Title: QCD and the Fabric of Space

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Abstract:

bj

MARCH '04

QCD & THE FABRIC OF SPACE

I INTRODUCTION

II. PADMANABHAN

III. DARK ENTROPY & FABRICS

IV. LOOP QUANTUM GRAVITY

V. FRW COSMOLOGY

VI. CONCLUDING REMARKS

1.1

I. INTRODUCTION

1967 OBSERVATION BY ZELDOVICH

USPEKHI 11, 381 (68)

QCD SCALE (PROTON MASS) IS

$$\Lambda_{QCD} \sim 10^{-20} M_{planck}$$

COSMOLOGICAL SCALE (HUBBLE PARAMETER H)

$$\lim_{\text{"}t\to\infty\text{"}} H(t) \sim 10^{-60} M_{planck}$$

$$\therefore \quad \Lambda_{QCD}^3 \sim H M_{planck}^2 \sim \mu^2 M_{pl}$$

[FRW $\quad H^2 = \frac{8\pi}{3} \frac{\mu^4}{M_{pl}^2} \quad$ μ = DARK ENERGY SCALE]

IS THIS AN ACCIDENT ?

☐ YES

☒ NO (IN THIS TALK)

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RELATED IDEAS:

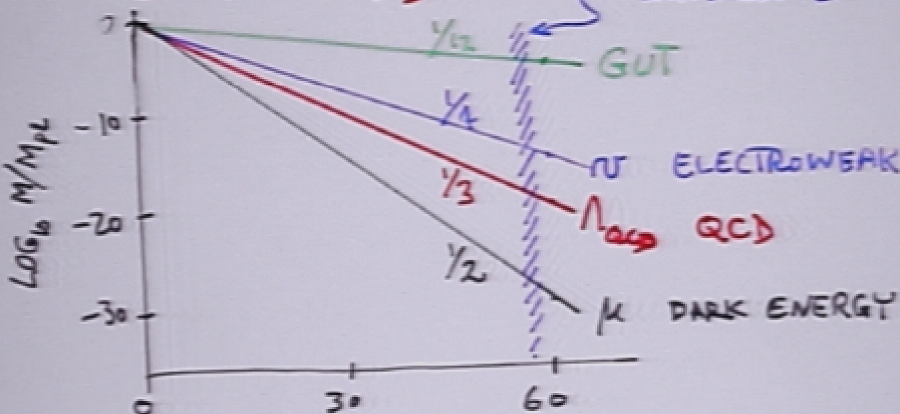
- SCHÜTZHOLD gr-qc/0204018
- CARNEIRO gr-qc/0305081
- NG

QCD VAC. ENERGY $\sim H\Lambda^3_{QCD}$

HOLOGRAPHY?

$$(\Delta x)^3 \gtrsim \frac{t}{m^2_{pl}} \sim \frac{1}{Hm^2_{pl}} \sim \frac{1}{\Lambda^3_{QCD}}$$

- BJORKEN RESCALING:



$\log_{10} M_{pl} R$ $\quad R = H_0^{-1}$ = SIZE OF UNIVERSE

- ALL HIERARCHY PROBLEMS "SOLVED"
- BEHAVIOR OF SCALING EXPONENTS NOT SOLVED

1.3

- AKHOURY & WEINSTEIN: hep-th/0312249

  - PUT THE PRESENT-DAY VISIBLE UNIVERSE IN A COMOVING BOX; VOLUME $= V(t) = V(0)a^3$

  - VACUUM OUT ALL THE DIRT

  - SEND THE UNIVERSE BACKWARD IN TIME TOWARD A BIG CRUNCH

  - IT BOUNCES WHEN $V(t) \sim \Lambda_{QCD}^{-3}$

WHY?

  - TOTAL ENERGY $U(t) = \mu^4 V(t)$

  - QUANTUM-EFFECTS WHEN $U \lesssim H$

$$V(t) \sim \frac{H}{\mu^4} \sim \left(\frac{\mu^2}{M_{Pl}}\right) \cdot \frac{1}{\mu^4} \sim \frac{1}{M_{Pl}\mu^2} \sim \frac{1}{\Lambda_{QCD}^3}$$

(2.1)

II. PADMANABHAN gr-qc/0205090 /0311036

- DIRECT, SIMPLE CONNECTION OF EINSTEIN-HILBERT LAGRANGIAN WITH THERMOGRAVITY

[MY OWN VARIATION ON HIS THEME]

