Title: Interpretation of Quantum Theory: Lecture 6

Date: Jan 20, 2005 02:15 PM

URL: http://pirsa.org/05010011

Abstract:

Are quantum states complete?

'*Einstein Attacks Quantum Theory. Scientist and Two Colleagues Find It Is Not "Complete" Even Though "Correct."*' New York Times, May 4th, 1935.

Are quantum states complete?

'Einstein Attacks Quantum Theory. Scientist and Two Colleagues Find It Is Not "Complete" Even Though "Correct."' New York Times, May 4th, 1935.

The EPR Argument against Completeness

"Can quantum-mechanical description of reality be considered complete?" A. Eintein, B. Podolosky, and N. Rosen, Physical Review 47, 777 (1935).

For EPR, a necessary condition for the completeness of a theory is:

> (i) "Every element of physical reality must have a counterpart in the physical theory."

This is not a sufficient condition for completeness: there may be other criteria that must be satisfied.

For EPR, a sufficient condition for the physical reality of a quantity is:

> (ii) "If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity."

This is not a necessary condition: there may be other ways to identify whether a physical quantity is real.

Following from the fact that quantum mechanical states did not permit simultaneous specification of definite properties for non-commuting observables, EPR deduced that: "either (1) *the quantum-mechanical description of reality given by the wave function is not complete*, or (ii) *when the operators corresponding to two physical quantities do not commute the two quantities cannot have simultaneous reality.*"

Both of these alternative inferences were appreciated already by von Neumann, who explicitly endorsed (2) and rejected (1), presumably due to his 'no go' theorem for hidden variables. The EPR argument concludes that (1) must be endorsed and (2) rejected.

EPR considered a system of two particles initially interacting such that they are produced in a joint eigenstate of their relative position and total linear momentum. Here we will consider a simpler system involving a two spin-1/2 particles (devised by Bohm (1951)) which illustrates the same features.

Consider two particles arranged to interact such that they are described by the singlet-state,

$$\psi = \frac{1}{\sqrt{2}}\left(|+\rangle_1 \otimes |-\rangle_2 - |-\rangle_1 \otimes |+\rangle_2\right).$$

This state has zero total angular momentum, so the spin of the first particle (system S_1) is anti-correlated with the spin of the second particle (system S_2).

Assume that after the state preparation *the two particles no longer interact.*

Both of these alternative inferences were appreciated already by von Neumann, who explicitly endorsed (2) and rejected (1), presumably due to his 'no go' theorem for hidden variables. The EPR argument concludes that (1) must be endorsed and (2) rejected.

EPR considered a system of two particles initially interacting such that they are produced in a joint eigenstate of their relative position and total linear momentum. Here we will consider a simpler system involving a two spin-1/2 particles (devised by Bohm (1951)) which illustrates the same features.

Consider two particles arranged to interact such that they are described by the singlet-state,

$$\psi = \frac{1}{\sqrt{2}}(|+\rangle_1 \otimes |-\rangle_2 - |-\rangle_1 \otimes |+\rangle_2).$$

This state has zero total angular momentum, so the spin of the first particle (system S_1) is anti-correlated with the spin of the second particle (system S_2).

Assume that after the state preparation *the two particles no longer interact.*

Observe that if measurement of particle 1, along, say, the z-axis, yields $+\hbar/2$ then measurement of particle 2 (along the same z-axis) must yield $-\hbar/2$, and vice versa. Similarly, if we measure instead S_x for particle 1, then we can predict with certainty the outcome of an S_x measurement for particle 2.

Hence we can predict with certainty the outcomes of measurements of either S_x or S_z of the second particle 'without in any way disturbing the second system".

In accordance with the EPR criterion of reality, there must therefore be elements of reality corresponding to both S_x and S_z for the second particle.

Hence option (2) is negated. Since the two options are considered mutually exclusive and jointly exhaustive possibilities, EPR were *forced to conclude that the quantum-mechanical description of physical reality given by wave functions is not complete.*

Bohm emphasized that the EPR argument relied on the additional assumptions that (iii) each element of physical reality must have a precisely defined counterpart in the mathematical theory [a stronger condition then the one EPR acknowledged] and (iv) the world can correctly be analyzed in terms of distinct and separately existing 'elements of reality'.

Bohm (1951, pp. 622-623) somehow concluded from his analysis that hidden variables were nonetheless impossible: "We can now use some of the results of the analysis of the paradox of [EPR] to help prove that quantum theory is inconsistent with the assumption of hidden causal variables . . . [Arguing from the apparent conflict with the uncertainty principle] . . . We conclude that no theory of mechanically determined hidden variables can lead to all of the results of the quantum theory."

In accordance with the EPR criterion of reality, there must therefore be elements of reality corresponding to both S_x and S_z for the second particle.

Hence option (2) is negated. Since the two options are considered mutually exclusive and jointly exhaustive possibilities, EPR were *forced to conclude that the quantum-mechanical description of physical reality given by wave functions is not complete.*

Bohm emphasized that the EPR argument relied on the additional assumptions that (iii) each element of physical reality must have a precisely defined counterpart in the mathematical theory [a stronger condition then the one EPR acknowledged] and (iv) the world can correctly be analyzed in terms of distinct and separately existing 'elements of reality'.

Bohm (1951, pp. 622-623) somehow concluded from his analysis that hidden variables were nonetheless impossible: "We can now use some of the results of the analysis of the paradox of [EPR] to help prove that quantum theory is inconsistent with the assumption of hidden causal variables . . . [Arguing from the apparent conflict with the uncertainty principle] . . . We conclude that no theory of mechanically determined hidden variables can lead to all of the results of the quantum theory."

One year later, Bohm published his causal hidden variable interpretation (1952): "A Suggested Interpretation of the Quantum Theory in Term of 'Hidden' Variables"

Hence option (2) is negated. Since the two options are considered mutually exclusive and jointly exhaustive possibilities, EPR were *forced to conclude that the quantum-mechanical description of physical reality given by wave functions is not complete.*

Bohm emphasized that the EPR argument relied on the additional assumptions that (iii) each element of physical reality must have a precisely defined counterpart in the mathematical theory [a stronger condition then the one EPR acknowledged] and (iv) the world can correctly be analyzed in terms of distinct and separately existing 'elements of reality'.

Bohm (1951, pp. 622-623) somehow concluded from his analysis that hidden variables were nonetheless impossible: "We can now use some of the results of the analysis of the paradox of [EPR] to help prove that quantum theory is inconsistent with the assumption of hidden causal variables ... [Arguing from the apparent conflict with the uncertainty principle] ... We conclude that no theory of mechanically determined hidden variables can lead to all of the results of the quantum theory."

One year later, Bohm published his causal hidden variable interpretation (1952): "A Suggested Interpretation of the Quantum Theory in Term of 'Hidden' Variables"

Additional observations:

If the collapse of the wave function is a physical process (as it is on the assumption that the wave function is complete), then the collapse must be an instantaneous change of physical properties throughout space. Recall from von Neumann's analysis of the Compton experiment that the process 1 transformation, with post-selection, is operationally demanded so that measurements of commuting operators yield consistent outcomes, even if the measurements are carried out 'simultaneously' in vastly separated spatial locations.

EPR did not linguistically distinguish, as we have, between the bare mathematical formalism (the abstract theory), and the set of bridge principles that specify correspondence rules between the elements of the mathematics and the elements of physical reality (the interpretation the theory). EPR referred to the combination of both the mathematical formalism and the orthodox interpretation as 'quantum theory.' So it is unclear if the argument for incompleteness is meant to imply the mathematical formalism itself, or merely the interpretive correspondence rules.

Bohr's response to EPR:

"The finite interaction between object and measuring agencies conditioned by the very existence of the quantum of action entails - because of the impossibility of controlling the reaction of the object on the measuring instruments if these are to serve their purpose - the necessity of a final renunciation of the classical ideal of causality and a radical revision of our attitude towards the problem of physical reality ... [While there is] no question of a mechanical disturbance of the system under investigation ... there is essentially the question of an influence on the very conditions which define the possible types of predictions regarding the future behavior of the system."

Bohr "Quantum Mechanics and Physical Reality" (1935)

"Recapitulating, the impossibility of subdividing the individual quantum effects and separating a behavior of the objects from their interaction with the measuring instruments serving to define the conditions under which the phenomena appear implies and ambiguity in assigning conventional attributes to atomic objects which calls for a reconsideration of our attitude towards the problem of physical explanation, in this novel situation, even the old question of an ultimate determinacy of natural phenomena has lost its conceptual basis, and it is against this background that the viewpoint of complementarity presents itself as a rational generalization of the very ideal of causality."

Bohr (1948)

Non-locality

The EPR argument presumes (implicitly) a notion of *separability*, i.e., that separately existing elements of reality may be attributed to each system, and a notion of *independence*, i.e., that it is possible to arrange that the elements of reality of one system can not be influenced by the elements of reality of another system. The assumption of independence can seemingly be well motivated by the 'locality' guaranteed by special relativity. Einstein later characterized this 'locality' assumption as follows (1949):

> "The real factual situation of the system S_2 is independent of what is done with the system S_1, which is spatially separated from the former."

This 'Einstein locality' assumption was put to a direct test by John Bell (1964) who devised a celebrated inequality that any local, realistic theory must satisfy, as we will see in a moment.

Incompleteness and hidden variables:

The conclusion that quantum mechanics is incomplete suggests that the quantum mechanical description may be supplemented by additional parameters, or 'hidden variables', in order to recover a (more) complete description.

As noted earlier, the possibility of hidden variables was considered and rejected by von Neumann, who produced an 'impossibility proof' based on a number of assumptions.

First, the hidden variables assignments were required to completely specify of reality of one system can not be influenced by the elements of reality of another system. The assumption of independence can seemingly be well motivated by the 'locality' guaranteed by special relativity. Einstein later characterized this 'locality' assumption as follows (1949):

> "The real factual situation of the system S_2 is independent of what is done with the system S_1, which is spatially separated from the former."

This 'Einstein locality' assumption was put to a direct test by John Bell (1964) who devised a celebrated inequality that any local, realistic theory must satisfy, as we will see in a moment.

Incompleteness and hidden variables:

The conclusion that quantum mechanics is incomplete suggests that the quantum mechanical description may be supplemented by additional parameters, or 'hidden variables', in order to recover a (more) complete description.

As noted earlier, the possibility of hidden variables was considered and rejected by von Neumann, who produced an 'impossibility proof' based on a number of assumptions.

