Title: Interpretation of Quantum Theory: Lecture 5

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Abstract:

What does it mean to interpret a theory?

Operational vs Ontological Bridge-Principles

Operational Bridge-Principles

Operational bridge-principles are operational rules that relate elements of the formalism to measurements that may be performed.

These rules provide the adequate information to use quantum theory in the lab, i.e., to explain experimental outcomes (observations).

Operational rules do not give insight into the nature of the underlying physical reality of the systems described by quantum theory.

An important example of an operational bridge-principle in quantum theory is the Born rule, which tells us the relative frequency (probability) with which outcome k is observed given the same measurement repeated on an ensemble of identically prepared systems:

$$\mathrm{Prob}(k|\rho) = \mathrm{tr}(\rho|\phi_k\rangle\langle\phi_k|)$$

Ontological Bridge-Principles

Ontological bridge-principles are a set of correspondence rules that relate elements of the mathematical formalism to elements of physical reality.

Bohr's "Copenhagen Interpretation" can be considered ontological, in spite of his denial of the meaningfulness of making statements about an independent reality,

> "An independent reality in the ordinary physical sense can neither be ascribed to the phenomena nor to the agencies of observation."

because he also insists that deducing additional information about what properties a system may have is impossible in principle.

The more complete analysis Einstein seeks "is *in principle* excluded."

The Orthodox (Dirac-von Neumann) Interpretation

1. Eigenvalue-eigenstate link:

> An observable has a determinate value *if and only if* the state is an eigenstate of that observable.

This is an ontological bridge-principle: it tells what properties a system possesses *independent of observation*.

The eigenvalue-eigenstate link implies that the *quantum state provides a complete description of a system's objective physical properties*, or put more boldly, of the objective elements of physical reality.

The completeness assumption implies that the unavoidable non-vanishing dispersion of outcomes for some observables (as demanded by Robertson's uncertainty principle) is due to a fundamental randomness (or stochasticity) in nature.

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Note that von Neumann was well aware that the existence of additional "hidden coordinates" provided another explanation for the non-vanishing dispersion associated with quantum states. However, he rejected this possibility because of an 'impossibility proof' that he devised against the possibility of dispersion-free assignments to all observables. However, von Neumann's proof was discredited much later by Bohm (1952), who constructed an explicit hidden variable model, and also by Bell (1966), who showed that one of von Neumann's assumptions was unreasonable in the sense that it could never be satisfied by a dispersion-free state.

2. The projection postulate:

> After an ideal measurement of an observable, the system state is transformed into [i.e., must be updated to] the eigenstate associated with the eigenvalue observed.

This postulate, also known as the collapse of the wavefunction, is *operationally demanded* for consistency with experiments involving sequential (ideal) measurements.

The eigenvalue-eigenstate link implies that the 'projection' is *a physical process* since it involves a transformation of the system's physical properties.

In contrast, if we reject the eigenvalue-eigenstate link, and if we reject that the quantum state is a complete description of a system's physical properties, then the 'projection' after measurement *does not correspond to not a physical process*.

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Projection Postulate with and without Post-Selection:

Consider the ideal measurement of a non-degenerate observable $R = \sum \lambda_k |\phi_k\rangle\langle\phi_k|$.

If the measurement outcome is ignored, then the following transformation,

$$\rho(t) \rightarrow \rho'(t) = \sum_k \langle\phi_k|\rho(t)|\phi_k\rangle \, |\phi_k\rangle\langle\phi_k|. \tag{1}$$

is required to describe the state after measurement.

If, on the other hand, the outcome is recorded, then consistency with subsequent measurements demands the following transformation:

$$\rho(t) \rightarrow \rho'(t) = |\phi_k\rangle\langle\phi_k|. \tag{2}$$

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Proof that projection (under post-selection) is not a unitary process

Conceptually it is clear that unitary evolution evolves any given state to a fixed final state. This is deterministic (in the sense of reproducible). In contrast, collapse is fundamentally stochastic: applying the same measurement to the same preparation produces different (apparently random) final states (depending on the outcome).

Is it possible that the final state outcome is not random but dependent on the quantum state associated with some additional degrees of freedom, and the whole process may be described by a unitary transformation?

