Title: Interpretation of Quantum Theory: Lecture 11

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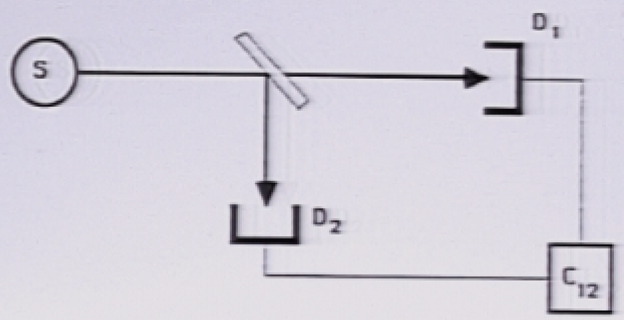
Abstract: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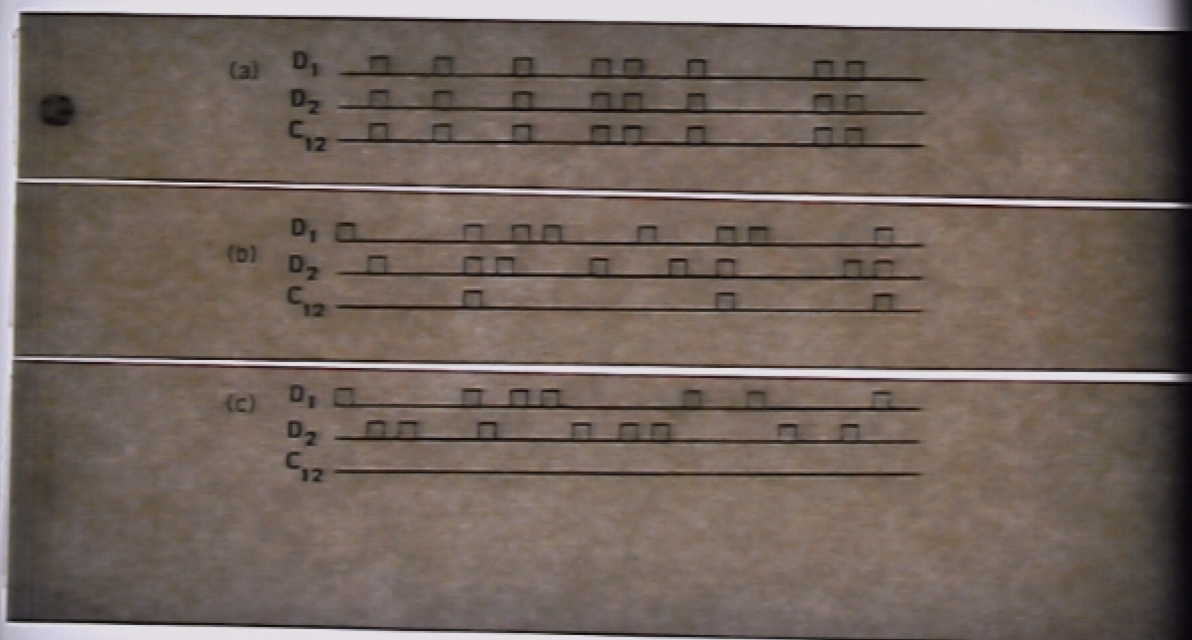
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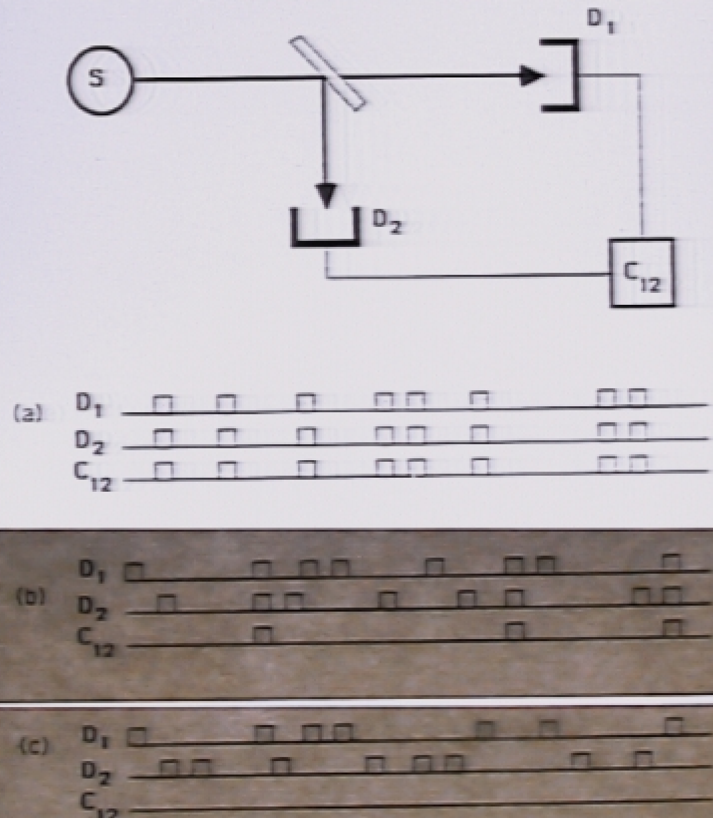
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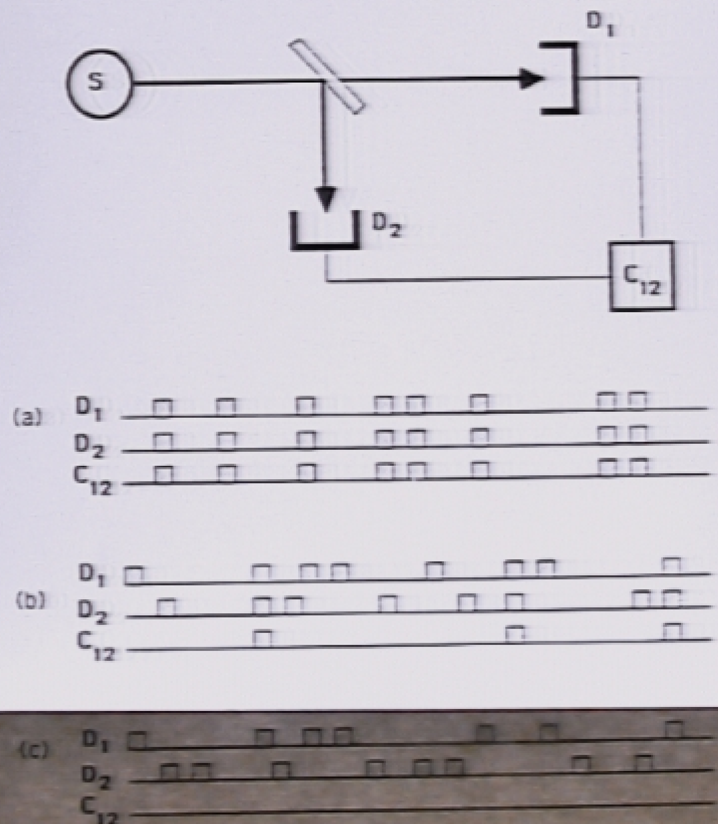
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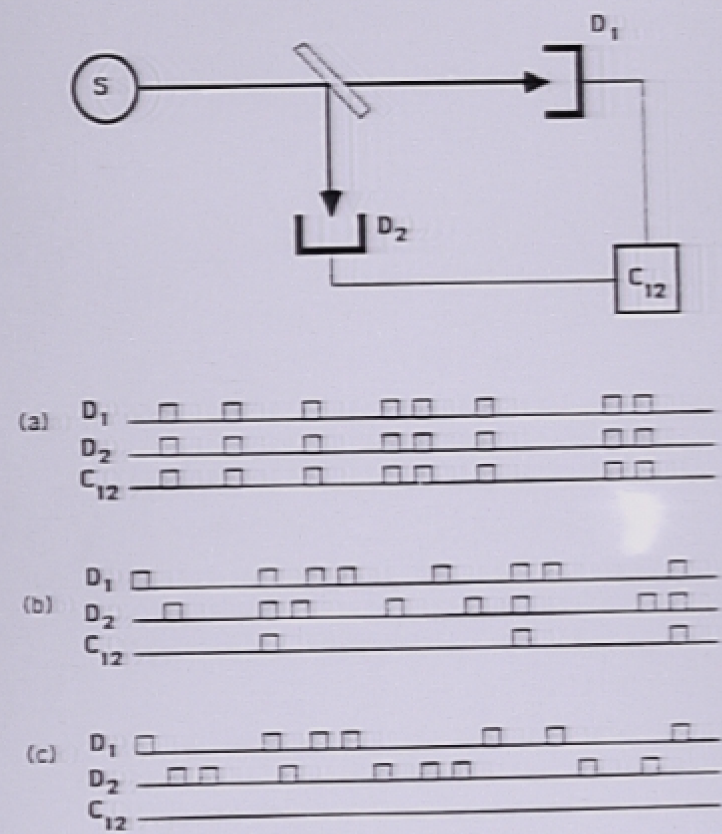
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An interpretation of the formalism must provide correspondence rules:

