Title: Entanglement, Critical Phenomena & RG Flows

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Abstract:

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19/01/05

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Entanglement, Critical Phenomena & RG Flows

•Entanglement in Quantum Critical Phenomena.
G. Vidal, J.I. Latorre, E. Rico, A. Kitaev. Phys.Rev.Lett. 90 (2003) 227902

•Ground State Entanglement in Quantum Spin Chains.
.I. Latorre, E. Rico, G. Vidal. Quant. Inf. & Comp. Vol.4 no.1 (2004) pp.048-092

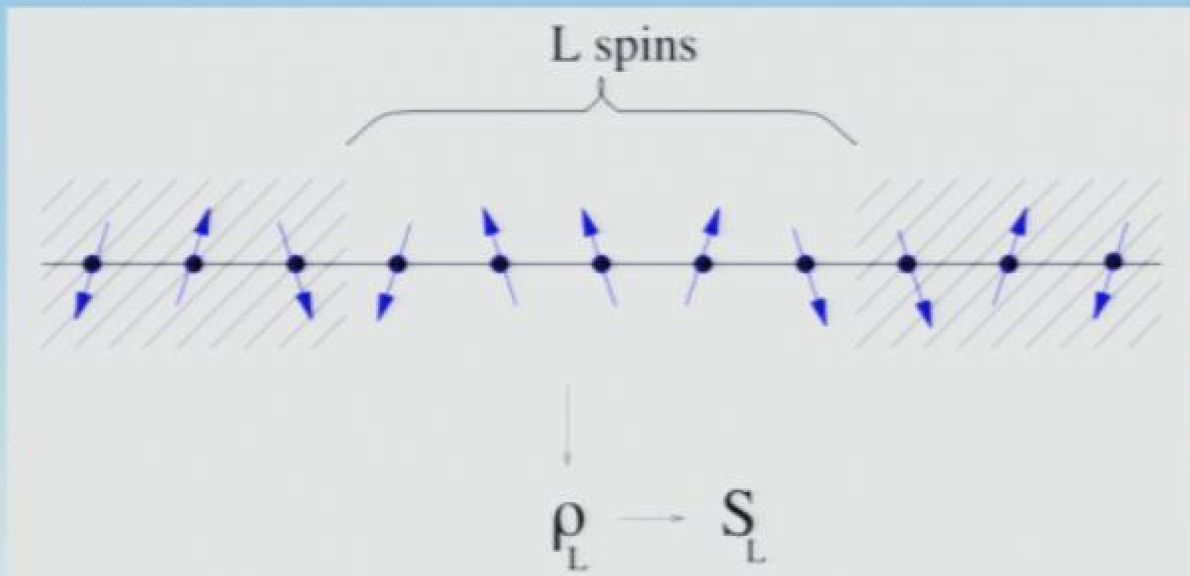
•Fine-Grained Entanglement Loss along Renormalization Group Flows.
J.I. Latorre, C.A. Lütken, E. Rico, G. Vidal. quant-ph/0404120

•Renormalization Group Transformation on quantum states.
F. Verstraete, J.I. Cirac, J.I. Latorre, E. Rico, M.M. Wolf, quant-ph/0410227

Outline

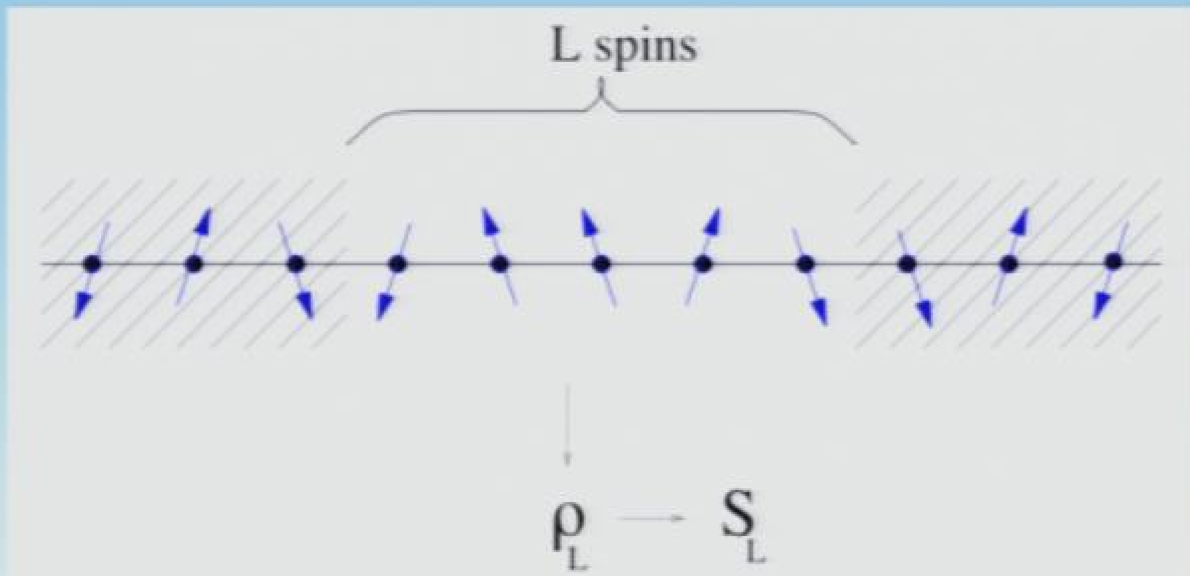
- Introduction (Motivation & Entanglement measures)
- Entanglement in Quantum Spin Models
  - Heisenberg and XY model
  - Analysis of the Ground State Entanglement
  - Connection with higher Dimension
  - Connection with Conformal Field Theory
- Entanglement loss along RG Flows
- Matrix Product States
  - Definition
  - Fixed points in Scale Transformation

Introduction & Motivation



$$\langle O_i \cdot O_j \rangle - \langle O_i \rangle \langle O_j \rangle \rightarrow \begin{cases} e^{-|i-j|/\xi} \Rightarrow \textit{Finite Correlation} \\ \dfrac{1}{|i-j|^{\nu}} \Rightarrow \textit{Phase Transition} \end{cases}$$

Introduction & Motivation



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$$\rho_L = Tr_{N-L}\left(\left| \Psi_G \right\rangle \left\langle \Psi_G \right| \right) \Rightarrow S_L = -Tr\left(\rho_L \log_2 \rho_L \right)$$

$$\rho_L(\lambda, \gamma) \succ \rho_{L'}(\lambda', \gamma')$$

Introduction & Entanglement measures

Schmidt Rank.- $|\psi\rangle = \sum_{i=1}^{D} \sqrt{\lambda_i} |i_A\rangle |i_B\rangle \quad \rho_A = tr_B(|\psi\rangle\langle\psi|) = \sum_{i=1}^{D} \lambda_i |i_A\rangle\langle i_A|$

Introduction & Entanglement measures

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ntropy of Entanglement.- $E(|\psi\rangle) = S(\rho_A) = -\sum_{i=1}^{D} \lambda_i \log_2 \lambda_i$

$$|\psi\rangle^{\otimes N} \xrightarrow{LOCC} |EPR\rangle^{\otimes M} \Leftrightarrow E(|\psi\rangle) = \frac{M}{N}$$

Bennett et al (1996)

Introduction & Entanglement measures

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Bennett et al (1996)

lajorization.- $|\psi\rangle \xrightarrow{LOCC} |\tilde{\psi}\rangle \Leftrightarrow |\psi\rangle \prec |\tilde{\psi}\rangle$

$$\lambda_1^{\downarrow} \le \tilde{\lambda}_1^{\downarrow}$$

Nielsen, Vidal (2000)

$$\lambda_1^{\downarrow} + \lambda_2^{\downarrow} \le \tilde{\lambda}_1^{\downarrow} + \tilde{\lambda}_2^{\downarrow}$$

$$\vdots$$

$$\lambda_1^{\downarrow} + \lambda_2^{\downarrow} + \cdots + \lambda^{\downarrow} = \tilde{\lambda}_1^{\downarrow} + \tilde{\lambda}_2^{\downarrow} + \cdots + \tilde{\lambda}^{\downarrow}$$

Quantum Spin Models

Heisenberg model

$$H_{XXZ} = \sum_n \left[\frac{1}{2} \left(\sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y + \Delta \sigma_n^z \sigma_{n+1}^z \right) - \lambda \sigma_n^z \right] = H_0 + \Delta H_1 + \lambda H_2$$

Quantum Spin Models

Heisenberg model

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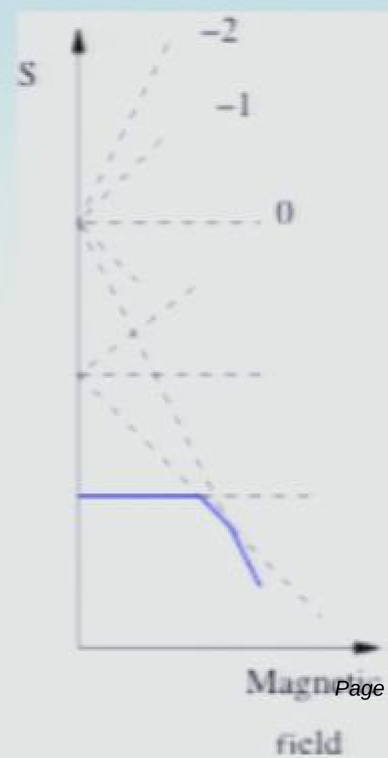
Solution of the model.- **Bethe Anstaz**:



