Title: Information-theoretic approach to quantum theory

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Abstract:

Thesis

- Quantum theory is a general theory of information constrained by several important information-theoretic principles. It can be formally derived from the corresponding information-theoretic axiomatic system.

Historical context

- **Information-theoretic approach:**

  Wheeler (1978, 1988), Rovelli (1996), Steane (1998), Fuchs (2001), Brukner and Zeilinger (2002), Clifton, Bub and Halverson (2003), Jozsa (2004).

- **Axiomatic approach:**

  La théorie physique moderne manifeste une tendance certaine à rechercher une présentation *axiomatique*, sur le modèle des axiomatiques mathématiques. L'idéal axiomatique, emprunté à la géométrie, revient à définir tous les « objets » initiaux d'une théorie uniquement par des *relations*, nullement par des qualités substantielles.

  Jean Ullmo (1958)

Outline

I. Philosophy

II. Quantum logical derivation of quantum theory

III. Information-theoretic interpretation of the C^*-algebraic approach

Four points

1. Epistemological attitude: one is concerned with *theories.*

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Science is the construction of theories. A theory is an *objective description of certain phenomena*, while selection criteria for phenomena and the understanding of objectivity of the description vary depending on the particular theory in question. All such phenomena, however, have *repeatable* traits. A theory is a description of repeatable traits of the observed phenomena with the goal of predicting these traits in unobserved (unknown, future) phenomena.

Four points

2. Universality: there is no repeatable trait of any phenomenon excluded from being described by a theory.

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3. Various theories can be depicted in the loop form.

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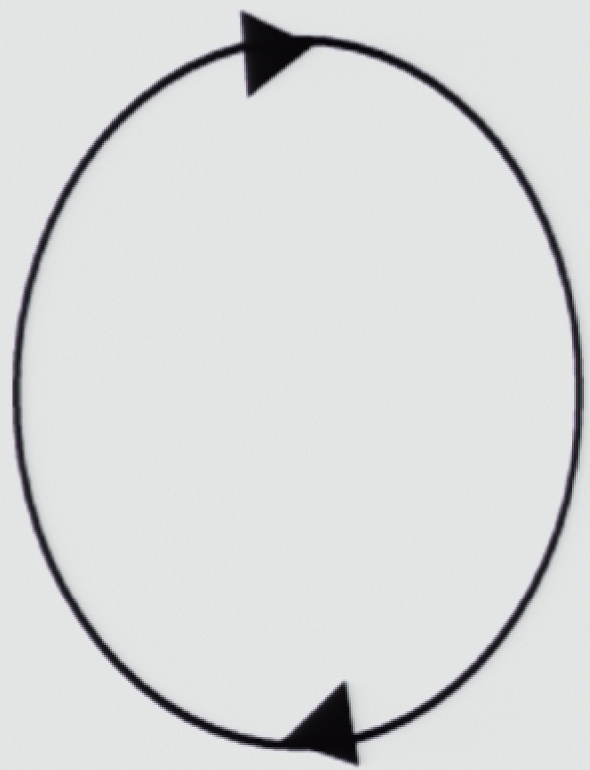
Theories can be classified by what they assume and by what they explain. Concepts assumed in one theory can be explained in another. This is a claim that, if depicted graphically and interconnected by their 'end concepts,' theories do not form a pyramid of infinite regress or another form; they form a circle, a loop.

Four points

3. The set of all theories can be depicted in the loop form.



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The cut separates *explanans* from *explanandum*, what is assumed from what is derived. A theory of the loop uncut is a logical circularity. Cutting the loop must be seen as a condition of possibility of the theoretical description as such.

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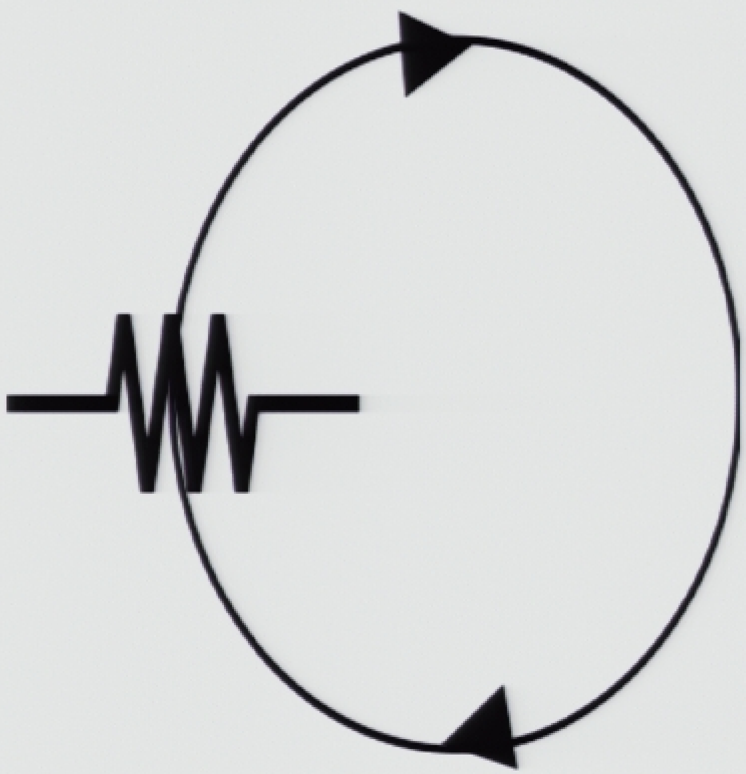

