{SU(2)_L}{U(1)} \times \frac{SU(2)_L}{U(1)} \right) / \mathbb{Z}_2$



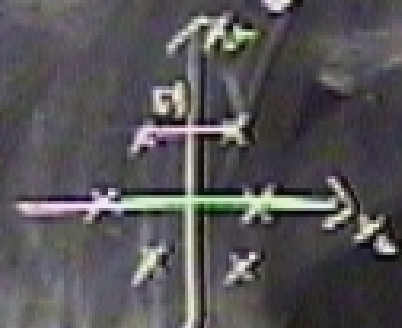
$$A = X^3 + iX$$

$$B = X^4 + iX^2$$

$$\langle A \rangle = 0$$

$$\langle B \rangle = \frac{1}{6}$$

$$\begin{pmatrix} 1 & e^{2\pi i/6} & & \\ & e^{4\pi i/6} & & \\ & & \ddots & \\ & & & e^{2\pi i/6} \end{pmatrix}$$



gauge $SO(k) \rightarrow U(1)^{k-1}$

global $SO(4) \rightarrow U(1) \times U(1)$

$$\frac{M_W}{M_S} - \text{differences plan.}$$



$$g_{SM} \rightarrow g_{SM} \rightarrow g_{SM}$$

$$\langle \text{tr}(B^2) \rangle \rightarrow \langle \text{tr}(B^2) \rangle \rightarrow 0$$

Dual background: $R^{SOL} \times \left(\frac{SU(2)_c}{U(1)} \times \frac{SU(2)_L}{U(1)} \right) / \mathbb{Z}_2$



$$g_S \ll g_S^{th} = \frac{M_{H_u}}{\Lambda}$$

$M_{H_u} \gg M_S \rightarrow$ weak coupling everywhere

gauge symmetry $\rightarrow$ unitary

$\langle \text{tr}(B^2) \rangle \rightarrow \langle \text{tr}(A^2) \rangle \rightarrow 0$

Dual background: $R^{SOL} \times \left(\frac{SU(2)_L}{U(1)} \times \frac{SU(2)_C}{U(1)} \right) // \mathbb{Z}_2 \times$

$$M_W \Rightarrow \frac{M_{H_u}}{2}$$



$$g_S \ll g_S^{H_u} = \frac{m_{H_u}}{M_W}$$

$$m_{H_u} \approx S^2$$

$M_{H_u} \gg M_W \rightarrow$ weak coupling everywhere

gauge symmetry → gauge



$\langle tr B^4 \rangle \neq 0$ to $\langle tr B^4 \rangle = \langle tr A^4 \rangle = 0$

via Lüscher

Dual Backgrounds $R^{S,1} \times \left(\frac{SU(2)_L}{U(1)} \times \frac{SU(2)_C}{U(1)} \right) / \mathbb{Z}_K$

$M_h = \frac{M_{Pl}}{\sqrt{N}}$



$g_S \ll g_S^{th} = \frac{M_{Pl}}{\Lambda}$

for all $x \in S^1$

$M_U \gg M_S \rightarrow$ weak coupling everywhere

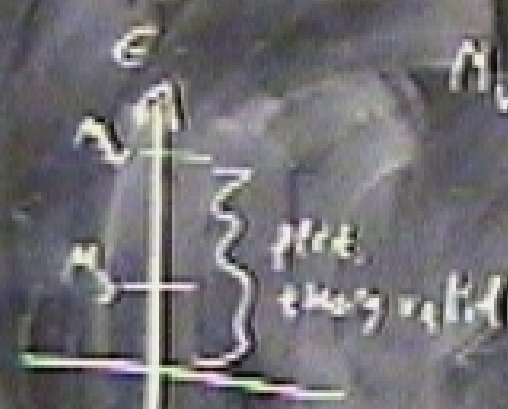
gauge symmetry → gauge bosons



$\langle \text{tr} B^4 \rangle \neq 0 \rightarrow \langle \text{tr} A^4 \rangle = 0$

no anomaly

Dual Lagrangian $R^{S+1} \times \left(\frac{SU(2)_L}{U(1)} \times \frac{SU(2)_L}{U(1)} \right) / \mathbb{Z}_2$



$M_W \approx \frac{M_{Pl}}{25}$



$g_S < g_S^{the} = \frac{m_{\nu}}{\Lambda}$

in all $\times S^1$

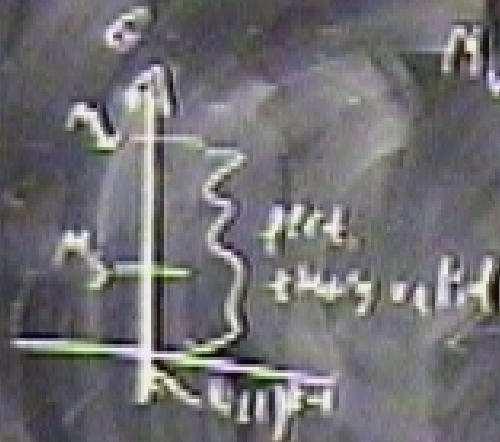
$M_{Pl} \gg M_S \rightarrow$ coupling future

gauge symmetry $\rightarrow$ unbroken

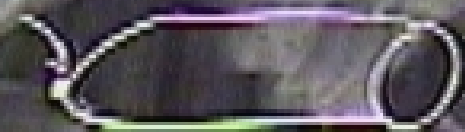
$$\langle T_{00} \rangle \neq 0 \quad \langle T_{0i} \rangle = \langle T_{i0} \rangle = 0$$

