Title: Interpretation of Quantum Theory: Lecture 12

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Abstract:

(2) Which wave function for a particle in a beam?

— a problem for the "individual" interpretation.
(based on R.H. Dicke, 1956)

Two hypotheses:

a. Each electron is emitted in an energy eigenstate (a plane wave), but the particular energy varies from one electron to the next.

$$\psi_k(x, t) = e^{i(kx - \omega t)} \qquad \hbar\omega = \hbar^2 k^2/2M,$$

b. Each electron is emitted as a wave packet that has an energy spread equal to the energy spread of the beam.

$$\psi_{t_0}(x, t) = \int A(\omega) e^{i[kx - \omega(t - t_0)]} \, d\omega$$

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Mixed state operator for a particle from the beam:

(a) Mixture of energies

$$\rho = \int |\psi_k\rangle\langle\psi_k| W(\omega)\, d\omega$$

$$\rho(x, x') \equiv \langle x|\rho|x'\rangle = \int \psi_k(x, t)\, \psi_k^*(x, t)\, W(\omega)\, d\omega$$

$$= \int e^{ik(x-x')} W(\omega)\, d\omega$$

(b) Mixture of emission times

$$\langle x|\rho|x'\rangle = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \psi_{t_0}(x, t)\, \psi_{t_0}^*(x, t)\, dt_0$$

$$= \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \int A(\omega)\, e^{i[kx-\omega(t-t_0)]}\, d\omega \int A^*(\omega')\, e^{-i[k'x'-\omega'(t-t_0)]}\, d\omega'\, dt_0$$

Performing the integral over t_0 first and then taking the limit $T \to \infty$ yields zero unless $\omega = \omega'$ (and so also $k = k'$). Therefore the state function reduces to

$$\rho(x, x') = \int e^{ik(x-x')} |A(\omega)|^2\, d\omega.$$

$$\lim_{T\to\infty}\int_{-T/2}^{+T/2} e^{i(\omega t_0 - \omega' t_0)}\, dt_0$$

$$\lim_{T\to\infty} \int_{-T/2}^{+T/2} e^{i(\omega t_0 - \omega' t_0)} \, dt_0 = 0 \quad \text{if } \omega \neq \omega'$$

from the beam:

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In the **Ensemble interpretation**:

— go directly to the state operator.

— "steady state" $\Longrightarrow \frac{d\rho}{dt} = 0$

$$\therefore \quad [H, \rho] = 0$$

So ρ must be diagonal in the eigenfunctions of H, which are

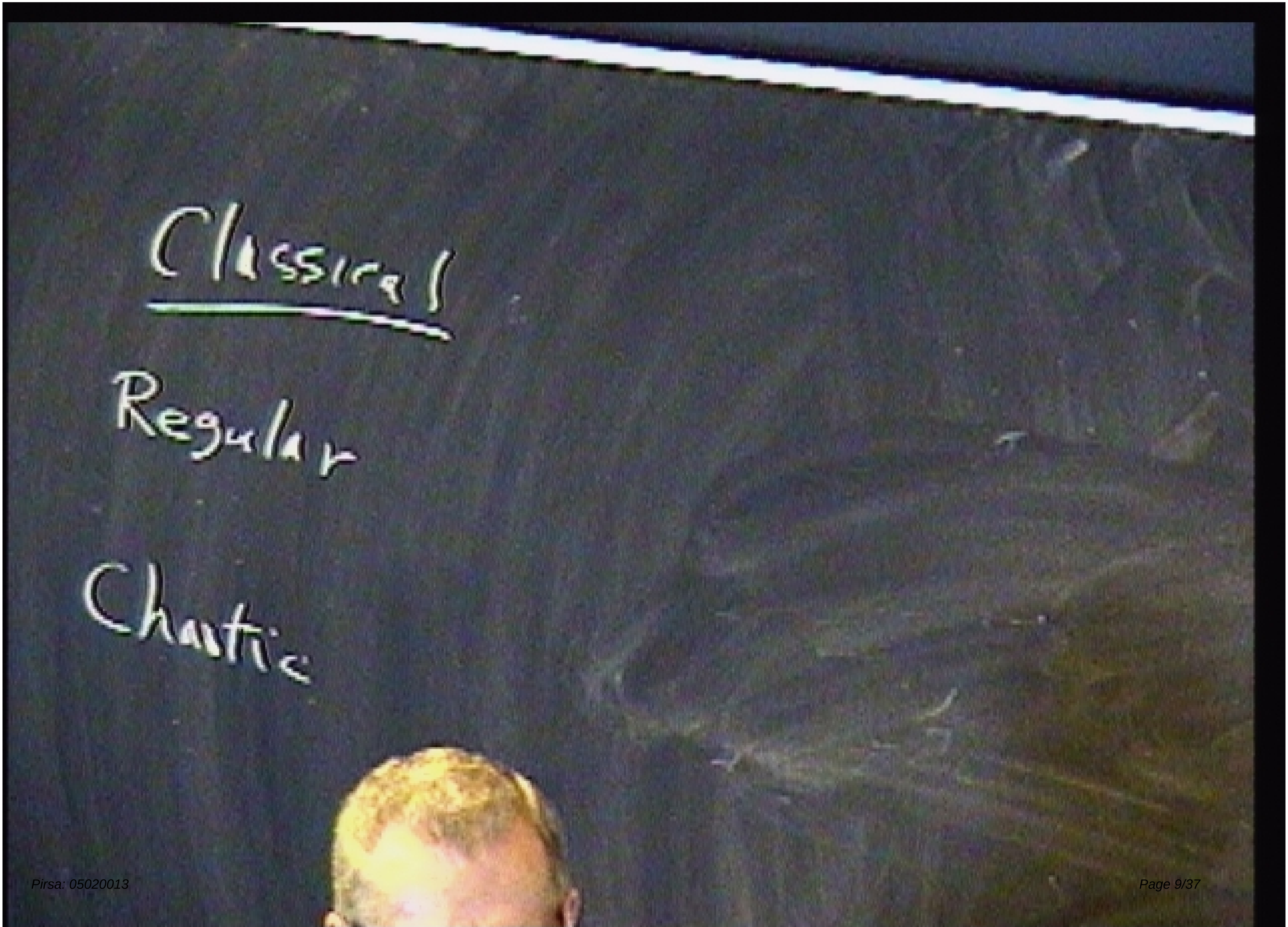
$$\psi_\omega(x) = e^{ikx} \,, \quad \hbar\omega = \frac{\hbar^2 k^2}{2m}.$$