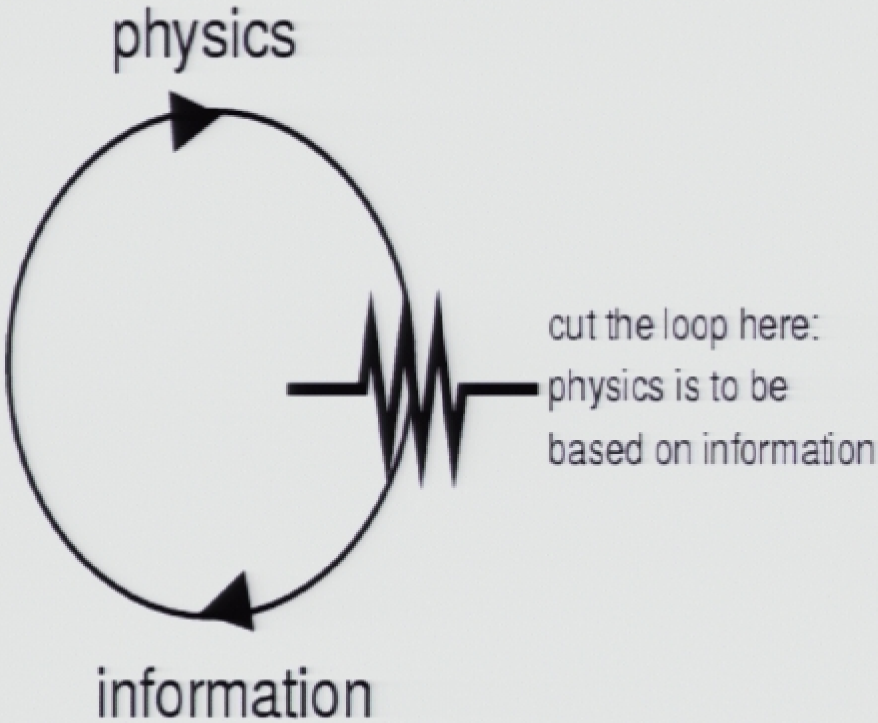
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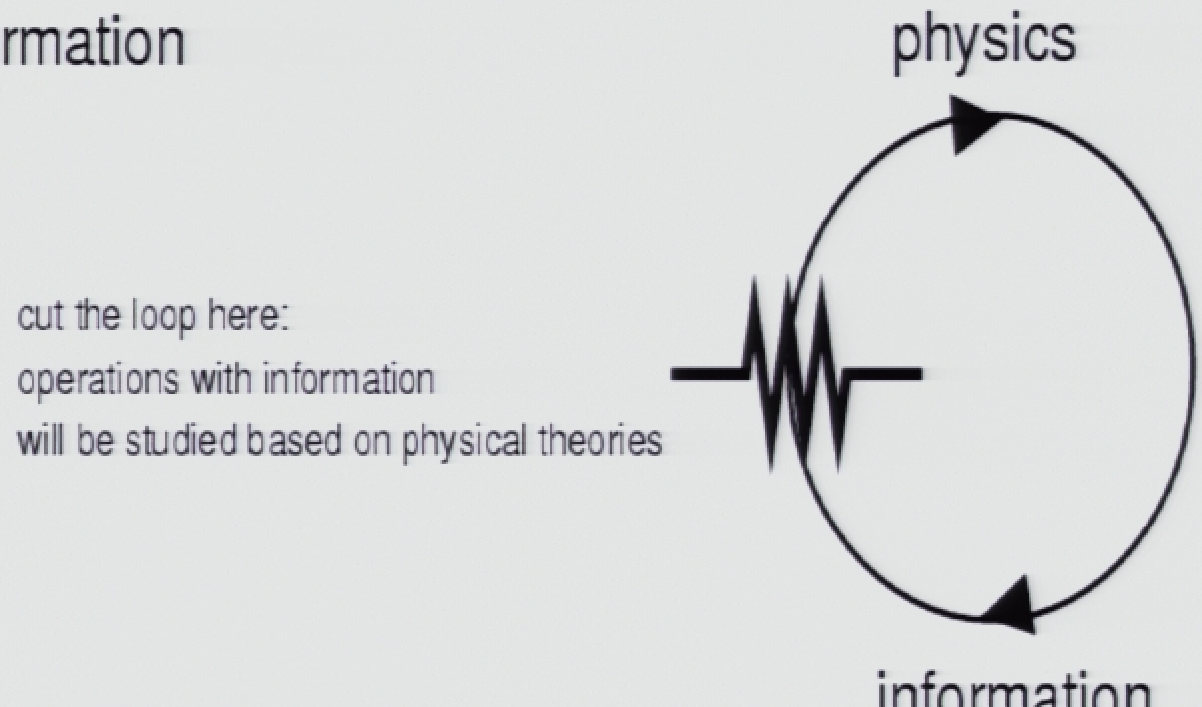