First, the hidden variables assignments were required to completely specify experimental outcomes (i.e., to produce dispersion-free states) for all Hermitian operators;

Second, the unique hidden value assignment to each operator was required to be one of the operator's eigenvalues;

"The real factual situation of the system S_2 is independent of what is done with the system S_1, which is spatially separated from the former."

This 'Einstein locality' assumption was put to a direct test by John Bell (1964) who devised a celebrated inequality that any local, realistic theory must satisfy, as we will see in a moment.

Incompleteness and hidden variables:

The conclusion that quantum mechanics is incomplete suggests that the quantum mechanical description may be supplemented by additional parameters, or 'hidden variables', in order to recover a (more) complete description.

As noted earlier, the possibility of hidden variables was considered and rejected by von Neumann, who produced an 'impossibility proof' based on a number of assumptions.

First, the hidden variables assignments were required to completely specify experimental outcomes (i.e., to produce dispersion-free states) for all Hermitian operators;

Second, the unique hidden value assignment to each operator was required to be one of the operator's eigenvalues;

Third, for any Hermitian operator $\hat{C} = a\hat{A} + b\hat{B}$ defined by a linear combination of arbitrary (e.g., non-commuting) Hermitian operators, the hidden value assignment for $\hat{C}$ was required to satisfy the same linear combination of the hidden value assignments for the operators $\hat{A}$ and $\hat{B}$.

However, the third assumption is incompatible with the first and second assumptions. Consider the Pauli operator defined by,

$$\sigma_n = \frac{1}{\sqrt{2}}(\sigma_x + \sigma_y).$$

The eigenvalues of all three Pauli operators are ± 1, but clearly, the eigenvalues of σ_n can not be expressed as any of the linear combinations,

$$\frac{(\pm 1 \pm 1)}{\sqrt{2}}.$$

von Neumann's third assumption is generally considered unjustified (even "silly" - by Mermin (1993)), since it imposes constraints on the hidden value assignments for incompatible experimental arrangements. His assumption appears to be inspired by the fact that this relation holds for quantum mechanical expectation values ($\langle C \rangle = \langle A \rangle + \langle B \rangle$) and maybe also the fact that it is expected in a trivial hidden variable model in which measurements of spin reveal the components of a pre-existing angular momentum vector.

If we drop the third assumption then hidden variable models can be, and indeed have been, constructed - see Bell (1966) for the complete analysis and a simple example. The most celebrated example is the de Broglie-Bohm hidden

The eigenvalues of all three Pauli operators are ± 1, but clearly, the eigenvalues of σ_n can not be expressed as any of the linear combinations,

$$\frac{(\pm 1 \pm 1)}{\sqrt{2}}.$$

von Neumann's third assumption is generally considered unjustified (even "silly" - by Mermin (1993)), since it imposes constraints on the hidden value assignments for incompatible experimental arrangements. His assumption appears to be inspired by the fact that this relation holds for quantum mechanical expectation values ($\langle C \rangle = \langle A \rangle + \langle B \rangle$) and maybe also the fact that it is expected in a trivial hidden variable model in which measurements of spin reveal the components of a pre-existing angular momentum vector.

If we drop the third assumption then hidden variable models can be, and indeed have been, constructed - see Bell (1966) for the complete analysis and a simple example. The most celebrated example is the de Broglie-Bohm hidden variable model (1927/1952), which has an explicit non-local character. This feature of the de Broglie-Bohm model interpretation and the role of locality in the EPR-Bohm argument led Bell to ask: is non-locality a necessary feature of any hidden variable theory reproducing the predictions of quantum theory? The answer is yes (Bell (1964)).

Bell's Theorem:

Bell considered a restriction on the correlations that can be exhibited between two systems in the EPR-Bohm set-up allowing for the fact that the outcomes could be determined by an arbitrary class of (deterministic) hidden variables.

Consider two spatially separated spin systems each subjected to measurement along directions a and b respectively. The results of the measurements, denoted A and B, can depend on arbitrary parameters (hidden variables) collectively denoted λ, and can take on the values $|A| \leq 1$ and $|B| \leq 1$. The outcome can of course depend on the local setting, but, by *assuming Einstein locality*, is not allowed to depend on the setting of the distant instrument. Hence $A = A(a, \lambda)$ and $B = B(b, \lambda)$ are allowed but $A = A(a, b, \lambda)$ and $B = B(a, b, \lambda)$ are excluded by the locality assumption.

The uncontrolled parameters are subject to an arbitrary probability density $\rho(\lambda)$, where,

$$\rho(\lambda) \geq 0, \quad \int \rho(\lambda) d\lambda = 1,$$

and hence we can define correlations of the form:

$$C(a, b) = \int A(a, \lambda) B(b, \lambda) \rho(\lambda) d\lambda$$

Each detector is allowed to have two *independently selected* settings $\{a, a'\}$ and $\{b, b'\}$. From these assumptions we can deduce Bell's inequality:

$$|C(a, b) - C(a, b')| + |C(a', b') + C(a', b)| \leq 2$$

Bell's Theorem:

Bell considered a restriction on the correlations that can be exhibited between two systems in the EPR-Bohm set-up allowing for the fact that the outcomes could be determined by an arbitrary class of (deterministic) hidden variables.

Consider two spatially separated spin systems each subjected to measurement along directions a and b respectively. The results of the measurements, denoted A and B, can depend on arbitrary parameters (hidden variables) collectively denoted λ, and can take on the values $|A| \leq 1$ and $|B| \leq 1$. The outcome can of course depend on the local setting, but, by *assuming Einstein locality*, is not allowed to depend on the setting of the distant instrument. Hence $A = A(a, \lambda)$ and $B = B(b, \lambda)$ are allowed but $A = A(a, b, \lambda)$ and $B = B(a, b, \lambda)$ are excluded by the locality assumption.

The uncontrolled parameters are subject to an arbitrary probability density $\rho(\lambda)$, where,

$$\rho(\lambda) \geq 0, \qquad \int \rho(\lambda) d\lambda = 1,$$

and hence we can define correlations of the form:

$$C(a, b) = \int A(a, \lambda) B(b, \lambda) \rho(\lambda) d\lambda$$

Each detector is allowed to have two *independently selected* settings $\{a, a'\}$ and $\{b, b'\}$. From these assumptions we can deduce Bell's inequality:

$$|C(a, b) - C(a, b')| + |C(a', b') + C(a', b)| \leq 2$$

Bell considered a restriction on the correlations that can be exhibited between two systems in the EPR-Bohm set-up allowing for the fact that the outcomes could be determined by an arbitrary class of (deterministic) hidden variables.

Consider two spatially separated spin systems each subjected to measurement along directions a and b respectively. The results of the measurements, denoted A and B, can depend on arbitrary parameters (hidden variables) collectively denoted λ, and can take on the values $|A| \leq 1$ and $|B| \leq 1$. The outcome can of course depend on the local setting, but, by *assuming Einstein locality*, is not allowed to depend on the setting of the distant instrument. Hence $A = A(a, \lambda)$ and $B = B(b, \lambda)$ are allowed but $A = A(a, b, \lambda)$ and $B = B(a, b, \lambda)$ are excluded by the locality assumption.

The uncontrolled parameters are subject to an arbitrary probability density $\rho(\lambda)$, where,

$$\rho(\lambda) \geq 0, \qquad \int \rho(\lambda) d\lambda = 1,$$

and hence we can define correlations of the form:

$$C(a, b) = \int A(a, \lambda) B(b, \lambda) \rho(\lambda) d\lambda$$

Each detector is allowed to have two *independently selected* settings $\{a, a'\}$ and $\{b, b'\}$. From these assumptions we can deduce Bell's inequality:

$$|C(a, b) - C(a, b')| + |C(a', b') + C(a', b)| \leq 2$$

... mechanical system satisfying Bell's assumption consists of two along directions a and b respectively. The results of the measurements, denoted A and B, can depend on arbitrary parameters (hidden variables) collectively denoted λ, and can take on the values $|A| \leq 1$ and $|B| \leq 1$. The outcome can of course depend on the local setting, but, by *assuming Einstein locality*, is not allowed to depend on the setting of the distant instrument. Hence $A = A(a, \lambda)$ and $B = B(b, \lambda)$ are allowed but $A = A(a, b, \lambda)$ and $B = B(a, b, \lambda)$ are excluded by the locality assumption.

The uncontrolled parameters are subject to an arbitrary probability density $\rho(\lambda)$, where,

$$\rho(\lambda) \geq 0, \qquad \int \rho(\lambda) d\lambda = 1,$$

and hence we can define correlations of the form:

$$C(a,b) = \int A(a,\lambda) B(b,\lambda) \rho(\lambda) d\lambda$$

Each detector is allowed to have two *independently selected* settings $\{a, a'\}$ and $\{b, b'\}$. From these assumptions we can deduce Bell's inequality:

$$|C(a,b) - C(a,b')| + |C(a',b') + C(a',b)| \leq 2$$

A quantum mechanical system satisfying Bell's assumption consists of two spin-1/2 particles (or generic two-level systems) in the singlet state,

$$\psi = \frac{1}{\sqrt{2}} (|+\rangle_A \otimes |-\rangle_B \; - \; |-\rangle_A \otimes |+\rangle_B).$$

Hence $A = A(a, \lambda)$ and $B = B(b, \lambda)$ are allowed but $A = A(a, b, \lambda)$ and $B = B(a, b, \lambda)$ are excluded by the locality assumption.

The uncontrolled parameters are subject to an arbitrary probability density $\rho(\lambda)$, where,

$$\rho(\lambda) \geq 0, \qquad \int \rho(\lambda) d\lambda = 1,$$

and hence we can define correlations of the form:

$$C(a, b) = \int A(a, \lambda) B(b, \lambda) \rho(\lambda) d\lambda$$

Each detector is allowed to have two *independently selected* settings $\{a, a'\}$ and $\{b, b'\}$. From these assumptions we can deduce Bell's inequality:

$$|C(a, b) - C(a, b')| + |C(a', b') + C(a', b)| \leq 2$$

A quantum mechanical system satisfying Bell's assumption consists of two spin-1/2 particles (or generic two-level systems) in the singlet state,

$$\psi = \frac{1}{\sqrt{2}} (|+\rangle_A \otimes |-\rangle_B \; - \; |-\rangle_A \otimes |+\rangle_B).$$

$$C(\mathbf{a},\mathbf{b}) = (2/\hbar)^2 \langle\psi|\mathbf{a}\cdot\mathbf{S_A}\otimes\mathbf{b}\cdot\mathbf{S_B}|\psi\rangle.$$

Define $\cos\theta_{\mathbf{a},\mathbf{b}} \equiv \mathbf{a}\cdot\mathbf{b}$, then,

$$C(\mathbf{a},\mathbf{b}) = -\cos\theta_{\mathbf{a},\mathbf{b}}$$

Choosing $\mathbf{a},\mathbf{b},\mathbf{a}',\mathbf{b}'$ to be four co-planar vectors with $\mathbf{a}$ and $\mathbf{b}$ parallel and $\phi \equiv \theta_{\mathbf{a},\mathbf{b}'} = \theta_{\mathbf{a}',\mathbf{b}}$, then the Bell inequality demands,

$$|1 + 2\cos(\phi) - \cos(2\phi)| \leq 2$$

but this is violated for a wide range of ϕ.