mathematics	→	physics
s.a. operator	→	dynamical variable
eigenvalue	→	possible value of d. v.
state vector	⇉	?

1918 – Planck

1921 – Einstein

1922 – Bohr

1929 – de Broglie

1932 – Heisenberg

1933 – Schrödinger & Dirac

1945 – Pauli

1954 – Born (w. Bothe)

The first paper,

Max Born – Z. Physik 37, 863 (1926)

"On the Quantum Mechanics of Collisions"
(translated title)

– probability interpretation was given only to the scattering amplitude.

For large r,

$$\psi(\vec{r}) \sim e^{i\vec{k}\cdot\vec{r}} + f(\theta,\phi)\,\frac{e^{ikr}}{r}$$

$|f(\theta,\phi)|^2 \propto$ probability of scattering from the direction of incidence $\vec{k}$ to the direction (θ,ϕ).

$$|\psi(\vec{x})|^2$$

$$|\langle r_i|\psi\rangle|^2$$

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Schrödinger's quantum mechanics therefore gives quite a definite answer to the question of the effect of the collision; but there is no question of any causal description. One gets no answer to the question, "what is the state after the collision," but only to the question, "how probable is a specified outcome of the collision" (where naturally the quantum mechanical energy relation must be fulfilled).

Max Born (1926)

A typical experiment consists of:

(1) Preparation

(2) Measurement

(3) Repeat 1 and 2 until you have enough data.

Essentially statistical nature:

- The same preparation may yield different measurement results.
- The same measurement result may be obtained from different preparations.
- Individual results are not reproducible, but
- statistical distributions of results are reproducible for the same preparation.

The Preparation is said to prepare a State.