- CONSIDER ~~STATIC~~ STATIONARY, SPHERICALLY SYMMETRIC PAINLEVE'-GULLSTRAND METRICS:

$$ds^2 = dt^2 - (d\vec{x} - \vec{v}(x,t)\, dt)^2 \quad \text{(GENERAL)}$$

$$\vec{v} = \hat{r}\, v(r) \qquad \text{(SPECIALIZATION HERE)}$$
$$u = v^2$$

- THIS DESCRIBES MANY SYSTEMS:

SCHWARZSCHILD $u = 2m/m_{P}^2 r$

DE SITTER $u = H^2 r^2$

REISSNER NORDSTROM $u = -8\pi Q^2/m_P^2 r^2$

- FOR MIXTURES, JUST ADD THE u's

- COORD TX TO $ds^2 = (1-u(r))\,dt^2 - (1-u(r))^{-1} dr^2 - r^2 d\Omega$

(2.2)

CALCULATE RIEMANN TENSOR $R^{\alpha\beta}_{\mu\nu}$ AS FUNCTIONAL OF $u(r)$:

$$u = V^2$$

- PETROV FORM:

$$
R = 2 \begin{array}{c}
\begin{array}{cccccc} tr & t\theta & t\varphi & \theta\varphi & r\varphi & r\theta \end{array} \\
\left(\begin{array}{ccc|ccc}
\frac{u''}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{u'}{2r} & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{u'}{2r} & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & \frac{u}{r^2} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{u'}{2r} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{u'}{2r}
\end{array}\right)
\end{array}
= \begin{pmatrix} R^+ & 0 \\ 0 & R^- \end{pmatrix}
$$

- MARVELOUSLY SIMPLE (IT STAYS SIMPLE FOR GENERAL $N(r,t)$)

- SELF-DUALITY CONDITION $\Leftrightarrow$ EINSTEIN EQ'NS

$$\frac{u''}{2} = \frac{u}{r^2}$$

- SOLUTION IS SCHWARZSCHILD-DE SITTER

$$u = \frac{c_1}{r} + c_2 r^2$$

- PURELY GEOMETRICAL CRITERION

• EINSTEIN-HILBERT LAGRANGIAN:

$$L = \frac{M_{pl}^2}{16\pi}\int d_x^3 \, \mathrm{Tr}\, R = \frac{M_{pl}^2}{8\pi}\int d_x^3 \left(\mathrm{Tr}\, R^+ + \mathrm{Tr}\, R^-\right)$$

$$= \frac{M_{pl}^2}{8\pi} \cdot 4\pi \int dr \, r^2 \left[\left(\frac{u''}{2} + \frac{u'}{r}\right) + \left(\frac{u}{r^2} + \frac{u'}{r}\right)\right]$$

$$= \frac{M_{pl}^2}{2}\int dr \, r^2 \left[\frac{1}{2r^2}\frac{d}{dr}(r^2 u') + \frac{1}{r^2}\frac{d}{dr}(ru)\right]$$

• L IS IN GENERAL A SURFACE INTEGRAL

EVALUATE ON A HORIZON:

$u = 1$ DEFINITION

$u' = 4\pi T$ SURFACE ACCELERATION (HAWKING) TEMPERATURE

radius

$$L_{\text{HORIZON}} = \frac{M_{pl}^2}{4}\left[\underbrace{4\pi R^2 T}_{TS} + \underbrace{2R}_{M}\right]$$

TS B-H ENTROPY

M MASS

- MASS DEFINITION APPROPRIATE FOR BOTH SCHWARZSCHILD & DE SITTER:

$$\frac{2M}{M_{pl}^2 R} = 1 \qquad \text{(SCHWARZSCHILD)}$$

$$M = \frac{4}{3}\pi R^3 \mu^4 = \frac{4}{3}\pi R^3 \left(\frac{3}{8\pi} H^2 M_{pl}^2\right) = \frac{M_{pl}^2 R}{2}$$

(DE SITTER)

MAIN MESSAGE FOR US:

FOR DE SITTER

$$TS(r) \sim \frac{M_{pl}^2}{4}\int_0^r dr \frac{d}{dr}(r^2 w') = \left(\frac{r}{R}\right)^3 (TS)_{Horizon}$$