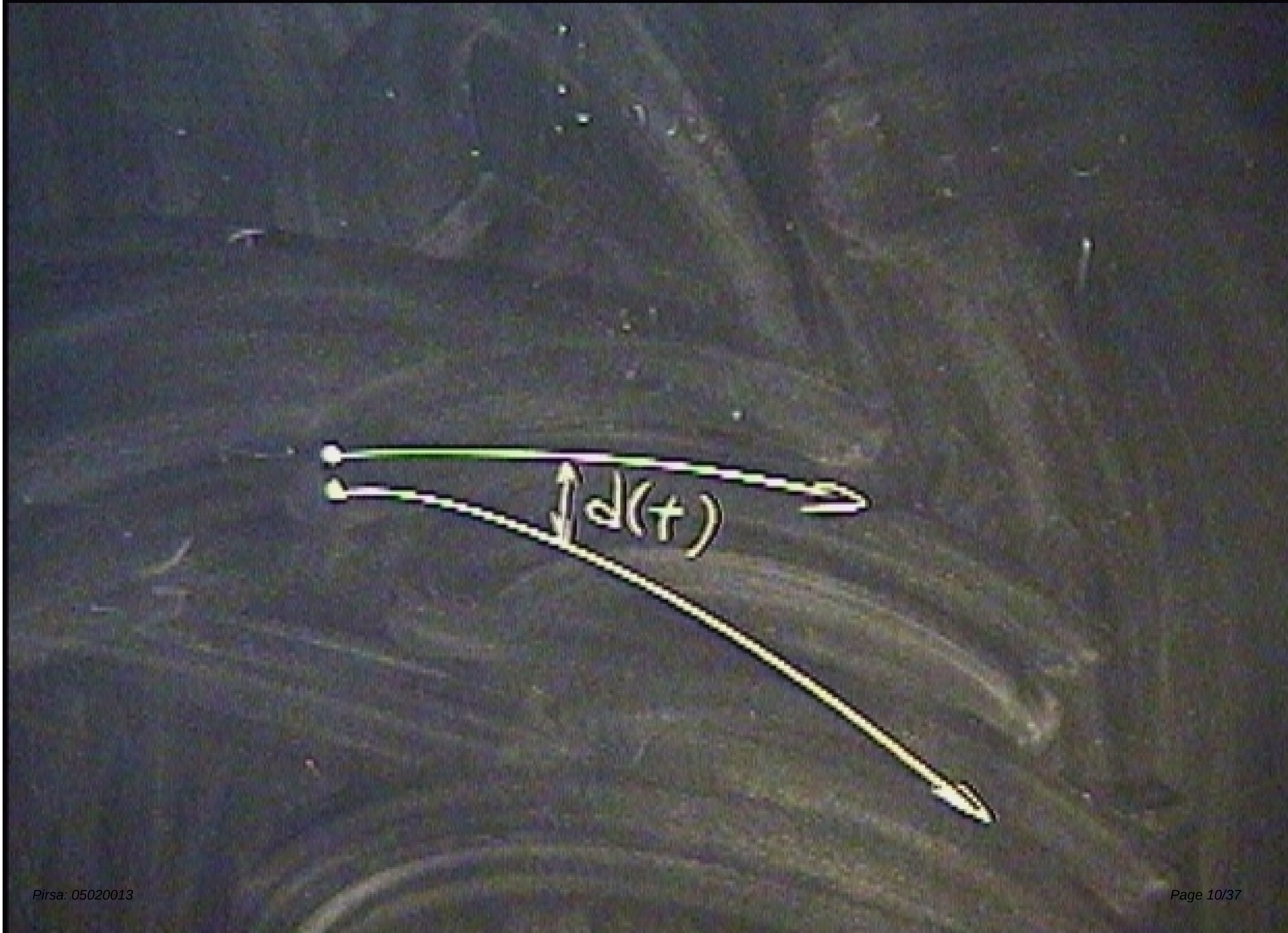
Hence

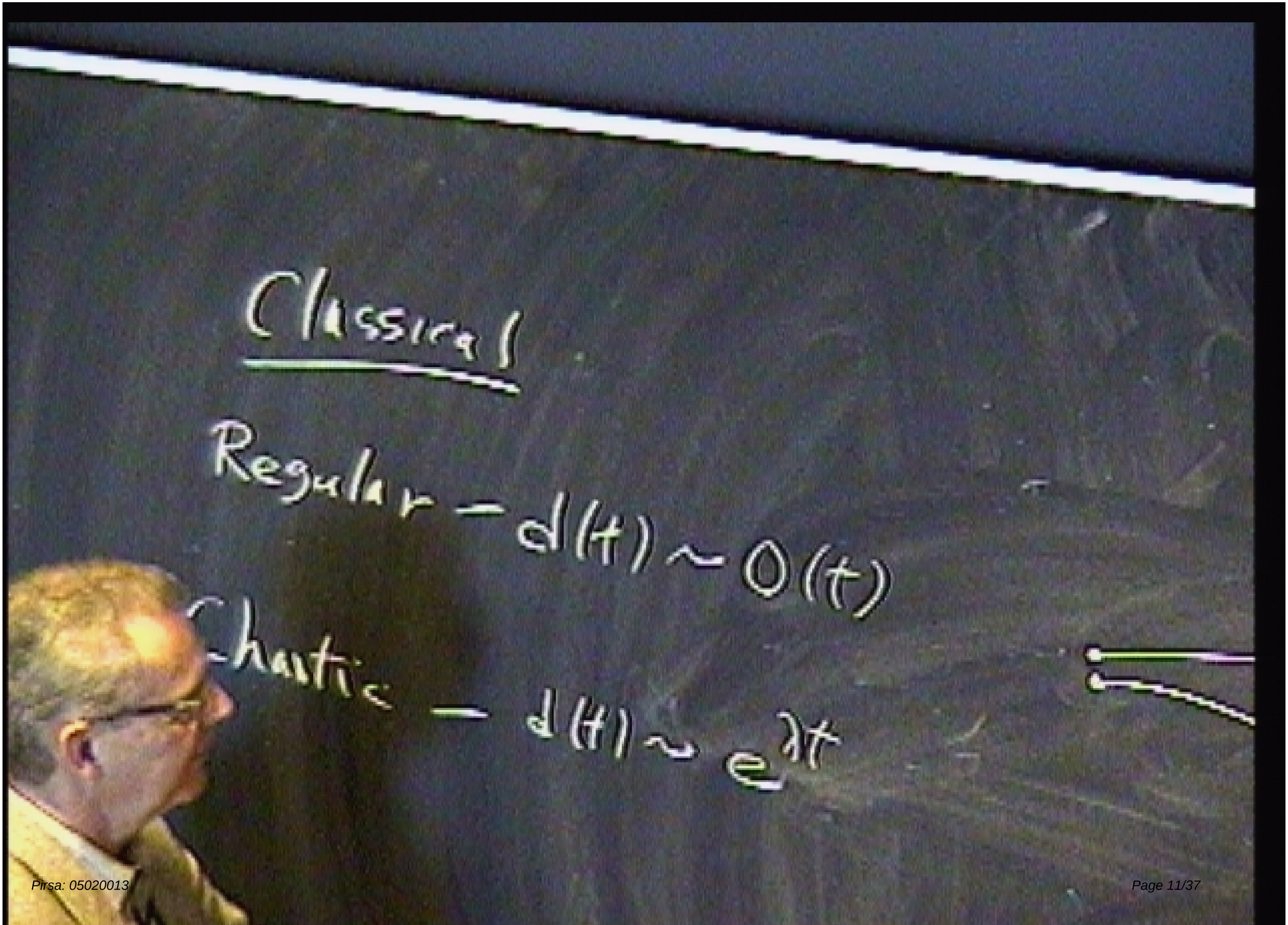
$$\rho = \int |\psi_\omega\rangle\langle\psi_\omega| W(\omega)\, d\omega$$

$$\rho(x, x') \equiv \langle x|\rho|x'\rangle = \int e^{ik(x-x')} W(\omega)\, d\omega$$

* No mention of "individual" wave functions.