Exercise 1 (Assignment 2): Derive Bell's inequality. Calculate $C(a,b)$ for the singlet state. Plot $|1 + 2\cos(\phi) - \cos(2\phi)|$ to show the range and degree of violation of Bell's inequality.

Observations:

The kind of locality that is violated by quantum mechanics is called *weak locality* because the violation does not permit 'super-luminal signaling.' That is, only a random sequence of outcomes are obtained at either location and the non-local correlations (on their own) can not be used to communicate information to the distant party. In contrast, a theory violating *strong locality* would allow the possibility of super-luminal signaling, e.g., rigid body mechanics.

Bell's argument relies also on an assumption of determinism: the outcomes are determined by the hidden variable $A = A(a,\lambda)$ and $B = B(b,\lambda)$. However, Clauser, Horne, Shimony, and Holt (1969), and Clauser and Horne

$\phi = \theta_{a,b'} - \theta_{a',b}$, then the Bell inequality demands,

$$|1 + 2\cos(\phi) - \cos(2\phi)| \leq 2$$

but this is violated for a wide range of ϕ.

Exercise 1 (Assignment 2): Derive Bell's inequality. Calculate $C(a, b)$ for the singlet state. Plot $|1 + 2\cos(\phi) - \cos(2\phi)|$ to show the range and degree of violation of Bell's inequality.

Observations:

The kind of locality that is violated by quantum mechanics is called *weak locality* because the violation does not permit 'super-luminal signaling.' That is, only a random sequence of outcomes are obtained at either location and the non-local correlations (on their own) can not be used to communicate information to the distant party. In contrast, a theory violating *strong locality* would allow the possibility of super-luminal signaling, e.g., rigid body mechanics.

Bell's argument relies also on an assumption of determinism: the outcomes are determined by the hidden variable $A = A(a, \lambda)$ and $B = B(b, \lambda)$. However, Clauser, Horne, Shimony, and Holt (1969), and Clauser and Horne (1974) developed inequalities (that assume local hidden variables and which are violated by quantum mechanics) that do not also rely on the determinism assumption - only a probabilistic dependence on the hidden variables is presumed.

but this is violated for a wide range of ϕ.

Exercise 1 (Assignment 2): Derive Bell's inequality. Calculate $C(a, b)$ for the singlet state. Plot $|1 + 2\cos(\phi) - \cos(2\phi)|$ to show the range and degree of violation of Bell's inequality.

Observations:

The kind of locality that is violated by quantum mechanics is called *weak locality* because the violation does not permit 'super-luminal signaling.' That is, only a random sequence of outcomes are obtained at either location and the non-local correlations (on their own) can not be used to communicate information to the distant party. In contrast, a theory violating *strong locality* would allow the possibility of super-luminal signaling, e.g., rigid body mechanics.

Bell's argument relies also on an assumption of determinism: the outcomes are determined by the hidden variable $A = A(a, \lambda)$ and $B = B(b, \lambda)$. However, Clauser, Horne, Shimony, and Holt (1969), and Clauser and Horne (1974) developed inequalities (that assume ~~local~~ hidden variables and which are violated by quantum mechanics) that do not also rely on the determinism assumption - only a probabilistic dependence on the hidden variables is presumed.

It is worth noting that Bell-type inequalities presume that the detector settings at the two separated locations may be selected independently, for example, by the 'free will' of the experimenters, or by some sufficiently pseudo-random function. Ultimately, in a fully deterministic conception of the world, all events could be traced back to a common cause, and are never truly independent.

Are hidden variables non-local or is quantum mechanics non-local?

Bell-type inequalities tell us that any hidden variable models reproducing quantum theory must be non-local. However, if we reject hidden variable models, and assume instead that the quantum state is a complete description of a system's physical properties, then the EPR analysis shows (implicitly) that because quantum states must be updated (collapsed) non-locally, it follows that physical properties of the world are exhibiting non-locality. *So whether one accepts or rejects that quantum states are a complete description, one is forced to accept non-locality.*Some authors (Stapp, 1985, 1988) even conclude that sequences of experimental outcomes which violate Bell-type inequalities *imply* that non-locality is a feature of the world, rather than just a feature of quantum mechanics. In this sense the violation of Bell's inequalities should not be viewed as a reason to reject hidden variables, but as a required constraint on hidden variable models.

... violation of Bell-type inequalities has been demonat the two separated locations may be selected independently, for example, by the 'free will' of the experimenters, or by some sufficiently pseudo-random function. Ultimately, in a fully deterministic conception of the world, all events could be traced back to a common cause, and are never truly independent.

Are hidden variables non-local or is quantum mechanics non-local?

Bell-type inequalities tell us that any hidden variable models reproducing quantum theory must be non-local. However, if we reject hidden variable models, and assume instead that the quantum state is a complete description of a system's physical properties, then the EPR analysis shows (implicitly) that because quantum states must be updated (collapsed) non-locally, it follows that physical properties of the world are exhibiting non-locality. *So whether one accepts or rejects that quantum states are a complete description, one is forced to accept non-locality.* Some authors (Stapp, 1985, 1988) even conclude that sequences of experimental outcomes which violate Bell-type inequalities *imply* that non-locality is a feature of the world, rather than just a feature of quantum mechanics. In this sense the violation of Bell's inequalities should not be viewed as a reason to reject hidden variables, but as a required constraint on hidden variable models.

The quantum mechanical violation of Bell-type inequalities has been demonstrated experimentally, in a number of distinct experiments. Some of the most important early experiments were performed by Alain Aspect and co-workers (1981,1982). Aspect will be giving us lectures on Bell-type inequalities and the experimental tests on March 15th and 17th.

Are hidden variables non-local or is quantum mechanics non-local?

Bell-type inequalities tell us that any hidden variable models reproducing quantum theory must be non-local. However, if we reject hidden variable models, and assume instead that the quantum state is a complete description of a system's physical properties, then the EPR analysis shows (implicitly) that because quantum states must be updated (collapsed) non-locally, it follows that physical properties of the world are exhibiting non-locality. *So whether one accepts or rejects that quantum states are a complete description, one is forced to accept non-locality.* Some authors (Stapp, 1985, 1988) even conclude that sequences of experimental outcomes which violate Bell-type inequalities *imply* that non-locality is a feature of the world, rather than just a feature of quantum mechanics. In this sense the violation of Bell's inequalities should not be viewed as a reason to reject hidden variables, but as a required constraint on hidden variable models.

The quantum mechanical violation of Bell-type inequalities has been demonstrated experimentally, in a number of distinct experiments. Some of the most important early experiments were performed by Alain Aspect and co-workers (1981,1982). Aspect will be giving us lectures on Bell-type inequalities and the experimental tests on March 15th and 17th.

Hidden Variable Assignments and the Contextuality Constraint

Consider again the idea that quantum statistics arise from incomplete knowledge of pre-existing values for all observables. On this assumption, for any observable A we wish to assign some pre-existing value $v_\psi(A) \in R$, where the subscript reminds us that the assignment will in general depend on the preparation.

What kinds of requirements should the value assignments satisfy?

Earlier we rejected von Neumann's requirement that the value assignments individually satisfy the same relations as the statistical averages (give some preparation).

A more innocent requirement is that any function of the value assigned to a commuting operators $\{A, B, C \ldots\}$ should be equal to the value of the function of the operators

$$f(v_\psi(A), v_\psi(B), v_\psi(C), \ldots) = v_\psi(f(A, B, C, \ldots)).$$

This requirement implies the properties:

$$\text{(i)} \quad v_\psi(A+B) = v_\psi(A) + v_\psi(B) \quad \text{if} \quad [A, B] = 0$$

$$\text{(ii)} \quad v_\psi(AB) = v_\psi(A)v_\psi(B) \quad \text{if} \quad [A, B] = 0$$

$$\text{(iii)} \quad v_\psi(\mathbb{1}) = 1.$$

Recall the spectral decomposition of a non-degenerate observable:

Hidden Variable Assignments and the Contextuality Constraint

Consider again the idea that quantum statistics arise from incomplete knowledge of pre-existing values for all observables. On this assumption, for any observable A we wish to assign some pre-existing value $v_\psi(A) \in R$, where the subscript reminds us that the assignment will in general depend on the preparation.

What kinds of requirements should the value assignments satisfy?

Earlier we rejected von Neumann's requirement that the value assignments individually satisfy the same relations as the statistical averages (give some preparation).

A more innocent requirement is that any function of the value assigned to a commuting operators $\{A, B, C \ldots\}$ should be equal to the value of the function of the operators

$$f(v_\psi(A), v_\psi(B), v_\psi(C), \ldots) = v_\psi(f(A, B, C, \ldots)).$$

This requirement implies the properties:

$$\text{(i)} \quad v_\psi(A+B) = v_\psi(A) + v_\psi(B) \quad \text{if} \quad [A, B] = 0$$

$$\text{(ii)} \quad v_\psi(AB) = v_\psi(A)v_\psi(B) \quad \text{if} \quad [A, B] = 0$$

$$\text{(iii)} \quad v_\psi(\mathbb{1}) = 1.$$

Recall the spectral decomposition of a non-degenerate observable:

$A = \sum_k \lambda_k \hat{P}_k$ where the projector $\hat{P}_k = |\phi_k\rangle\langle\phi_k|$ satisfies $\hat{P}_k^2 = \hat{P}_k$.

preparation.

What kinds of requirements should the value assignments satisfy?

Earlier we rejected von Neumann's requirement that the value assignments individually satisfy the same relations as the statistical averages (give some preparation).