COMPARE WITH MASS DENSITY:

$$M(r) \sim \frac{M_{pl}^2}{2}\int_0^r dr \frac{d}{dr}(r u) = \left(\frac{r}{R}\right)^3 M_{HORIZON}$$

$$\boxed{\text{ENTROPY IS AN INTEGRAL OVER A DENSITY}}$$

RICCI TENSOR:

$$R^t_t = R^r_r = \frac{4}{M_{pl}^2 r^2}\frac{d}{dr}(TS)$$

$$R^\theta_\theta = R^\varphi_\varphi = \frac{2}{M_{pl}^2 r^2}\frac{d}{dr}M$$

EINSTEIN TENSOR

$$G^t_t = G^r_r = -\frac{2}{M_{pl}^2 r^2}\frac{d}{dr}M$$

$$G^\theta_\theta = G^\varphi_\varphi = \frac{-4}{M_{pl}^2 r^2}\frac{d}{dr}(TS)$$

THERMOGRAVITY INFERENCE

$$d(TS) \overset{?}{=} T\,dS = \frac{M_{pl}^2}{4} G^\theta_\theta\, r^2 dr = p\, dV$$

$$dM = \frac{M_{pl}^2}{2} G^t_t\, r^2 dr = \rho\, dV = dU$$

- WE DO NOT YET TRY TO INTERPRET.
- MOVE ON —

2.3

EINSTEIN-HILBERT LAGRANGIAN:

$$L = \frac{M_{pl}^2}{16\pi}\int d^3x \, \mathrm{Tr}\, R = \frac{M_{pl}^2}{8\pi}\int d^3x \left(\mathrm{Tr}\, R^+ + \mathrm{Tr}\, R^-\right)$$

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$$= \frac{M_{pl}^2}{2}\int dr\, r^2 \left[\frac{1}{2r^2}\frac{d}{dr}(r^2 u') + \frac{1}{r^2}\frac{d}{dr}(r u)\right]$$

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radius

$$L_{\text{HORIZON}} = \frac{M_{pl}^2}{4}\left[4\pi R^2 T + 2R\right]$$

TS B-H ENTROPY

M_1 MASS

SCHWARZSCHILD & DE SITTER:

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(DE SITTER)

MAIN MESSAGE FOR US:

FOR DE SITTER

$$TS(r) \approx \frac{M_{pl}^2}{4}\int_0^r dr \, \frac{d}{dr}(r^2 w) = \left(\frac{r}{R}\right)^3 (TS)_{\text{Horizon}}$$

COMPARE WITH MASS DENSITY:

$$M(r) \sim \frac{M_{pl}^2}{2}\int_0^r dr \, \frac{d}{dr}(r\omega) = \left(\frac{r}{R}\right)^3 M_{\text{HORIZON}}$$

$$\boxed{\text{ENTROPY IS AN INTEGRAL OVER A DENSITY}}$$

2.7

GRAVITATIONAL TOPOLOGICAL TERM:
(GAUSS–BONNET)

$$I = \frac{1}{32\pi^2}\int d^3x\, \mathrm{Tr}\, R\tilde{\tilde{R}} = \frac{4\pi\cdot 2\cdot 4}{32\pi^2}\int r^2 dr\, \mathrm{Tr}\, R^+ R^-$$

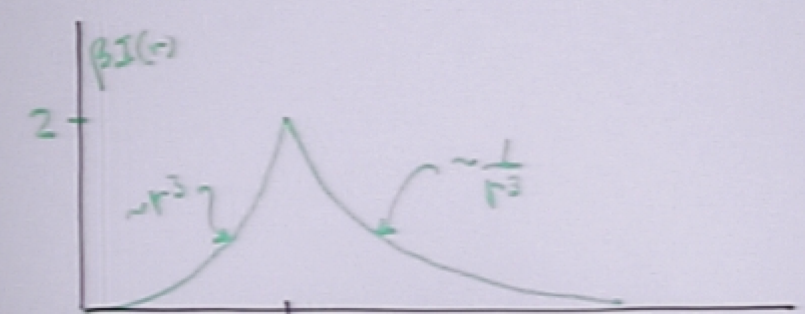
$$= \frac{1}{\pi}\int r^2 dr\left[\frac{u u''}{2r^2} + \frac{u'^2}{2r^2}\right] = \frac{1}{2\pi}\int dr \frac{d}{dr}(u u')$$