- <u>Rotational symmetry</u>

about the z-axis in the spins space

- <u>Translational invariance</u>

by any number of lattice spacing

$$|F\rangle = |\uparrow\uparrow\uparrow\cdots\uparrow\uparrow\rangle \Rightarrow |\psi\rangle = \sum_n e^{ikn} \sigma_n^- |F\rangle$$

Quantum Spin Models

XY model

$$H_{XY} = \sum_{l=1}^{N}\left(\frac{1+\gamma}{2}\sigma_l^x\sigma_{l+1}^x + \frac{1-\gamma}{2}\sigma_l^y\sigma_{l+1}^y - \lambda\sigma_l^z\right)$$

Solution of the model.- **Spinless fermions** Lieb, **Schultz, Mattis** (1961)

Quantum Spin Models

XY model

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Solution of the model.- **Spinless fermions** Lieb, **Schultz, Mattis** (1961)

Canonical Transformations:

- **Jordan-Wigner**

$$a_l = \left(\prod_{m<l} \sigma_m^z \right) \frac{\sigma_l^x - i\sigma_l^y}{2} \qquad \{a_l, a_m^+\} = \delta_{l,m}$$

- **Fourier**

$$d_k = \frac{1}{\sqrt{N}} \sum_{l=-\frac{N}{2}+1}^{\frac{N}{2}-1} a_l Exp\left[i \frac{2\pi}{N} kl \right] \qquad \{d_k, d_p^+\} = \delta_{k,p}$$

- **Bogoliubov**

$$b_k^+ = u_k d_k^+ + i v_k d_{-k} \qquad \{b_k, b_p^+\} = \delta_{k,p}$$

Quantum Spin Models

XY model

$$H_{XY} = \sum_{l=1}^{N} \left(\frac{1+\gamma}{2} \sigma_l^x \sigma_{l+1}^x + \frac{1-\gamma}{2} \sigma_l^y \sigma_{l+1}^y - \lambda \sigma_l^z \right)$$

$$\xrightarrow[N\to\infty]{} \int_{-\pi}^{\pi} \frac{d\phi}{2\pi} \sqrt{(\lambda - \cos\phi)^2 + \gamma^2 \sin^2\phi}\, b_\phi^+ b_\phi$$

$$\xrightarrow[low-energy]{} \int_{-\infty}^{+\infty} \frac{d\phi}{2\pi} \sum_{s=R,L} \sqrt{m^2 + c^2 \phi_s^{\ 2}}\, b_{s,\phi}^+ b_{s,\phi}$$

Quantum Spin Models

XY model

Ground State:

$$b_\phi|\Psi_G\rangle = 0, \forall\phi \Rightarrow n_\phi = \langle b_\phi^+ b_\phi \rangle = 0 \qquad \rho = \prod_{\otimes} \rho_\phi \Rightarrow \rho_\phi = \begin{pmatrix} n_\phi & 0 \\ 0 & 1 - n_\phi \end{pmatrix}$$

$$\Gamma_{mn} = \frac{1}{2\pi}\int_{-\pi}^{\pi} d\phi \cdot Exp[-i\phi(m-n)]U_\phi \rho_\phi U_\phi^+$$

Quantum Spin Models

XY model

Ground State:

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Wick's Theorem:

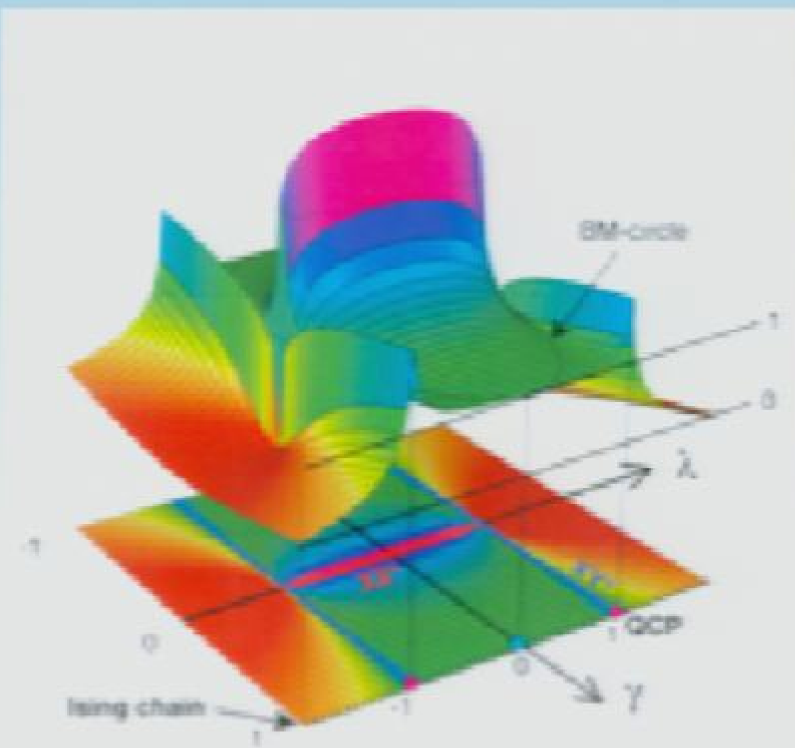
$$\Gamma_{mn} = \frac{1}{2\pi}\int_{-\pi}^{\pi} d\phi \cdot Exp[-i\phi(m-n)]U_\phi \rho_\phi U_\phi^+$$

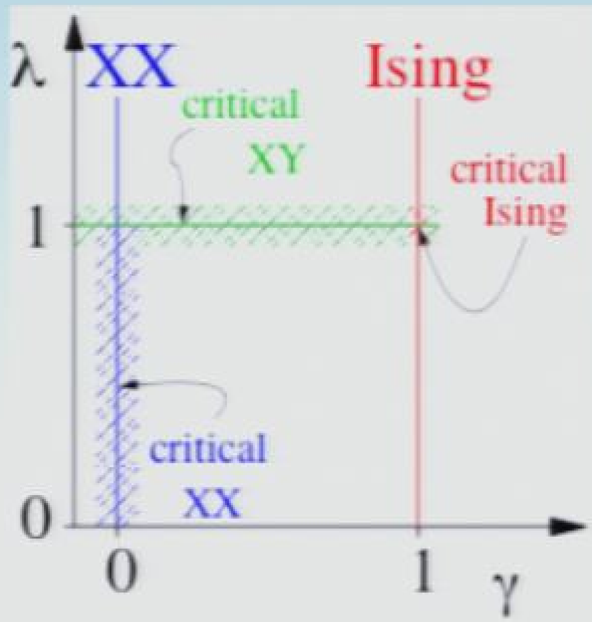
$$\langle a_{m1} a_{m2} a_{m3}^+ a_{m4}^+\rangle = \langle a_{m1} a_{m2}\rangle\langle a_{m3}^+ a_{m4}^+\rangle - \langle a_{m1} a_{m3}^+\rangle\langle a_{m2} a_{m4}^+\rangle + \langle a_{m1} a_{m4}^+\rangle\langle a_{m2} a_{m3}^+\rangle$$

$$\rho_L = \lim_{N\to\infty} Tr_{N-L}|\Psi_G\rangle\langle\Psi_G| = \prod_{\otimes}\begin{pmatrix} \upsilon_\varphi & 0 \\ 0 & 1-\upsilon_\varphi \end{pmatrix}$$

Peschel(2002)

Analysis of the Ground State Entanglement

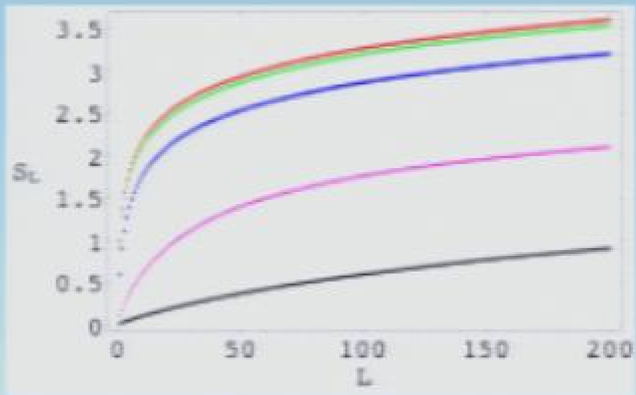




Analysis of the Ground State Entanglement

XX Model:

$$S_L^{XX} = \frac{1}{3}\log_2 L + \frac{1}{6}\log_2\left(1-\lambda^2\right)$$



Analysis of the Ground State Entanglement

Heisenberg Model:

$$S_L \cong \frac{1}{3}\log_2 L$$

Entanglement in higher Dimensions

Srednicki (1993):

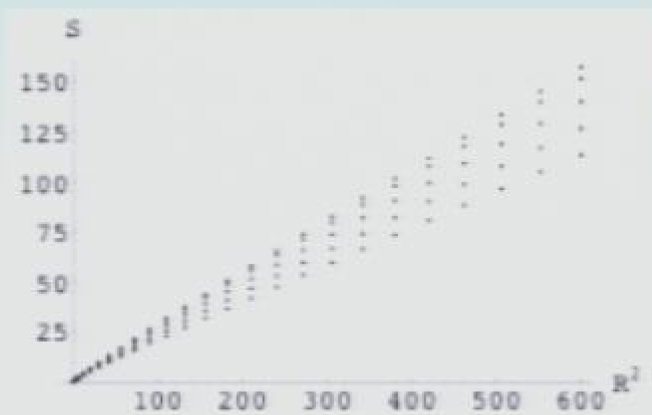
$$H = \frac{1}{2}\int d^3x\left(\Pi^2(x) + \left|\nabla\phi(x)\right|^2 + m^2\left|\phi(x)\right|^2\right)$$

Entanglement in higher Dimensions

Srednicki (1993):

$$H = \frac{1}{2}\int d^3x\left(\Pi^2(x) + |\nabla\phi(x)|^2 + m^2|\phi(x)|^2\right)$$

$$\phi = \sum \mu \cdot \phi_{in} \cdot \phi_{out}$$



$$S_{d=3} \propto R^2 \Rightarrow S_d \propto R^{d-1}$$

Kabat, Shenker, Strassler, Bombelli,...