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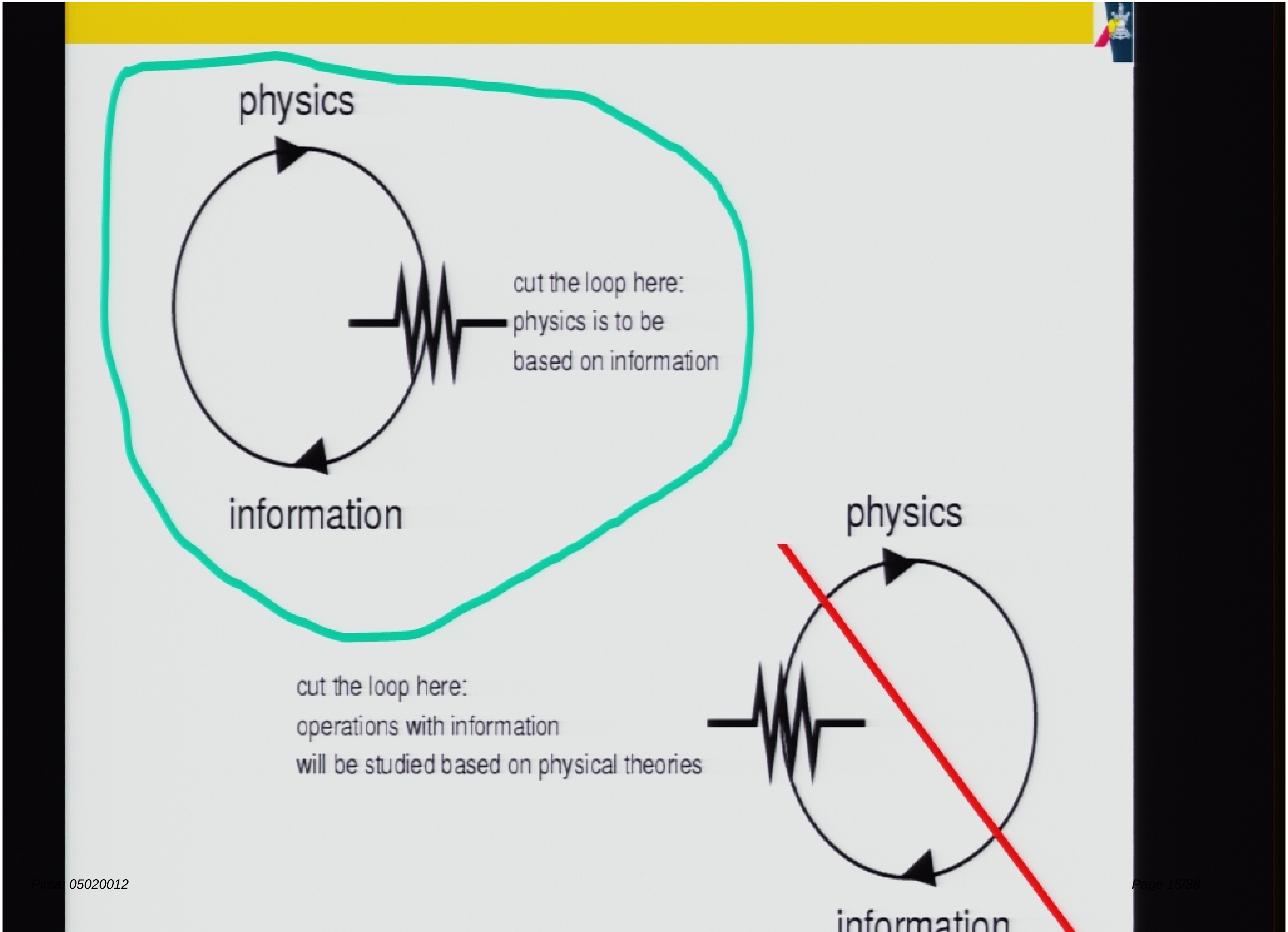
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Language