Classical
Regular
Chaotic


d(t)


Classical
Regular – d(t) ~ O(t)
Chaotic – d(t) ~ e^{λt}

"Proof" that there is no Quantum Chaos:

Let $|\Psi_1(0)\rangle$ and $|\Psi_2(0)\rangle$ be two initially close states, such that

$$\langle\Psi_1(0)|\Psi_2(0)\rangle = 1 - \epsilon.$$

Unitary time evolution implies

$$\langle\Psi_1(t)|\Psi_2(t)\rangle = 1 - \epsilon.$$

The states do not diverge!

"Proof" that there is no Classical Chaos:

Let $f_1(q,p,0)$ and $f_2(q,p,0)$ be two initially close classical phase space distributions.

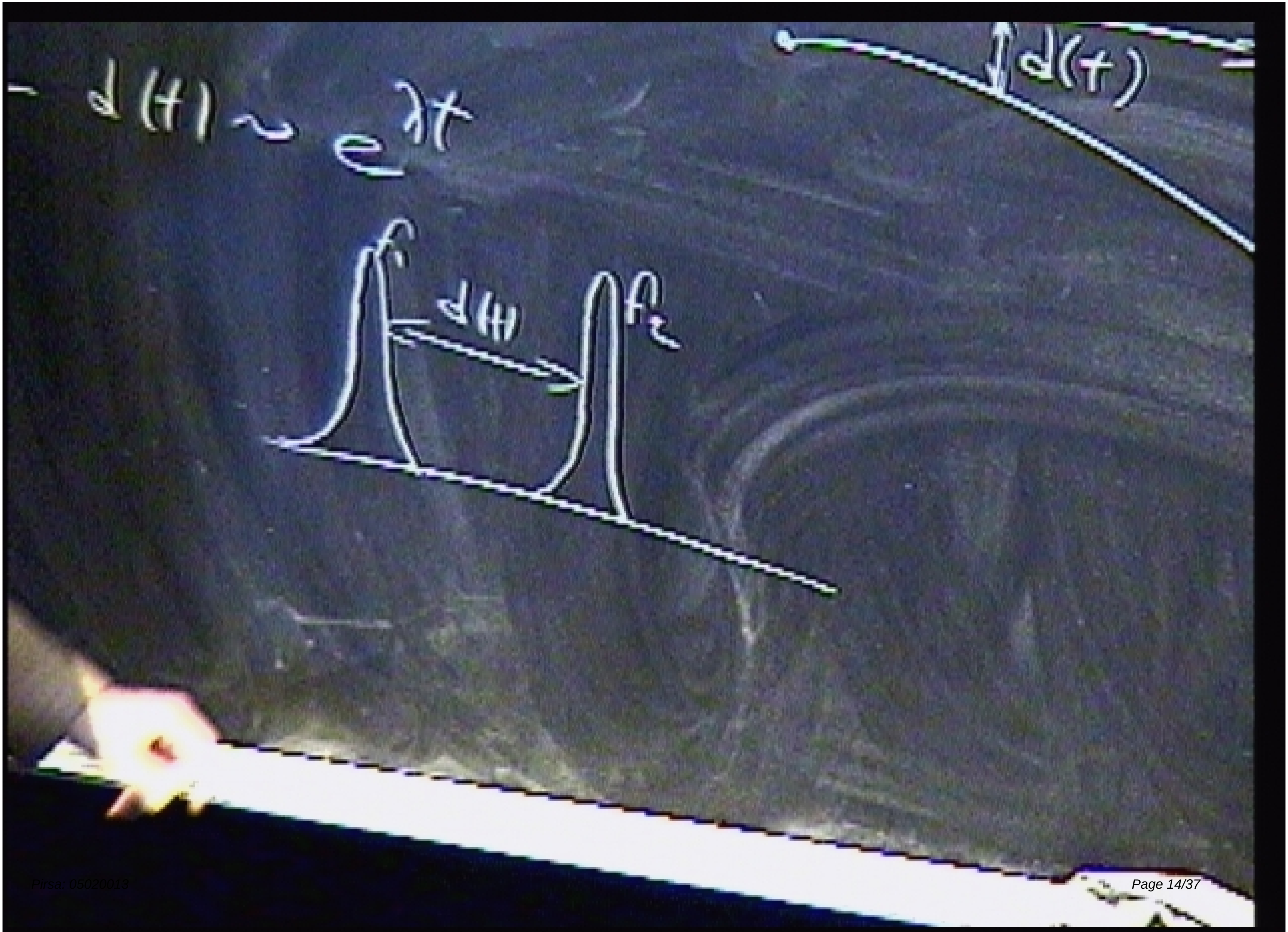
Denote $\{f_1(t)|f_2(t)\} =$

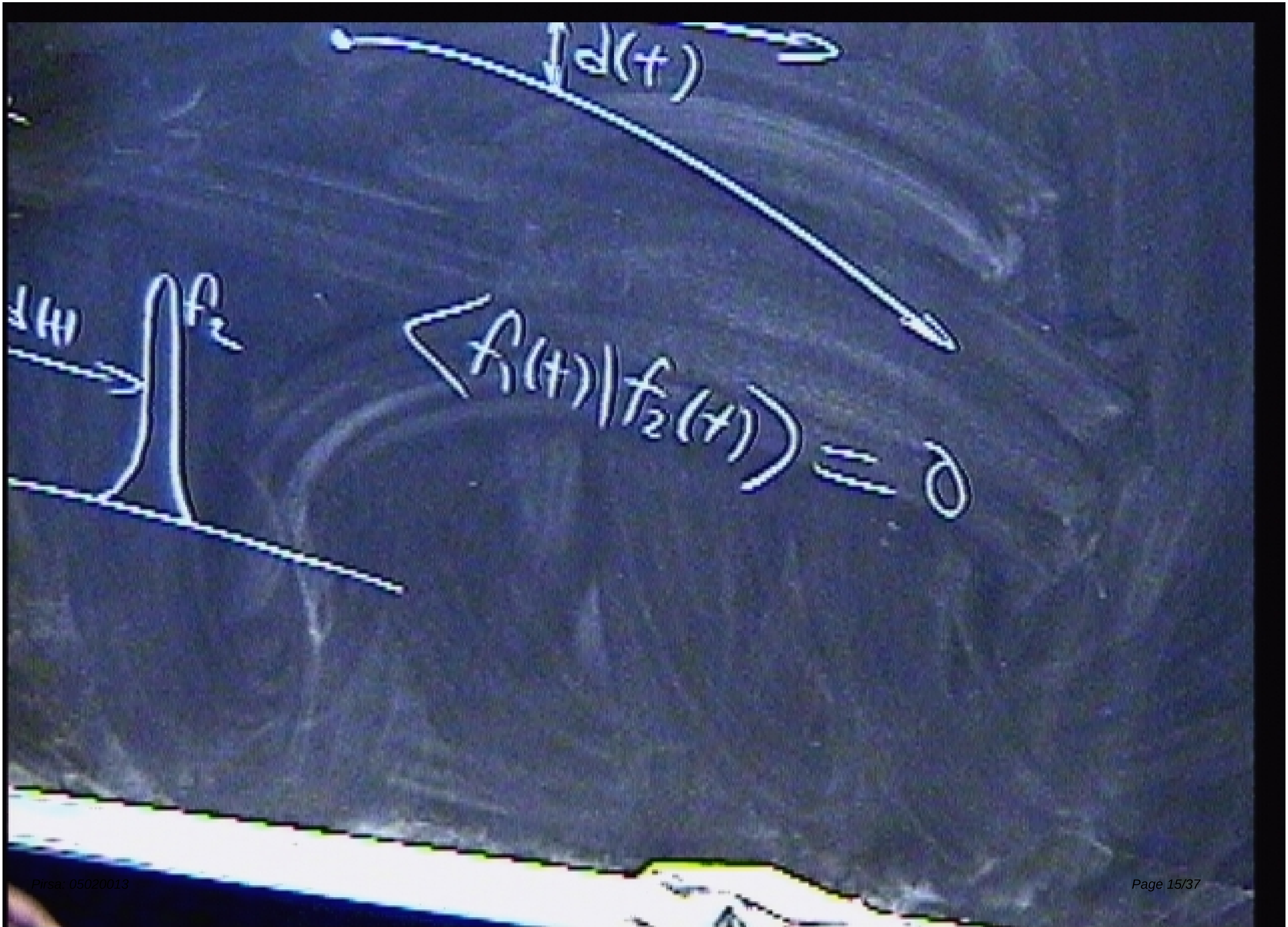
$$\iint f_1(q,p,t)\, f_2(q,p,t)\, dq\, dp.$$

Liouville's eqn. implies that

$$\{f_1(t)|f_2(t)\} = \{f_1(0)|f_2(0)\}.$$

The states do not diverge!


d(t) ~ e^{λt}
d(t)
d(t)
f₁
f₂


d(t)
f2
<f1(t)|f2(t)> = 0

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The states do not diverge!