Consider an atom described by a pure state corresponding to a coherent superposition of moving along two distinct trajectories. We arrange so that both trajectories pass through a detector such that a macroscopic pointer is moved to the 'left' if the atom is on the 'up' trajectory and to the 'right' if the atom is on the 'down' trajectory. We want to model the measurement process with a unitary transformation and for complete generality we extend the quantum system to include additional degrees of freedom denoted by a state $|\chi\rangle$. If we demand *faithful measurements* this means that we must have, for any $|\chi\rangle$,

$$U|\text{up}\rangle \otimes |\text{ready}\rangle \otimes |\chi\rangle = |\text{up}\rangle \otimes |\text{left}\rangle \otimes |\chi'\rangle$$

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\begin{aligned}
U|\mathrm{up}\rangle \otimes |\mathrm{ready}\rangle \otimes |\chi\rangle &= |\mathrm{up}\rangle \otimes |\mathrm{left}\rangle \otimes |\chi'\rangle \\
U|\mathrm{down}\rangle \otimes |\mathrm{ready}\rangle \otimes |\chi\rangle &= |\mathrm{down}\rangle \otimes |\mathrm{right}\rangle \otimes |\chi''\rangle
\end{aligned}
$$

where $|\chi'\rangle$ and $|\chi''\rangle$ are allowed to be independent of $|\chi\rangle$.

Now if we prepare a coherent superposition over atomic trajectories, and allow for both possible outcomes, then by linearity it follows that, for any χ,

$$
\begin{aligned}
U(\alpha|\mathrm{up}\rangle + \beta|\mathrm{down}\rangle) \otimes |\mathrm{ready}\rangle \otimes |\chi\rangle &= \alpha|\mathrm{up}\rangle \otimes |\mathrm{left}\rangle \otimes |\chi'\rangle \\
&\quad + \beta|\mathrm{down}\rangle \otimes |\mathrm{right}\rangle \otimes |\chi''\rangle
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so it is impossible that after the interaction the state is driven to one or the other outcome. Hence the transformation (2) can not be modeled by a unitary transformation.

Proof that projection (without post-selection) can be represented by a unitary process

If we describe only the state of the system and pointer, then we must take a partial trace over the ancillary degrees of freedom represent by $|\chi\rangle$. This partial trace produces the following state (after measurement),

$$\rho = |\alpha|^2 \, |\text{up}\rangle\langle\text{up}| \; + \; |\beta|^2 \, |\text{down}\rangle\langle\text{down}| \; + \\ \alpha\beta^* \, \langle\chi''|\chi'\rangle \, |\text{up}\rangle\langle\text{down}| + \alpha^*\beta \, \langle\chi'|\chi''\rangle \, |\text{down}\rangle\langle\text{up}|$$

If the ancillary states are orthogonal $\langle\chi'|\chi''\rangle = 0$, then we recover the projection postulate describing the final state when the outcome is ignored or unknown:

$$\rho = |\alpha|^2 \, |\text{up}\rangle\langle\text{up}| \; + \; |\beta|^2 \, |\text{down}\rangle\langle\text{down}|.$$

Hence the projection transformation (1) (without post-selection) can indeed be modeled by a unitary transformation.

This important process is called "decoherence." It shows us that the non-classical features of a coherent superposition, such as interference, are eliminated if the system or apparatus is allowed to interact with ancillary degrees

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This important process is called "decoherence." It shows us that the non-classical features of a coherent superposition, such as interference, are eliminated if the system or apparatus is allowed to interact with ancillary degrees of freedom which are either ignored or unknown.

Consider the ancillary states to describe microscopic degrees of freedom associated with ever-present "environment" systems, such as dust particles, or the cosmic microwave background. These environment systems are unavoidably interacting with systems such as a macroscopic measurement apparatus, since they can never be fully isolated. Moreover, it is reasonable to infer that their states will become orthogonal after interacting with (reflecting off of) the macroscopically pointer states.

Does decoherence solve the measurement problem?

Can we interpret the final mixed state as an ordinary classical mixture of the two possible pointer positions?

A first problem with this approach is the “ambiguity of mixtures” (discussed last week). While the state describing the final pointer state may be interpreted as a classical mixture of the two possible pointer positions, this is a non-unique decomposition of the mixed state. It is possible also to re-express the state as a mixture of two very non-classical states that have nothing to do with well-defined pointer positions. This is called the “preferred basis problem”.

A second problem is that the ‘total system’ is still in a pure state (coherent entangled superposition). The state for the combined system clearly does not allow the assignment of definite position properties for elements of the combined system (consisting of the pointer and the atom and the ancillary degrees of freedom). Is it self-consistent to deny definite properties for the combined system while asserting definite properties for a subsystem?

Can we conclude that there is no longer a conflict between the orthodox interpretation and the existence of definite position property for the macroscopic pointer?

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Can we conclude that there is no longer a conflict between the orthodox interpretation and the existence of definite position property for the macroscopic pointer?

Recall that the eigenvalue-eigenstate link of the orthodox interpretation tells us that definite properties for the positions of the atom and pointer should be assigned if and only if the combined state is a factorable state of the form,

$$|\psi\rangle = |\text{up}\rangle \otimes |\text{left}\rangle.$$

A mixed state obtained by partial tracing over the environment (or over the environment and the atom) is not in this form and therefore can not be assigned a definite property.

Hence, even if decoherence effects are taken into account, the orthodox interpretation still needs the projection postulate to explain the existence of macroscopic facts.