Fundamental notions

Language

Fundamental notions
System
Information
Fact (act of bringing about information)

Language

Fundamental notions	Formal representation
System	
Information	
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Language

Fundamental notions	**Formal representation**
System	Systems S, O, P...
Information	Yes-no questions
Fact (act of bringing about information)	Answer to a yes-no question (given at time *t*)

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What do we need to reconstruct?

- Obtain Hilbert space and prove quantumness
- Obtain Born rule with the state space
- Obtain unitary dynamics

Quantum logical reconstruction of the Hilbert space

1. Definition of the lattice of yes-no questions.
2. Definition of orthogonal complement.
3. Definition of relevance and proof of orthomodularity.
4. Introduction of the space structure.
5. Lemmas about properties of the space.
6. Definition of the numeric field.
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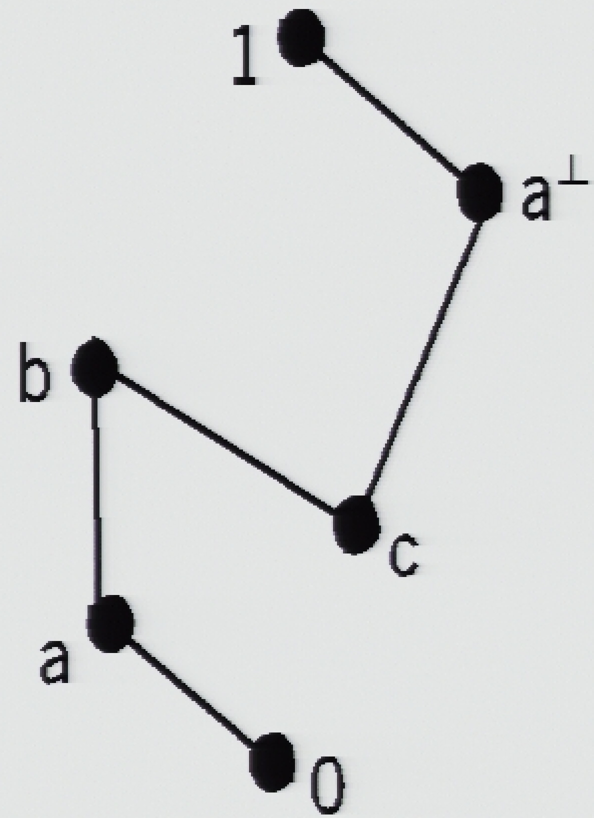

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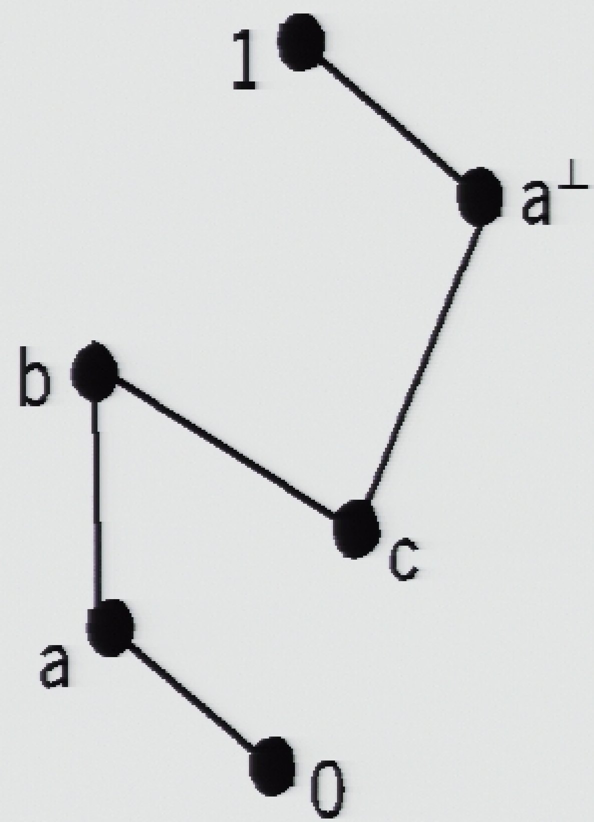

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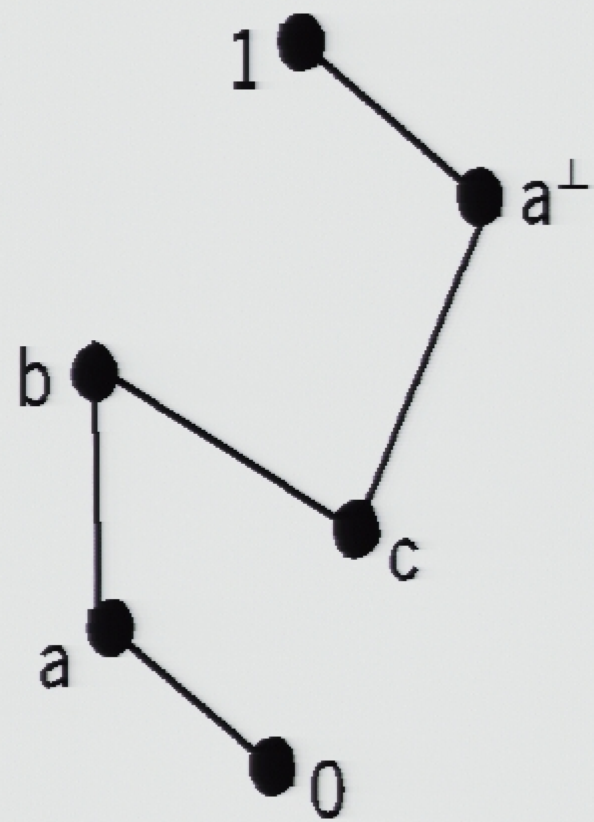