Connection with Conformal Field Theory

Connection with Conformal Field Theory

Wilczek, Callan, Holzhey, Larsen (1994):

Calabrese, Cardy (2004)

$$S_L = \frac{c + \bar{c}}{6} \log_2 L$$

Heisenberg & XX model:

$$c = 1$$

Ising model:

$$c = \frac{1}{2}$$

Entanglement Loss along RG-Flows

$$H_{Ising} \xrightarrow[low-energy]{} \int_{-\infty}^{+\infty} \frac{dk}{2\pi} \sqrt{\left|\frac{1-\lambda}{a}\right|^2 + c^2k^2}\, b_k^+ b_k = \int_{-\infty}^{+\infty} \frac{dk}{2\pi} \sqrt{\left(\frac{m}{a}\right)^2 + c^2k^2}\, b_k^+ b_k$$

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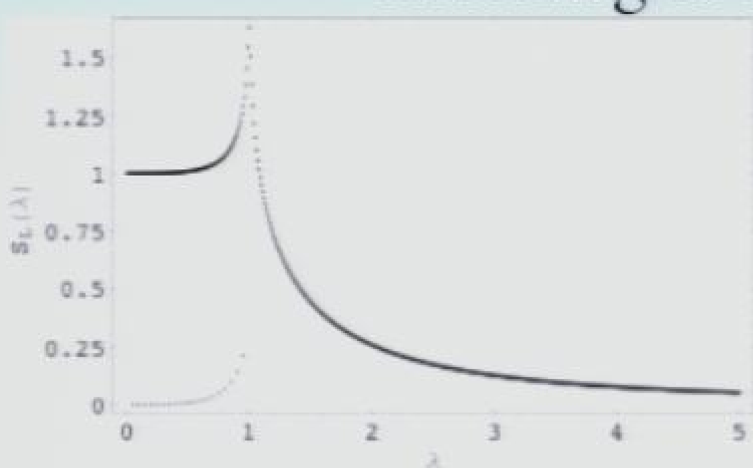
Renormalization Group Transformation

- Integration of the high-energy modes
- Rescaling of the parameters



$$\begin{aligned} a &\rightarrow ae^{-l} \\ m &\rightarrow me^{l} \end{aligned} \Rightarrow \quad \frac{\partial a}{\partial l} = -a \qquad \frac{\partial \lambda}{\partial l} = |1-\lambda| \qquad \frac{\partial c}{\partial l} = 0$$

Entanglement Loss along RG-Flows

- **Global loss.-** Direct consequence of universal scaling of entanglement in 1+1D systems and conformal symmetry

c-theorem: There is a monotonic nonincreasing function C along the Renormalization Group Flow.

Zamolodchikov (1986)

$$S = \frac{c + \bar{c}}{6} \log_2 L \qquad c^{UV} > c^{IR} \Rightarrow S^{UV} > S^{IR}$$

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Casini, Huerta (2004)

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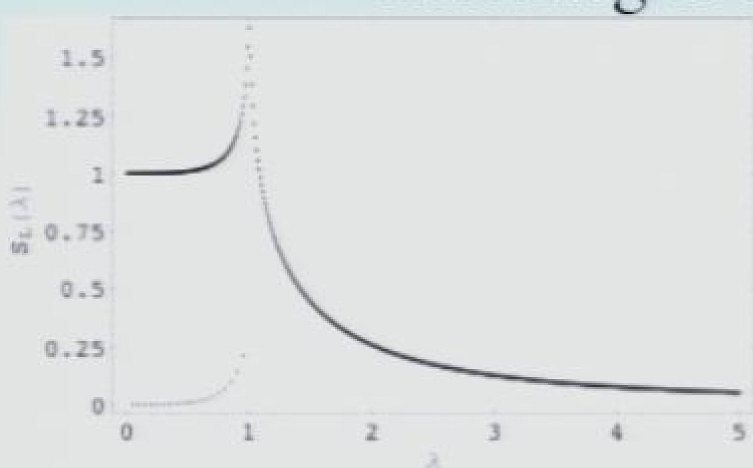
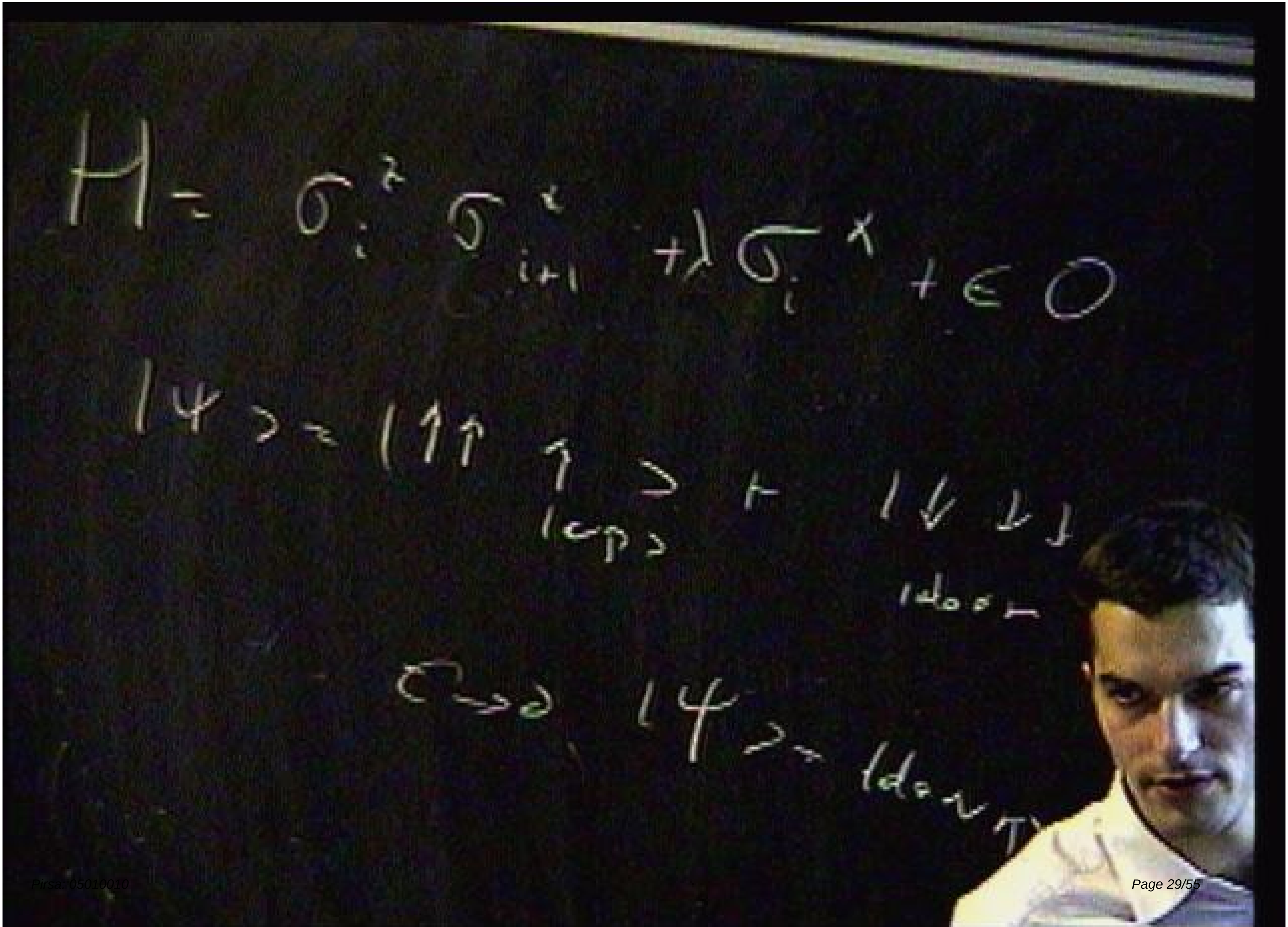
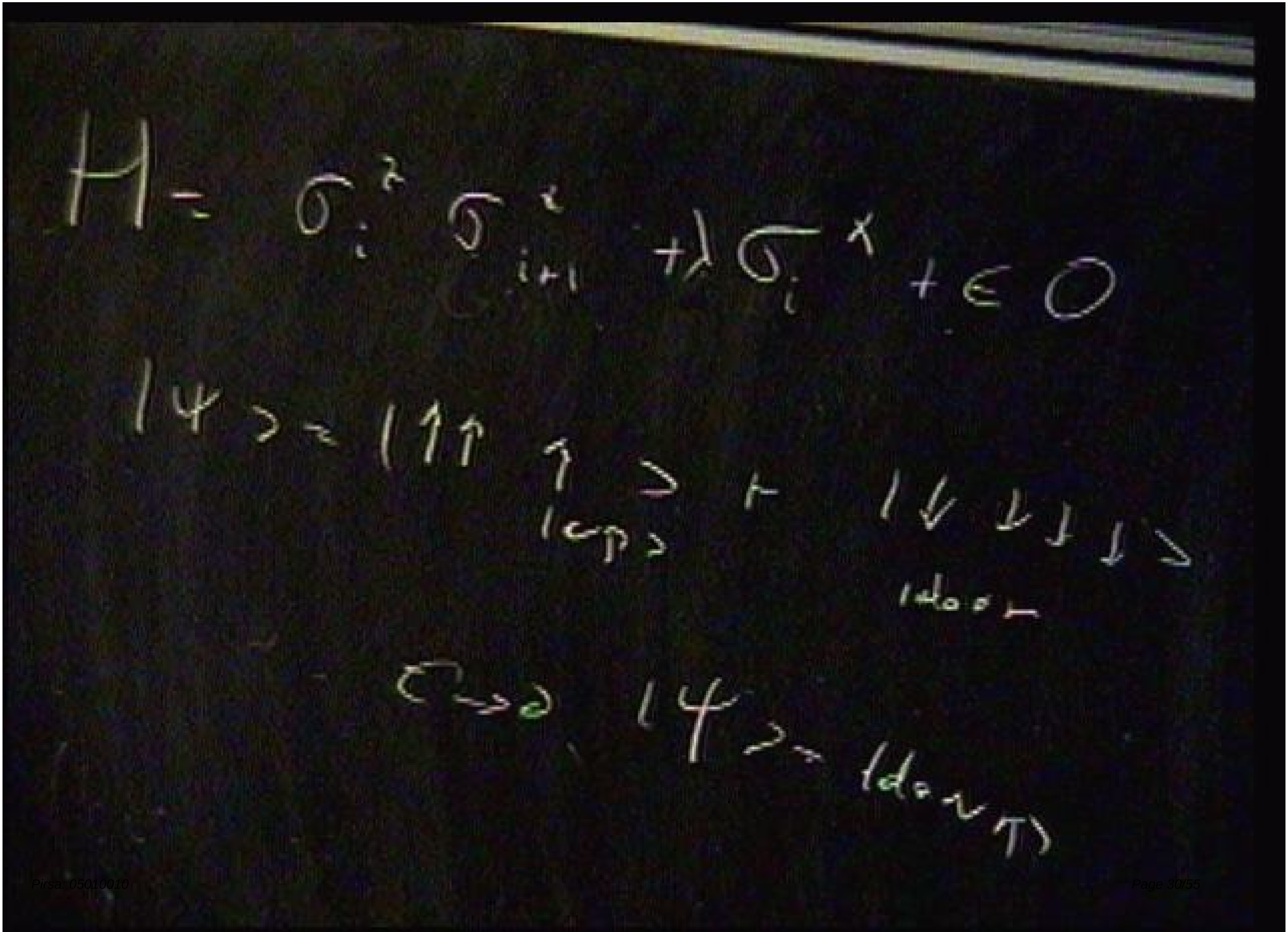



