Title: Counting Enhanced Symmetries in Flux Vacua

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Abstract:

Counting Enhanced Symmetries in Flux Vacua

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Based on:

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& O. DeWolfe hep-th/0503???

Flux vacua

To make contact with phenomenology, need to make ten-dimensional superstring theory effectively four-dimensional.

"Traditional" compactifications:

Calabi-Yau manifolds.

Some good points:
- 4D theory
- $N=1$ SUSY @ high scale
- Gauge groups, chiral fermions...

Major drawback: UNFIXED MODULI.

Set of scalar fields with no-potential:

- Complex structure
- Kähler structure

"Shapes" and "sizes" of compact CY space.

- Dilaton (gives string coupling)

Unfixed moduli are phenomenological distaster. What to do about them?

Besides spacetime metric, 10D effective theory of massless modes has additional fields:

"fluxes" p-form gauge fields.

Our example, type IIB string theory:

$g_{\mu\nu}$	$C_{(4)}$	$B_{(2)}$ $C_{(2)}$	$\tau \equiv C_{(0)} + i e^{-\phi}$
Metric	4-form	2-form	dilaton

Compactifications with nontrivial fluxes can generate potential for moduli!

Can lead to isolated FLUX VACUA.

A very significant improvement.

Instead of infinite families of effective theories with unwanted scalars, discrete vacua without such scalars.

But still - very many of them

What are their properties? And how many?

Questions we'll ask regarding flux vacua

1. How many are there? (How to quantify?)

2. How are they distributed on (what used to be) moduli space?

Questions about particular properties of sets of vacua:

3. How many preserve SUSY @ tree level?
(Suggestions related to naturalness of small cos const)

4. Do some vacua have enhanced (discrete) symmetries for low-energy theory? How many?

Meta question:

What are the right tools to answer these questions?

(Very) important questions I won't address today:

- How could the SM be embedded?
- Is inflation or some other cosmology possible?

} Work being done

☆ Possibly related to these two:
What selects a single vacuum? ☆

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Imaginary self-dual flux vacua in IIB strings

Choose a certain construction of flux vacua to investigate. (Outstanding question: are these representative of other kinds of flux vacua?)

Recipe:

- Start with old-fashioned Calabi-Yau compact; choice of CY M with b_3 3-cycles $\{A_i, B^i\}$
- Declare that three-form fluxes $F_3 \equiv dC_2$, $H_3 \equiv dB_2$ are nonvanishing integrated over these cycles:

$$f_i = \int_{A^i} F_3 \quad (\text{or } B^i) \qquad h_i = \int_{B^i} H_3 \quad (\text{or } A^i)$$

quantized.

Response of system:

1. CY metric is "warped"

$$ds^2 = e^{-2A(y)} g_{\mu\nu} dx^\mu dx^\nu + e^{2A(y)} g^{CY}_{mn} dy^m dy^n$$

Can naturally generate a hierarchy of scales given simple choices of f_i, h_i. (Not our focus)

2 Superpotential generated for dilaton ϕ & cx structure moduli, z^i.

$$W[\phi, z^i] = \int_{CY} (F_3 - \phi H_3) \wedge \Omega[z^i]$$

Ω is holomorphic 3-form of CY: determines cx structure and so depends on all z^i.

Potential for low-energy action

$$V = e^K \left(|D_\phi W|^2 + |D_{z^i} W|^2 + |D_{\rho_a} W|^2 - 3|W|^2 \right)$$

Kähler moduli ρ_a: not in W. Kähler potential is such that $|D_{\rho_a} W|^2 = 3|W|^2$

$$\rightarrow V = e^K \left(|D_\phi W|^2 + |D_{z^i} W|^2 \right)$$

"No-scale structure"

Minimum at $D_\phi W = D_{z^i} W = 0$

$\longrightarrow$ generically freezes ϕ, z^i.