Contemporary Interpretations

A consistent description of macroscopic facts requires either expanding upon or rejecting the interpretative postulates of the orthodox interpretation.

'Dynamical collapse' interpretations specify the exact conditions under which collapse occurs by adding a non-linear term to the Schrodinger equation. Strictly speaking these interpretations are actual modifications of the mathematical formalism and not just interpretations in the sense of specifying ontological bridge-principles. P. Pearle will describe spontaneous collapse models to us in March.

The many-worlds interpretation developed by Everett (1957) rejects the projection postulate and imagines reality dividing into alternate but equally valid branches. This interpretation will be described to us next week by D. Wallace.

In many contemporary interpretations the effects of decoherence play a pivotal role in defining the ontology.

> One example is the "existential interpretation" advocated by W. Zurek (1993), which is a variation of the many-worlds interpretation.
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On the other hand there are interpretations which seek to explicitly identify the additional 'hidden variables' needed for a complete specification of the system's properties. The most important example of this kind of interpretation is the de Broglie-Bohm (1927/1952) pilot wave theory, which will be introduced to us by S. Goldstein in February, and further elaborated by A. Valentini in March.

On Thursday we will spend our last introductory lecture discussing the constraints on hidden variables.


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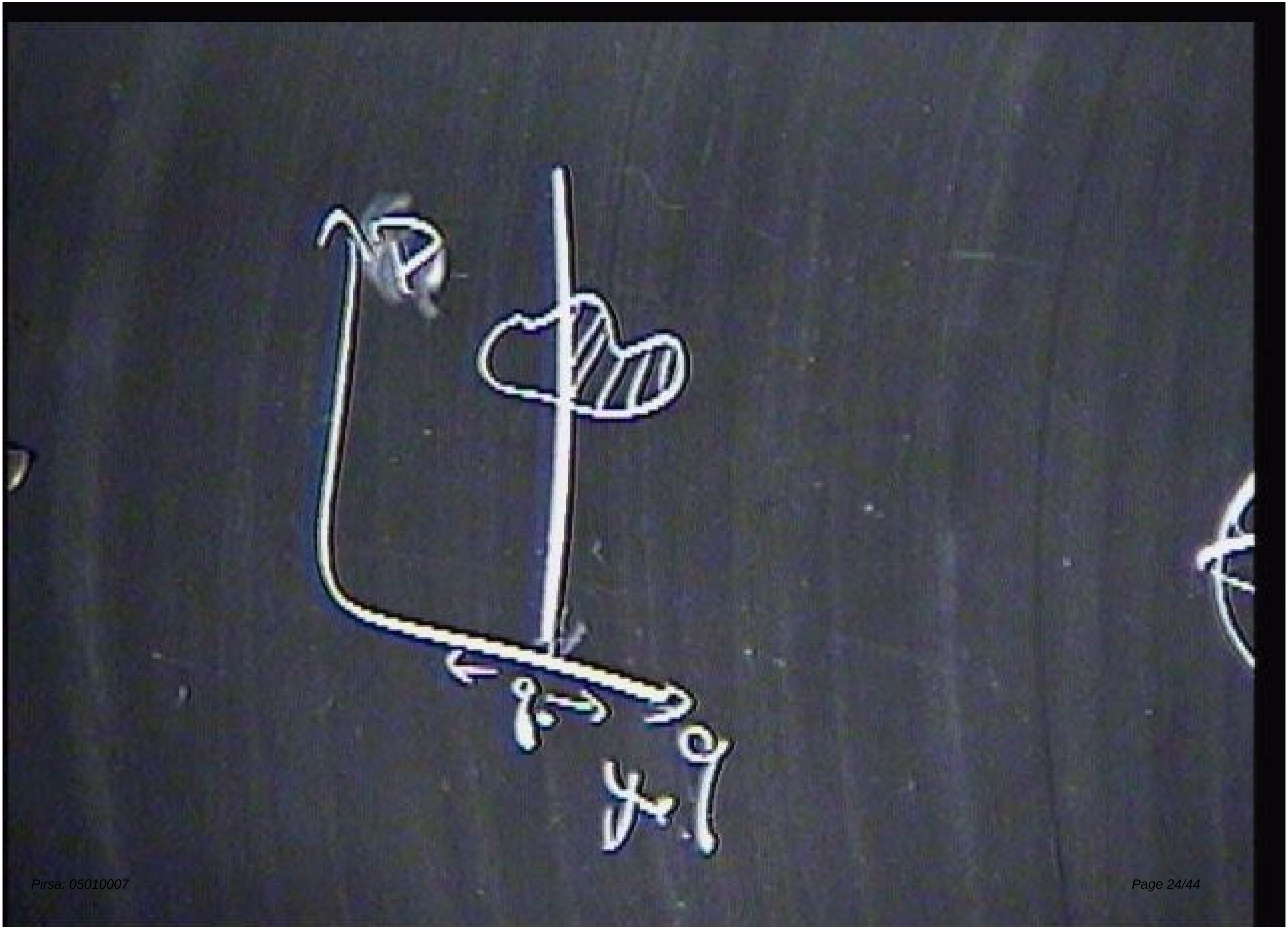
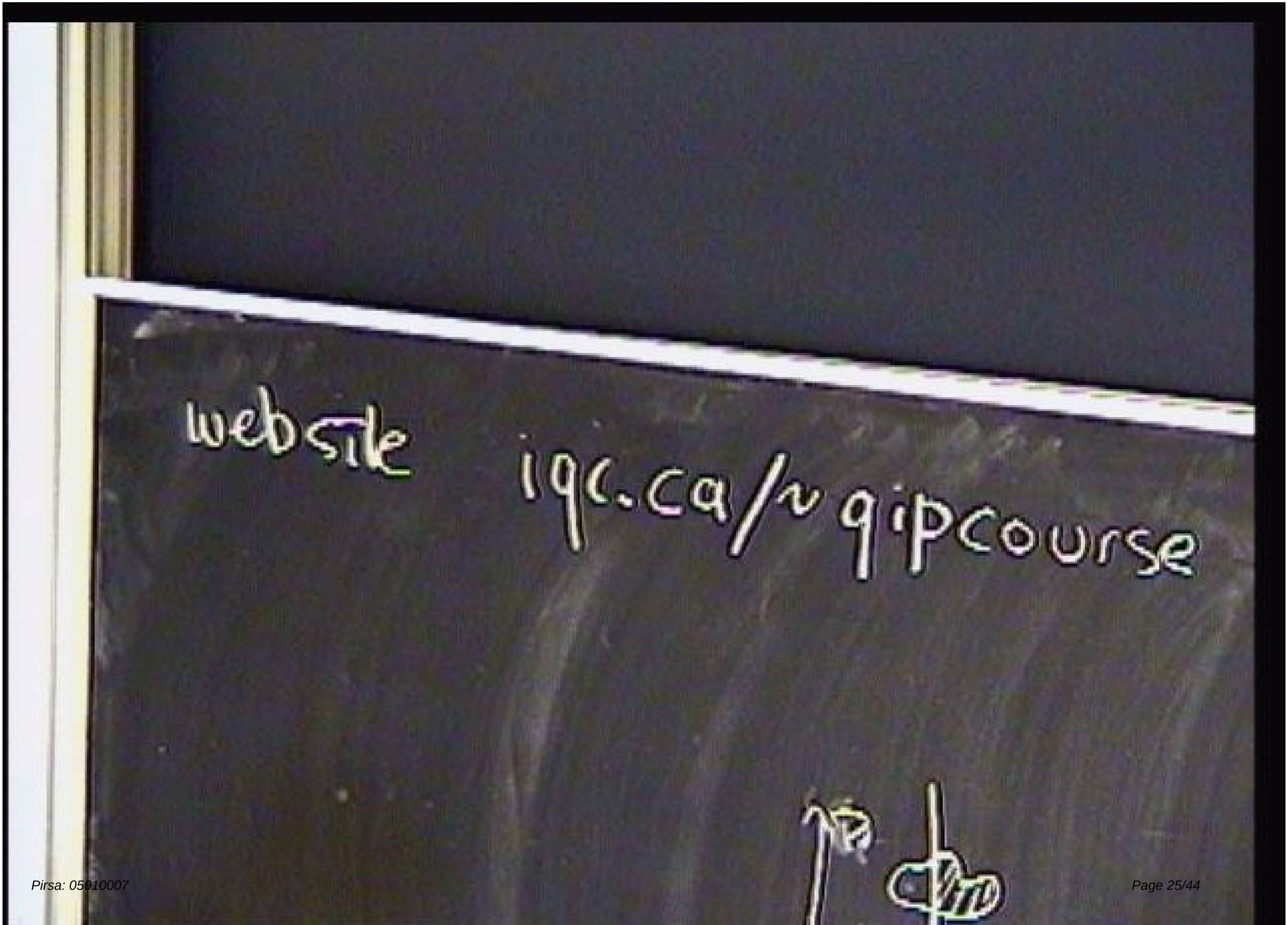
Yes
x
Yes