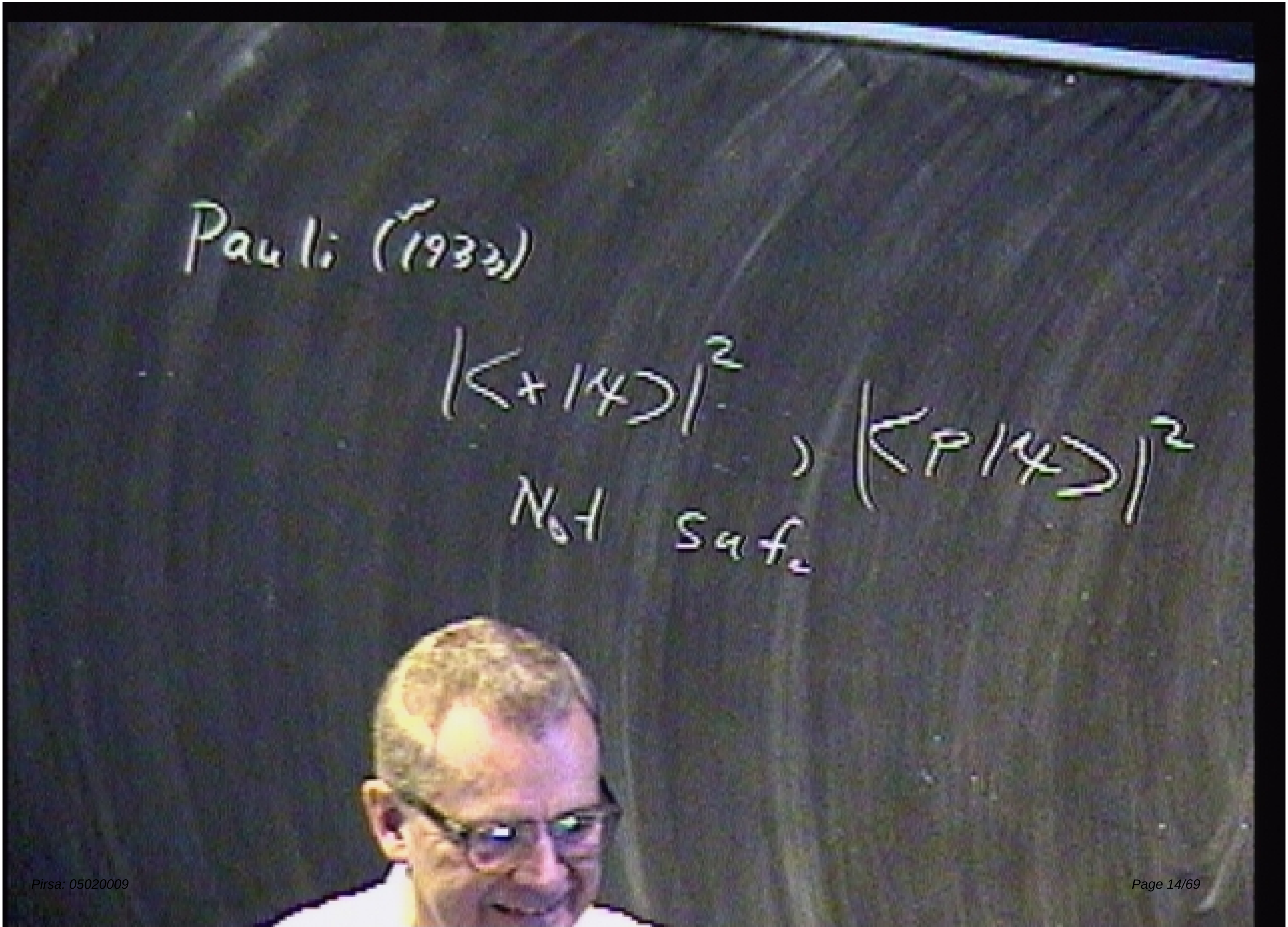
The term "state" may be identified with:

- a set of probability distributions for each observable
- an ensemble of similarly prepared systems, on which measurements can be made to reveal the probabilities as relative frequencies.

Note: The probabilities for different observables, which characterize a state, can not all be specified independently.

- Some compatibility conditions must be satisfied. (ex.: the uncertainty relations)
- A state is uniquely determined by specifying data on a "quorum" of non-commuting observables.

(necessary and sufficient conditions?)


Pauli (1933)
|<x|ψ>|², |<p|ψ>|²
Not Suf

What's in a name?

"Copenhagen interpretation"

"Orthodox interpretation"

"Statistical interpretation"

from "Resource Letter IQM2: Foundations of QM since the Bell Inequalities"
— L.E.B., Am. J. Phys. 55, 785 (1987).

what is meant by the "Copenhagen interpretation."? I shall use the term in its popular, but not necessarily historically accurate sense, as including the following propositions:

(a) The state vector provides a complete description of an individual system (that is, the views of Bohr rather than of Einstein in their famous controversy);

(b) The state vector evolves according to the Schrödinger equation while the system is isolated, but changes discontinuously during measurement to an eigenstate of the observable that is measured (Von Neumann's "*projection postulate*"). Of course this does not fully characterize the Copenhagen interpretation.

Some will not like my terminology. L. Rosenfeld has said (quoted in Ref. 21), "We in Copenhagen do not like (the phrase Copenhagen interpretation) at all. Indeed, this expression was invented, and is used by people wishing to suggest that there may be other interpre-

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"I am convinced that you have made a decisive advance with your formulation of the quantum condition, just as I am equally convinced that the Heisenberg–Born route is off the track."[7]

Einstein to Schrödinger
26 April 1926

"Quantum mechanics is certainly imposing. But an inner voice tells me that it is not yet the real thing. The theory says a lot, but does not really bring us any closer to the secret of the 'old one.' I, at any rate, am convinced that He is not playing at dice."