ON THE HORIZON $u' = 4\pi T$

$$I_{Hor} = \frac{1}{2\pi}(4\pi T) = 2T$$

$$\beta I_{HoR} = 2 = \text{INTEGER}$$

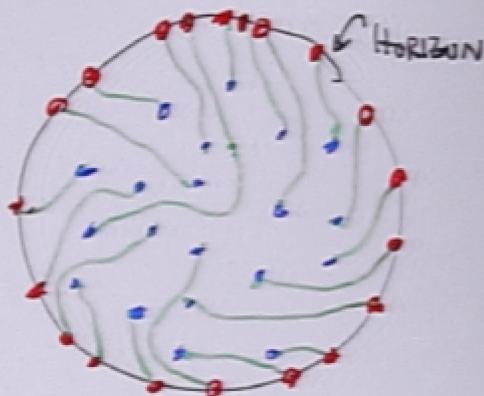
FOR A GRAVASTAR:



III. BULK ENTROPY & FABRICS

(3.1a)

- PADMANABHAN SUGGESTS ENTROPY LIVES IN THE VOLUME.



- USUAL PICTURE IS THAT IT COUNTS BITS ON THE HORIZON

- MAYBE IT IS BOTH

$$\frac{S}{V} = \frac{\pi R^2 M_{pl}^2}{\frac{4}{3}\pi R^3} = \frac{3 M_{pl}^2}{4R} = \frac{3}{4} H M_{pl}^2 \sim \Lambda_{QCD}^3$$

- ENTROPY MEASURES $\left\{ \begin{array}{l} \text{BITS ON HORIZON} \\ \text{STRING ENDS IN VOLUME} \\ \text{STRINGS THEMSELVES} \end{array} \right.$

- MY PERSONAL CHOICE:

  × A LATTICE OF STRINGS: 3D FABRIC

  × ONLY POSSIBLE IN FRW (TRANSLATION GROUP EXISTS)

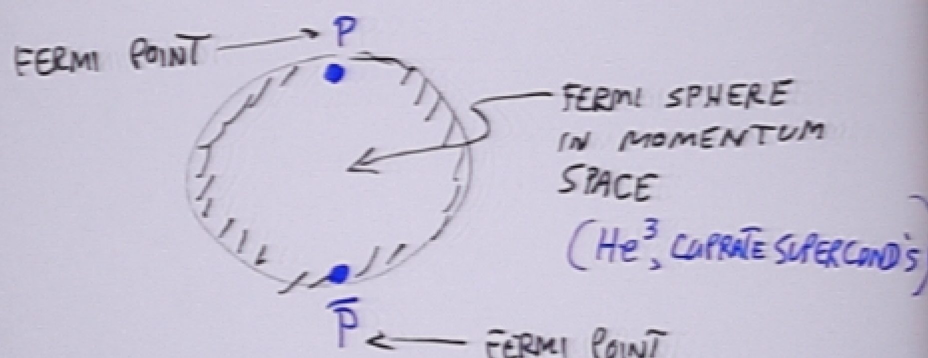
WHY A LATTICE ?? (CONDENSED-MATTER ANALOGUES)

VOLOVIK ("The Universe in a Helium Droplet")

1. WHY NOT ?

   Not all condensed-matter systems are liquids or polymers.

2. INTERNAL SYMMETRIES & FERMI POINTS:



3. AT THE FERMI-POINTS $P = \bar{P}$:

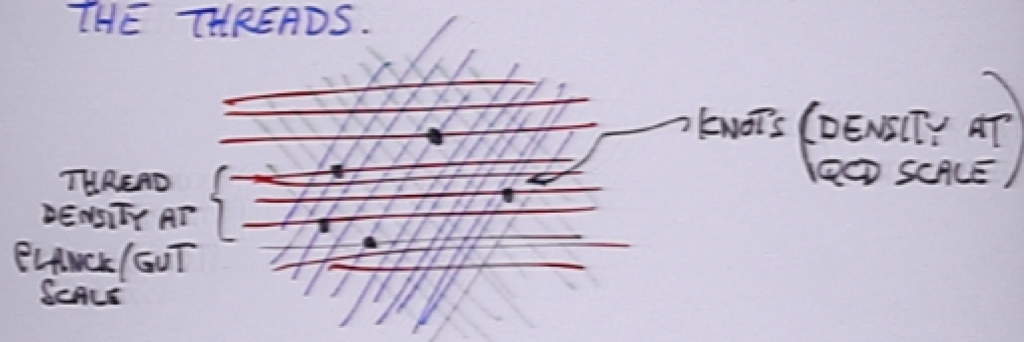
   - DIRAC-WEYL EXCITATIONS:

$$H_{eff} = \psi^\dagger \sigma \cdot p \, \psi$$

   - GAUGE-BOSON EXCITATIONS

     Fluctuations in position of P & $\bar{P}$

- GO TO FRW BASIS (GAUGE CHOICE)
- ASSUME 1) STRINGS ARE CARTESIAN FABRIC
  2) "THREAD COUNT" (OR SPACING) IS PLANCK/GUT SCALE
  3) PERHAPS THREADS GO BY EACH OTHER IN GENERAL — NO JUNCTIONS, A "WEAVE".
- ENTROPY COUNTS NONTRIVIAL KNOTTINGS OF THE THREADS.