The classical limit of a quantum state is not a single classical orbit, but rather an **ensemble of classical orbits**.

Classical limit:

$$\hbar \to 0\,,$$
$$n \to \infty\,,$$
$$\hbar n = \text{constant}$$

Ehrenfest's Theorem

Let $\hat{H} = \hat{p}^2/2m + V(\hat{q})$. Then

$$d\hat{q}/dt = (i/\hbar)[H,\hat{q}] = \hat{p}/m$$

$$d\hat{p}/dt = (i/\hbar)[H,\hat{p}] = F(\hat{q}),$$

where $F(x) = -\nabla V(x)$

Take the average in some state:

$$d\langle\hat{q}\rangle/dt = \langle\hat{p}\rangle/m$$

$$d\langle\hat{p}\rangle/dt = \langle F(\hat{q})\rangle$$

If $\langle F(\hat{q})\rangle \approx F(\langle\hat{q}\rangle)$, then $\langle\hat{q}\rangle$ will follow a classical orbit.

Corrections to Eherenfest's Theorem

Expand in powers of deviation operators,

$$\delta\hat{q} = \hat{q} - \langle\hat{q}\rangle$$

$$\delta\hat{p} = \hat{p} - \langle\hat{p}\rangle$$

Write $P = \langle\hat{p}\rangle$ and $Q = \langle\hat{q}\rangle$.

Then

$$dQ/dt = P/m$$

$$\frac{dP}{dt} = F(Q) + \frac{1}{2}\langle(\delta\hat{q})^2\rangle \frac{\partial^2}{\partial Q^2}F(Q) + \ldots$$

classical orbit + corrections

not all due to QM

Classical Ensemble, Liouville equation:

$$\frac{\partial}{\partial t}\rho(q,p,t) = -\frac{p}{m}\frac{\partial}{\partial q}\rho(q,p,t) - F(q)\frac{\partial}{\partial p}\rho(q,p,t)$$

$$\langle q\rangle_c = \iint q\,\rho(q,p,t)\,dq\,dp$$

$$\langle p\rangle_c = \iint p\,\rho(q,p,t)\,dq\,dp$$

Differentiate w.r.t. t:

$$d\langle q\rangle_c/dt = \langle p\rangle_c/m$$

$$d\langle p\rangle_c/dt = \iint F(q)\,\rho(q,p,t)\,dq\,dp$$

Expand in powers of $\delta q = q - \langle q\rangle_c$:

$$\frac{d}{dt}\langle p\rangle_c = F(\langle q\rangle_c) + \frac{1}{2}\langle(\delta q)^2\rangle_c \frac{\partial^2}{\partial\langle q\rangle_c{}^2}F(\langle q\rangle_c) + \dots$$

The centroid of a classical ensemble need not follow a classical trajectory.

Two regimes of quantum-classical correspondence.

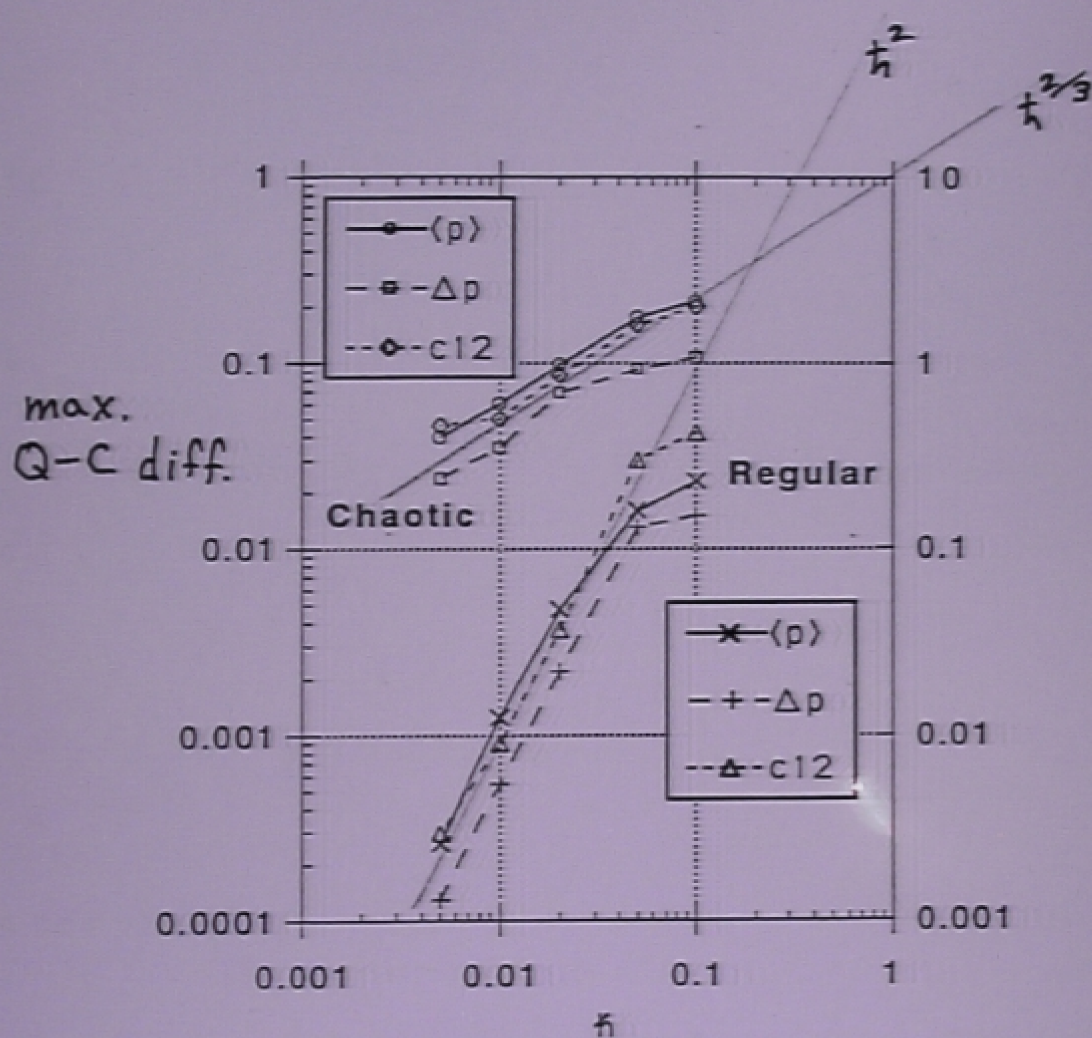
(a) Ehrenfest regime:

- The widths of the quantum and classical probability distributions are small compared to the scale of the system.
- The centroid of the quantum state, and also of the classical ensemble, approximately follow a classical trajectory. (Ehrenfest's Theorem)

(b) Liouville regime:

- The quantum and classsical probability distributions are approximately equal.
- The states need not be narrow; Ehrenfest's theorem does not apply.
- Some modest coarse graining of the quantum probability distribution may be needed in order to reveal the underlying classical background.

The breakdown of Ehrenfest correspondence does **not** signal the end of classical behavior. *



$$c_{12} = \langle p_1 p_2 \rangle - \langle p_1 \rangle \langle p_2 \rangle$$

The Logic of Inductive Inference

(E.T. Jaynes, R.T. Cox, H. Jeffreys)

Probability is assigned to *propositions.*

P(A|C) is the probability that A is true, given the information C.

Ensemble and Frequency

(Kolmogorov, Bernoulli, von Mises, de Moivre)

Probability identified with a *limit frequency* in an ordered sequence
-- (Frequency theory).

Probability identified with a *measure* on a set (which need not be ordered)
-- (Ensemble theory).

Propensity

- a form of causality weaker than determinism.
  (K.R. Popper)

Probability is ascribed to *events.*

P(A|C) is the propensity for A to occur under the condition C.

Subjective and Personal probabilities.

(de Finnetti, L.J. Savage, I.J. Good)

- Incomplete knowledge.
- Degrees of reasonable belief.

Reference for the *Theory of Inductive Inference*
(proposed as the master theory):

E. T. Jaynes - *"Probability Theory: The Logic of Science"*,
(Cambridge University Press, ~~to be~~ published ~~soon~~); 2003

~~-- incomplete work available at http://bayes.wustl.edu/~~

1. Probability is *a logical relation among propositions that is weaker than entailment.*
It is an *objective* relation, and should not be confused with degrees of belief.

The propositions may have any particular content.

- Specialize to propositions about repeated experiments:
--> ensemble-frequency theory.

- Specialize to propositions about personal belief:
--> subjective probability.

- Specialize to propositions about indeterministic or unpredictable events:
--> propensity theory.

2. Although P(A|C) is a *logical* relation between the proposition A and the conditioning information C, it is *not* merely a *formal*, syntactic relation. The content (meaning) of A and C must be invoked to evaluate P(A|C).
There is no magic formula to translate arbitrary information into

The Axioms of Probability:

R.T. Cox, *The Algebra of Probable Inference*, (Johns Hopkins University Press, Baltimore MD, 1961). -- [hard to obtain?]

R.T. Cox, *Probability, Frequency, and Reasonable Expectation* - Amer. J. Phys. **14**, 1-13 (1946).

Axiom 1.	$0 \le P(A\|B) \le 1$
Axiom 2.	$P(A\|A) = 1$
Axiom 3.	$P(\sim A\|B) = 1 - P(A\|B)$
Axiom 4.	$P(A\&B\|C) = P(A\|C)\, P(B\|A\&C)$

Notation: ~A means "not A",
A&B means "A and B",
A∨B means "either A or B".