H = σi z σi+1 z + λσi x + εO
|ψ> = (|↑↑↑> + |↓↓↓>)
|up>
|down>
and |ψ> = |down>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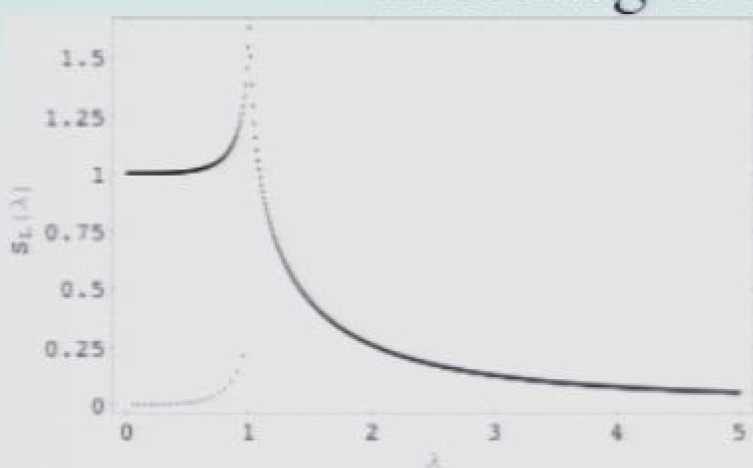

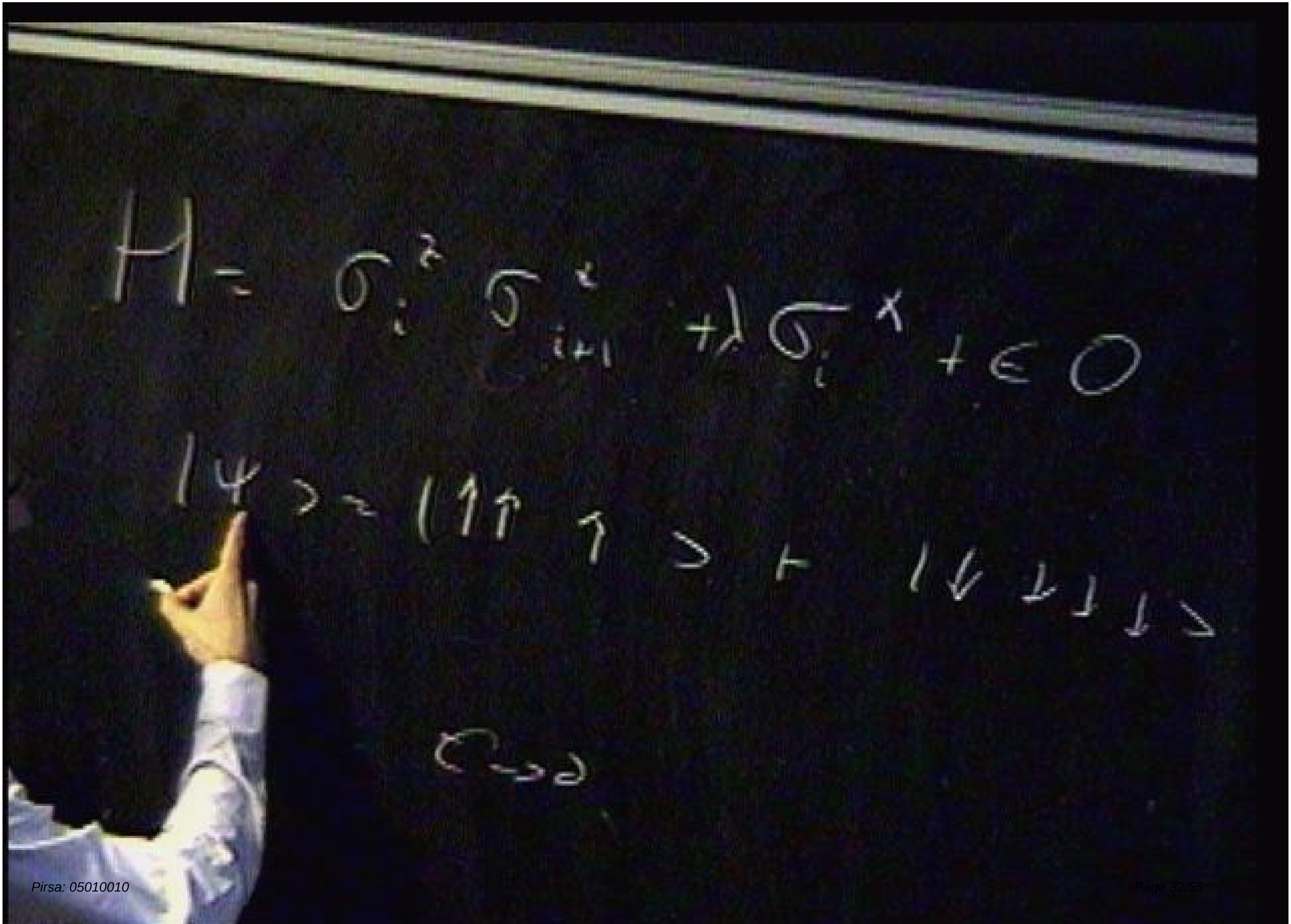
H = σ_i^z σ_{i+1}^z + λ σ_i^x + ε O
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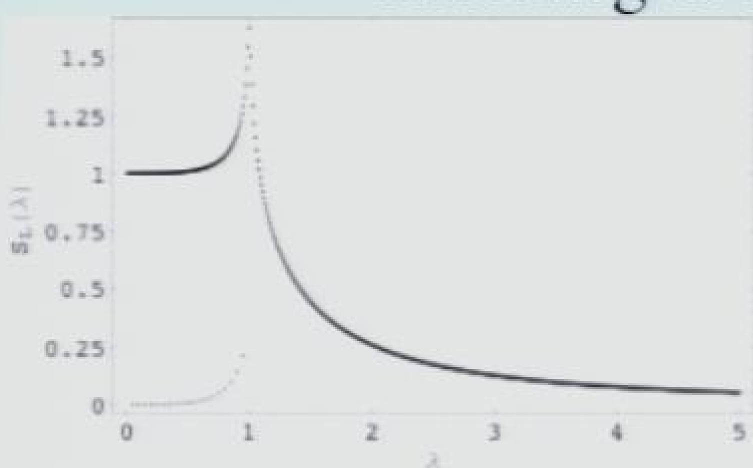