- ONE KNOT PER THREAD PER HUBBLE SCALE
- ENERGY PER KNOT ϵ IS VERY SMALL:

$$\epsilon = \frac{\mu^4 V}{S} = \left(\frac{3}{8\pi} H^2 M_{pl}^2\right)\left(\frac{4}{3M_{pl}^2 H}\right) = \frac{H}{2\pi} = T$$

- KNOTS ARE CONTINUOUSLY CREATED

WHAT ARE THE KNOTS?

3.3

- THEY TALK TO QCD
- THEY ARE ESSENTIALLY "PURE GAUGE"
- THEY ARE TOPOLOGICAL

- ASSUME IT IS TOPOLOGICAL CHARGE N

$$L_{CP}^{(QCD)} = \frac{\theta}{24\pi^2}\int d^3x\, E\cdot B \equiv \theta \dot{N}$$

$$\frac{E\cdot B}{24\pi^2} = \partial_\mu K_\mu$$

$$N = \int d^3x\, K_0$$

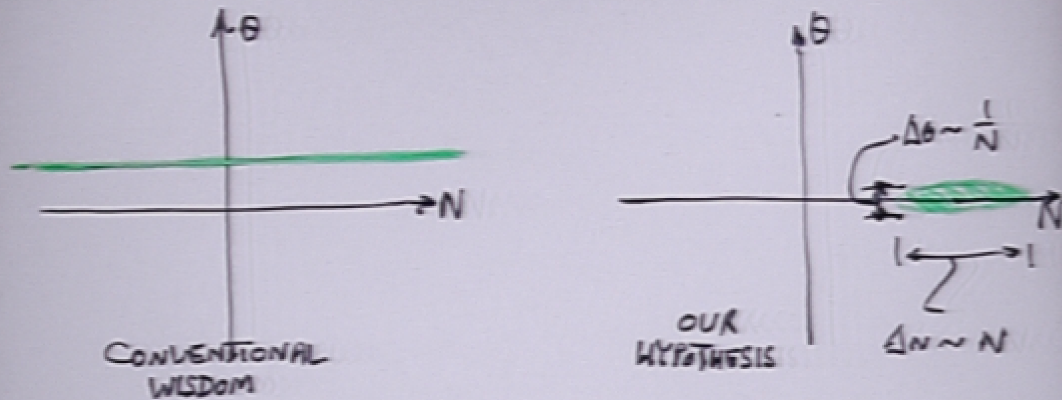
$$\langle N \rangle = 10^{120}$$

- INSTANTONS CHANGE N BY ± 1 ; RATE IS Λ_{QCD}^4

$$\sqrt{(\Delta N)^2} = \sqrt{\text{TOTAL NUMBER OF INSTANTON TRANSITIONS}} = \sqrt{\left(\frac{\Lambda_{QCD}}{H}\right)^4} = \frac{\Lambda_{QCD}^2}{H^2} \sim 10^{80}$$

- INSTANTON NOISE IS NEGLIGIBLE

- Θ & N ARE CONJUGATE VARIABLES
  LOOK AT PHASE SPACE



- CAVEAT : NO DYNAMICS OF Θ, N SYSTEM — YET

IS N PHYSICAL ??

- ANALOGY: 1+1 DIMENSIONAL QED

$$L^{ANOM}_{QED} = \frac{\epsilon}{2\pi}\int dx\, E = \frac{\epsilon}{2\pi}\frac{d}{dt}\int A\,dx = \epsilon \dot{N}_{QED}$$

$$(A_0 = 0)$$

$$N_{QED} = \oint A\,dx$$

- IT CAN BE GAUGE INVARIANT
- TOPOLOGICAL

- THE PHYSICS IS BACKGROUND E FIELD

$$L_{QED} = \int dx \left(\frac{1}{2}\dot{A}^2 + \epsilon \dot{A}\right)$$

$$= \int dx\, \frac{1}{2}(\dot{A}+\epsilon)^2 - \frac{1}{2}\epsilon^2$$

- θ_{QCD} ANALOGOUS TO THE BACKGROUND E FIELD

IV. LOOP QUANTUM GRAVITY

(DRAFT OF BOOK BY CARLO ROVELLI. GO TO HIS HOME PAGE)

- THE CORRESPONDENCE

$$\int E\cdot B = \Theta \tilde{N} \Longleftrightarrow \int R\tilde{R}$$

IMPLIES YANG-MILLS $\Longleftrightarrow$ GRAVITY

- LOOP QUANTUM GRAVITY IS YANG-MILLS
CAN IT BE IDENTIFIED WITH A QCD SUBGROUP ??