Negation (~A) requires an axiom.

Conjunction (A&B) requires an axiom.

Disjunction (A∨B) needs no axiom because A∨B = ~(~A & ~B):
"A or B" is the negation of "neither A nor B".

Additivity follows from a theorem:

$$P(A\vee B|C) = P(A|C) + P(B|C) - P(A\&B|C)\,.$$

If A and B are *mutually exclusive*, then

$$P(A\vee B|C) = P(A|C) + P(B|C)\,.$$

Remark:

The notion of *randomness* plays no fundamental role in probability theory.

Remark:

These axioms are not arbitrary.

They are determined by conditions of *plausibility* and *consistency* (R.T. Cox):

(i) The probability of A on some given evidence determines the probability of "not A" on the same evidence.

(ii) The probability on given evidence that *both* A and B are true is determined by their separate probabilities, one on the given evidence, and the other on that evidence plus the assumption that the first is true.

(iii) If a complex proposition can be composed in more than one way

[ex.: (A&B)&C , A&(B&C)]

then all ways of computing its probability must lead to the same answer.

Remark:

Anyone who proposes an inequivalent alternative to Cox's axioms (ex.: negative probabilities) has an obligation to explain *how* and *why* he departs from these conditions of plausibility and probability theory.

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Very important remark!

All probabilities are conditional.

Use of the single-varible notation **P(A)**,
instead of **P(A|C)**,
is permissible only if:

- the conditional information **C** is *obvious from the context*,
- and is *unchanging* throughout the problem.

Which *interpretations* of probability are relevant to QM?

ensemble-frequency -- Yes

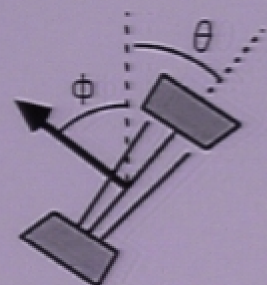
propensity -- Yes

subjective -- No

The *propensity* interpretation:

- allows meaningful statements about individual events.
- is in accord with the *ensemble-frequency* interpretation when applied to repeated experiments.

The *propensity* interpretation is more natural when one considers time-dependent states, and time-dependent probabilities.



Ex.(i): A source produces $s=\frac{1}{2}$ particles polarized at an angle ϕ (relative to some coordinate axis). A Stern-Gerlach magnet has its field gradient axis oriented at an angle θ.

What is the probability that such a particle, incident on the apparatus, will emerge with spin "up" (relative to the magnet)?

Formal answer: $p=\{\cos[(\theta-\phi)/2]\}^2$.

Interpretation of this answer:

Propensity interpretation:

-- The propensity (chance) of the particle emerging with spin "up" is p.

Ensemble-frequency interpretation:

-- In a long run of similar experiments the fraction of particles

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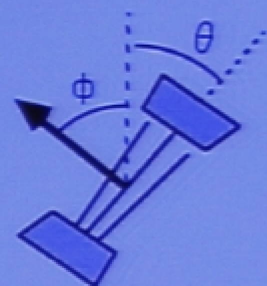

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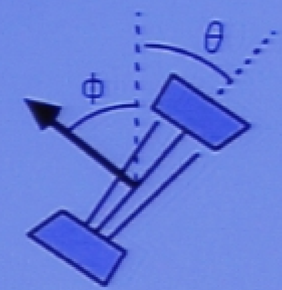

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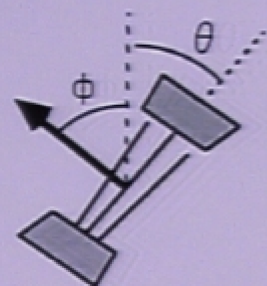

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(ii) Now let the magnet be reoriented in some arbitrary manner before each particle is released, so that θ is different in each case.

Propensity interpretation:

-- The propensity (chance) has a different value, $p = p_\theta$, in each case.

Ensemble-frequency interpretation:

-- One must conceptually embed each event in an imaginary long run of experiments having the same value of θ, in order to make a frequency statement.

(iii) Suppose that the polarization direction ϕ of the particles is unknown.

Can it be inferred from the data of (ii), where each event corresponded to a different orientation θ of the magnet?

Ensemble-frequency interpretation:

-- No. A long run of events for each value of θ would be necessary to estimate p_θ as a frequency, and hence to determine its dependence on θ.

Propensity interpretation:

-- Yes. Bayesian inference (equivalent to maximum likelyhood if the prior probability distribution for ϕ is uniform) can determine the most probable value of ϕ, even if there is only one event for each value of θ.

I have never seen a coherent exposition of quantum mechanics based on a *subjective* interpretation of quantum probabilities as representing *knowledge!*