A more innocent requirement is that any function of the value assigned to a commuting operators $\{A, B, C \ldots\}$ should be equal to the value of the function of the operators

$$f(v_\psi(A), v_\psi(B), v_\psi(C), \ldots) = v_\psi(f(A, B, C, \ldots)).$$

This requirement implies the properties:

$$\text{(i)} \quad v_\psi(A+B) = v_\psi(A) + v_\psi(B) \quad \text{if} \quad [A, B] = 0$$

$$\text{(ii)} \quad v_\psi(AB) = v_\psi(A) v_\psi(B) \quad \text{if} \quad [A, B] = 0$$

$$\text{(iii)} \quad v_\psi(\mathbb{1}) = 1.$$

Recall the spectral decomposition of a non-degenerate observable:
$A = \sum_k \lambda_k \hat{P}_k$ where the projector $\hat{P}_k = |\phi_k\rangle\langle\phi_k|$ satisfies $\hat{P}_k^2 = \hat{P}_k$.
These properties allow us to deduce that projectors must be assigned values according to:

$$v_\psi(P_k) \in \{1, 0\}.$$

Also note that the value assignment to a general Hermitian operator must be one of its eigenvalues:

$$v_\psi(A) \in \{\lambda_k\}$$

Earlier we rejected von Neumann's requirement that the value assignments individually satisfy the same relations as the statistical averages (give some preparation).

A more innocent requirement is that any function of the value assigned to a commuting operators $\{A, B, C \ldots\}$ should be equal to the value of the function of the operators

$$f(v_\psi(A), v_\psi(B), v_\psi(C), \ldots) = v_\psi(f(A, B, C, \ldots)).$$

This requirement implies the properties:

$$\text{(i)} \quad v_\psi(A+B) = v_\psi(A) + v_\psi(B) \quad \text{if} \quad [A, B] = 0$$

$$\text{(ii)} \quad v_\psi(AB) = v_\psi(A)v_\psi(B) \quad \text{if} \quad [A, B] = 0$$

$$\text{(iii)} \quad v_\psi(\mathbb{1}) = 1.$$

Recall the spectral decomposition of a non-degenerate observable:
$A = \sum_k \lambda_k \hat{P}_k$ where the projector $\hat{P}_k = |\phi_k\rangle\langle\phi_k|$ satisfies $\hat{P}_k^2 = \hat{P}_k$.
These properties allow us to deduce that projectors must be assigned values according to:

$$v_\psi(P_k) \in \{1, 0\}.$$

Also note that the value assignment to a general Hermitian operator must be one of its eigenvalues:

$$v_\psi(A) \in \{\lambda_k\}$$

Bell-Kochen-Specker theorem: If the Hilbert space dimension is greater than two, a consistent value assignment constrained by these properties is not possible.

This result was obtained by Bell (1966) and, independently, by Kochen and Specker (1967). Bell's proof assumed value assignments to a continuum of projectors, whereas the Kochen-Specker theorem required only a finite set (actually, 117 of them). The approach of the KS-proof was to consider sets of commuting triads which share a single vector in common, e.g., although $[A, B] = [B, C] = 0$, it does not follow that $[A, C] \neq 0$.

A simpler proof, requiring a 4-dimensional Hilbert space is due to Mermin (1993).

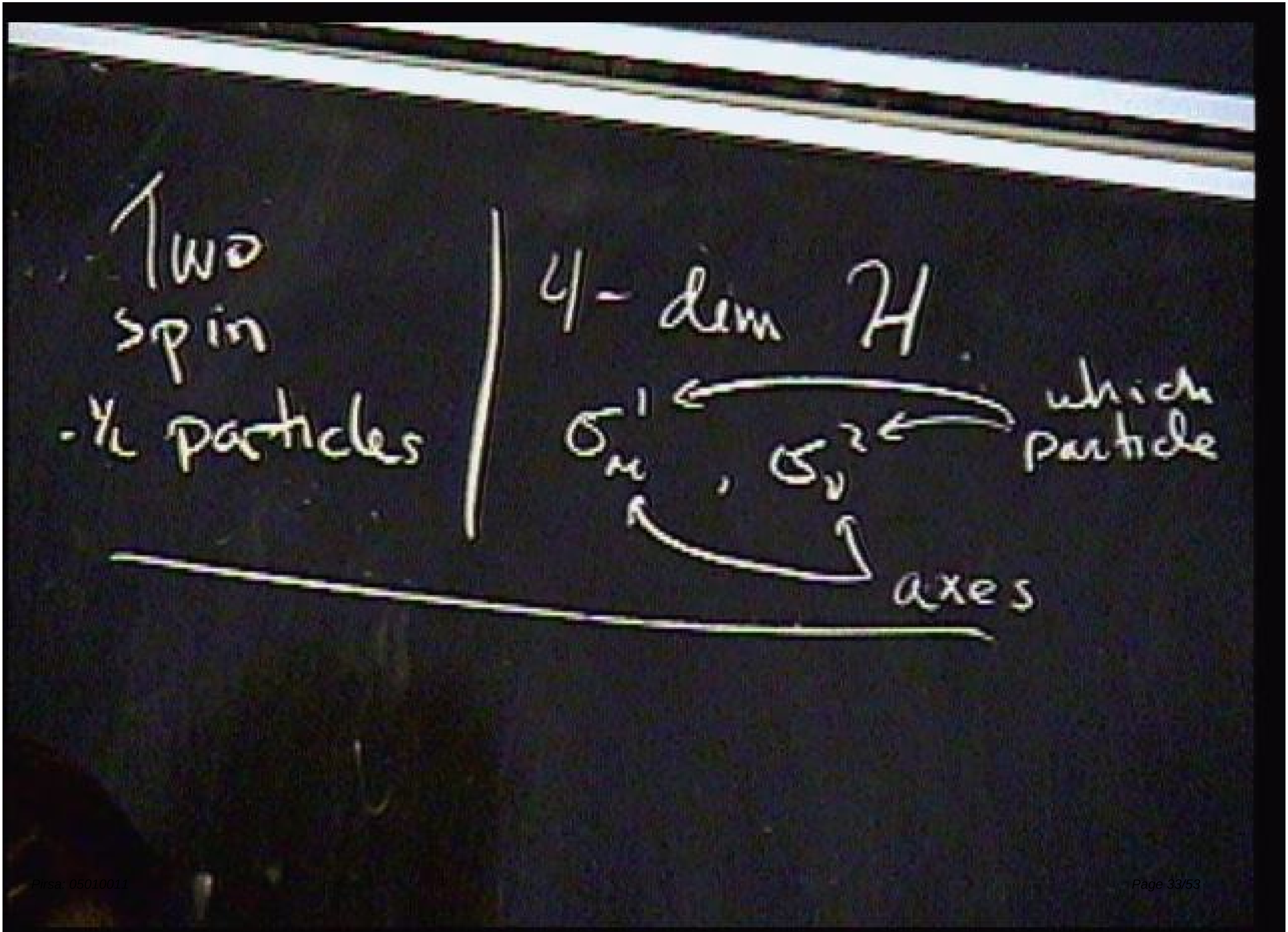
The consequence of the Bell-KS-type theorems is expressed by saying that value assignments to quantum mechanical observables are *contextual*. This means that the value assigned to any observable must depend on the specification of which other commuting observables are being assigned values along with it.

4-dim $\mathcal{H}$

$\sigma^1_\mu \;,\; \sigma^2_\nu$

which particle

axes


Two spin-½ particles
4-dim $\mathcal{H}$
$\sigma_\mu^1 , \sigma_\nu^2$
which particle
axes

$\Rightarrow$

$\sigma_x^1 \quad \sigma_x^2 \quad \sigma_x^1 \sigma_x^2$

$\sigma_y^2 \quad \sigma_y^1 \quad \sigma_y^1 \sigma_y^2$

$\sigma_x^1 \sigma_y^2 \quad \sigma_x^2 \sigma_y^1 \quad \sigma_z^1 \sigma_z^2$

$\sigma_y^1 \sigma_y^2$

$\sigma_x^1 \sigma_y^2 \quad \sigma_x^2 \sigma_y^1 \quad \sigma_z^1 \sigma_z^2$

$[\sigma_x^1 \sigma_y^2 , \sigma_x^2 \sigma_y^1]$

$= \sigma_x^1 \sigma_y^1 \sigma_y^2 \sigma_x^2 - \sigma_x^2 \sigma_y^1 \sigma_x^1 \sigma_y^2$

$= i\sigma_z^1 (-) i\sigma_z^2 -$

$$\sigma_x^1 \sigma_y^2 \qquad \sigma_x^2 \sigma_y^1 \qquad \sigma_z^1 \sigma_z^2$$

$$[\sigma_x^1 \sigma_y^2 , \sigma_x^2 \sigma_y^1]$$

$$= \sigma_x^1 \sigma_y^1 \sigma_y^2 \sigma_x^2 - \sigma_x^2 \sigma_y^1 \sigma_x^1 \sigma_y^2$$