no Lorentz

Dual background: $R^{sol} \times \left(\frac{SU(2)_L}{U(1)} \times \frac{SU(2)_R}{U(1)} \right) / \mathbb{Z}_k$



$$M_W \rightarrow \frac{M_W}{g}$$



$$g_S \ll g'_S = \frac{M_W}{M_U}$$

$$\frac{1}{M_W} \ll \frac{1}{M_U} \approx g'_S$$

$M_U \rightarrow$ weak energy

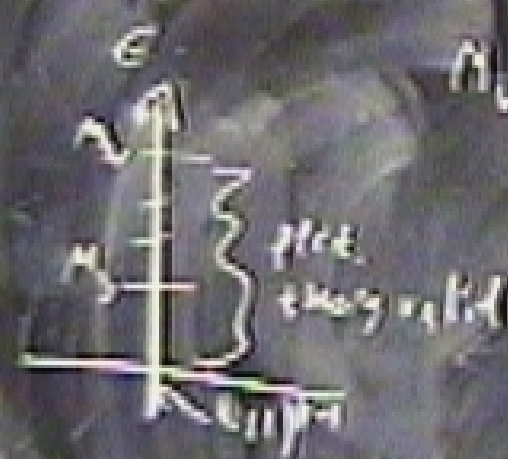
gauge symmetry $\rightarrow$ unbroken



$\langle \text{tr} B^4 \rangle \neq 0$ to $\langle \text{tr} B^4 \rangle = \langle \text{tr} A^4 \rangle = 0$

via Lüscher

Dual Backgrounds $R^{S+1} \times \left(\frac{SU(2)_L}{U(1)} \times \frac{SU(2)_C}{U(1)} \right) / \mathbb{Z}_K$

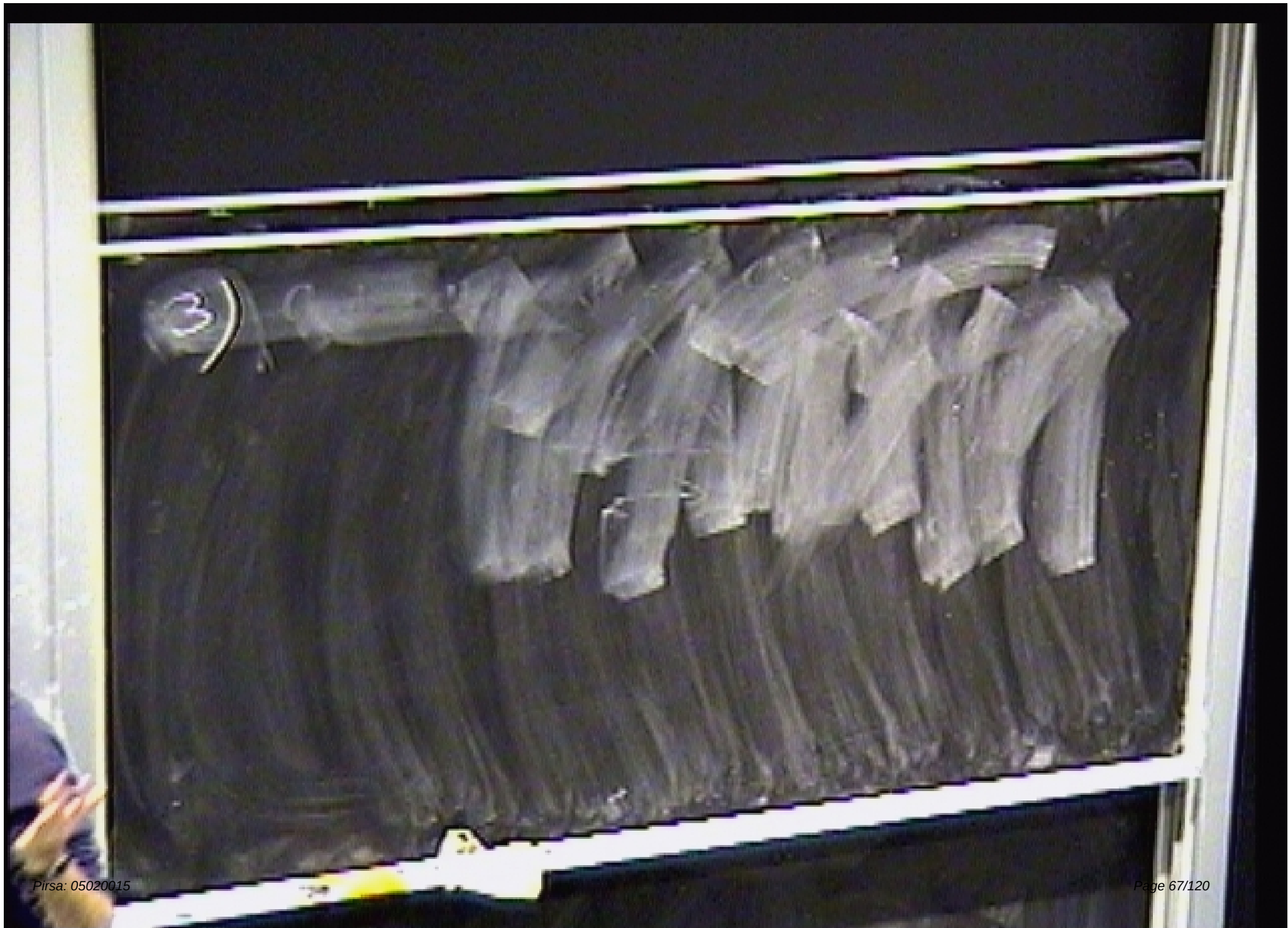


$M_4 = \frac{M_5}{\sqrt{2}}$



$g_S \ll g_S^{\text{IR}} = \frac{M_5}{\Lambda}$

$M_4 \gg M_5 \rightarrow$ weak coupling everywhere



3)