Einstein to Born

28 April 1926

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Einstein to Born
4 December 1926

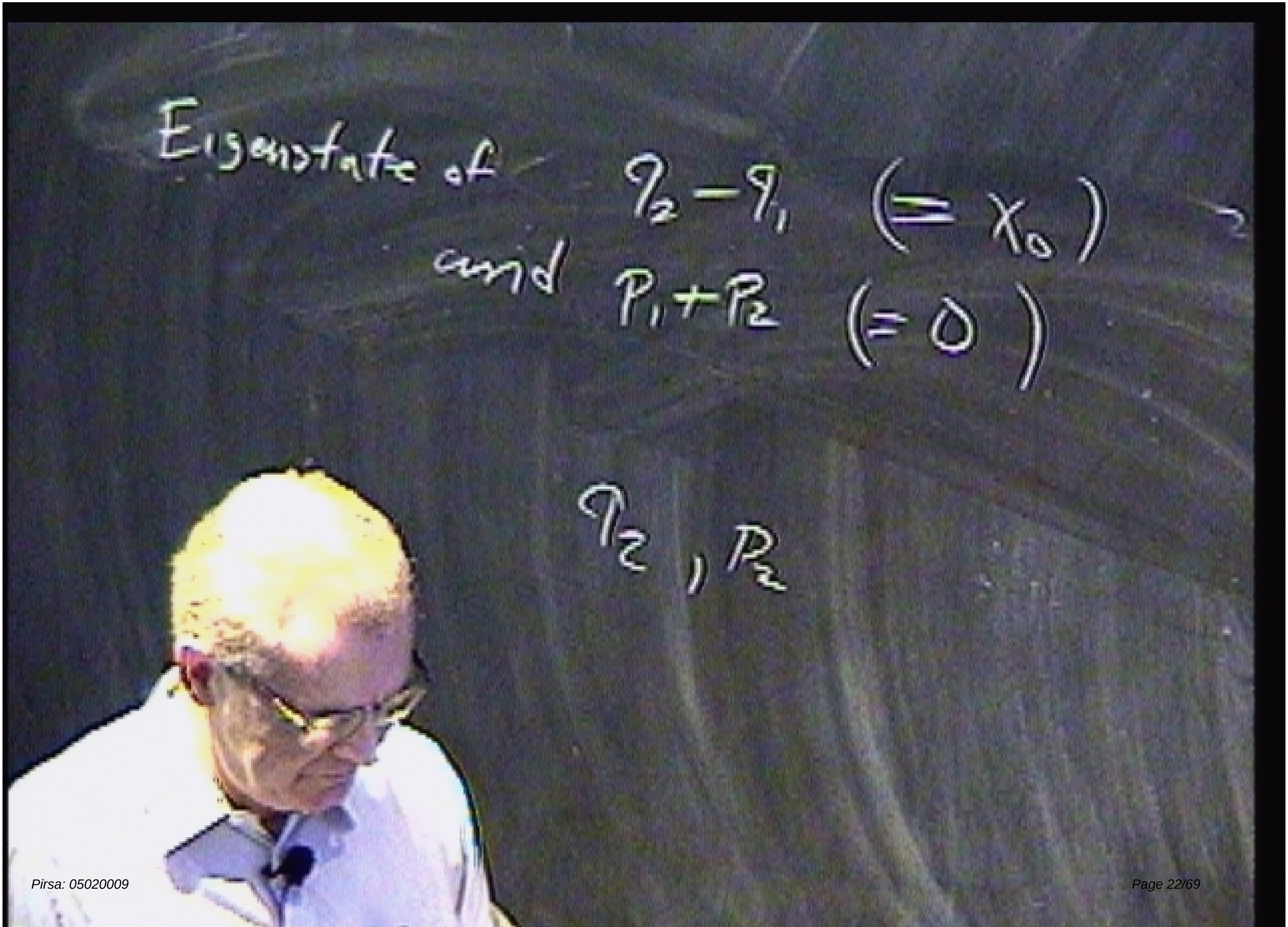
Einstein, Podolsky, Rosen (1935)

D1. A *necessary* condition for a *complete* description is that *"every element of the physical reality must have a counterpart in the physical theory."*

D2. A *sufficient* criterion for identifying an *element of reality* is, *"If, without in any way disturbing a system, we can predict with certainty* (*i.e.*, *with probability equal to unity*) *the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity."*

Eigenstate of $q_2 - q_1 \ (= x_0)$

and $p_1 + p_2 \ (= 0)$


Eigenstate of
q2 − q1 (= x0)
and p1 + p2 (= 0)
q2 , p2

It seems to be clear, therefore, that Born's statistical interpretation of quantum theory

is the only possible one.[20] The ψ function does not in any way describe a state which could be that of a single system; it relates rather to many systems, to "an ensemble of systems" in the sense of statistical mechanics. If, except for certain special cases, the ψ function furnishes only *statistical* data concerning measurable magnitudes, the reason lies not only in the fact that the *operation of measuring* introduces unknown elements, which can be grasped only statistically, but because of the fact that the ψ function does not, in any sense, describe the state of *one* single system.

Einstein (1936)

"I am as convinced as ever that the wave representation of matter is an incomplete representation of the state of affairs. . . . The prettiest way to show this is by your example with the cat."