Note that $W \neq 0$ in general $\rightarrow$ broken SUSY @

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Note that $W \neq 0$ in general $\rightarrow$ broken SUSY @ tree level
(but sometimes $W=0$ possible)

We will focus on distribution of vacua on dilaton, cx structure moduli space.

A few words on Kähler moduli:

~~At~~ this level they are not fixed.

Examples of nonperturbative corrections to superpotential that depend on Kähler

eg. $W_{nonpert}(\rho) \sim e^{-A\rho}$ from brane instantons

KKLT

No-scale structure not expected to survive corrections. (also to Kähler potential)

Philosophy of Douglas: study (tractable) system of τ & z^i to represent full, nonperturbatively fixed sspt of moduli.

Questions we are interested in:

- symmetries
- W=0 vacua @ ~~tree~~ level

can be expected to give meaningful answers regardless of how Kähler moduli ultimately stabilized

Hence we ignore Kähler moduli, study vacua where dilaton, cx str are frozen via:

$$D_\phi W = D_{z^i} W = 0.$$

Can show these conditions imply fluxes are imaginary-self dual: $*_6 G_3 = i G_3$, $G_3 \equiv F_3 - \phi H_3$

D3 charge cancelation:

Gauss's law requires all D3 charge to cancel on compact space. Possible contributions:

- D3 branes filling spacetime, N_{D3}
- Crossed 3-form fluxes:

$$N_{flux} \equiv \int_M F_3 \wedge H_3$$

In general indefinite, but for ISD $\Longrightarrow$ positive definite.

Both of above, if compatible with same SUSY, have positive charge. Need a charge sink:

- 7-branes & O7-planes wrapped on 4-cycles give negative charge compatible with SUSY; $-L$.

Rescues flux compactifications, can satisfy Gauss's law:

$N_{D3} + N_{flux} = L.$

Note since $W = \int (F_3 - \tau H_3) \wedge \Omega$ is linear in fluxes,

if fluxes (f_i, h_i) satisfy $DW = 0$,

so do $(n f_i, n h_i)$ for $n \in \mathbb{Z}$

$\Longrightarrow$ In principle an infinite number of flux vacua.

But then $N_{flux} \equiv \int_M F \wedge H \Longrightarrow n^2 N_{flux}.$

Finite number of flux vacua compatible with fixed 7-brane charge L.

Instead of looking for explicit 7-brane configurations, treat L as parameter, to tune freely.

How to count IIB ISD flux vacua: see how many exist with $N_{flux} \leq L$

- In a given CY space
- with fixed extra properties ($W=0$, symmetries...)

Questions about flux vacua

1. How many are there?

$N_{flux}(f_i, h_i) \leq L$ Quadratic Form

For large L: ball in $2b_3$-dimensional space.

points inside ball:

$N_{vac} \sim (\sqrt{L})^{2b_3} \sim \underline{L^{b_3}}$

Expectation is that for generic fluxes, $DW=0$ eqns have soln @ some moduli.

Note for reasonable models this can get pretty big. $N_{vac} \sim 10^{100s}$.

What can we do with all these vacua?

Instead of looking at one or another, try to understand them in sets w/ certain properties. A <u>statistical</u> approach

2. How are vacua distributed on moduli space?

Ashok & Douglas: analytic expression for # of vacua (actually index)

$$N_{vac} \sim L^{b_3} \int_{\text{Moduli space}} \det(-R-\omega)$$

ω, R are metric, curvature on moduli space.

Natural to promote $\det(-R-\omega) \cdot L^{b_3}$ to a density on moduli space, and integrate over subregions.

AD approximation: continuous approximation for fluxes $f_i\, h_i$

Density has several successes, though it must break down in certain cases.

Questions that follow cannot be addressed in continuous approx. for fluxes.

3. W=0 vacua (at tree level)

Motivation: debate over "naturalness" of small cos const.