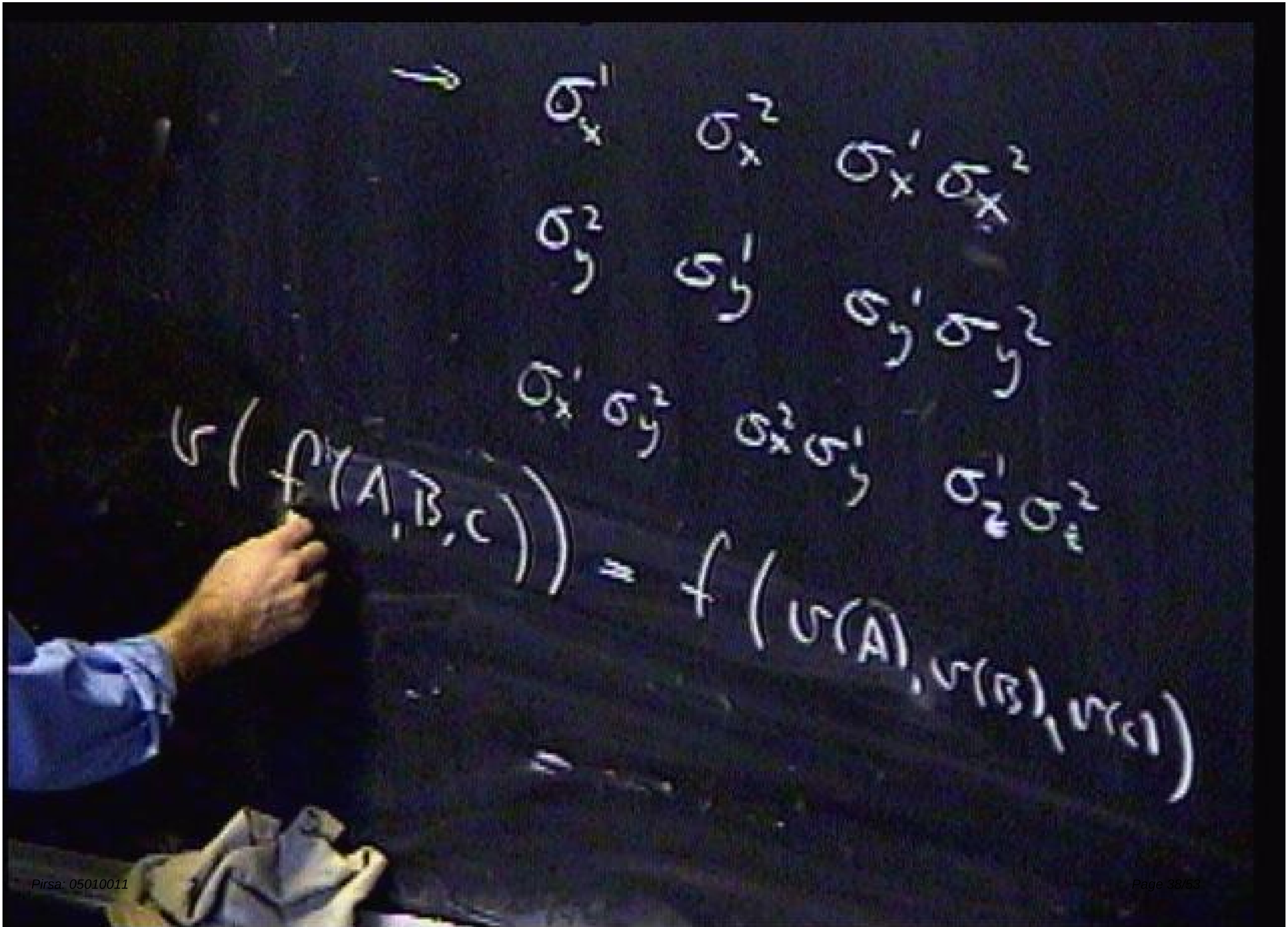
$$= (i\sigma_z^1 (-)(i\sigma_z^2 - (\sigma_z^2 (-1) i \sigma_z^2$$

$$\sigma_x^1 \sigma_y^2 \quad \sigma_x^2 \sigma_y^1 \quad \sigma_z^1 \sigma_z^2$$

$$[\sigma_x^1 \sigma_y^2, \sigma_x^2 \sigma_y^1]$$

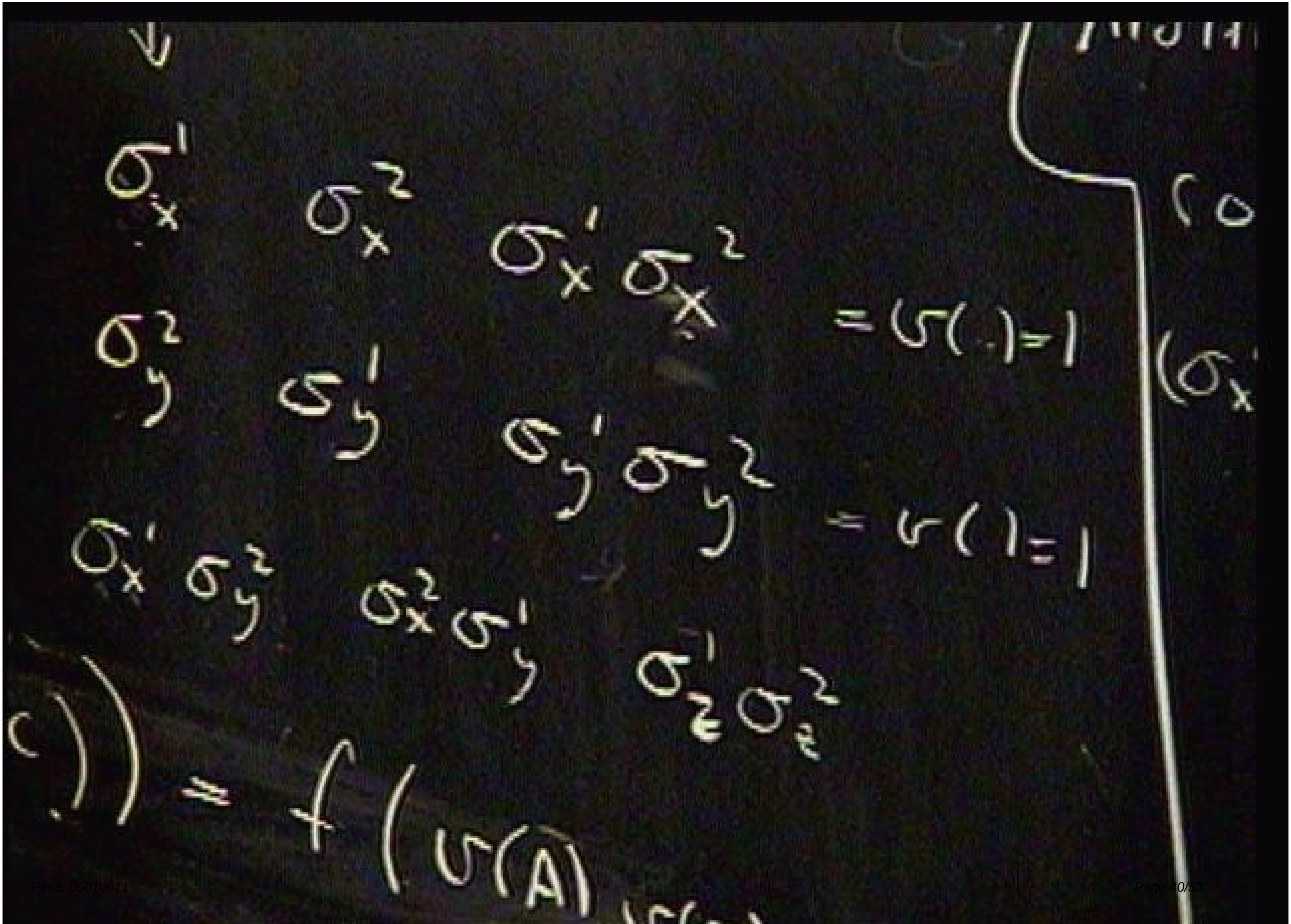
$$= \sigma_x^1 \sigma_y^1 \sigma_y^2 \sigma_x^2 - \sigma_x^2 \sigma_y^1 \sigma_x^1 \sigma_y^2$$

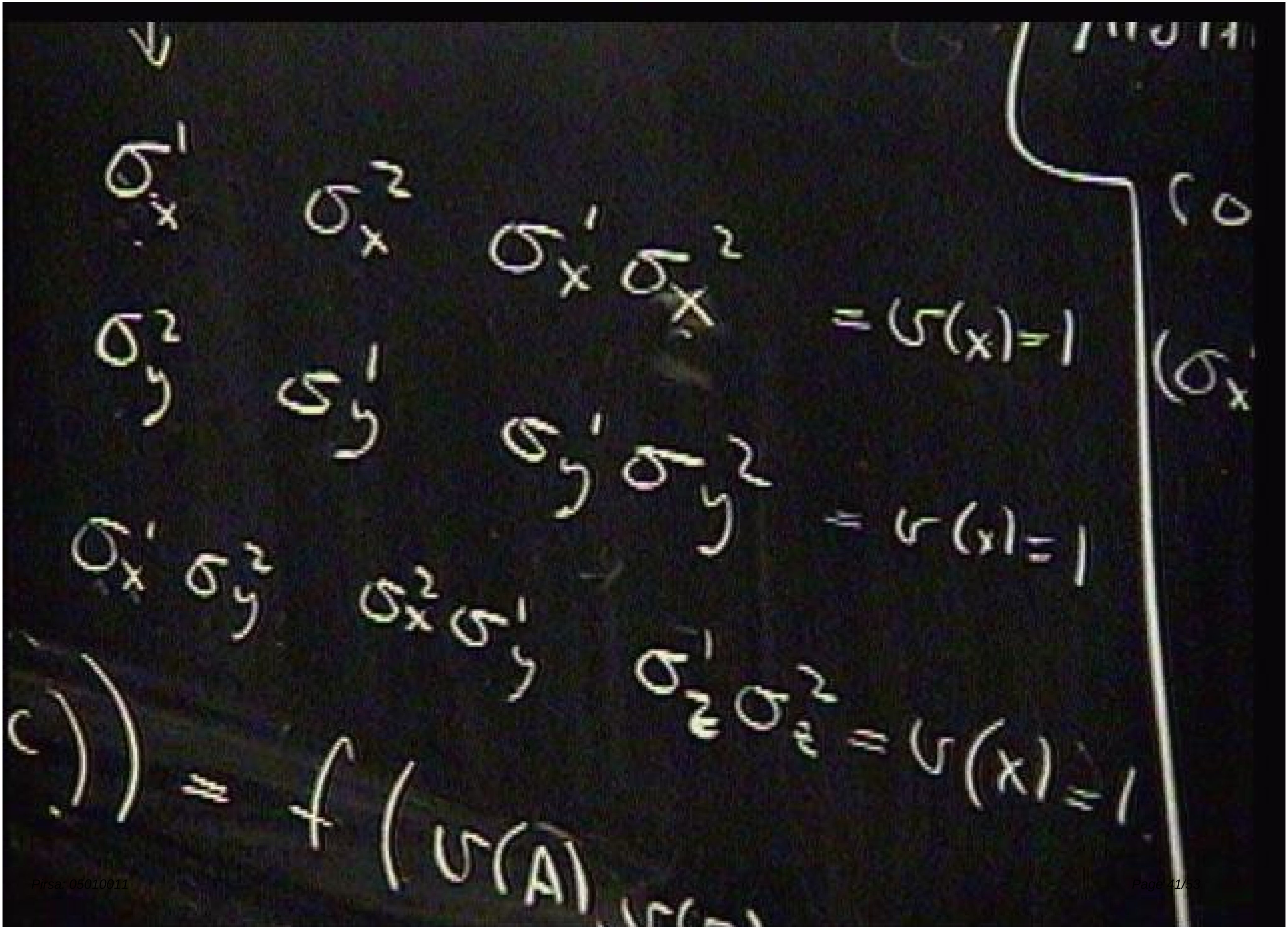
$$= i\sigma_z^1 (-) i\sigma_z^2 - i\sigma_z^2 (-1) i\sigma_z^2 = 0$$