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Einstein to Schrödinger (9 Aug. 1939)

The problem with quantum theory has never been a "measurement problem".
We have long known how to describe measurements.
The problem has been how to describe reality.

--- Philip Pearle

Measurement Problem:

Let *initial state* of *object* be $|q_i\rangle$;

initial state of measurement *apparatus* be $|A_0\rangle$

Design the *interaction* between them to have the effect

$$|q_i\rangle \otimes |A_o\rangle \rightarrow |q_i\rangle \otimes |A_i\rangle \quad \text{(if value of q unchanged)}$$

or

$$|q_i\rangle \otimes |A_o\rangle \rightarrow |\phi_i\rangle \otimes |A_i\rangle \quad \text{(if value of q changed)}$$

Theorem:

For an *initial state* of the object that is a *superposition* , $|\Psi\rangle = (|q_1\rangle + |q_2\rangle)/\sqrt{2}$,

we obtain a final state of the whole system

$$|\Psi\rangle \otimes |A_o\rangle \rightarrow (|\phi_1\rangle \otimes |A_1\rangle + |\phi_2\rangle \otimes |A_2\rangle)/\sqrt{2}$$

that is a *coherent superposition* of *macroscopically distinct* apparatus *"pointer position" states.*

The value of the analysis of "measurement" is to show that *entangled superpositions of macroscopically distinct states are not pathological or rare;*

rather, it is the factored states that are rare.

Interpretation (i):

$|\psi\rangle$ provides a complete description of an individual system.
A dynamical variable Q has a value (q, say) if and only if $Q|\psi\rangle = q|\psi\rangle$.

Interpretation (ii):

$|\psi\rangle$ describes the probability distributions of the observables in an ensemble of similarly prepared systems.

"But we never observe macroscopic objects to be in a superposition state!" says a supporter of (i).

"How do you know? Have you ever looked?" says a supporter of (ii).

- - - - -

Suppose the state is a **coherent superposition of two non-overlapping wave packets:**

$$\psi(\vec{x}) = \phi(\vec{x}) + \phi(\vec{x} + \vec{\mathbf{a}})$$

The **<u>position</u> probability density** is

$$|\psi(\vec{x})|^2 = |\phi(\vec{x}) + \phi(\vec{x} + \vec{\mathbf{a}})|^2$$

$$= |\phi(\vec{x})|^2 + |\phi(\vec{x} + \vec{\mathbf{a}})|^2$$

-- No interference because the wave packets do not overlap.

-- No difference from an *incoherent* (classical) mixture.

The **<u>momentum</u> probability density** is

$$|\langle\vec{\mathbf{p}}|\psi\rangle|^2 = |\langle\vec{\mathbf{p}}|\psi\rangle + \exp(i\vec{\mathbf{p}}\cdot\vec{\mathbf{a}}/\hbar)\langle\vec{\mathbf{p}}|\psi\rangle|^2$$

non-overlapping wave packets:

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-- It contains a very **fine-grained interference pattern.**

-- Observable in principle, but very difficult to detect if the separation a is macroscopic.

The "Measurement Problem"

is insoluble.

The "Measurement Problem"

is solved.

The "Measurement Problem"

viewed as the problem of explaining how the "reduction of the state vector" comes about,

is insoluble.

The "Measurement Problem"

viewed as seeking an interpretation of the formalism that is compatible with the existence of *entangled macroscopic superpositions* (Measurement Theorem),

is solved

by adopting an *ensemble interpretation* , which does not require any "state reduction" process.

Myths about Quantum Measurement

(1) Reduction (collapse) of the state vector is caused by an uncontrollable disturbance of the object by the measuring apparatus. [Heisenberg]

Fact -- The interaction between object and apparatus leads to the macroscopically entangled state.

(2) The observer causes the "reduction" when he reads the apparatus. [Wigner]

Fact -- Wigner did not believe this to be true; he intended it as a reductio ad absurdum..

(3) The apparatus must be classical. [Bohr]

Fact -- There is no boundary between the "classical" and "quantum" domains. Presumably, QM is the more general theory.

(4) Environmental decoherence causes an effective reduction of the entangled state. [Zurek]

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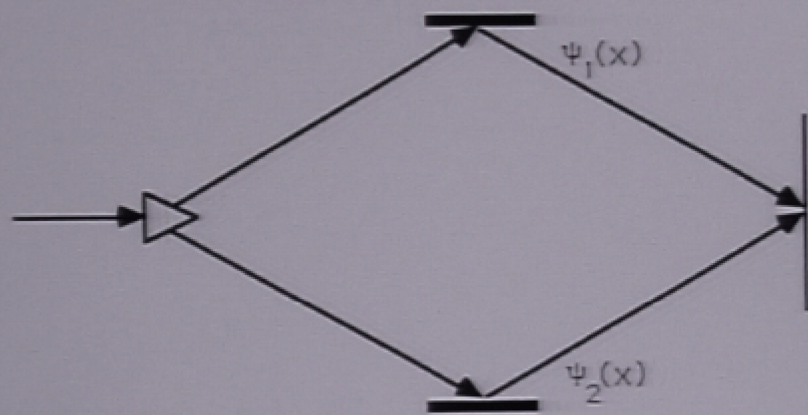
Fact -- Interaction with the environment only makes the entanglement even more macroscopic.