$$B_L = \text{tr}(B^L)$$

$$\langle B_L(\rho) B_L^*(1-\rho) \rangle$$

3)

$$B_L = \text{tr}(B^L)$$

$$\langle B_L(p) B_L^*(1-p) \rangle$$

low-energy sym, at leading



1



3)

$$B_L = \text{tr}(B^L)$$

$$\langle B_L(p) B_L^*(1-p) \rangle$$

low-energy sym, at leading

$$\frac{M_W^2}{p^2}$$

3)

$$B_L = \text{tr}(B^L)$$

low-energy sym, at leading

$$\langle B_L(p) B_L^*(-p) \rangle$$

$$= \frac{M_W^{2(L-1)}}{p^2}$$

$$\langle \hat{B}_L(p) \hat{B}_L^*(-p) \rangle$$

$$=$$

3)

$$B_\ell = \text{tr}(B^\ell)$$

$$\langle B_\ell(p) B_\ell^*(-p) \rangle$$

low-energy sym, at leading

$$\downarrow \frac{M_W^{2(L-1)}}{\rho^2}$$

$$\langle \hat{B}_\ell(p) \hat{B}_\ell^*(-p) \rangle$$

$$\Rightarrow \frac{M_W^{2(L-1)}}{\rho^2}$$

$$A_\ell = \text{tr}(A^\ell)$$

$$\langle A_\ell(p) A_\ell^*(-p) \rangle =$$

3)

$$B_\ell = \text{tr}(B^\ell)$$

$$\langle B_\ell(p) B_\ell^*(-p) \rangle$$

low-energy sym, at leading

$$\frac{M_W^{2(\ell-1)}}{p^2}$$

$$\langle \hat{B}_\ell(p) \hat{B}_\ell^*(-p) \rangle$$

$$\frac{M_W^{2(\ell-1)}}{p^2}$$

$$A_\ell = \text{tr}(A^\ell)$$

$$\langle A_\ell(p) A_\ell^*(-p) \rangle$$

$$= M_W^{2(\ell-1)} (p^2)$$

$$\langle \hat{A}_\ell(p) \hat{A}_\ell^*(-p) \rangle$$

$$\frac{M_W^{2(\ell-1)}}{p^2}$$

3)