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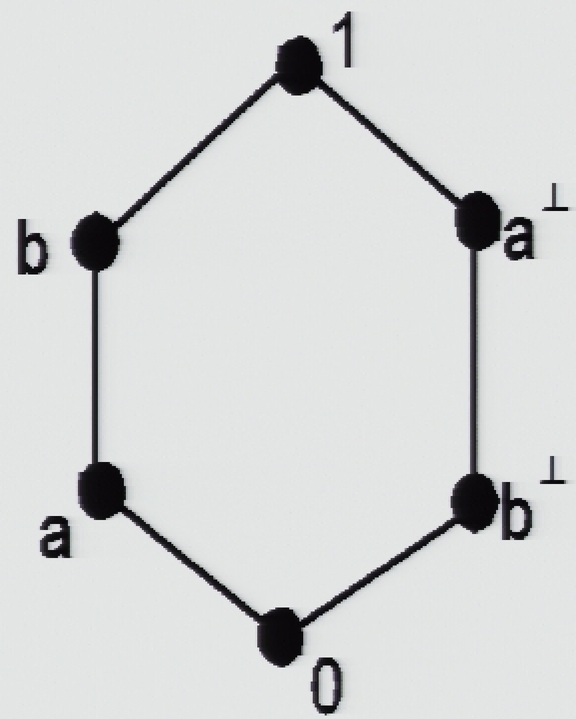
- Non-trivial if used to *derive* what in Hilbert

Non-trivial notion of relevance



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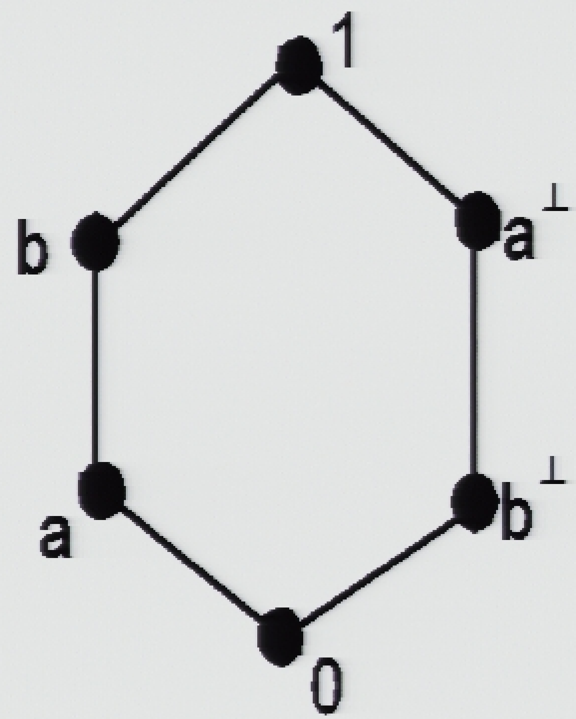

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- Question *b* is relevant with respect to *a*; and question *ã* is relevant with respect to *b*.
- Consider {a,b,ã}. If $b > a$, this sequence preserves relevance and brings about strictly more than N bits of relevant information.

Kalmbach's theorem

Infinite-dimensional Hilbert space characterization theorem:

Let *H* be an infinite-dimensional vector space over real or complex numbers or quaternions. Let *L* be a complete orthomodular lattice of subspaces of *H* which satisfies:

(i) Every finite-dimensional subspace of *H* belongs to *L*.

(ii) For every element *U* of *L* and for every finite-dimensional subspace *V* of *H*, linear sum *U*+*V* belongs to *L*.

Then there exists an inner product *f* on *H* such that (*H*,*f*) is a Hilbert space with *L* as its lattice of closed subspaces.

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- From the contradiction follows a = b.

Amount of information

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Step 6: Definition of the numeric field

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- Substitute: Solèr's theorem assuming existence of an infinite orthonormal sequence of vectors

Step 7: Construction of the Hilbert space

- **Theorem**:

  Let W(P) be an ensemble of yes-no questions that can be asked to a physical system and V a vector space over real or complex numbers or quaternions such that a lattice of its subspaces L is isomorphic to W(P).

  Then there exists an inner product f on V such that V together with f form a Hilbert space.

Quantumness

Axiom II: It is always possible to acquire new information about a system.

Criterion: Orthomodular lattice, in order to describe a quantum mechanical system, must be nondistributive.