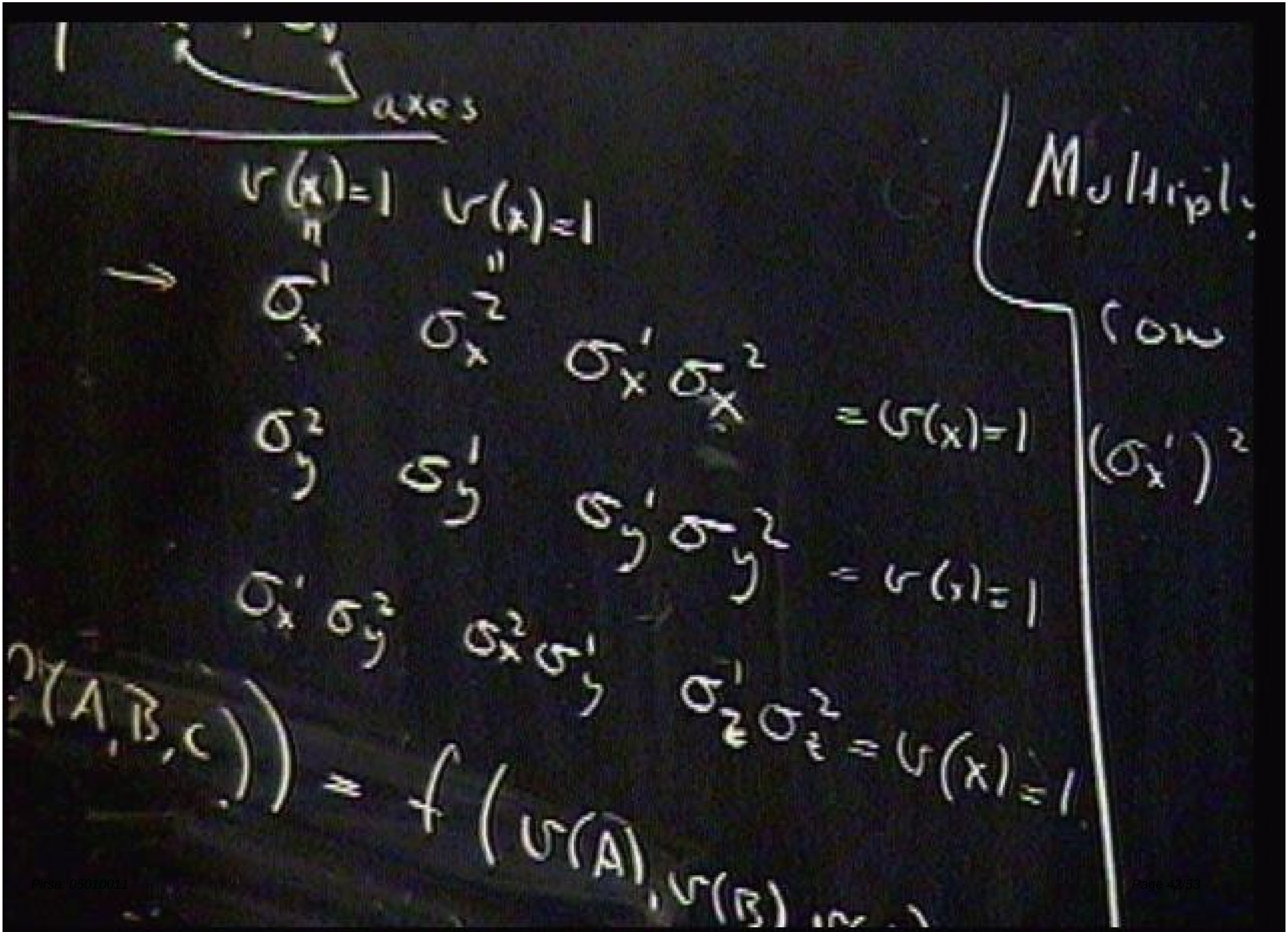


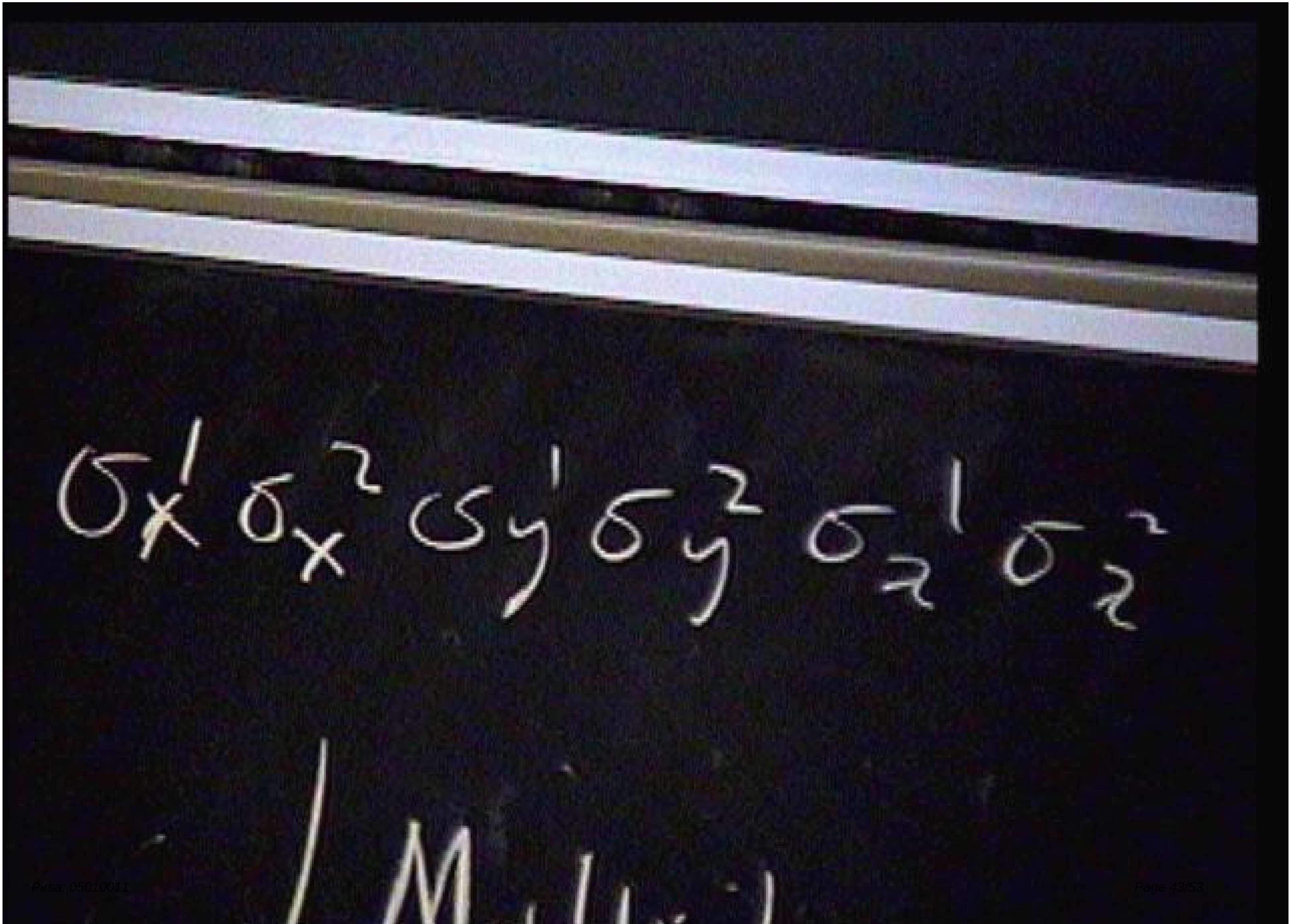
σx1 σx2 σx1σx2
σy2 σy1 σy1σy2
σx1σy2 σx2σy1 σz1σz2
v(f(A,B,C)) = f(v(A),v(B),v(C))

Multiply operators in each row and each column




σ_x^1 σ_x^2 σ_x^1 σ_x^2 = v(x) = 1
σ_y^2 σ_y^1 σ_y^1 σ_y^2 = v(x) = 1
σ_x^1 σ_y^2 σ_x^2 σ_y^1 σ_z^1 σ_z^2 = v(x) = 1
= f(v(A)