With many fluxes, small Λ can appear somewhat naturally as generic step size of mismatch between various fluxes in high-dimensional space. Bousso Polchinski

Suggestions (Susskind, Arkani-Hamed & Dimopoulos, Douglas) that small Λ vacua naturally have high SUSY breaking-scale.

$$V \sim \sum |DW|^2 - 3|W|^2 + \sum D_a^2$$

Many more vacua with large values of $|W|$, $|F| \sim DW$

→ not so good.

uniform distribution

Dine, Gorbatov & Thomas argued this could be misleading.

If $W_{tree} = 0$, nonperturb. generated W could have different distribution:

$$N_{vacua}\left(\frac{|W|^2}{M_P^6} \le \epsilon\right) \sim -\frac{1}{\log \epsilon}$$

eg $W \sim e^{-1/g_s}$ uniform distrib for g_s.

Then small Λ would favor small SUSY breaking!

Hence interesting to see how common are W=0 vacua.

4. Enhanced Discrete Symmetries

Discrete symmetries can be useful for phenomenology:
model building, cosmology...

Symmetries acting on dilaton, cx str moduli should be robust to how Kähler moduli are stabilized.

What is meant by having discrete sym:

Each choice of fluxes determines not just vacuum but complete low-energy theory for moduli, with fluxes appearing in role of coupling constants.

Discrete sym present if $\exists$ transformation of moduli preserving Lagrangian — with couplings invt.

Modular group:

$\exists$ symmetries acting naturally on moduli space:

- S-duality of IIB ($SL(2,\mathbb{Z})$) on dilaton +
- Geometrical modular gp of cx structure mod space.

These are gauge syms: for proper counting of vacua, quotient by them (otherwise find ∞ # of copies!)

But we find in some cases, descend to symmetries acting on moduli alone. Fluxes must be such that vacuum sits at fixed point of modular gp.

When do $W=0$, enhanced ~~d~~discrete symmetry vacua occur?

How common are they? Constrain fluxes - power law supressed in L.

Solving moduli eqns for fluxes

Recall: with fixed fluxes, $D_\tau W = D_{\bar{z}} W = 0$

is $h_{21}+1$ complex eqns for $h_{21}+1$ unknowns - generically solns.

$W=0$ is an additional complex eqn; overdetermined.

But eqns depend also on choice of fluxes. If they were continuously tunable, could adjust them & always find $W=0$ vacua!

Integrality makes more subtle: sometimes can solve, sometimes not.

Central issue:

Viewed as eqn for fluxes, single complex eqn can become many eqns for integers.

$W = (f - \tau h) \cdot \Pi$, periods Π in general live in some vector space over integers (or rationals)

Dimension of this vector space tells us how many equations come from single complex constraint.

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ex. "Rigid Calabi-Yau"

$h_{2,1}=0$, only dilaton $\Longrightarrow$ $b_3 = 2$

Take periods $\int_A \Omega = i$ $\quad \int_B \Omega = 1$

$$W = A\phi + B \qquad A = -(h_1 + i h_2),\quad B = f_1 + i f_2$$

Vacuum eqn: $D_\phi W = 0 \longrightarrow \phi = -B^*/A^*$

Count vacua:

$$N_{flux} \equiv \int_M F_3 \wedge H_3 = f_1 h_2 - h_1 f_2 \overset{?}{\leq} L$$

gauge fix $SL(2,\mathbb{Z})$: $f_2 = 0$, $0 \le h_1 < f_1$

then

$$N_{vac}(\le L) = \sum_{m=1}^{L} \sum_{k|m} k \approx \frac{\pi^2}{12} L^2 \approx L^{b_3} \checkmark$$

(Ashok-Douglas)

Distribution on ϕ fundamental domain:

$1/(\mathrm{Im}\,\phi)^2$ @ large L

Finer scale: integrality of fluxes → "fractal" structure

$\phi \sim i$ ok @ low L, $\phi \sim \frac{100}{99} i$ needs larger L —

W=0 vacua in rigid CY?