$$B_L = \text{tr}(B^P)$$

$$\langle B_L(p) B_L^*(-p) \rangle$$

$$\langle \hat{B}_L(p) \hat{B}_L^*(-p) \rangle$$

$$A_L = \text{tr}(A^P)$$

$$\langle A_L(p) A_L^*(-p) \rangle = M_W^0(p^P)$$

$$\langle \hat{A}_L(p) \hat{A}_L^*(-p) \rangle \sim$$

low-energy sum, at leading order in

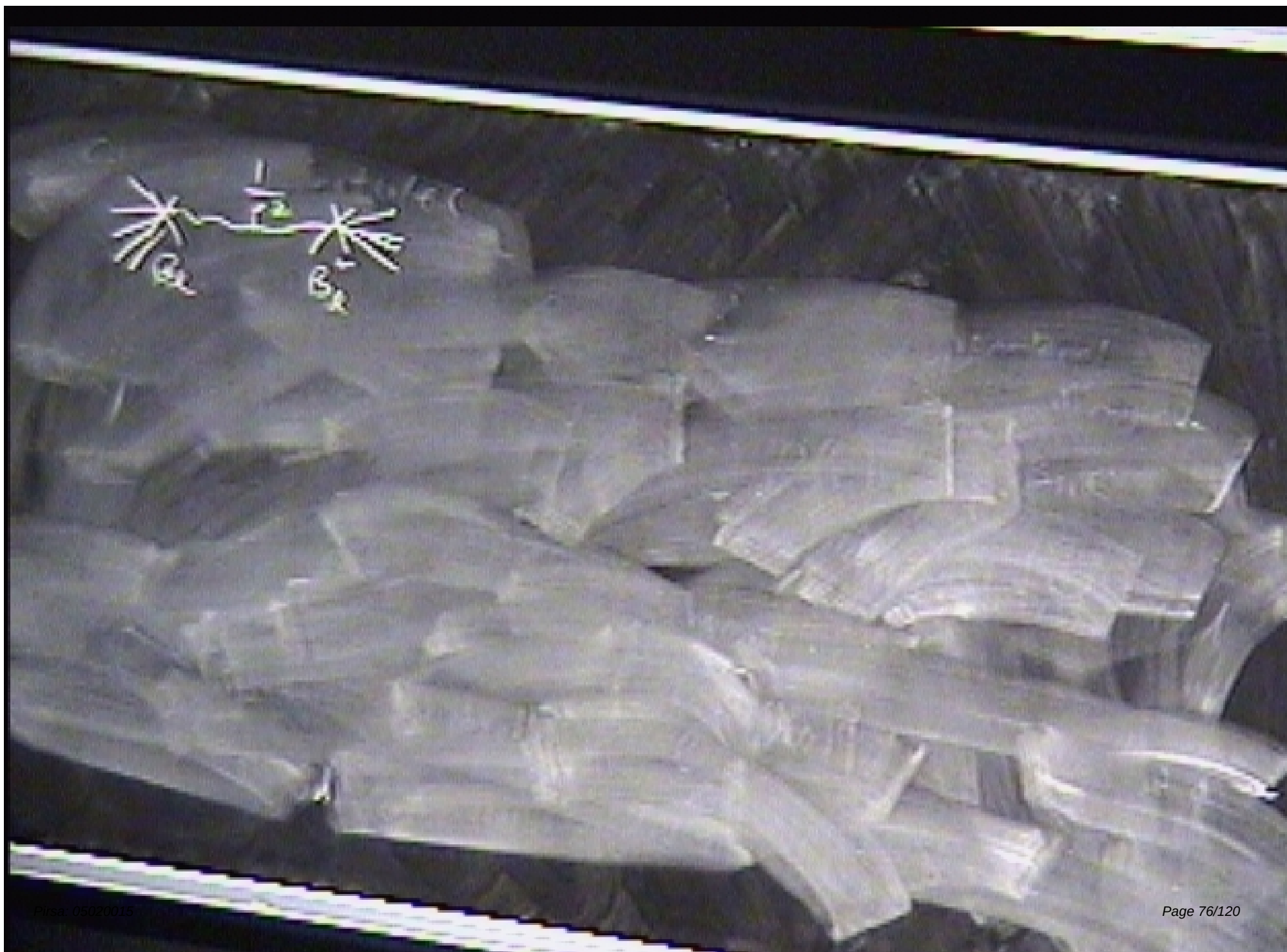
$$\frac{M_W^{2(L-1)}}{p^2}$$

$$\frac{M_W^{2(L-1)}}{p^2}$$

$$\frac{M_W^{2(L-1)}}{p^2}$$

$p \rightarrow 0$







$$\langle B(p) \theta_1 \dots \theta_n \rangle \sim \frac{\#}{p^2 + m^2}$$

$$\underline{S_L(\gamma)} = \frac{S_L(2)}{\underline{U(1)}} \times \underline{U(1)}$$





$$\langle B(p) \theta_1 \dots \theta_n \rangle \sim \frac{1}{p^2 + m^2} + \dots$$



$$\underline{SL(2)} = \frac{SL(2)}{U(1)} \times \underline{U(1)}$$

$$SL(2)_L = AHS_2$$





$$\underline{SL(2)} = \frac{SL(2)}{U(1)} \times \underline{U(1)}$$

$$\langle B(p) \theta_1 \dots \theta_n \rangle \sim \frac{\#}{p^{\frac{n}{2}} + n g}$$