Entanglement Loss along RG-Flows

•**Global loss.-** Direct consequence of universal scaling of entanglement in 1+1D systems and conformal symmetry

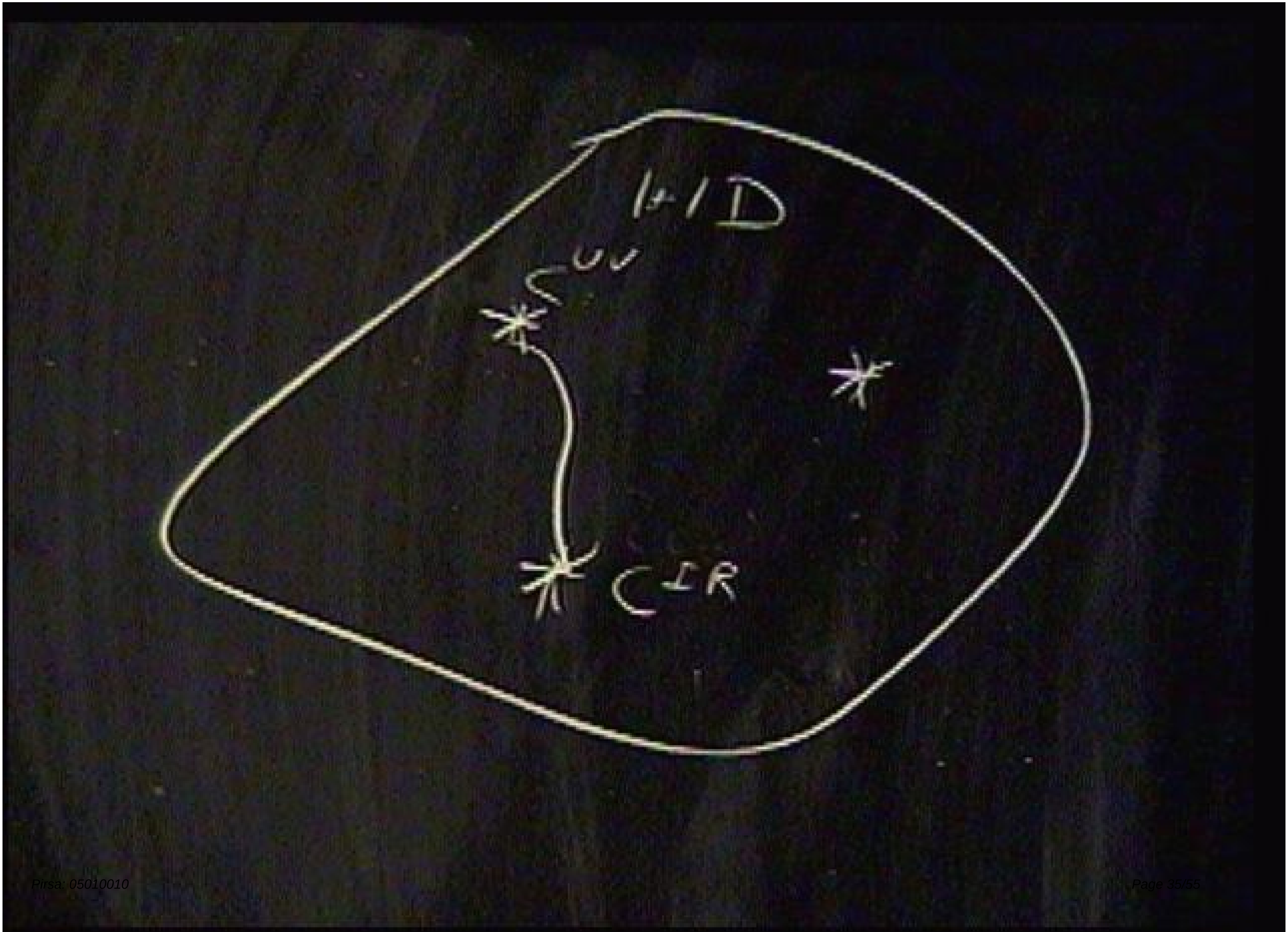
c-theorem: There is a monotonic nonincreasing function C along the Renormalization Group Flow.

Zamolodchikov (1986)

$$S = \frac{c+\bar{c}}{6}\log_2 L \qquad c^{UV} > c^{IR} \Rightarrow S^{UV} > S^{IR}$$

•**Monotonic loss of entanglement.-** Entanglement is nonincreasing step by step under renormalization group flows.

Casini, Huerta (2004)


1+1D
C UV
C IR

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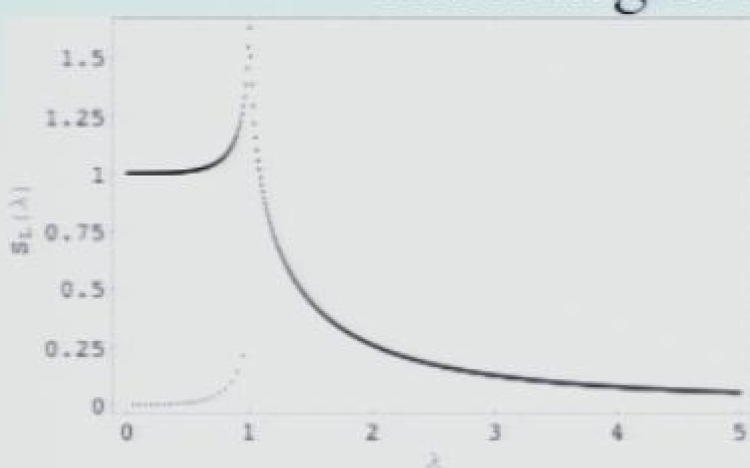

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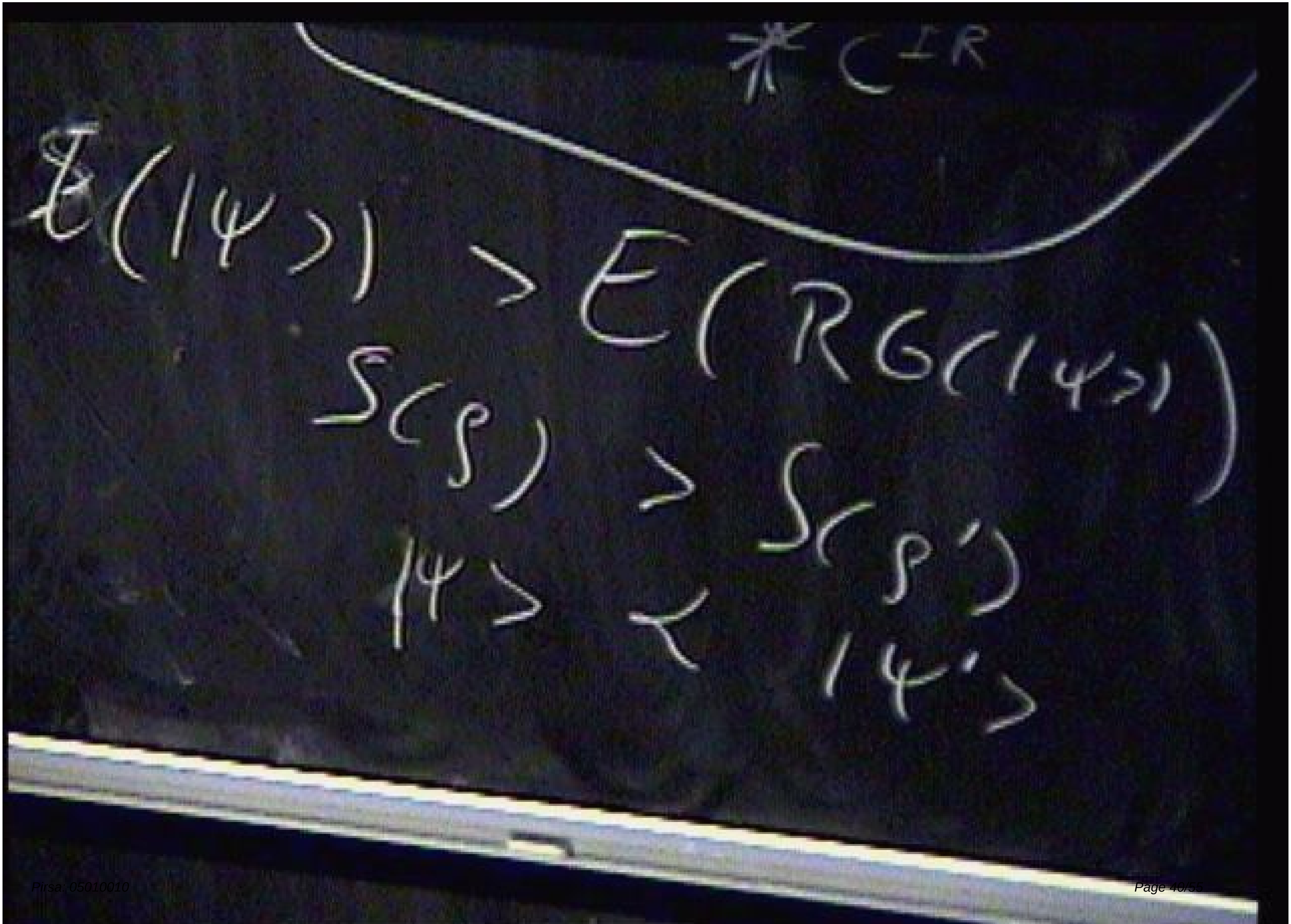
Casini, Huerta (2004)

$$E(|\psi\rangle) > E(RG(|\psi\rangle))$$

$$S(\rho) > S(\rho')$$


* C IR
E(|ψ⟩) > E(RG(|ψ⟩))
S(ρ) > S(ρ')
|ψ⟩ < |ψ'⟩

Entanglement Loss along RG-Flows

Majorization.-Partial order relation $\vec{p} \prec \vec{q}$

$$\vec{p} = D\vec{q} = \left(\sum_j r_j \Pi_j\right)\vec{q}$$

$$\sum_j r_j = 1$$

$$p_1 \leq q_1$$

$$p_1 + p_2 \leq q_1 + q_2$$

$$\cdots\cdots\cdots\cdots\cdots$$

$$\sum_i p_i \leq \sum_i q_i$$

•**Fine-grained Loss of Entanglement.-**Majorization at any infinitesimal step in the RG-Flow.