position probability density =

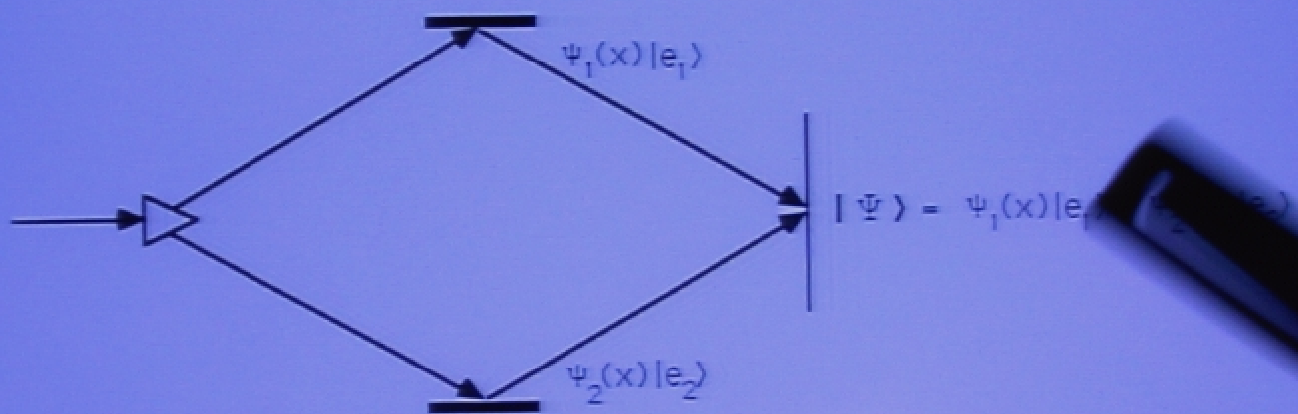
$$\langle x|\Psi\rangle\langle\Psi|x\rangle = |\psi_1(x)|^2 + |\psi_2(x)|^2$$

$$+ \psi_1^*(x)\psi_2(x) \qquad + \psi_2^*(x)\psi_1(x)$$

Interference terms

Interference

with environmental decoherence



position probability density =

$$Tr_{(e)}\ \langle x|\Psi\rangle\langle\Psi|x\rangle = |\psi_1(x)|^2 + |\psi_2(x)|^2$$

$$+\ \psi_1^*(x)\,\psi_2(x)\,\langle e_1|e_2\rangle + \psi_2^*(x)\,\psi_1(x)\,\langle e_2|e_1\rangle$$

Interference terms

reduced by decoherence

Interference

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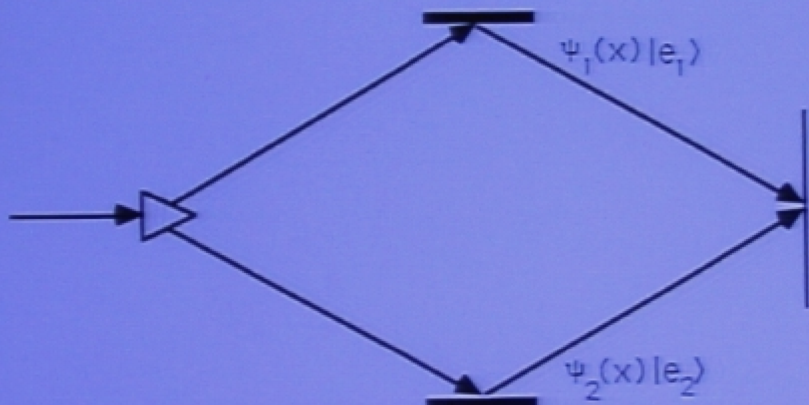

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Interference terms

reduced by decoherence

Schrödinger's cat problem

(a.k.a. the quantum measurement problem)

System:

unstable atom + cat

state:

$|\text{undecayed}\rangle|\text{live}\rangle + |\text{decayed}\rangle|\text{dead}\rangle$

Problem:

What is the meaning of a state vector that is a coherent superposition of macroscopically different terms?

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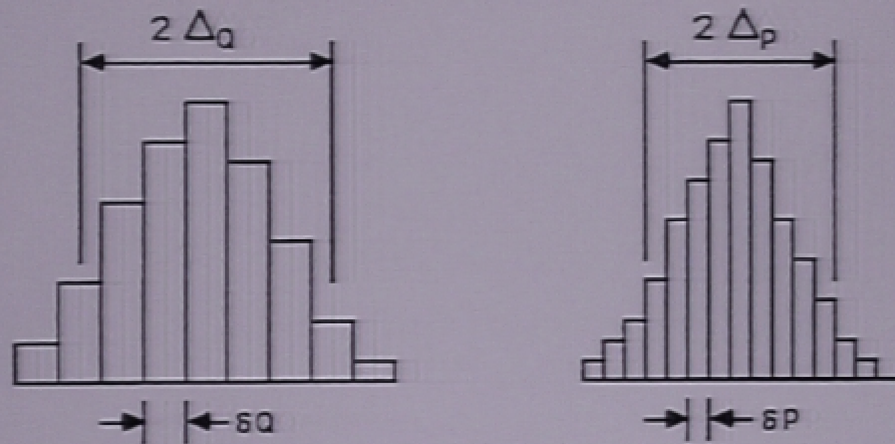
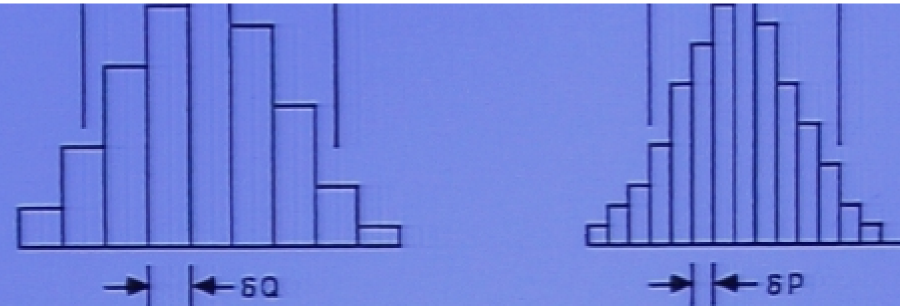
The Indeterminacy Relations



$$\Delta_Q \ \Delta_P \geq \frac{\hbar}{2} \qquad (1)$$

- The quantities Δ_Q and Δ_P are not errors of measurement.