$$SL(2)_k = AHS_3$$

$$\phi_j(x, z)$$





$$\underline{SL(2)} = \frac{SL(2)}{U(1)} \times \underline{U(1)}$$

$$\langle B(p) \theta_1 \dots \theta_n \rangle \sim \frac{\#}{p^2 + m^2}$$


$$SL(2)_k = AHS_2$$

$$\underline{\phi_3(x, z)}$$





$$\underline{SU(2)} = \frac{SU(2)}{U(1)} \times \underline{U(1)}$$

$$\langle D_1(f) \theta_1 \dots \theta_n \rangle \sim \frac{\#}{p} + \dots$$


$$SU(6)_c = AHS_3$$

$$\underline{\phi_j(x, z)} \quad \Delta_{\phi_j} = -\frac{j(j+1)}{k}$$

$$\Delta_{\psi_i} = j+1$$





$$\underline{SL(\gamma)} = \frac{SL(2)}{U(1)} \times \underline{U(1)}$$

$$\langle B_1(p) \theta_1 \dots \theta_n \rangle \sim \frac{\#}{p^2 + c_0}$$


$$SL(b)_k = AHS_3$$

$$\phi_3(x, z) \Delta_{\text{scat}} = \frac{\phi_3(x)}{z}$$

$$-\frac{1}{2} \int dx \frac{z(x)}{2} \quad \Delta_{\text{scat}} = \int dx$$





$$\langle B_2(p) \theta_1 \dots \theta_n \rangle \sim \frac{\#}{p^2 + m^2}$$

$$SU(7) = \frac{SU(2)}{U(1)} \times U(1)$$

$$\Phi_{j,m,n}(z) = \int d^2x \, x^{j+m} \bar{x}^{j-n} \phi_j(x,z)$$

$$SU(6)_c = AHS_3$$

$$\Phi_j(x,z) \quad \Delta_{-1} = -\frac{j(j+1)}{2} \quad \Delta_{+1} = j+1$$





$$\langle B_1(p) \theta_1 \dots \theta_n \rangle \sim \frac{\#}{p^2 + m^2}$$


$$\underline{SL(z)} = \frac{SL(z)}{U(1)} \times U(1)$$

$$\Phi_{j,m,n}(z) = \int d^2x \, x^{j-m} \bar{x}^{m-n} \phi_j(x, z)$$

$\downarrow$
 $V_{j,m,n} = 0$
 $\downarrow$
 $U(1)$

$$SL(6)_k = AHS_3$$

$$\Phi_j(x, z) \quad \Delta_{SL} = \frac{SL(6)_k}{k}$$

$\Delta_{SL} = \frac{SL(6)_k}{k}$

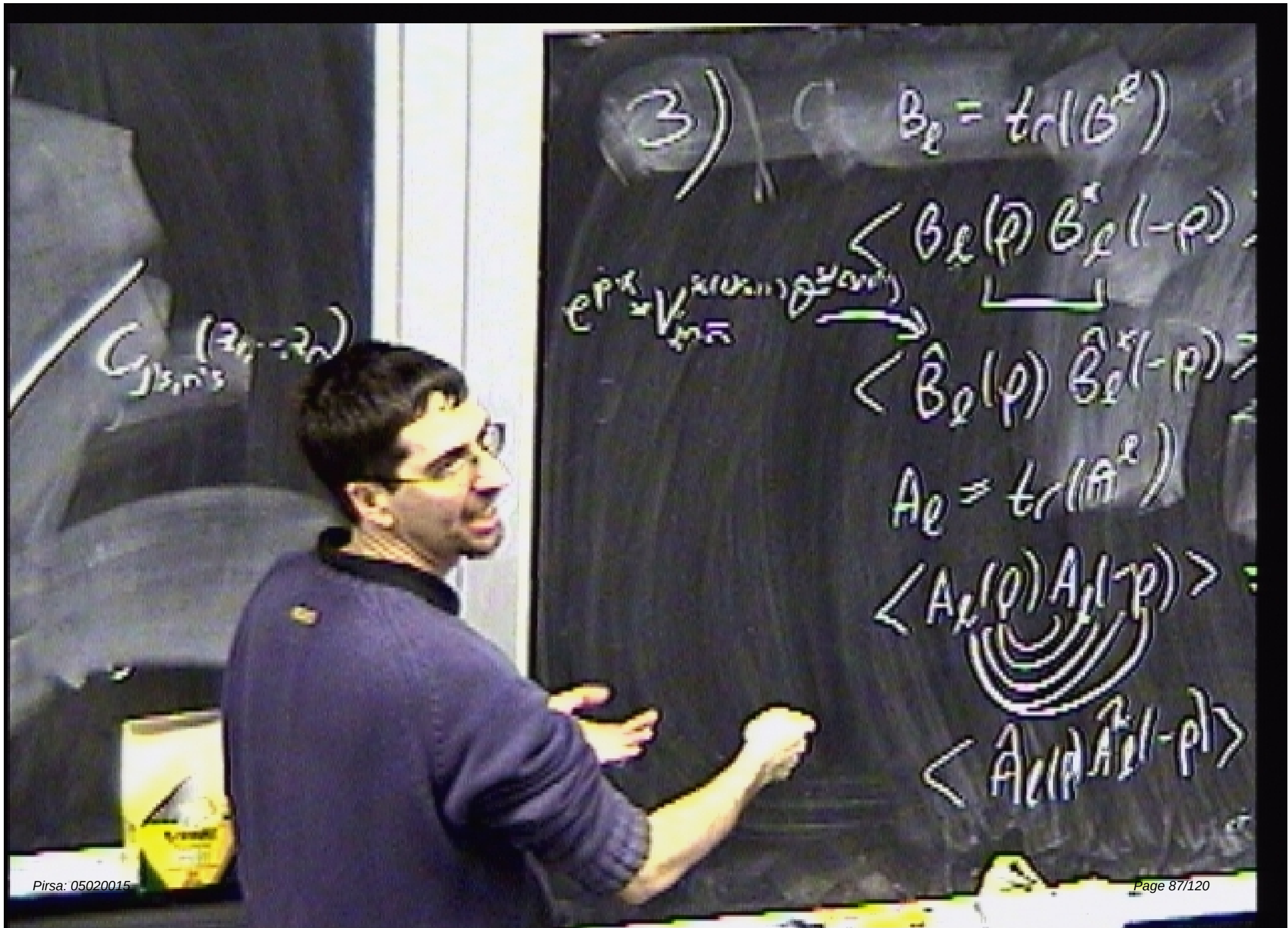


$$\langle V_{\text{kin},\vec{a}}(z_1) V_{\text{kin},\vec{a}}(z_2) - \dots \rangle_{\text{SUSY}} \leftarrow$$

$$\langle V_{j_1 m_1, \bar{j}_1}(\vec{z}_1) V_{j_2 m_2, \bar{j}_2}(\vec{z}_2) \rangle_{\text{super}} =$$

$$\int d^2 x_1 x_1^{j_1 m_1} \bar{x}_1^{\bar{j}_1 m_1} \int d^2 x_2 x_2^{j_2 m_2} \bar{x}_2^{\bar{j}_2 m_2} \dots$$

$$\langle \Phi_j(x_1; \vec{z}_1) \Phi_j(x_2; \vec{z}_2) \rangle_{\text{SL}(2)}$$



$$C_{j_1, j_2}(\vec{r}_1, \vec{r}_2)$$

3) $B_\rho = \text{tr}(B^\rho)$

$\langle B_\rho(\rho) B_\rho^*(-\rho) \rangle$

$\xrightarrow{\text{tr} \times V_{\text{tr}}}$

$\langle \hat{B}_\rho(\rho) \hat{B}_\rho^*(-\rho) \rangle$

$A_\rho = \text{tr}(A^\rho)$

$\langle A_\rho(\rho) A_\rho(-\rho) \rangle$

$\langle \hat{A}_\rho(\rho) \hat{A}_\rho(-\rho) \rangle$

$$\lim_{\epsilon \rightarrow 0} \langle V_{\text{kin}, \epsilon}(z) \rangle_{\text{vac}} =$$

$$\int d^3x_1 x_1^{j_1} \bar{x}_1^{j_1} \int d^3x_2 x_2^{j_2} \bar{x}_2^{j_2} \dots$$

$$\langle \Phi_{j_1}(x_1, z) \Phi_{j_2}(x_2, z) \dots \rangle_{\text{vac}}$$

$$x_1 \rightarrow 0$$

$$\langle \psi_{j_1, m_1}(z_1) \psi_{j_2, m_2}(z_2) \rangle_{\text{sup}} =$$

$$\int d^2x_1 x_1^{j_1+m_1} \bar{x}_1^{j_1-m_1} \int d^2x_2 x_2^{j_2+m_2} \bar{x}_2^{j_2-m_2} =$$