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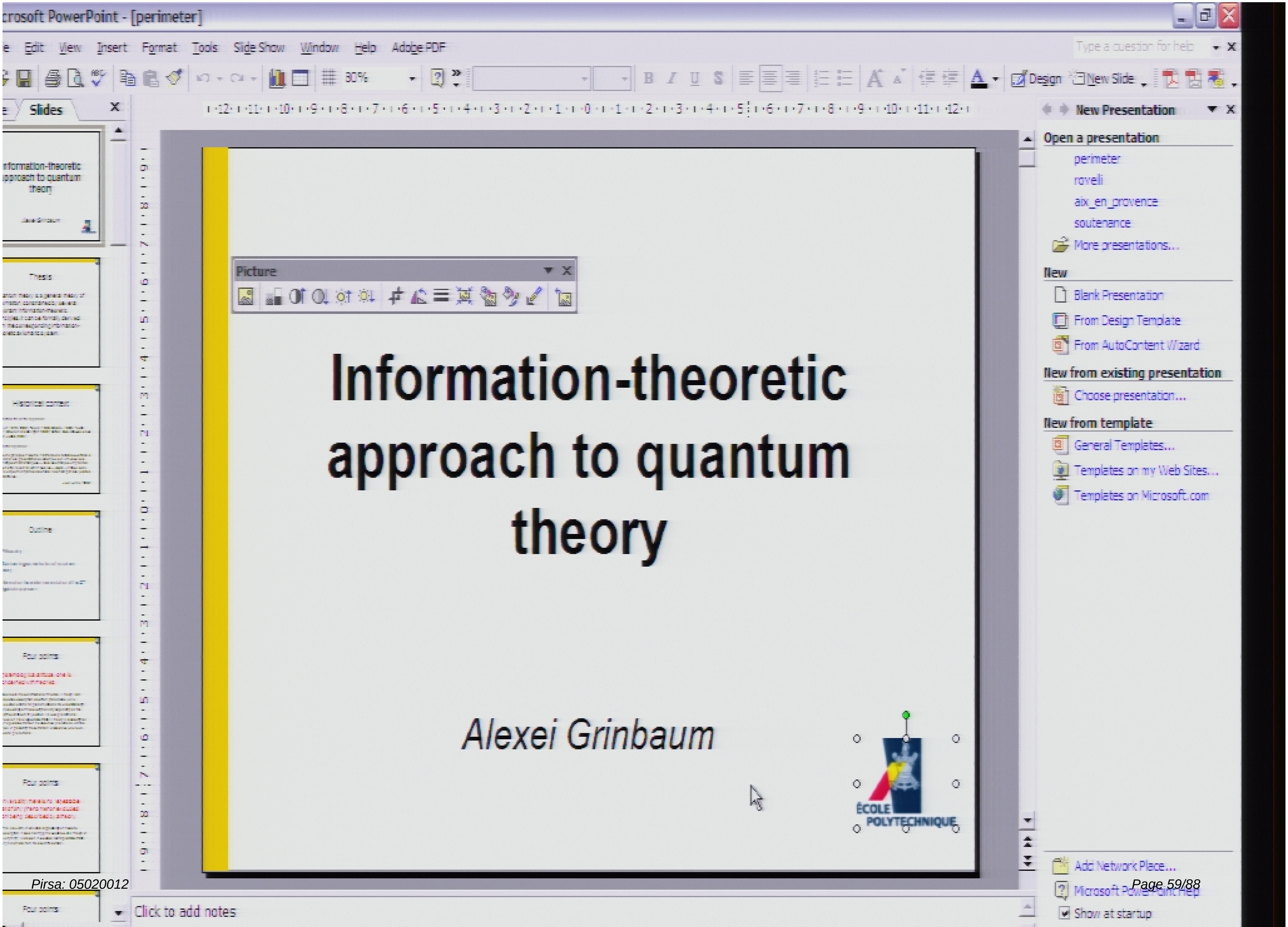
- Lemma: all Boolean subalgebras of L(V) are proper.
- Corollary: W(P) is non-Boolean.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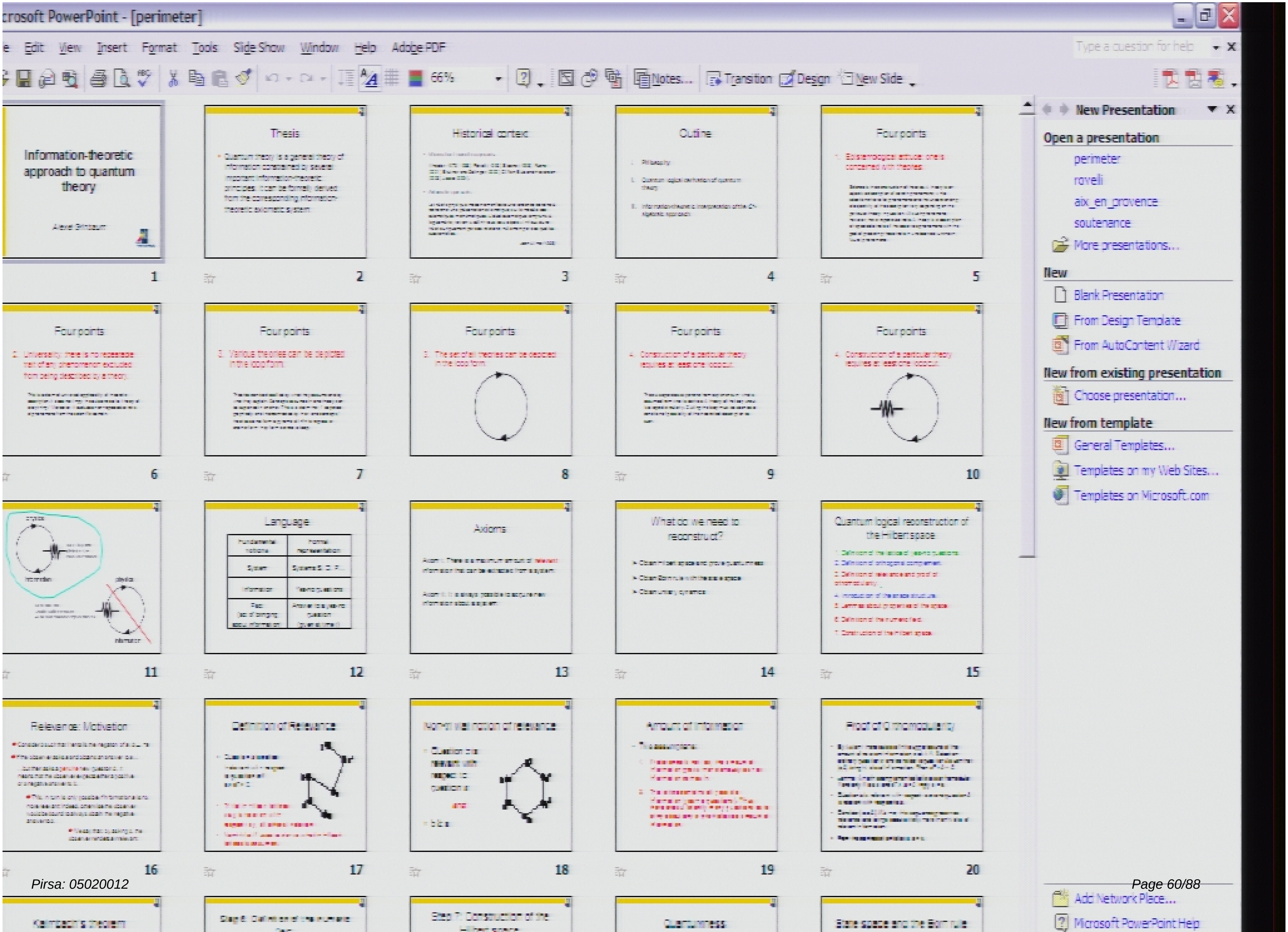
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New Slide
Information-theoretic approach to quantum theory
Thesis
Historical context
Outline
Four points
Four points
Four points
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Four points
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Language
Axioms
What do we need to reconstruct?
Quantum logical reconstruction of the Hilbert space
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Amount of Information
New Presentation
Open a presentation
perimeter
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New
Blank Presentation
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State space and the Born rule