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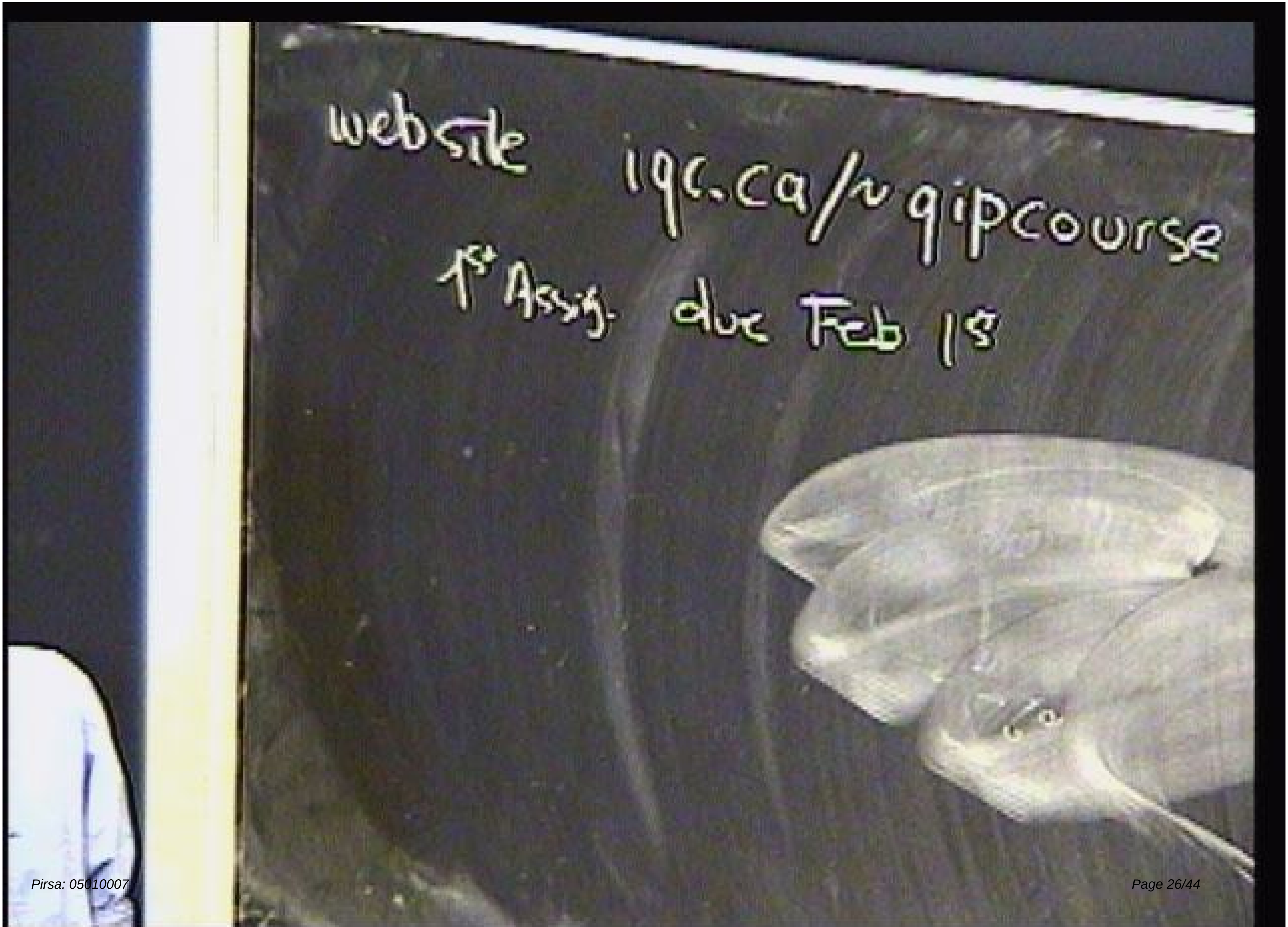
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