New eqn: $W = f_1 + i f_2 - \tau (h_1 + i h_2) = 0.$

Period vector $\Pi = \begin{pmatrix} \int_A \Omega \\ \int_B \Omega \end{pmatrix} = \begin{pmatrix} i \\ 1 \end{pmatrix}$ lives in

$\mathbb{Z}[i]$: extention of integers by i

(obviously) has dimension 2 as vector space over $\mathbb{Q}$,

basis $\{1, i\}$

Hence 1 complex eqn $W=0$

$\Longrightarrow$ 2 eqns for integer fluxes.

Since 4 fluxes here, might guess by counting

$W=0$ vacua exist.

Actually constraint nongeneric:

$D_\tau W = W = 0$ forces $N_{flux} = 0$.

No nontrivial flux vacua w/ $W=0$ for rigid CY.

Discrete symmetries of rigid CY

Can descend from modular op (here S-duality) when vacuum sits at a fixed point.

2 fixed pts of $SL(2,\mathbb{Z})$: $\mathbb{Z}_2$ @ $\phi = i$
$\mathbb{Z}_3$ @ $\phi = e^{i\pi/3}$

$\phi = \frac{f_1 - i f_2}{h_1 - i h_2}$: can't realize $\mathbb{Z}_3$ pt. but $\mathbb{Z}_2$ point possible.

Vacuum @ $\phi = i$ is condition on fluxes $f_1 = h_2$, $f_2 = -h_1$

Under these conditions: $W = -(h_1 + i h_2)(\phi + i)$

$\mathbb{Z}_2$ transform on moduli only : $\phi \to -1/\phi$
✓ is a symmetry.

$\mathbb{Z}_2$ always sym of low-energy theory for this subclass of vacua!

Count them: $N_{flux} = h_1^2 + h_2^2 \leq L$

$\#$ vacua $\sim \pi L$.

$\frac{N(\mathbb{Z}_2 \text{ vacua})}{N(\text{all vac})} \sim \frac{1}{L}$.

Symmetry arose w/ 2 constraints on fluxes; power law suppression.

Final ex[...]

$(T^2)^3$. Can consid[...]
certain "diagonal" fluxes [...]
3 tori having same modular pa[...]

→ Another effective one-parameter model.

$$F_3 = a^0\, dx^1\wedge dx^2\wedge dx^3 + a\,(dx^1\wedge dx^2\wedge dy^3 + \cdots) + b\,(dx^1\wedge dy^2\wedge dy^3 + \cdots) + b^0\, dy^1\wedge dy^2\wedge dy^3$$

$$H_3 = c^0\, dx^1\wedge dx^2\wedge dx^3 + c\,(dx^1\wedge dx^2\wedge dy^3 + \cdots) + d\,(dx^1\wedge dy^2\wedge dy^3 + \cdots) + d^0\, dy^1\wedge dy^2\wedge dy^3$$

Generic vacua: can analyze numerically

Confirm $N_{vac}(\leq L) \sim L^{b_3} = L^4$.

Hard to get a good method to solve exactly generic vacua.

Period $\Pi \sim (\tau^3, \tau^2, \tau, 1)$ → all real coeffs

Can show for any fluxes, $\mathrm{Re}\,\tau$ obeys cubic, $\mathrm{Im}\,\tau$ square root of this (many cancellations)

→ τ lives in at largest degree six extension (vector space) over $\mathbb{Q}$

~~Final~~ example: Symmetric torus.

$(T^2)^3$. Can consider more general T^6 compacts: certain "diagonal" fluxes lead to all 3 tori having same modular parameter. τ.