Axiom III (intratheoretic non-contextuality):

If information is obtained by an observer, then it is obtained independently of how the measurement was eventually conducted, i.e. independent of the measurement context.

- Gleason's theorem builds the state space.

Time and unitary dynamics

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- Stone's theorem: Hamiltonian description $U(t_2 - t_1)=\exp[-i(t_2 - t_1)H]$.

POVM description

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- Twofold role of the observer:
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- Starting with an orthogonal projector description of measurement and factoring out P-observer, one obtains the general POVM description of measurement

List of axioms

Key information-theoretic axioms:

I. There is a maximum amount of relevant information that can be extracted from a system.

II. It is always possible to acquire new information about a system.

III. If information I about a system has been brought about, then it happened independently of information J about the fact of bringing about information I.

Supplementary axioms:

IV. For any two yes-no questions there exists a yes-no question to which the answer is positive if and only if the answer to at least one of the initial question is positive.

V. For any two yes-no questions there exists a yes-no question to which the answer is positive if and only if the answer to both initial questions is positive.

VI. The lattice of questions is complete.

VII. The underlying field of the space of the theory is one of the numeric fields R, C or

C*-algebraic approach

C*-algebraic approach

- Information-theoretic interpretation of the local algebra theory
- Information-theoretic approach to time

Language

Fundamental notions	Formal representation
System	C*-algebra
Information	State over algebra
Fact (act of bringing about information)	Change of state over algebra

Hyperfiniteness

Hyperfiniteness

- Hyperfinitess is a unique balance between two constraints: that there be non-equivalent representations defining different folia and that one could get information with any degree of precision from a finite sequence of facts.

Hyperfiniteness

KMS condition at all β

⇕ ↘

nuclearity

⇓

split property

⇓

hyperfinite type III_1 factor

Hyperfiniteness

KMS condition at all β

- generalized Wick rotation
- generalized Gibbs condition

nuclearity

split property

hyperfinite type III_1 factor

Time

- Using KMS formalism via Tomita-Takesaki theorem, define time as modular flow. It is state-dependent. Unless the state is changed, time does not change. A change in the state means a change in information. A change in information can be brought about in a new fact. At each fact state-dependent time "restarts." Thus, the temporality of facts (variable *t* that indexes facts) is unconnected with the state-dependent notion of time.
- Assume the information-theoretic interpretation of the local algebra theory. Then, if no new information is brought about, and if the algebra is a type III_1 factor, the spectrum of *t* is from 0 to $+\infty$. We obtain that the internal, state-dependent time behaves "correctly": it is a real positive one-dimensional parameter.
- Factorization by inner automorphisms leads to the state-independent notion of time. This factorization corresponds to neglecting the difference between states. Thus, the concept of time arises due to the possibility to neglect certain information, i.e. to treat it as irrelevant.