$$\langle \phi_{j_1}(x_1, z_1) \phi_{j_2}(x_2, z_2) \rangle_{\text{SU}(2)}$$

$$x_1 \rightarrow 0 \quad \frac{1}{j_1+m_1+1}$$

$$\langle \phi_{j_1, m_1}(z_1) \phi_{j_2, m_2}(z_2) \dots \rangle_{\text{supp}} =$$

$$\int d^2x_1 x_1^{j_1, m_1} \bar{x}_1^{j_1, m_1} \int d^2x_2 x_2^{j_2, m_2} \bar{x}_2^{j_2, m_2} \dots$$

$$\langle \phi_{j_1}(x_1, z) \phi_{j_2}(x_2, z) \dots \rangle_{\text{supp}(z)}$$

$$x_1 \rightarrow 0$$

$$\frac{1}{j_1 + m_1 + 1}$$

pole at $j_1 = -m_1 - 1$

$$= -m_1 - 2$$

$$= -m_1 - 3$$

$$\int_{S^{m_1-1}} V_{j_1 m_1, \bar{j}_1}(\bar{z}_1) V_{j_2 m_2, \bar{j}_2}(\bar{z}_2) \dots \int_{S^{m_n-1}} =$$

$$\int d^2 x_1 x_1^{j_1 m_1} \bar{x}_1^{\bar{j}_1 m_1} \int d^2 x_2 x_2^{j_2 m_2} \bar{x}_2^{\bar{j}_2 m_2} \dots$$

$$\langle \Phi_{j_1}(x_1; \bar{z}_1) \Phi_{j_2}(x_2; \bar{z}_2) \dots \rangle_{SU(2)}$$

$$x_1 \rightarrow 0 \quad \frac{1}{j_1 + m_1 + 1} \quad \text{pole at } j_1 = -m_1 - 1$$

$$-m_1 - 2$$

$$-m_1 - 3$$

$$\vdots$$

$$\text{Mass-shell condition} \quad \frac{1}{2} \rho^2 = \frac{j(j+1)}{k}$$

$$\langle \psi_{j_1, m_1}(\vec{z}_1) V_{j_2, m_2, \bar{a}}(\vec{z}_2) \dots \rangle_{\text{supra}} =$$

$$\int d^2x_1 x_1^{j_1, m_1} \bar{x}_1^{j_1, m_1} \int d^2x_2 x_2^{j_2, m_2} \bar{x}_2^{j_2, m_2} \dots$$

$$\langle \Phi_{j_1}(x_1, \tau) \Phi_{j_2}(x_2, \tau) \dots \rangle_{\text{SU}(2)}$$

$$x_1 \rightarrow 0 \quad \frac{1}{j_1 + m_1 + 1} \quad \text{pole at } j_1 = -m_1 - 1$$

$$= -m_1 - 2$$

$$= -m_1 - 3$$

$$\text{Mass-shell condition} \quad \frac{1}{2} p^2 - \frac{j(j+1)}{k} + \# = 1$$



$$\sum_i \langle \hat{\theta}_i(p_i) - \hat{\theta}_i(n) \rangle \approx \frac{1}{(p_i/n) - (1/n)} \sum_i (p_i - n)$$

$$S \sum \langle \hat{\theta}_1(p_1) - \hat{\theta}_1(p_2) \rangle \sim \frac{1}{(p_1 \hbar) \cdot (p_2 \hbar)} S(p_1 - p_2)$$

$\hat{p}_1 \rightarrow -\hat{p}_2$

$$S \sim \langle \hat{\theta}_1(p_1) - \hat{\theta}_1(p_2) \rangle \sim \frac{1}{(p_1 - p_2) - (A + i\epsilon)} S(p_1, \dots, p_n)$$

Poles : $\hat{\theta}_l(A) = t_l(B^l)$ Poles at $1/\epsilon = 0, \frac{2}{\epsilon}(l-1), \frac{4}{\epsilon}(l-2)$.

$$SZ \langle \hat{\theta}_1(p_1) - \hat{\theta}_1(p_2) \rangle \sim \frac{1}{(p_1 p_2) - (A^2 + \eta^2)} S(p_1, \dots, p_n)$$



Poles : $\hat{B}_l(A) = \text{tr}(B^l)$ Poles at $1/p^2 = 0, \frac{2}{K}(l-1), \frac{4}{K}(l-2)$
 $\hat{A}_l(p) = \text{tr}(A^l)$ No poles of this type

$$SZ \langle \hat{\theta}_l(p_1) \dots \hat{\theta}_l(p_n) \rangle \approx \frac{1}{(\hat{p}_l^2 - (A + \hat{A}))} S(p_1, \dots, p_n)$$



Poles : $\hat{B}_l(p) = \text{tr}(B^l)$ Poles at $1/p^2 = 0, \frac{2}{\kappa}(l-1), \frac{4}{\kappa}(l-2)$.
 $\hat{A}_l(p) = \text{tr}(A^l)$ No poles of this type



Other poles come from IR divergences

$$V_{\text{hard}} \sim e^2$$

Other poles come from IR divergences

$$V_{j_1 \dots j_n} \sim e^{Q_j \phi} (\dots) + O(e^{-\phi})$$

$$\int d\phi. e^{Q_j \phi} e^{Q_j \phi} e^{Q_j \phi} \dots e^{Q_j \phi} e^{Q_j \phi}$$