→ Another effective one-parameter model.

$$F_3 = a^0\, dx^1\wedge dx^2\wedge dx^3 + a\,(dx^1\wedge dx^2\wedge dy^3 + \dots) + b\,(dx^1\wedge dy^2\wedge dy^3 + \dots) + b^0\, dy^1\wedge dy^2\wedge dy^3$$

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Eqns for SUSY vacua on symmetric torus:

$$D_\phi W = \underbrace{D_\tau W = W}_{} = 0.$$

reduce to 2 cubic eqns:

$$P_1(\tau) = a^0\tau^3 - 3a\tau^2 - 3b\tau - b_0 = 0$$
$$P_2(\tau) = c^0\tau^3 - 3c\tau^2 - 3d\tau - d_0 = 0.$$

Then use $D_\phi W = 0$ eqn to solve for dilaton.

Constraint for physical dilaton: $\operatorname{Im}\tau \neq 0$

so τ complex. Coefficients real; hence solns come in pairs.

$\Rightarrow$ Implies $P_1(\tau)$, $P_2(\tau)$ have common quadratic factor

$$P_1(\tau) = (f\tau + g)\,P(\tau)$$
$$P_2(\tau) = (h\tau + k)\,P(\tau)$$

for $\underline{P(\tau) = l\tau^2 + m\tau + n}$.

for some integers $f, g \ldots n$.

τ obeys quadratic equation: extension of degree 2 for special case of $W=0$ vacua.

Here to solve for $W=0$ vacua:

How many cubic eqns in integer coefficients have a common quadratic factor?

Number theoretic; estimate using heights:

consider $f(x) = a_d x^d + \cdots + a_1 x + a_0$

w/ $a_i \in \mathbb{Z}$, relatively prime.

height of polynomial f is

$$H \equiv \max(|a_d|, \ldots, |a_0|)$$

Useful: $H(f_1 f_2) = H(f_1) H(f_2)$

$$N_{\substack{\text{polys,}\\ \text{degree } d}}(\leq H) = \frac{2^n}{\zeta(d+1)} H^{d+1} + \cdots$$

Then average value for fluxes: $f, h \sim \sqrt{L}$

Height of polynomials given by these fluxes.

Count # of common quadratic polynomials, linear parts:

$N_{vac}(W=0) \sim L^2$. Down by $1/L^2$.

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Flux vacua in genuine CYs - hypersurfaces in weighted projective space

Four "one-parameter" models: $M_5 \in \mathbb{P}^4$, $M_6 \in \mathbb{P}^4_{11112}$, $M_8 \in \mathbb{P}^4_{11114}$, $M_{10} \in \mathbb{P}^4_{11125}$

$$P = x_0^5 + x_1^5 + x_2^5 + x_3^5 + x_4^5 - 5\psi x_1 x_2 x_3 x_4$$

2 similar.

ψ-plane includes conifold point and Landau-Ginzburg point ← fixed point of $\mathbb{Z}_d$ in modular sp: $d = 5, 6, 8, 10$

Expect $N_{vac} \sim L^{b_3} = L^4$ → confirmed numerically for M_8.

General vacua difficult to compute as periods $\Pi(\psi)$ complicated.

Specialize: vacua @ $\psi = 0$ (LG) point?

$W=0$ vacua? Perhaps remnant of $\mathbb{Z}_d$ as discrete symmetry?

Imposing $\psi = 0$ as constraint: one cx equation too many even for $W \neq 0$ vacua now.

How many constraints on fluxes from are equ of type $D_\tau W = 0$ or $D_\phi W = 0$?

Recall for rigid, periods lived in $\mathbb{Z}[i]$, with dimension 2.

One-param CYs: periods live in $\mathbb{Q}[\alpha] \equiv \mathcal{F}_d$ where $\alpha^d = 1$ is a root of unity: $\mathbb{Q}[\alpha]$ is "cyclotomic field"

Dimension of $\mathcal{F}_d$? euler totient function $\phi(d)$

$\phi(6) = 2$ 6th roots of unity: eg basis $\{1, i\sqrt{3}\}$

$\phi(5) = \phi(8) = \phi(10) = 4$

eg 8th roots of unity basis $\{1, i, \sqrt{2}, i\sqrt{2}\}$

Can show that 2 new constraints is OK, but 4 too many $\Longrightarrow$

LG vacua for M_6, not M_5, M_8, M_{10}

Careful height-related counting: L^2 LG vacua in M_6. Down by $1/L^2$.