$$(\lambda,\gamma) \xrightarrow[RG-flow]{} (\lambda',\gamma')$$

$$\rho(\lambda,\gamma) \prec \rho(\lambda',\gamma') \Rightarrow S(\rho(\lambda,\gamma)) > S(\rho(\lambda',\gamma'))$$

Entanglement Loss along RG-Flows

- **Fine-grained Loss of Entanglement.-**

Heisenberg & XY model:

$$\rho(L,\gamma,\lambda) = \prod_{\otimes k} \rho_k(L,\gamma,\lambda)$$

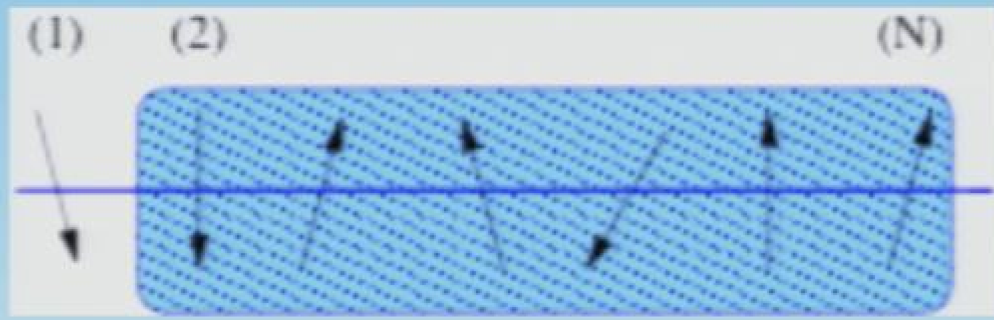
$$\rho_k[L,\gamma,\lambda] = \frac{\exp(-n_k \varepsilon_k[L,\gamma,\lambda])}{Z_k[L,\gamma,\lambda]}$$

Peschel (2004)

$$\rho_1 \prec \rho_1' \Rightarrow \rho_1 = D\rho_1' \qquad \rho_2 \prec \rho_2' \Rightarrow \rho_2 = \tilde{D}\rho_2'$$

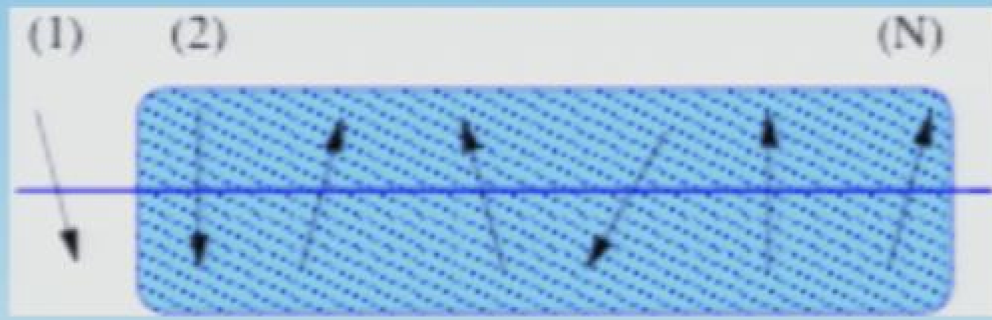
$$(\rho_1 \otimes \rho_2) = (D \otimes \tilde{D})(\rho_1' \otimes \rho_2') \Rightarrow \rho_1 \otimes \rho_2 \prec \rho_1' \otimes \rho_2'$$

Matrix Product States



$$\left|\psi\right\rangle = \sum_{a=1}^{D} \lambda_a^{(1)} \left|\phi_a^{(1)}\right\rangle \left|\phi_a^{(2,\ldots N)}\right\rangle = \sum_{a=1}^{D} \sum_{s_1=1}^{d} \lambda_a^{(1)} \left|s_1\right\rangle \left\langle s_1 \middle| \phi_a^{(1)}\right\rangle \left|\phi_a^{(2,\ldots N)}\right\rangle$$

Matrix Product States



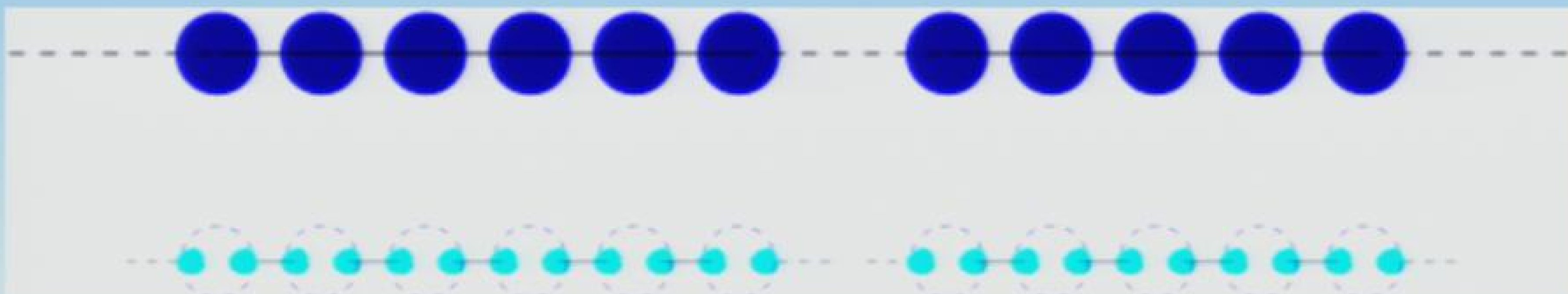
$$\left|\psi\right\rangle = \sum_{a=1}^{D} \lambda_a^{(1)} \left|\phi_a^{(1)}\right\rangle \left|\phi_a^{(2,\ldots N)}\right\rangle = \sum_{a=1}^{D}\sum_{s_1=1}^{d} \lambda_a^{(1)} \left|s_1\right\rangle\left\langle s_1 \middle| \phi_a^{(1)}\right\rangle \left|\phi_a^{(2,\ldots N)}\right\rangle$$

$$= \sum_{a=1}^{D}\sum_{s_1=1}^{d} \left|s_1\right\rangle A_a[s_1] \left|\phi_a^{(2,\ldots N)}\right\rangle = \sum_{a,b=1}^{D}\sum_{s_1,s_2=1}^{d} \left|s_1\right\rangle A_a[s_1] \left|s_2\right\rangle A_{ab}[s_2] \left|\phi_b^{(3,\ldots N)}\right\rangle$$

$$= \cdots = \sum_{\{s_j\}=1}^{d} tr\left(A[s_1]\cdots A[s_N]\right)\left|s_1\right\rangle\ldots\left|s_N\right\rangle$$