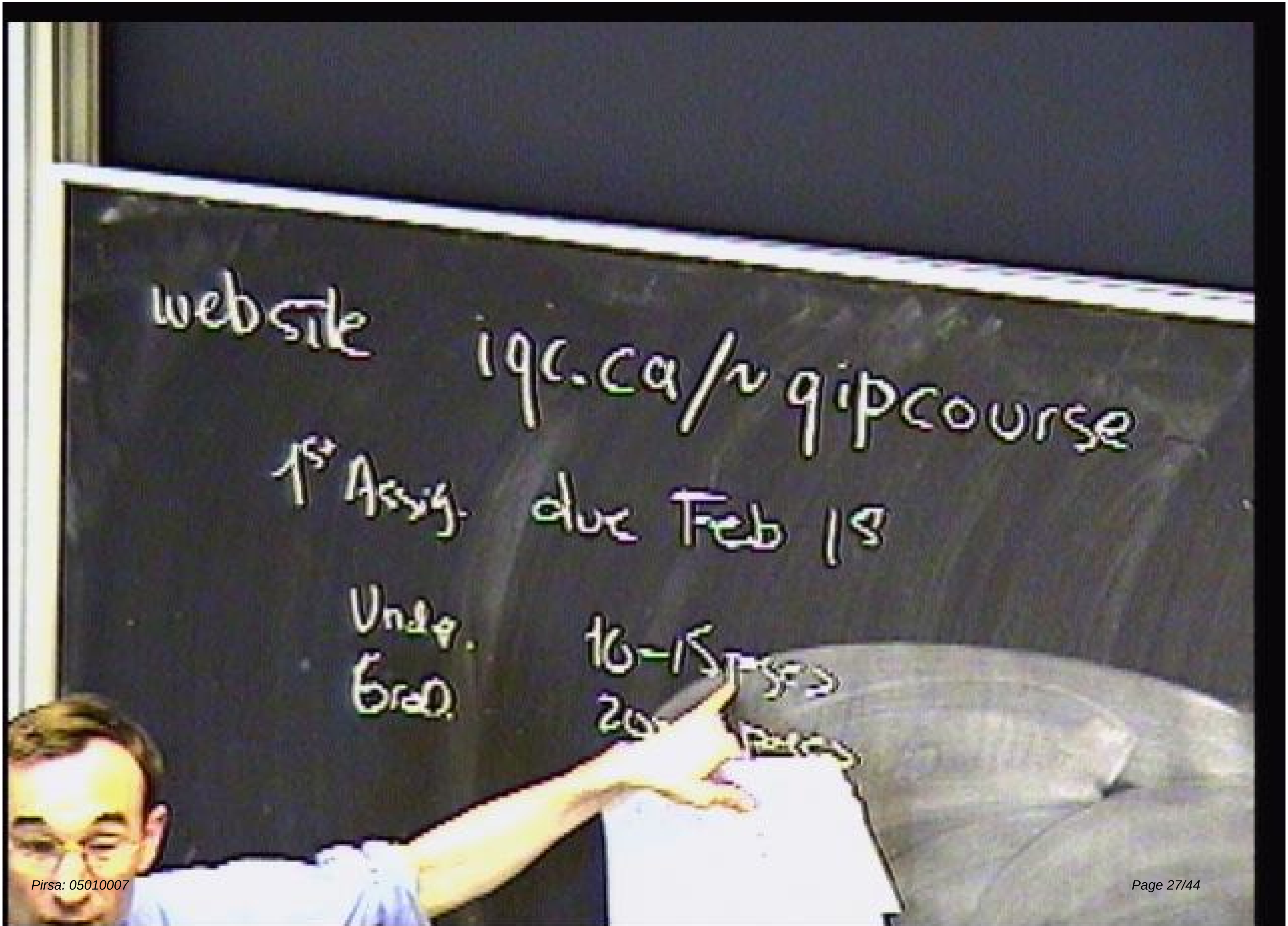
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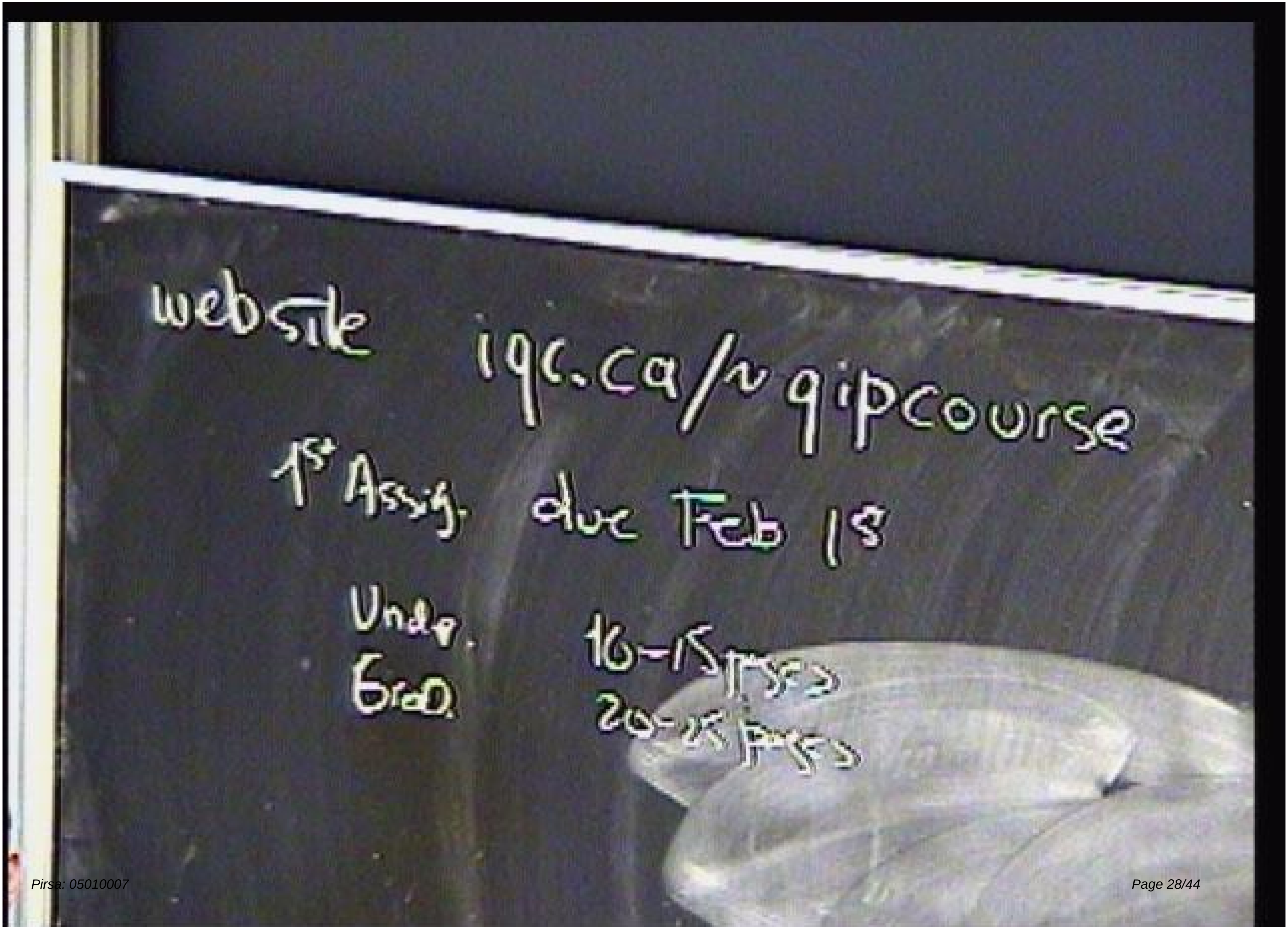