- The experimental test of the inequality ([redacted]) (1) does not involve *simultaneous measurements* of Q and P, but rather it involves the measurement of one or the other of these dynamical variables on each independently prepared representative of the particular state being



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- Eq. (1) refers to state preparation, not to measurement.

Simultaneous measurement of Q and P

ref.: S. Stenholm, Ann. Phys. 218, 233 (1992)

Couple a Q-measuring device and a P-measuring device to the Hamiltonian of the system.

For optimal choices of the various parameters, the statistical distribution of the results is given by the Husimi distribution,

$$\rho_H(q,p) = (2\pi\hbar)^{-1} \, |\langle q,p|\Psi\rangle|^2 \ ,$$

where $|q,p\rangle$ is a miminum-uncertainty state.

The half-widths of the Husimi distribution satisfy

$$(\Delta Q)_H \, (\Delta P)_H \ \geq \ \hbar \qquad (2)$$

Note: Lower bound in (2) is twice as high as the bound in (1).

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$$e^{iP_0 q}$$

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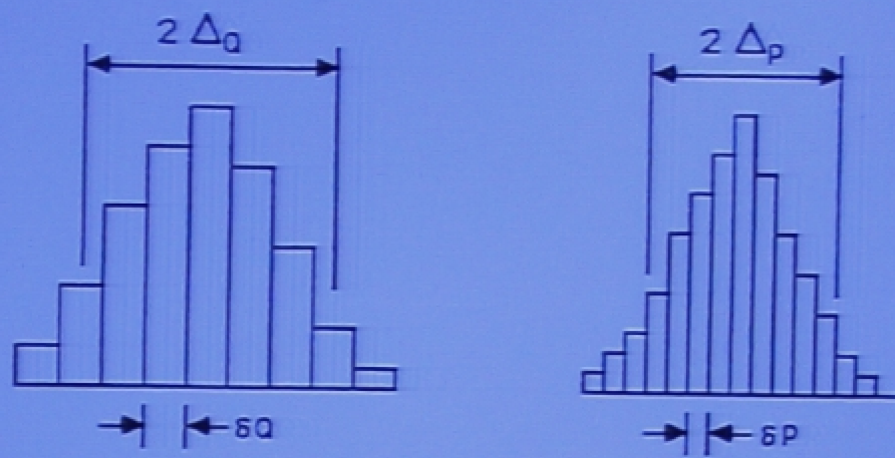

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Some Practical Cases where the Interpretation of QM Makes a Difference

(1) "Limitations of the Projection Postulate"
L.E.B., Foundations of Physics 20, 1329 (1990).

(original title: "The Projection Postulate is either Redundant or Wrong!")

P.P. = "Upon measurement, the state of the system collapses to an eigenstate of the observable that was measured"

Measurements on a Correlated 2-component system (Q and R)

$$Q|q_i\rangle = q_i |q_i\rangle \quad , \quad R|r_j\rangle = r_j |r_j\rangle$$

$$|\Psi\rangle = \sum_i \sum_j a_{ij} \, |q_i\rangle \otimes |r_j\rangle$$

Joint probability for the 2 measurements:

$$P(Q=q_i \,\&\, R=r_j \,|\, \Psi) = |a_{ij}|^2 \, . \qquad (1)$$

Probability for 2nd (R) measurement conditional on the result $Q=q_i$ in the first:

$$P(R=r_j \,|\, Q=q_i \,\&\, \Psi) = \frac{|a_{ij}|^2}{\sum_{j'} |a_{ij'}|^2} \, . \qquad (2)$$

Projection Postulate after first $(Q=q_i)$ measurement "collapses" the state to

$$|\psi'\rangle \quad \sum a_{ij} |q_i\rangle \otimes |r_j\rangle$$

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$$|\Psi'\rangle = \frac{\sum_j a_{ij}\, |q_i\rangle \otimes |r_j\rangle}{\sum_{j'} |a_{ij'}|^2}$$

2nd (R) measurement on $|\Psi'\rangle$ yields the

2-component system (Q and R)

$$Q|q_i\rangle = q_i|q_i\rangle \quad , \quad R|r_j\rangle = r_j|r_j\rangle$$

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Joint probability for the 2 measurements:

$$P(Q=q_i + R=r_j \mid \Psi) = |a_{ij}|^2 . \quad (1)$$

Probability for 2nd (R) measurement conditional on the result $Q=q_i$ in the first:

$$P(R=r_j \mid Q=q_i + \Psi) = \frac{|a_{ij}|^2}{\sum_{j'} |a_{ij'}|^2} . \quad (2)$$

Projection Postulate after first $(Q=q_i)$ measurement "collapses" the state to

$$|\Psi'\rangle = \frac{\sum_j a_{ij} |q_i\rangle \otimes |r_j\rangle}{\sum_{j'} |a_{ij'}|^2}$$

2nd (R) measurement on $|\Psi'\rangle$ yields the correct conditional probability (2).