- LQG IS SO(3) (OR SU(2))
BUT QCD IS SU(3).

- THE GUESS: IDENTIFY LQG WITH REAL SUBGROUP OF QCD:

$$\begin{pmatrix} r \\ y \\ b \end{pmatrix} \to U \begin{pmatrix} r \\ y \\ b \end{pmatrix} \qquad U = U^*$$

FOR SO(3) SUBGROUP

$$T_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & i \\ 0 & -i & 0 \end{pmatrix} \quad T_2 = \begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{pmatrix} \quad T_3 = \begin{pmatrix} 0 & i & 0 \\ -i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$U = e^{i\vec{T}\cdot\vec{\theta}} \text{ IS REAL}$$

CAN LQG E & B BE IDENTIFIED WITH QCD E & B?

- IT'S NOT EASY.
  I HAVE NO ANSWER NOW.

- A & B LOOK FAMILIAR: CORRECT UNITS & SMALL

- E IS A "DENSITIZED TRIAD"; IT DEFINES THE COORDINATE GRID & IS ORDER 1 (DIMENSIONLESS)

- $[A(x), E(y)] \sim \frac{1}{M_{pl}^2} \delta^3(x-y)$

- PROBLEM:
  - LACK OF SIMPLE EXAMPLES
  - NOTHING DONE FOR PAINLEVE-GULLSTRAND METRICS

- BOJOWALD HAS APPLIED LQG TO FRW COSMOLOGY. MORE LATER.

5.4

LQG COSMOLOGY (BOJOWALD):

gr-qc/0303072

CARTESIAN:
$$E_i^a = E\delta_i^a$$
$$A_i^a = A\delta_i^a$$
$$B_i^a = B\delta_i^a$$
etc

i = SPACE

a = YANG-MILLS

FRW IDENTIFICATIONS:

$$A \sim \gamma K \sim \gamma \dot{a}$$

K IS EXTRINSIC CURVATURE
γ IS IMMIRZI PARAMETER

$$E \sim a^2$$

E MEASURES AREA IN PLANCK UNITS.

$$[A, E] \sim \frac{\gamma}{M_{pl}^2}$$

$$B \sim A^2 \sim \gamma^2 K^2 \sim \gamma^2 \dot{a}^2$$

HAMILTONIAN
$$H \sim M_{pl}^2 \left[-\frac{B\sqrt{E}}{\gamma^2} + \mu^4 E\sqrt{E} \right]$$

"DARK ENERGY IS ELECTRIC"

THERMOGRAVITY & FRW

6.2

- SEMICLASSICAL LIMIT SHOULD SUFFICE

- FRW IN A COMOVING BOX SHOULD SUFFICE

- INTERNAL STRUCTURE OF BOX "EXPOSED" WHEN THE BOX IS EXTRACTED FROM ENVIRONMENT. PARAMETERS:

    - NUMBER OF STRINGS IN FABRIC WHICH ARE EXPOSED

    - "FLUX" IN EACH STRING ??

    - STRUCTURE OF THE KNOTS

- IS A CLASSICAL GENERATING FUNCTIONAL SUFFICIENT FOR DEFINING THE CONTENTS OF THE FRW BOX ??

6.4

LOOP GRAVITY

- STUDY 1+1 AND 2+1 LQG
- KEEP TRYING TO IDENTIFY $(E,B)_{LQG} \neq (E,B)_{QD}$
- MUNDANE EXAMPLES OF LQG AT WORK (ESPECIALLY PAINLEVE - GULLSTRAND)
- SHOULD E & B BE INTERCHANGED?