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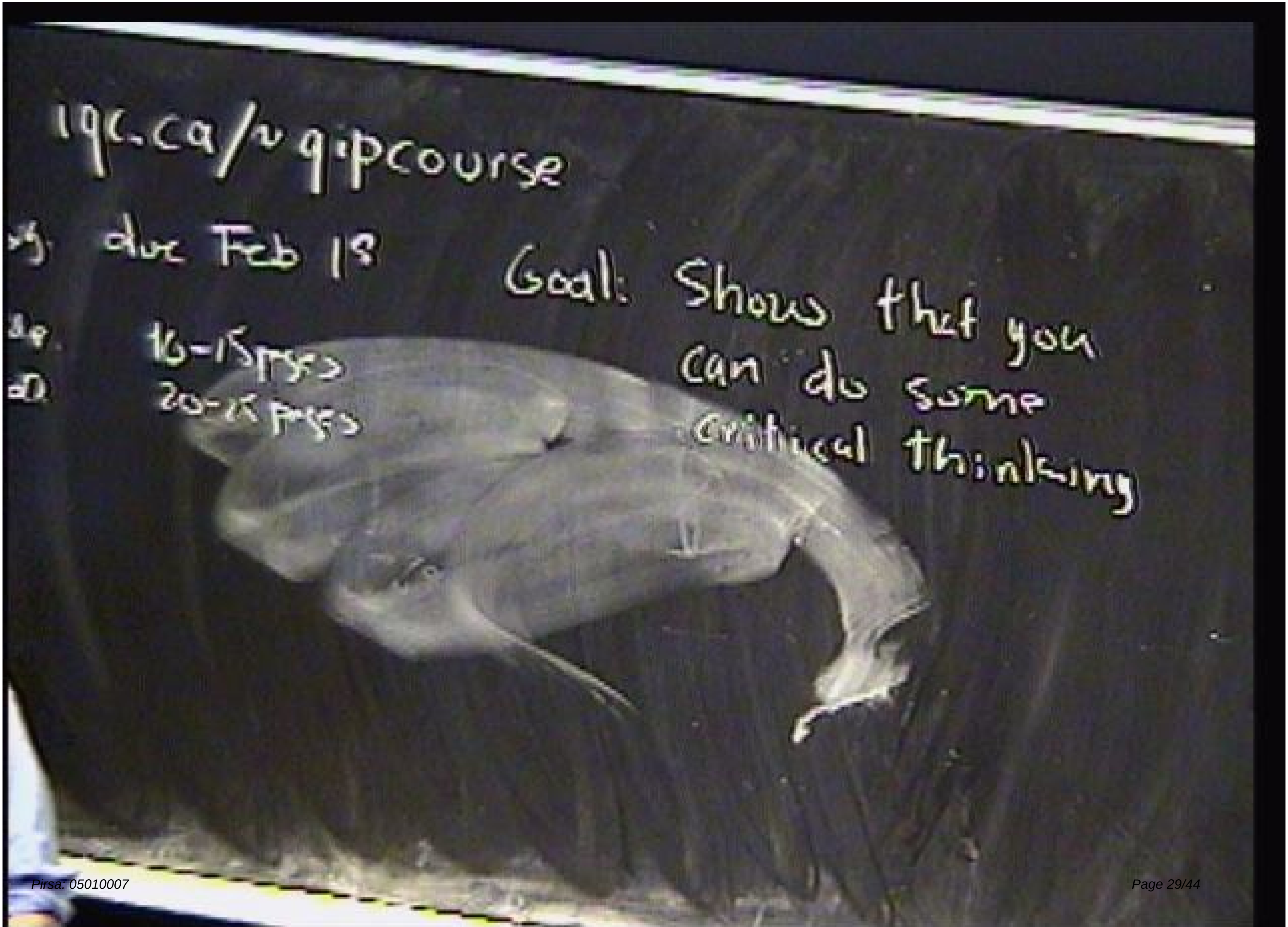