Matrix Product States

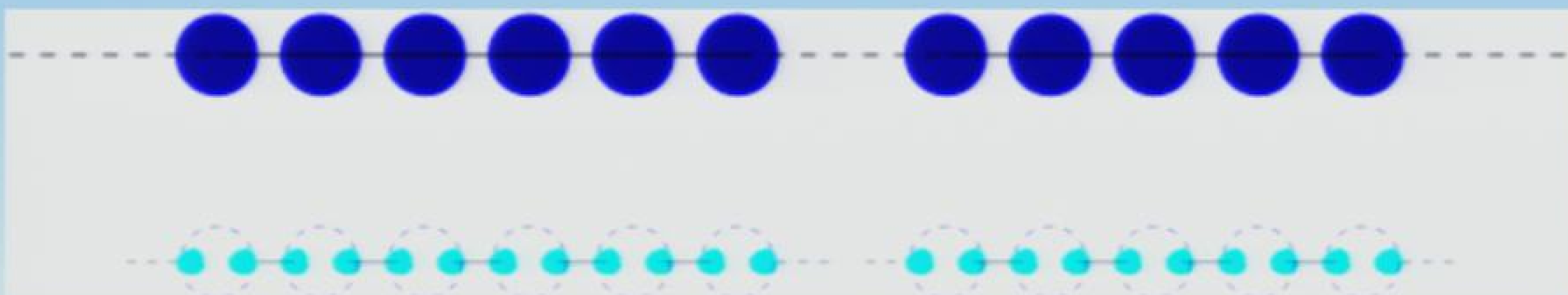
$$A_{D\times D}[s]|s\rangle_{s=1}^{d}$$



$$C^D \otimes C^D \xrightarrow{A} C^d$$

Matrix Product States

$$A_{D\times D}[s]\left|s\right\rangle_{s=1}^{d}$$



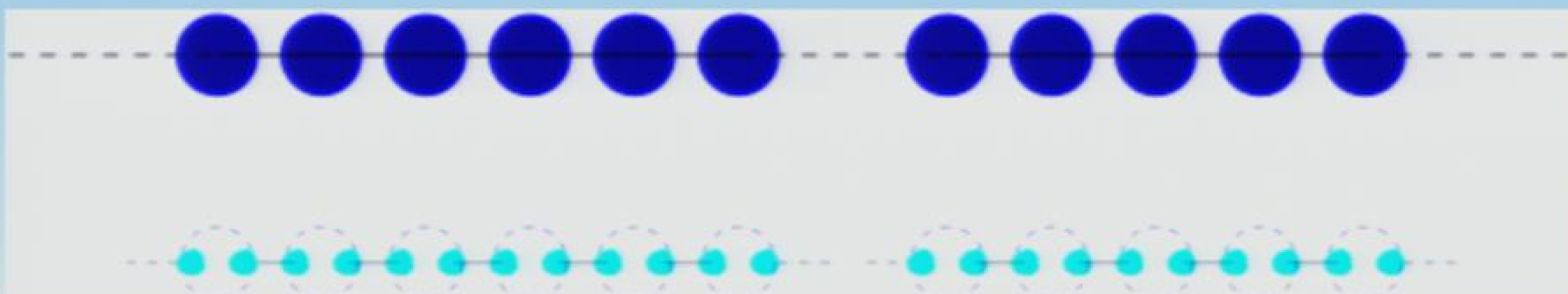
$$C^D \otimes C^D \xrightarrow{A} C^d$$

Properties of MPS by definition

ariant

Matrix Product States

$$A_{D\times D}[s]|s\rangle_{s=1}^{d}$$

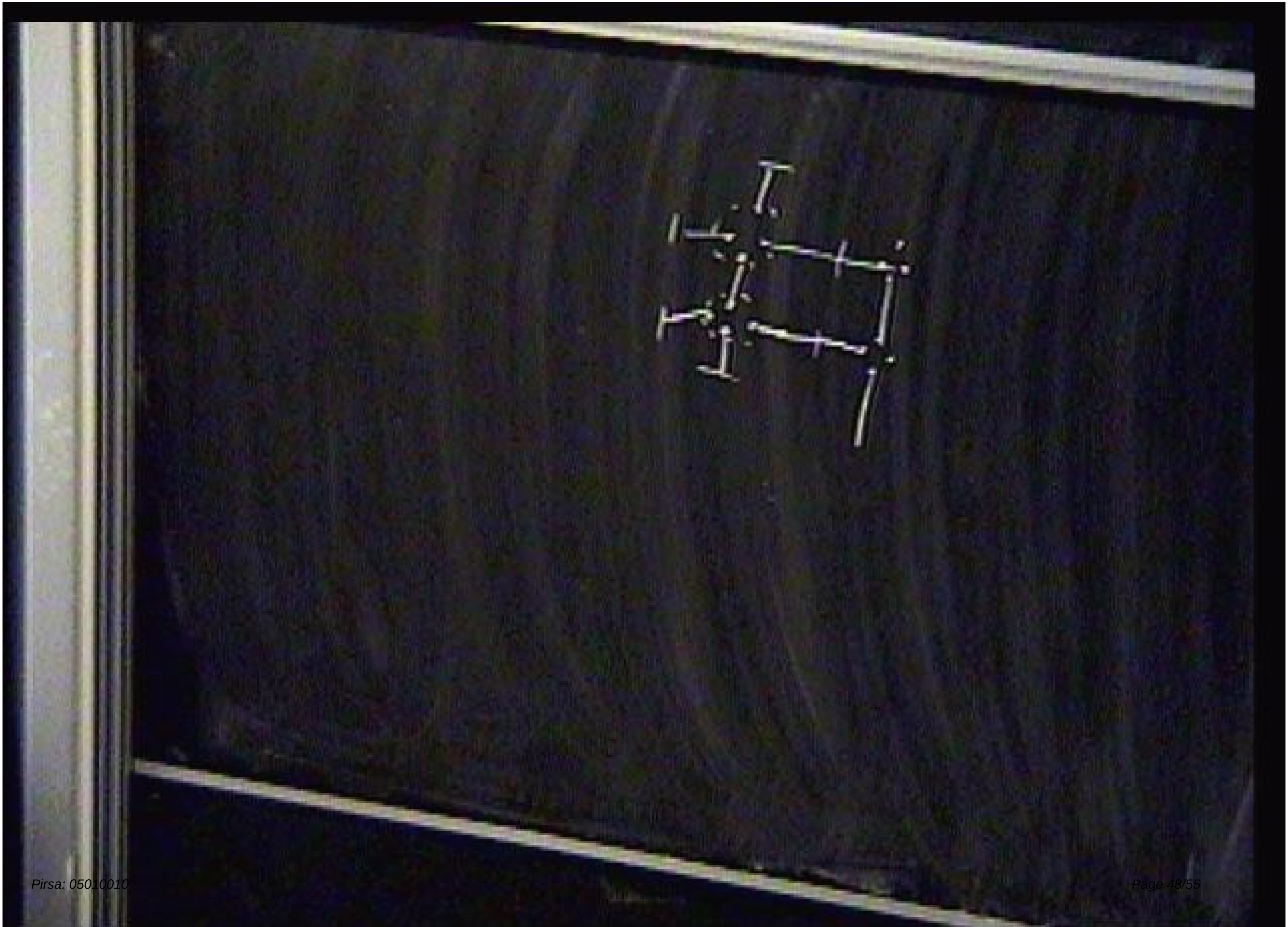


$$C^D \otimes C^D \xrightarrow{A} C^d$$

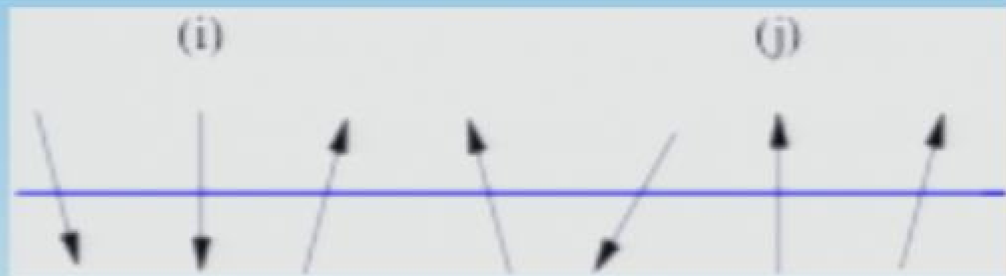
Properties of MPS by definition

1. Translational invariant

picture)



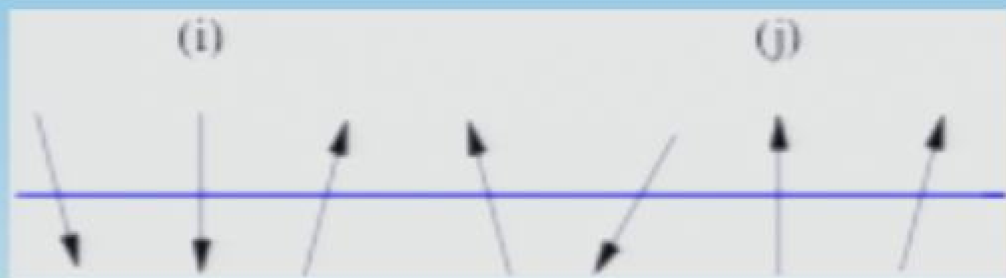
Fixed Points in Matrix Product States



$$\langle O_i O_j \rangle = tr\left(E^{N-j+i-2}\tilde{O}_i E^{j-i}\tilde{O}_i\right)$$

$$E = \sum_{s=1}^{d} A^*[s] \otimes A[s] \qquad \tilde{O} = \sum_{s,s'=1}^{d} A^*[s] \otimes A[s']\langle s|O|s'\rangle$$