2-photon state of a 2-mode field

$$E^{(+)}(x) = c\left\{a_1 e^{ik_1\cdot x} + a_2 e^{ik_2\cdot x}\right\}$$

$$E^{(-)}(x) = c\left\{a_1^{\dagger} e^{-ik_1\cdot x} + a_2^{\dagger} e^{ik_2\cdot x}\right\}$$

Probability of detecting a photon at x is

$$G^{(1)}(x) = \langle E^{(-)}(x)\, E^{(+)}(x)\rangle$$

Probability of detecting a photon at each of x_1 and x_2 is

$$G^{(2)}(x_1, x_2) =$$

$$\langle E^{(-)}(x_1)\, E^{(-)}(x_2)\, E^{(+)}(x_2)\, E^{(+)}(x_1)\rangle$$

In the 2-photon state $|1,1\rangle$

$$G^{(1)}(x) = (\text{constant})$$

$$G^{(2)}(x_1, x_2) \propto \left\{ 1 + \cos\left[(k_1 - k_2)\cdot(x_1 - x_2)\right] \right\}$$

Singles count rate is spatially uniform, but the pairs are spatially correlated.

What would the Projection Postulate yield?

— ambiguous.

— If the detection of the first photon "collapses" the 2-photon state $|1,1\rangle$ to a one-photon state, $|1,0\rangle$ or $|0,1\rangle$, then the conditional probability for detection of the second photon would be spatially uniform.

X wrong!

— If it "collapses" to a superposition like $\alpha|1,0\rangle + \beta|0,1\rangle$

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— If it "collapses" to a superposition like $\alpha|1,0\rangle + \beta|0,1\rangle$ then the phase of the "interference pattern" for detection of the 2nd photon, conditional on detection of the first, would remain arbitrary.

X wrong!

(the phase in $G^{(2)}$ is definite)

Quantum Zeno Effect
(Watched Pot paradox)

The survival probability of an unstable initial state $|\psi_0\rangle$ is

$$P_s(t) = \left| \langle \psi_0 | e^{-iHt/\hbar} | \psi_0 \rangle \right|^2$$

$$\cong 1 - \left(\frac{\sigma t}{\hbar}\right)^2 \quad \text{for small } t.$$

$$\sigma^2 = \langle H^2 \rangle - \langle H \rangle^2$$

Observe (and "collapse") the system at n times $\frac{T}{n}, \frac{2}{n}T, \frac{3}{n}T, \ldots T$.

The probability of surviving all n observations is

$$P_s(T) = \left[1 - \left(\frac{\sigma}{\hbar}\frac{T}{n}\right)^2\right]^n$$

$\to 1$ in the …

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$\rightarrow 1$ in the "continuous" limit $n \rightarrow \infty$. *

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* Absurd conclusion — Watched pots do boil!

A simple measurement model

Object states: $|r_1\rangle$, $|r_2\rangle$

Apparatus states: $|A_1\rangle$, $|A_2\rangle$

Effect of the interaction:

$$U|r_1, A_1\rangle = |r_1 A_1\rangle$$
$$U|r_1, A_2\rangle = |r_1, A_2\rangle$$

] if r_1, no change in A_i

$$U|r_2, A_1\rangle = |r_2, A_2\rangle$$
$$U|r_2, A_2\rangle = |r_2, A_1\rangle$$

] if r_2, flip A_i

$$U = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{matrix} (1,1) \\ (1,2) \\ (2,1) \\ (2,2) \end{matrix}$$

Eigenvectors are:

$|r_1, A_1\rangle$; $|r_1, A_2\rangle$, Apparatus states: $|A_1\rangle$, $|A_2\rangle$

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Eigenvectors are:

$$|r_1, A_1\rangle \;,\; |r_1, A_2\rangle \;,$$

$$|+\rangle = \left(|r_2, A_1\rangle + |r_2, A_2\rangle\right)/\sqrt{2}$$

$$|-\rangle = \left(|r_2, A_1\rangle - |r_2, A_2\rangle\right)/\sqrt{2}$$

Choose the initial state

$$|r_2, A_1\rangle = \frac{|+\rangle + |-\rangle}{\sqrt{2}}$$

Then

$$|\Psi(t)\rangle = \left\{ e^{-iE_+ t/\hbar} |+\rangle + e^{-iE_- t/\hbar} |-\rangle \right\} / \sqrt{2}$$

Take $E_+ = 0$, $E_- = -V$.

$$|\Psi(t)\rangle = \left\{ |+\rangle + e^{iVt/\hbar} |-\rangle \right\} / \sqrt{2}$$

At $t = \tau$ (completion of measurement)

we must have $|\psi(\tau)\rangle = |r_2, A_2\rangle$,

hence $\frac{V\tau}{\hbar} = \pi$.

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The limit of continuous measurement,

$\tau \to 0$,

requires $V \to \infty$.

This infinite interaction swamps the Hamiltonian, causing the (unphysical) Zeno effect.

- Continuous "von Neumann" measurement is impossible.

- For bounded interaction matrix elements in the measurement, the Zeno effect does not happen (P. Pearle, unpub.)

- See A. Peres for a realistic theory of continuous measurement the Hamiltonian, causing the (unphysical) Zeno effect.

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— See A. Peres for a realistic theory of continuous measurement [PRA **42**, 5720 (1990)]