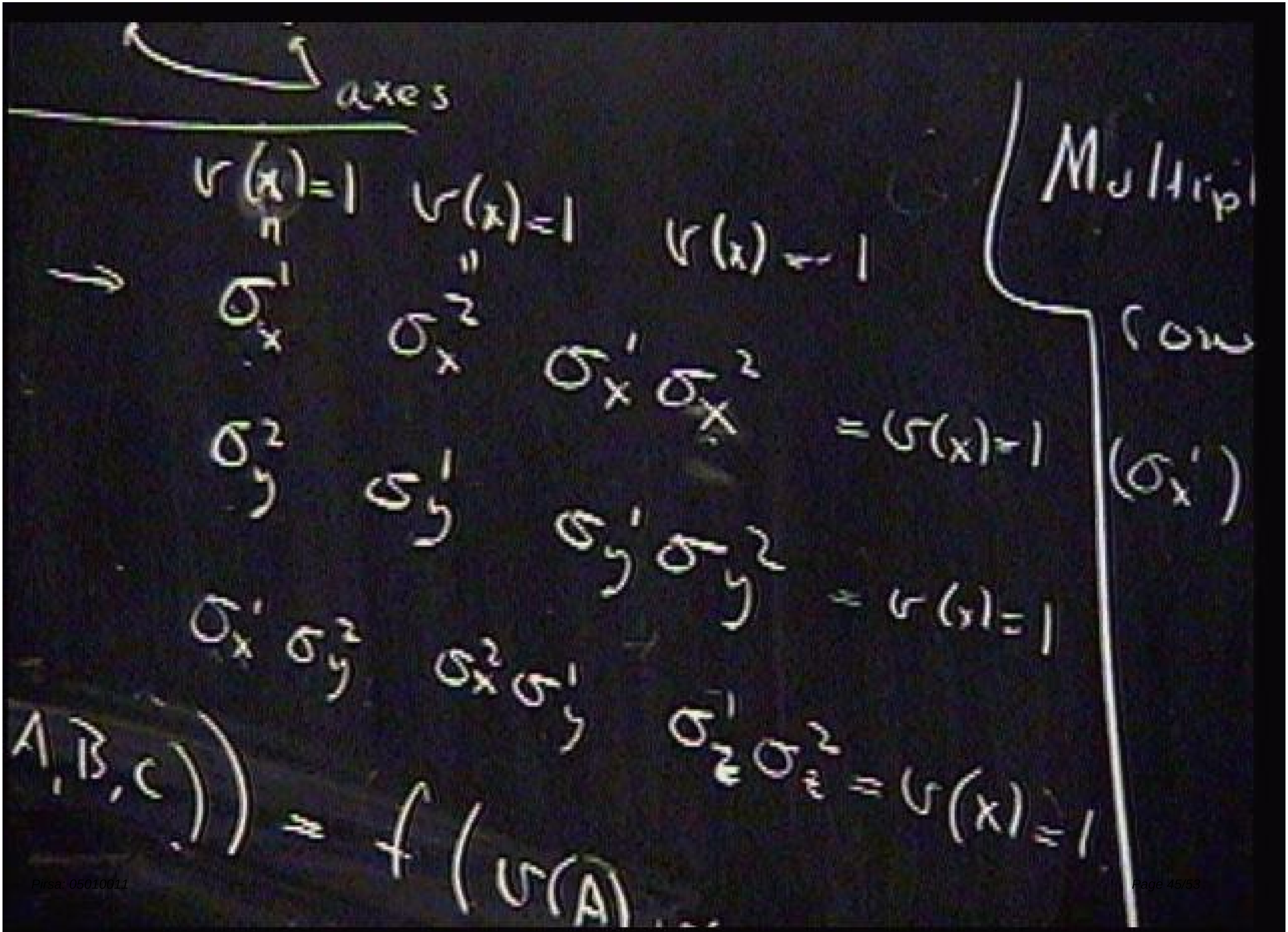

axes
v(x)=1 v(x)=1
Multipl
com
= v(x)=1
= v(y)=1
= v(x)=1
(A,B,C)) = f(v(A), v(B)


$\sigma_x^1 \sigma_x^2 \sigma_y^1 \sigma_y^2 \sigma_z^1 \sigma_z^2$

$$\sigma_x^1 \sigma_x^2 \sigma_y^1 \sigma_y^2 \sigma_z^1 \sigma_z^2 = i\sigma_z^1 \, i\sigma_z^2 \, \sigma_z^1 \sigma_z^2$$

$$= (-1)\, \mathbb{1}$$

Multiply operators in each row and each


axes
Multipl
com

$\sigma_x \quad \sigma_x \quad \sigma_x^1 \sigma_x^2 = v(x) = 1$

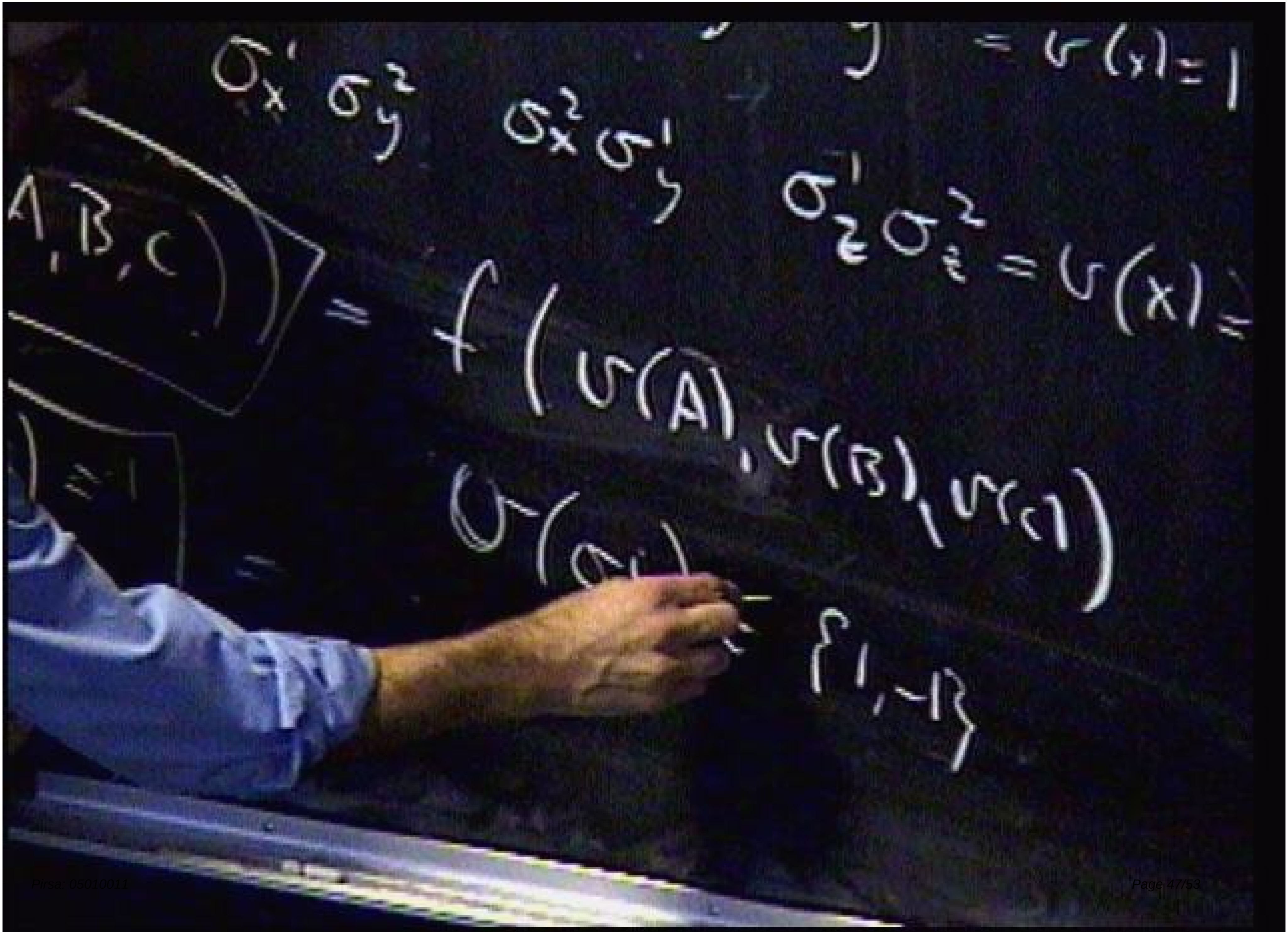
$\sigma_y^2 \quad \sigma_y^1 \quad \sigma_y^1 \sigma_y^2 = v(y) = 1$

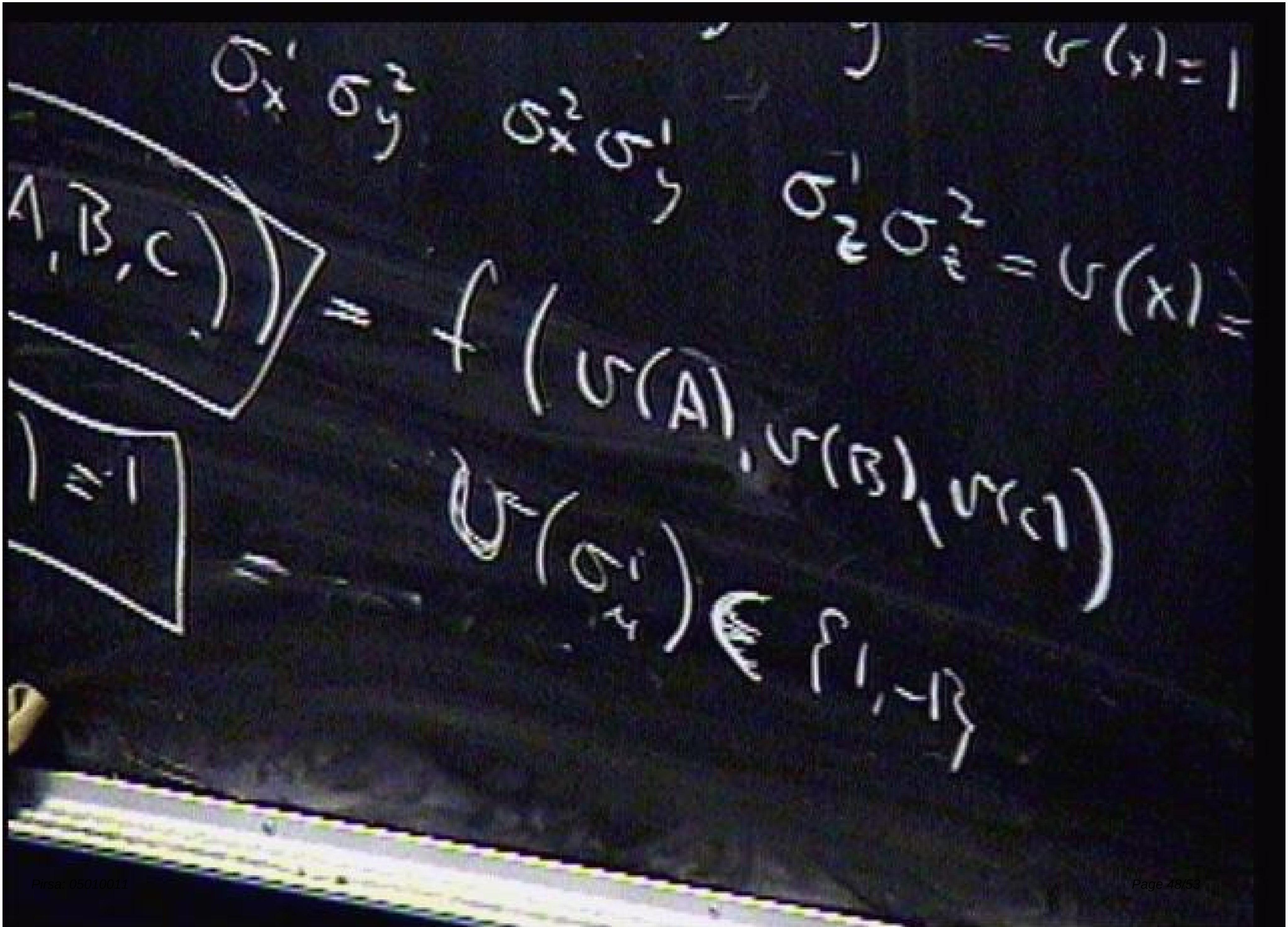
$\sigma_x^1 \sigma_y^2 \quad \sigma_x^2 \sigma_y^1 \quad \sigma_z^1 \sigma_z^2 = v(x) = 1$