Other poles come from IR divergences

$$V_{\text{IR}} \sim e^{a_j \phi} (\dots) + O(e^{-a_j \phi})$$

$$\int d\phi \, e^{a_1 \phi} e^{a_2 \phi} e^{a_3 \phi} \dots e^{a_{l-1} \phi} \frac{1}{\int d\phi \, e^{a_l \phi}}$$

$$\phi \rightarrow \infty$$

Other poles come from IR divergences

$$V_{j_1 n, \bar{n}} \sim e^{a_{j_1} \phi} (\dots) + O(e^{-a_{j_1} \phi})$$

$$\int d\phi. e^{a_{j_1} \phi} e^{a_{j_2} \phi} e^{a_{j_3} \phi} \dots$$

2-pt. function

$$\langle \theta(p) \theta(-p) \rangle$$

$$\frac{1}{p^2 + m^2}$$

$$\frac{1}{j+j_1+\dots}$$

$$\phi \rightarrow \infty$$

$$\frac{1}{p^2 + m^2}$$

Other poles come from IR divergences

$$V_{j_1 \dots j_n} \sim e^{Q_j \phi} (\dots) + O(e^{-Q_j \phi})$$

$$\int d\phi. e^{Q_{j_1} \phi} e^{Q_{j_2} \phi} e^{Q_{j_3} \phi} \dots e^{Q_{(1-n)} \phi} \sim \frac{1}{j_1 + j_2 + \dots + j_{1-n}} \int \frac{1}{\phi^{j_1 + j_2 + \dots + j_{1-n}}} d\phi$$

2-pt. function

$$\langle \theta(p) \theta(-p) \rangle \sim \frac{1}{p^2}$$

poles come from IR divergences

$$n, \bar{n} \sim e^{Q(\phi)} (\dots) + O(e^{-Q(\phi)})$$

$$\int d\phi \cdot e^{Q_1(\phi)} e^{Q_2(\phi)} e^{Q_3(\phi)} \dots e^{Q_{l-1}(\phi)} \sim \frac{1}{\beta + \beta_1 + \dots + \beta_{l-1} + \beta}$$

$\phi \rightarrow \infty$

2-pt. function
 $\langle \theta(p) \theta(-p) \rangle$

$$\frac{1}{p^2}$$

$$\frac{1}{p^2 + \#}$$

$$\frac{1}{p^2 + \beta_1 + \beta_2 + \dots + \beta_l + \#}$$

poles come from IR divergences

$$n, \bar{n} \sim e^{Q(\phi)} (\dots) + O(e^{-Q(\phi)})$$

$$\int d\phi \cdot e^{Q_1(\phi)} e^{Q_2(\phi)} e^{Q_3(\phi)} \dots e^{Q_{(1-n)}(\phi)} \sim \frac{1}{j+j_1+\dots+j_{n+1}-j}$$

$\phi \rightarrow \infty$

2-pt. function
 $\langle \theta(p) \theta(-p) \rangle$

$$\langle \hat{A}_1(p) \hat{A}_2(-p) \rangle \sim \frac{1}{p^2}$$

fish
 (only
 large ϕ !)

$$\frac{1}{p^2}$$

$$\frac{1}{p^2 + \#}$$

$$\frac{1}{p_1^2 + p_2^2 + \dots + p_n^2 + \#}$$

poles come from IR divergences

$$n, \bar{n} \sim e^{Q(\phi)} (\dots) + O(e^{-\phi})$$

$$\rightarrow \int d\phi \cdot e^{Q_1(\phi)} e^{Q_2(\phi)} e^{Q_3(\phi)} \dots e^{Q_{l-1}(\phi)} \sim \frac{1}{j_1 + j_2 + \dots + j_{l-1} + j}$$

$\phi \rightarrow \infty$

2-pt. function
 $\langle \theta(p) \theta(-p) \rangle$

$$\langle \hat{A}_1(p) \hat{A}_2(-p) \rangle \sim \frac{1}{p^2}$$

from large ϕ !

$$\frac{1}{p^2}$$

$$\frac{1}{p^2 + \#}$$

$$\frac{1}{p^2 + p_1^2 + p_2^2 + \#}$$

$$\langle \text{tr}(\hat{B}^n(A)) \text{tr}(\hat{B}^n(A)) \rangle = \dots \text{tr}(\hat{B}^n(p)) \text{tr}(\hat{B}^n(p)) \rangle$$

$$n = \sum n_i, \quad p = \sum p_i$$

$$\langle \text{tr}(\hat{B}^{\alpha_1}(A)) \text{tr}(\hat{B}^{\alpha_2}(A)) \dots \text{tr}(\hat{B}^{\alpha_p}(A)) \text{tr}(\hat{B}^{\alpha_{p+1}}(A)) \rangle$$

$$n = \sum n_i, \quad p = \sum p_i$$

$$\langle \text{tr}(\hat{B}^1(A)) \text{tr}(\hat{B}^2(p)) \dots \text{tr}(\hat{B}^n(p)) \text{tr}(\hat{B}^1(p)) \rangle$$

$$n = \sum n_i, \quad p = \sum p_i$$

$$\langle \hat{B}^1(p) \rangle = \frac{1}{Z} \int \mathcal{D}A \mathcal{D}p \hat{B}^1(p) e^{-\int A^2 - p^2}$$

$$\langle \text{tr}(\hat{B}^n(A)) \text{tr}(\hat{B}^n(E)) \rangle = \dots \text{tr}(\hat{B}^n(E)) \text{tr}(\hat{B}^n(E))$$

$$n = \sum n_i, \quad p = \sum p_i$$

$$\langle \hat{B}^n(E) \rangle = \dots \frac{1}{\text{tr}(\hat{B}^n)} \text{tr}(\hat{B}^n(E))$$