(LG vacua in M_8 actually exist due to nongeneric redundancy of some equs. Not clear if just coincidence or telling us something...)

W=0 vacua & discrete syms

$W = D_\psi W = 0 \longrightarrow \boxed{f \cdot \Pi(\psi) = h \cdot \Pi(\psi) = 0}$

dilaton-independent eqns.

Solve @ $\psi=0$, then solve
$D_\psi W = 0$ for dilaton.

$\mathbb{Z}_d$ monodromy around $\psi = 0$: matrix A

$$A\,\Pi(\psi) = \alpha\,\Pi(\alpha\psi), \quad \text{so} \quad A^d = \mathbb{I}$$

M_6: $f \cdot A^3 = f$ has nontrivial soln

so $f \cdot \Pi(\psi=0) = f \cdot A^3 \cdot \Pi(\psi=0) = -f \cdot \Pi(\psi=0) = 0$.

impose $h \cdot A^3 = h$ as well.

$\Longrightarrow$ W=0 vacua.

Discrete R-symmetry: $\psi \to \alpha^3\psi = -\psi$:

$$W(\psi) = (f - \tau h)\cdot\Pi(\psi) = -(f - \tau h)\cdot A^3 \cdot \Pi(-\psi)$$
$$= -W(-\psi)$$

$\mathbb{Z}_2$ R-sym "enforcing" $W=0$.

Can recover full $\mathbb{Z}_6$ @ special values of dilaton.

Generalization to multiparameter models:

$M_d \in \mathbb{P}^4_{k_0 k_1 k_2 k_3 k_4}$ $\qquad \sum k_i = d$

More cx str moduli: φ^a

Near $\psi = 0$, $\Pi = \sum_{n=1}^{\infty} c_n(\alpha_d)\, \psi^{n-1}\, U_n(\varphi^a)$

Functions $U_n(\varphi^a)$ in general don't live in $\mathcal{F}_d$: transcendental values → looks bad.

Surprisingly, can show:

* $f \cdot \Pi = h \cdot \Pi = 0$ ($D_\psi W = W = 0$) imply all $D_{\varphi^a} W = 0$ trivially!

* vector space over $\mathbb{Q}$ generated by $\alpha \in \mathcal{F}_d$, $U_n(\varphi^a)$ often no bigger than $\mathcal{F}_d$, and not sensitive to value of φ^a!

⇒ Can solve $f \cdot \Pi = h \cdot \Pi = 0$ via general monodromy A in many examples: $f \cdot A^n = \pm f$

Classify eigenvalues of A: faithful or unfaithful reps of $\mathbb{Z}_d$ – (scaling given by # of unfaithful.

(Includes info about size of $t(d)$!)

Discrete R-syms always appear by construction.

Summary

Generic flux vacua scale as L^{b_3} in given model.

Interesting & useful features such as discrete symmetries & $W=0$ vacua appear power law suppressed $\quad N_{vac} \sim L^{b_3 - m}$

Natural question: how does m grow with b_3?

weighted projective space examples:

$$\sqrt{d} < m \sim f(d) \leq b_3 \lesssim d$$

At best we have $\quad \dfrac{N_{vac}(\text{special})}{N_{vac}(\text{generic})} \sim L^{-\sqrt{b_3}}$

Right tools for these problems: number theory techniques, heights, algebraic extensions

Combination of these with analytic, numerical techniques promising.

More questions: Standard Model, inflation ...

vacuum selection? or statistics?

Much still to explore.