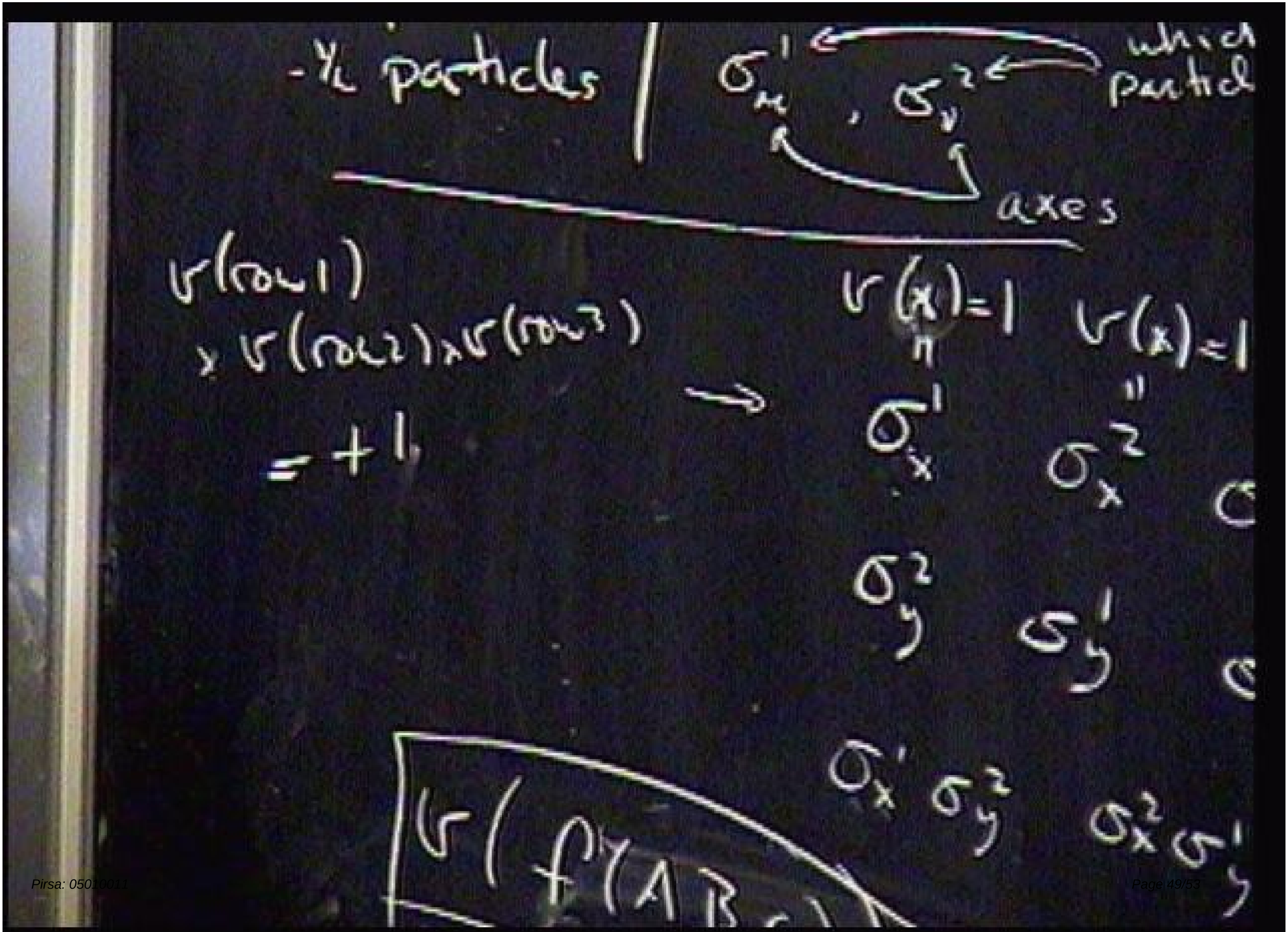
$$v(f(A,B,C)) = f(v(A), v(B), v(C))$$

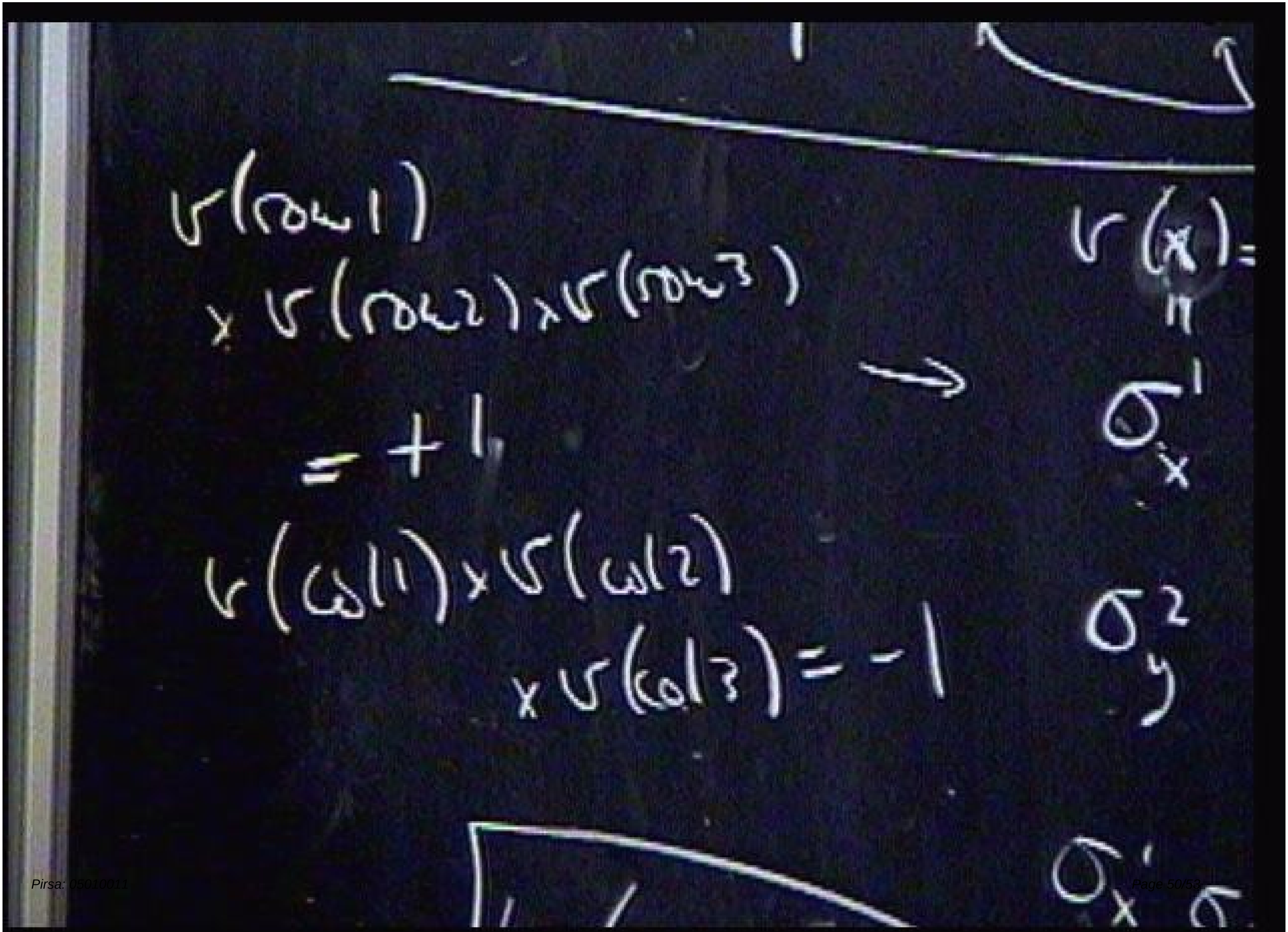
$$v(I) = 1$$

$=$




$= v(x) = 1$
$\sigma_x^1 \sigma_y^2 \quad \sigma_x^2 \sigma_y^1 \quad \sigma_z^1 \sigma_z^2 = v(x) =$
$A, B, C)) = f(v(A), v(B), v(C))$
$) = 1$
$v(\sigma_x^1) \in \{1, -1\}$


-½ particles
σ¹ₓ , σ²ᵧ
which particle
axes
v(row1) x v(row2) x v(row3) = +1
v(x) = 1
σ¹ₓ
v(x) = 1
σ²ₓ
σ²ᵧ
σ¹ᵧ
σ¹ₓ σ²ᵧ
σ²ₓ σ¹ᵧ
v(f(ABC


v(row1)
x v(row2) x v(row3)
= +1
v(col1) x v(col2)
x v(col3) = -1
v(x) =
σx1
σy2
σx1 σ

⇒ Non-contextual assignment of values is not possible.

Bell's (self-)Criticism

"[we have] tacitly assumed that the measurement of an observable must yield the same value independently of what other measurements must be made simultaneously." Since some observables in each set (row or column in Mermin's proof, triads in the KS proof) do not commute with the additional observables in the other, the measurements of each complete set are incompatible. "These different possibilities require different experimental arrangements; there is no a priori reason to believe that the results . . . should be the same."

Bell (1966)

Bohr's Prescience

"[The] measuring instruments . . . serve to define the conditions under which the phenomena appear."

Bohr (1949)

Assigned Reading for Next Week's Lectures:

Many Worlds Interpretation by D. Wallace:

"Everett and Structure", quant-ph/0107144.

"Quantum Probability from Subjective Uncertainty", quant-ph/0312157.

Anyone particularly keen (all of you right?) should also look at:

Lev Vaidman's encyclopedia article "The Many-Worlds Interpretation of Quantum Mechanics", Stanford Encyclopedia of Philosophy, available at http://www.tau.ac.il/ vaidman/mwi/mw2.html.

Adrian Kent, "Against many-worlds interpretations", gr-qc/9703089

If you want a better understanding of decoherence read:

W. Zurek, quant-ph/0306072.