Open questions

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1. Information-theoretic meaning of Axioms IV, V, and VI.
2. Information-theoretic meaning of Axiom VII or of Solèr's theorem.
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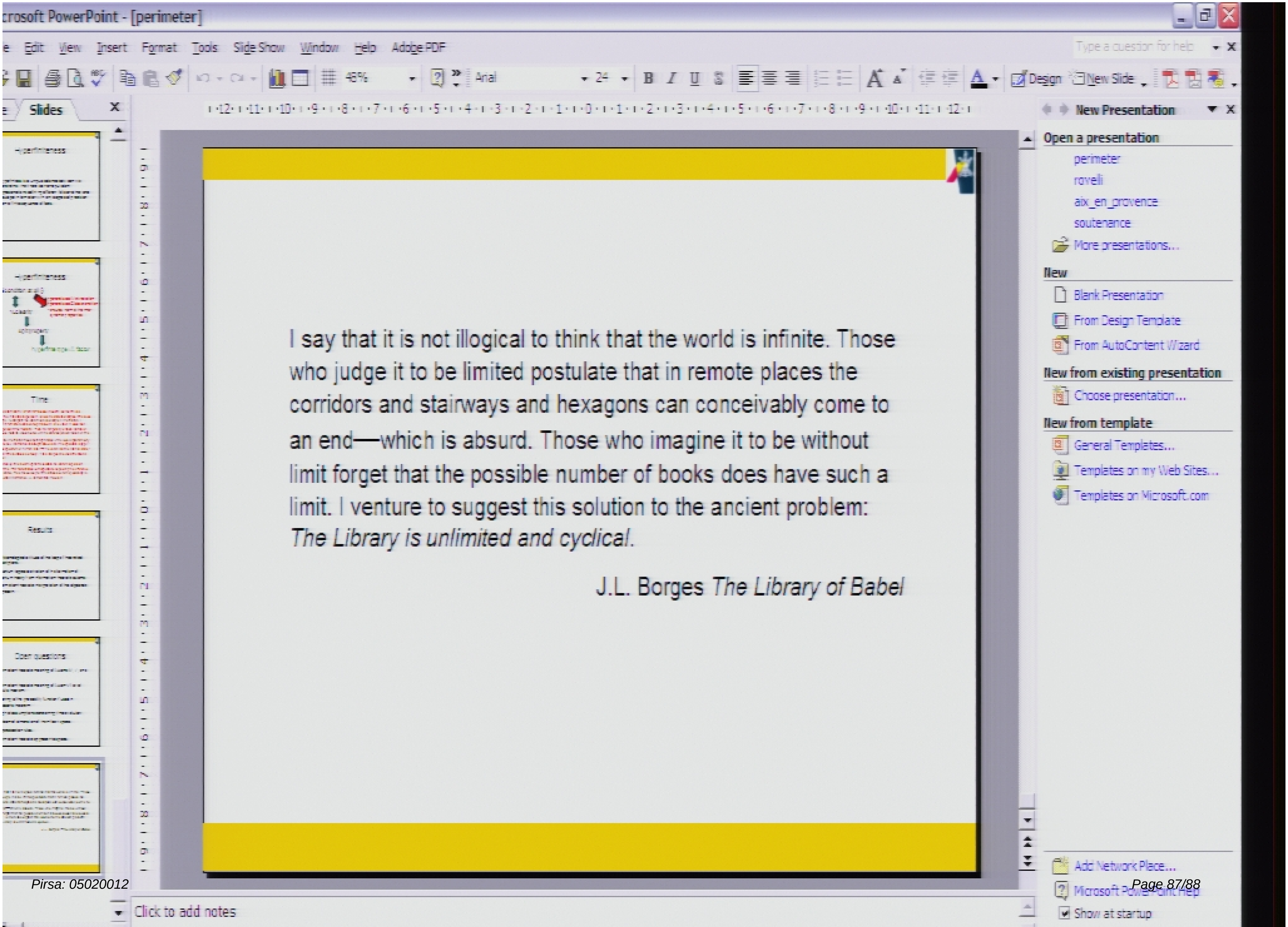
6. Superselection rules.
7.

I say that it is not illogical to think that the world is infinite. Those who judge it to be limited postulate that in remote places the corridors and stairways and hexagons can conceivably come to an end—which is absurd. Those who imagine it to be without limit forget that the possible number of books does have such a limit. I venture to suggest this solution to the ancient problem: *The Library is unlimited and cyclical*.

J.L. Borges *The Library of Babel*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