STANDARD GRAVITY (PAINLEVE - GULLSTRAND)

- GO TO SMALL VOLUMES (CONST. CURVATURE APPROX)
- FLUID ANALOGY (FISCHER & VISSER COND-MAT/0205139)
- IDENTIFY AKHOURY - WEINSTEIN VARIABLES: PROBABLY A CONDENSATE WAVE FUNCTION

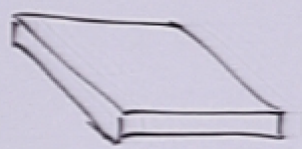
FABRIC MODELS

6.5

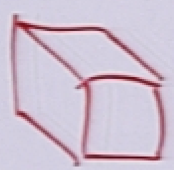
- ASYMMETRIC FRW BOXES FOR EFFECTIVE DIMENSIONAL REDUCTION:

VOLUME AT $S=1$

$M_{pl} \times M_{pl} \times H$



$\mu \times \mu \times M_{pl}$



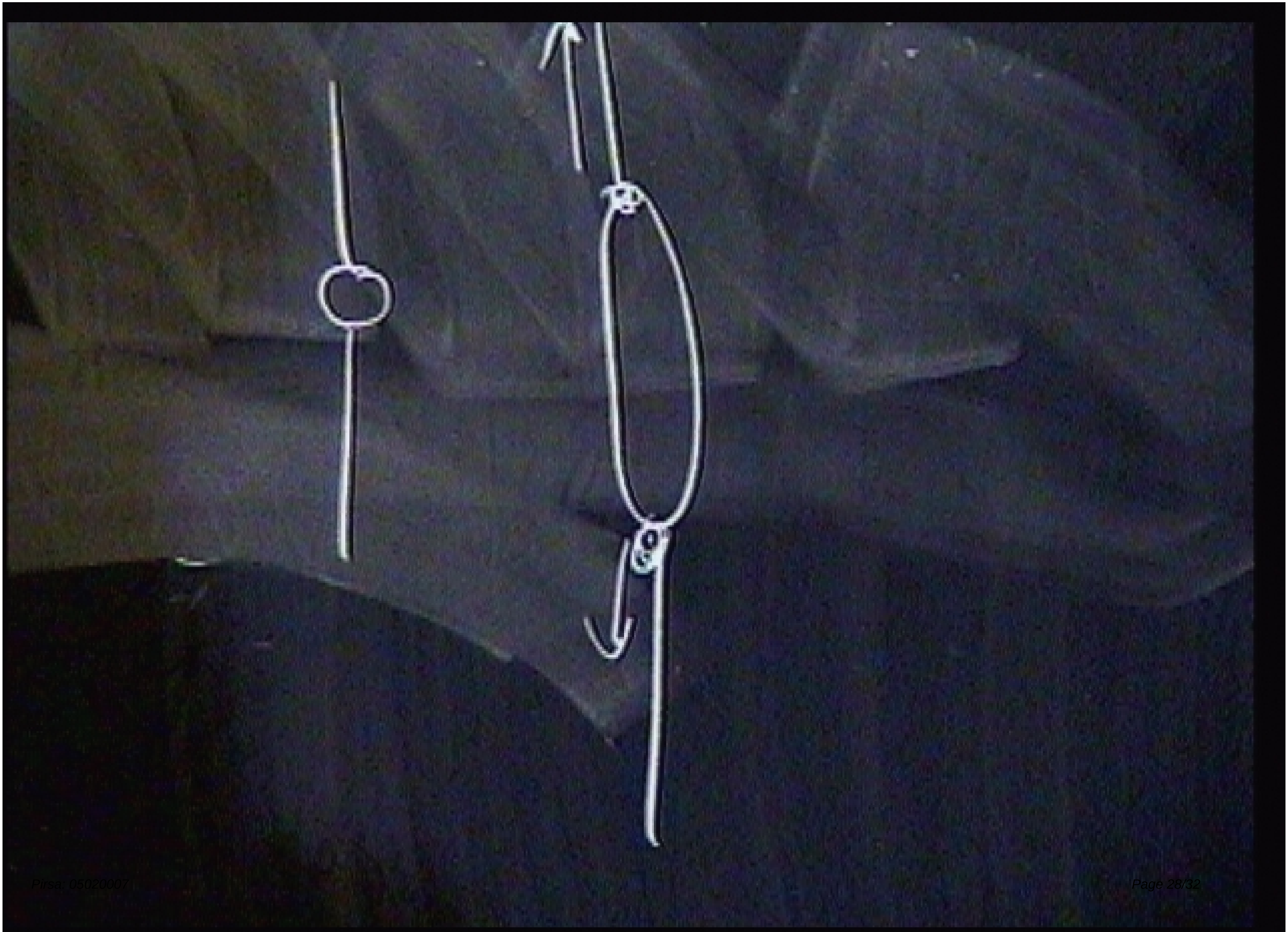
$\Lambda_{QCD} \times \Lambda_{QCD} \times \Lambda_{QCD}$