Fixed Points in Matrix Product States



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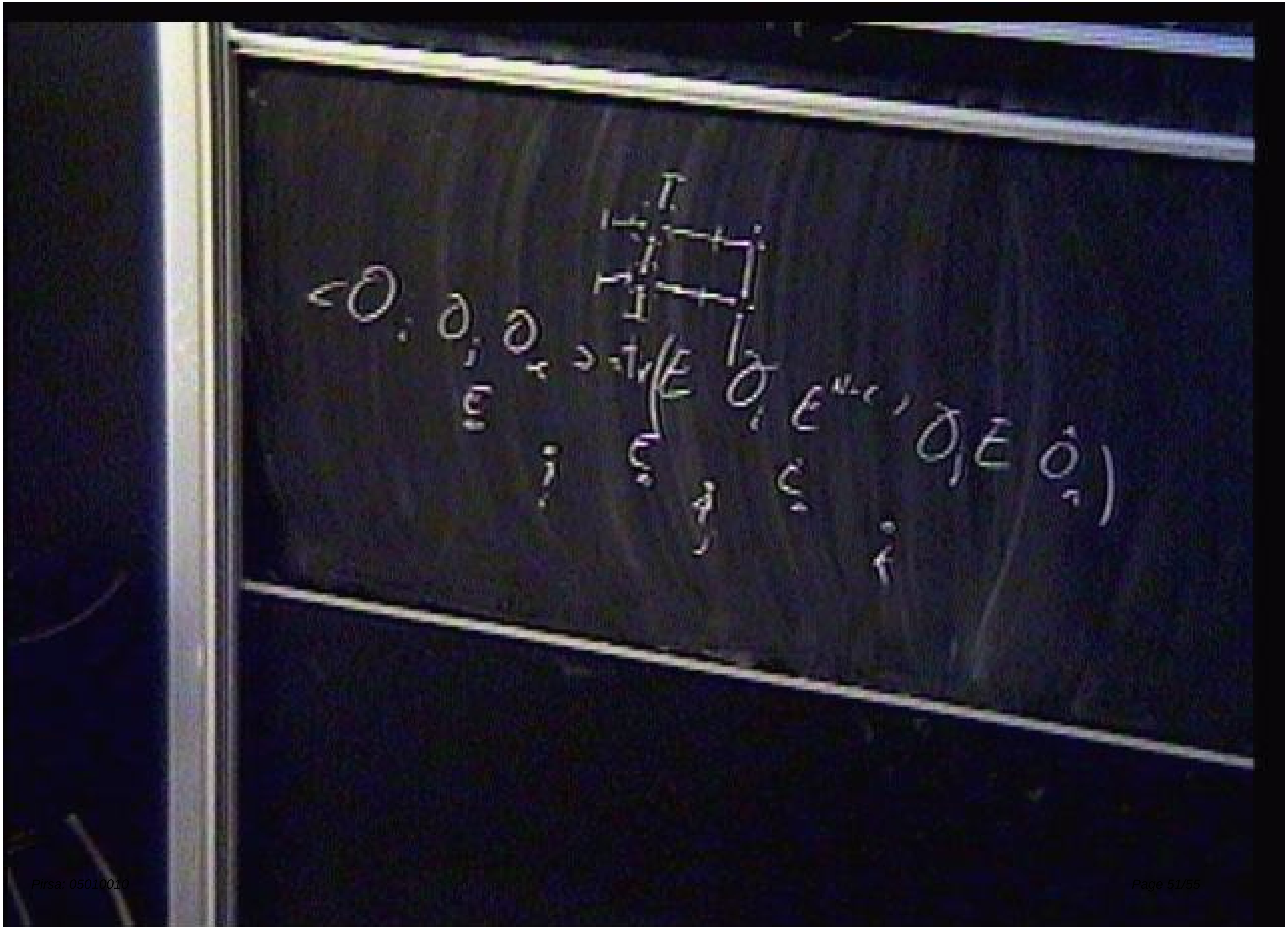
Scale Transformation

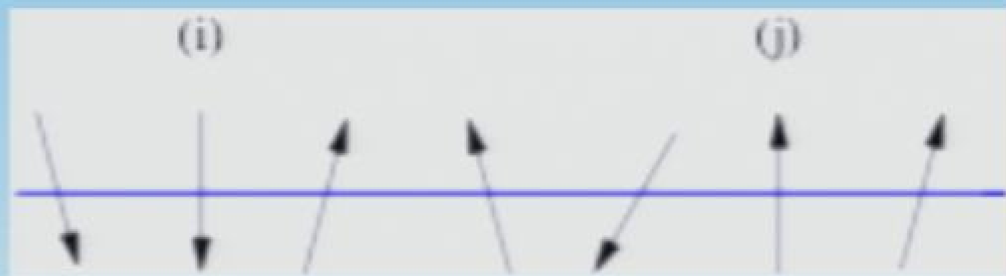
$$E \rightarrow E' = E \cdot E = \sum_{s'=1}^{d'} A'^*[s'] \otimes A'[s']$$

Fixed Points

$$E = E' = E \cdot E$$



Fixed Points in Matrix Product States



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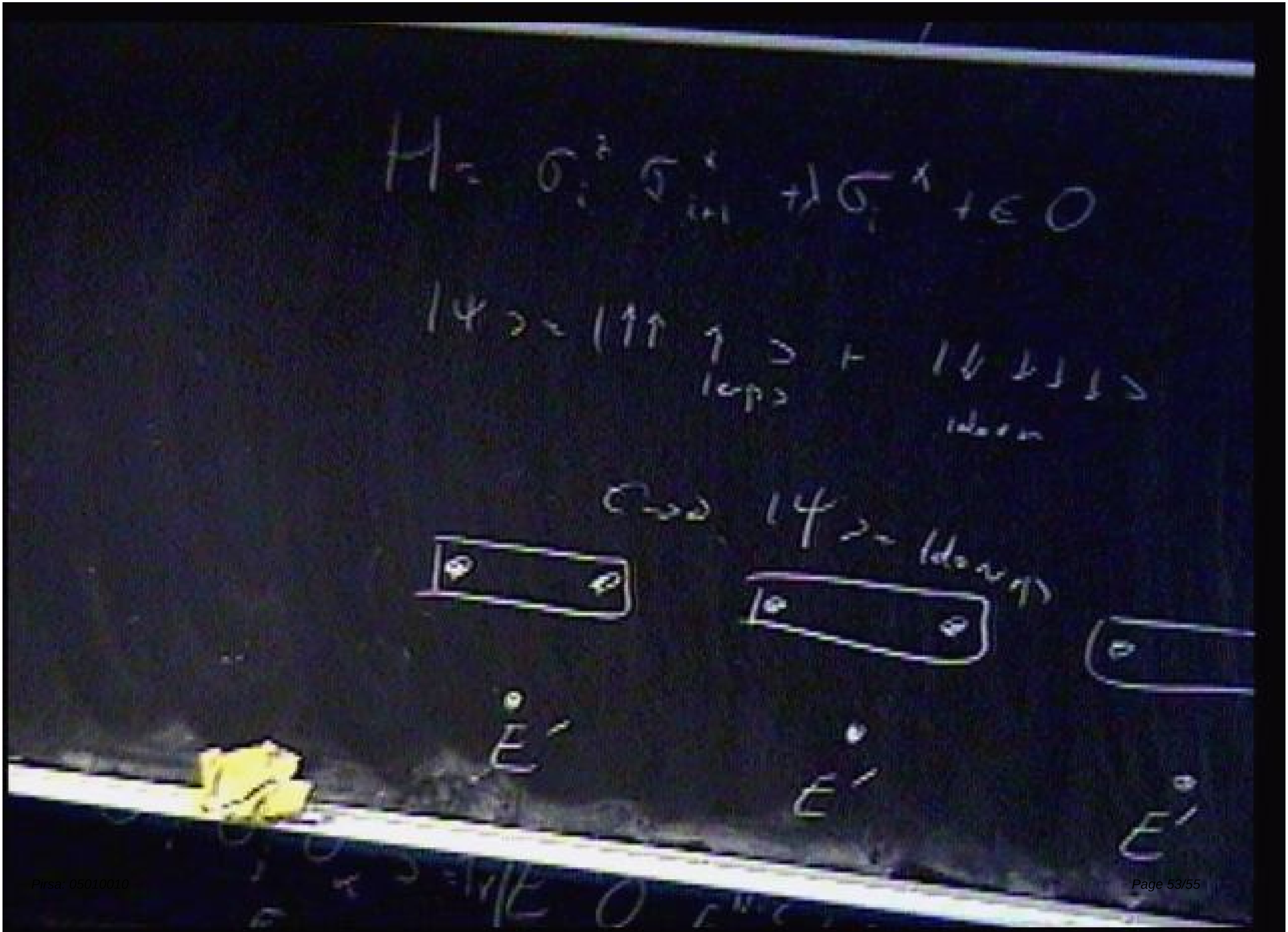
$H = \sigma_i^z \sigma_{i+1}^z + \lambda \sigma_i^x + \epsilon O$
$|\psi\rangle = |\uparrow\uparrow\uparrow\uparrow\rangle + |\downarrow\downarrow\downarrow\downarrow\downarrow\rangle$
$|ups\rangle$
$|down\rangle$
$|\psi\rangle = |downs\rangle$
E'
E'
E'

Conclusions

- Universal Scaling Law of Entanglement in 1+1 D Systems in the critical intervals
- Entanglement behaviour is controlled by Conformal Symmetry
- Away from the critical points Entanglement is saturated by a "mass" scale
- DMRG (Density Matrix Renormalization Group) failure in higher Dimensions
- Entanglement is no increasing under RG Flows
- Relation between Majorization and C-functions
- Description of Real Space Renormalization Group Transformations within Matrix Product States

Entanglement, Critical Phenomena & RG Flows

•Entanglement in Quantum Critical Phenomena.
G. Vidal, J.I. Latorre, E. Rico, A. Kitaev. Phys.Rev.Lett. 90 (2003) 227902

•Ground State Entanglement in Quantum Spin Chains.
.I. Latorre, E. Rico, G. Vidal. Quant. Inf. & Comp. Vol.4 no.1 (2004) pp.048-092

•Fine-Grained Entanglement Loss along Renormalization Group Flows.
J.I. Latorre, C.A. Lütken, E. Rico, G. Vidal. quant-ph/0404120

•Renormalization Group Transformation on quantum states.
F. Verstraete, J.I. Cirac, J.I. Latorre, E. Rico, M.M. Wolf, quant-ph/0410227

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