$$\hat{B}_n \leftrightarrow \text{tr}(\hat{B}^n), \text{tr}(\hat{B}^n) \text{tr}(\hat{B}^{n+m})$$

$$\langle \text{tr}(\hat{B}^n(a)) \text{tr}(\hat{B}^n(a)) \rangle = \dots \text{tr}(\hat{B}^n(e)) \text{tr}(\hat{B}^n(e)) \rangle$$

$$n = \sum n_i, \quad p = \sum p_i$$

$$\langle \hat{B}^n(e) \rangle = \dots \frac{p_1 p_2 \dots p_n}{p_1^2 p_2^2 \dots p_n^2}$$

$$\hat{B}_n \leftrightarrow \text{tr}(\hat{B}^n), \text{tr}(\hat{B}^n) \text{tr}(\hat{B}^{n+1})$$

$$\hat{B}_n = \# \text{tr}(\hat{B}^n) + \#_1 \text{tr}(\hat{B}^n) \text{tr}(\hat{B}^{n+1}) + \dots$$

$$\begin{aligned}
 \langle \hat{B}^n(p) \rangle &= \frac{1}{\mathcal{Z}} \sum_i |p_i| \langle \hat{B}^n(p) \rangle \sim \frac{0}{p_1 \cdots p_i} \\
 \hat{B}_n &\leftrightarrow \text{tr}(\hat{B}^n), \text{tr}(\hat{B}^n) \text{tr}(\hat{B}^{n-1}) \\
 \hat{B}_n &= \# \text{tr}(\hat{B}^n) + \# \text{tr}(\hat{B}^n) \text{tr}(\hat{B}^{n-1}) + \# \text{tr}(\hat{B}^n) \text{tr}(\hat{B}^{n-2}) \cdots \text{tr}(\hat{B}^1) \\
 \frac{1}{n} \hat{B}_n &= 1
 \end{aligned}$$

$$\begin{aligned}
 \langle \hat{B}(p) \rangle &= \frac{1}{N} \langle \hat{B}(p) \rangle = \frac{1}{N} \langle \hat{B}(p) \rangle \\
 \hat{B}_n &\leftrightarrow \hat{B}(p), \hat{B}(p), \hat{B}(p) \\
 \hat{B}_n &= \hat{B}(p) + \hat{B}(p) + \hat{B}(p) \\
 \frac{1}{N} \hat{B}_n &= \frac{1}{N} \sum_{i=1}^N \frac{1}{L} \sum_{j=1}^L (\frac{1}{L} + \hat{B}(p)) = (\frac{1}{L} + \hat{B}(p)) \hat{B}(p)
 \end{aligned}$$

$$\begin{aligned}
 \langle \hat{B}^n(p) \rangle &= \sum_{i_1, \dots, i_n} |i\rangle \hat{B}^n(p) |i\rangle \sim \frac{0}{p^1 \dots p^l} \quad l \geq 2 \\
 \hat{B}_n &\leftrightarrow \text{tr}(\hat{B}^n), \text{tr}(\hat{B}^n) \text{tr}(\hat{B}^{n+1}) \\
 \hat{B}_n &= \# \text{tr}(\hat{B}^n) + \# \text{tr}(\hat{B}^n) \text{tr}(\hat{B}^{n+1}) + \dots \\
 \frac{1}{n} \hat{B}_n &= \sum_{l=1}^{\infty} \frac{1}{l!} \sum_{n_1, \dots, n_l} \left(\frac{1}{n_1} \text{tr}(\hat{B}^{n_1}) \right) \dots \left(\frac{1}{n_l} \text{tr}(\hat{B}^{n_l}) \right) s(n_1, \dots, n_l)
 \end{aligned}$$

$$1) \quad k \rightarrow \infty$$

$$\sim \frac{1}{k^{d-1}}$$

1) $k \rightarrow \infty$

$\sim \frac{1}{k^{d-1}}$

✓

2) $k \rightarrow \infty$
 $n \sim k$

large $S(n)$
angular momentum

$$1) \quad k \rightarrow \infty \quad \sim \frac{1}{k^{l-1}} \quad \checkmark$$

$$2) \quad \begin{array}{l} k \rightarrow \infty \\ n \sim k \end{array} \xrightarrow{\text{giant gravitons}} \text{F1 on } S^1 \times S^3 \xrightarrow{\text{sublattice limit}} \frac{\left(\frac{1-n}{k}\right)^{l-1}}{(-1)^{l-1}}$$

large spin angular-momentum

$$1) \quad k \rightarrow \infty$$

$$\sim \frac{1}{k^{l-1}}$$

✓

$$2) \quad k \rightarrow \infty$$

$$n \sim k$$

giant gravitons
FD on $S^1 \subset S^3$

Sublattice

large S^2
angular-momentum

$$\left(\frac{1-n}{k} \right)^{l-1} \sim (-1)^{l-1}$$

Sublat.

$$1) \quad k \rightarrow \infty \quad \sim \frac{1}{k^{l-1}} \quad \checkmark$$

$$2) \quad \begin{array}{l} k \rightarrow \infty \\ n \sim k \end{array} \rightarrow \begin{array}{l} \text{giant gravitons} \\ \text{FD on } S^1 \subset S^3 \end{array} \rightarrow \text{Sublattice} \\ \begin{array}{l} \uparrow \\ \text{large spin} \\ \text{angular momentum} \end{array} \quad \left(\frac{1-n}{k} \right)^{l-1} \sim (-1)^{l-1} \quad \text{Sublattice}$$