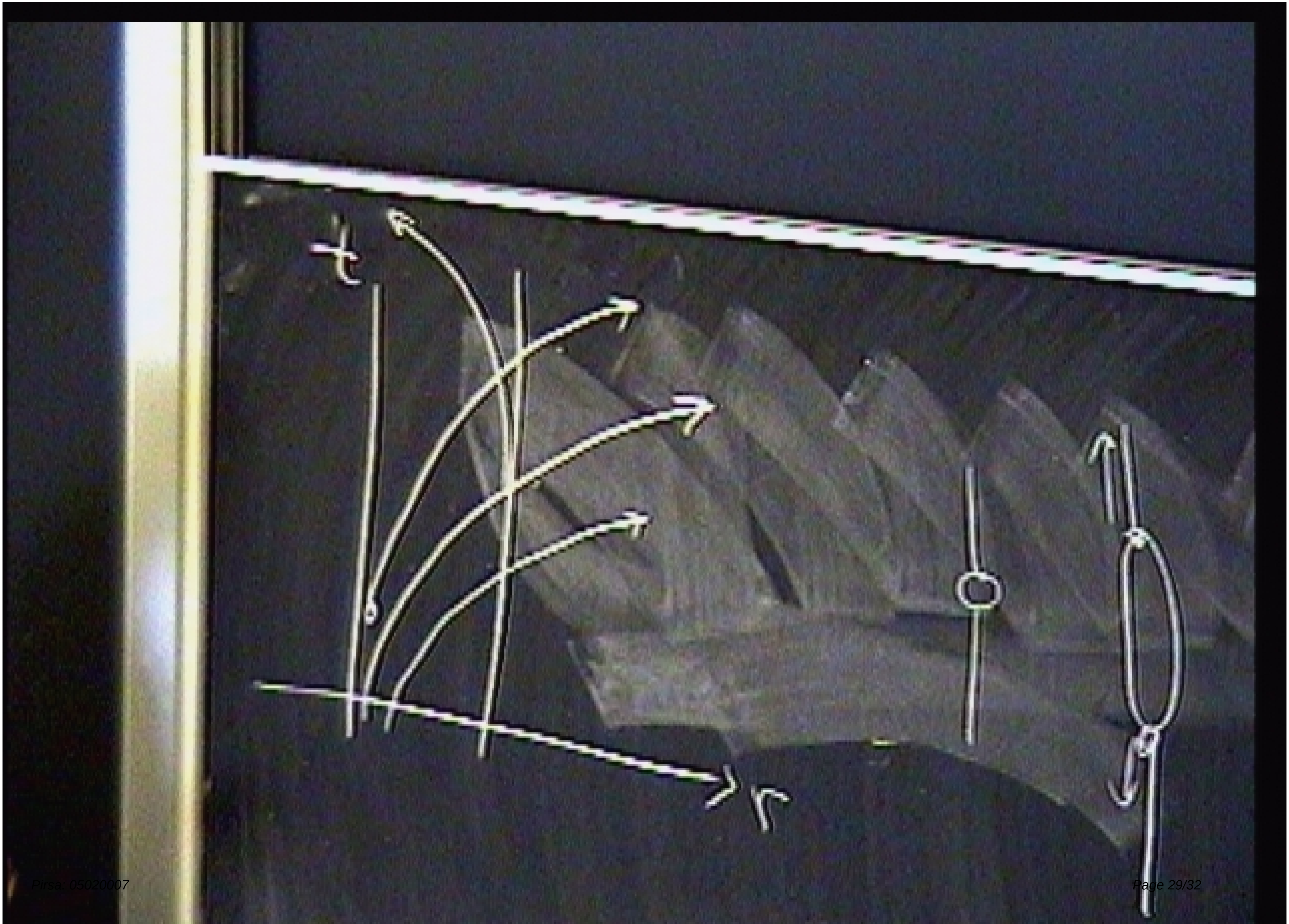
- START WITH SIMPLE SMALL BOXES: $S \ll 1$

- GET TO MOMENTUM SPACE ASAP

  FERMI POINTS
  SPECTRAL FLOW
  HELP FROM VOLOVIK ("THE UNIVERSE IN A HELIUM DROPLET")

- LEARN BY DOING

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