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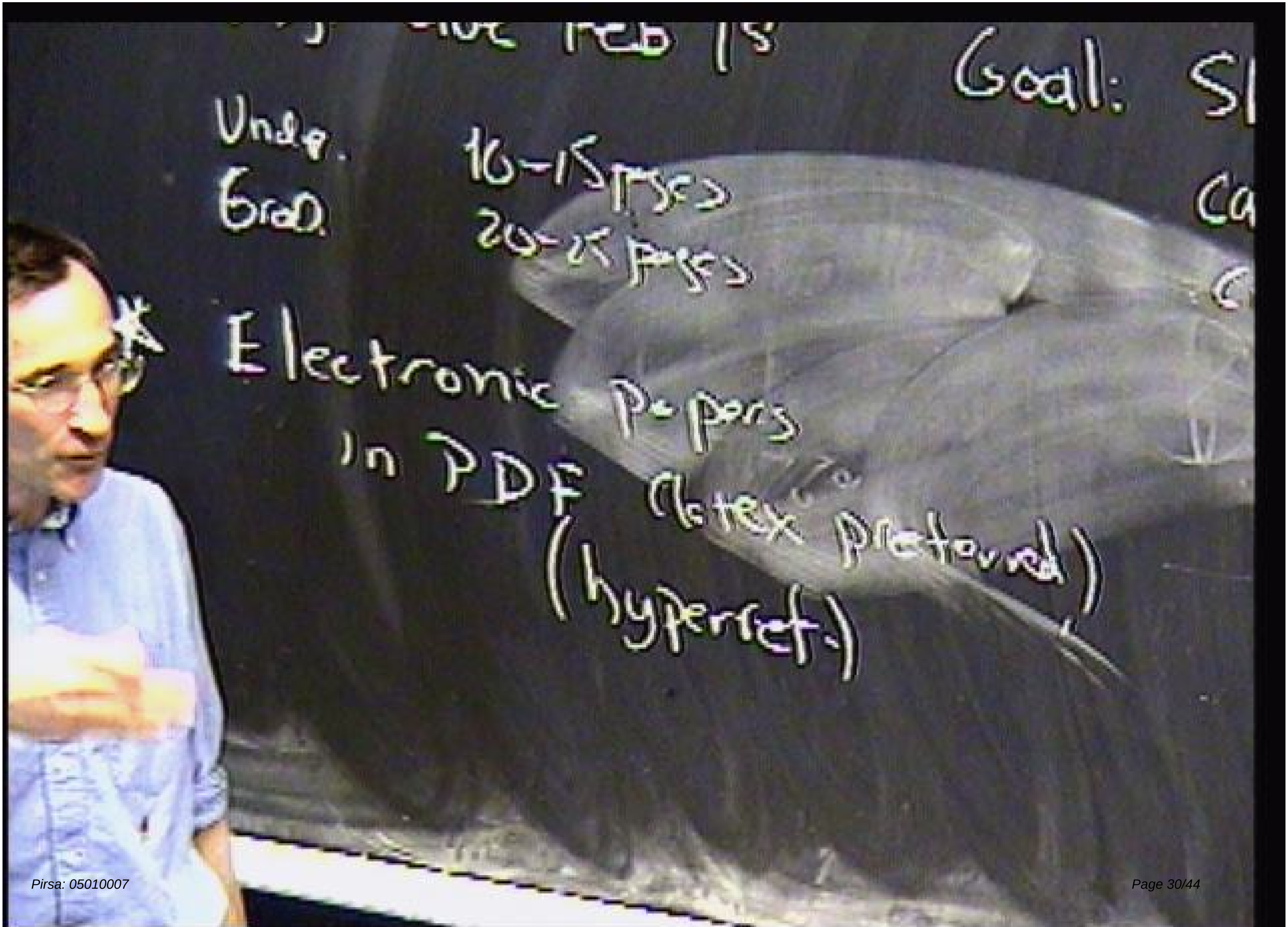
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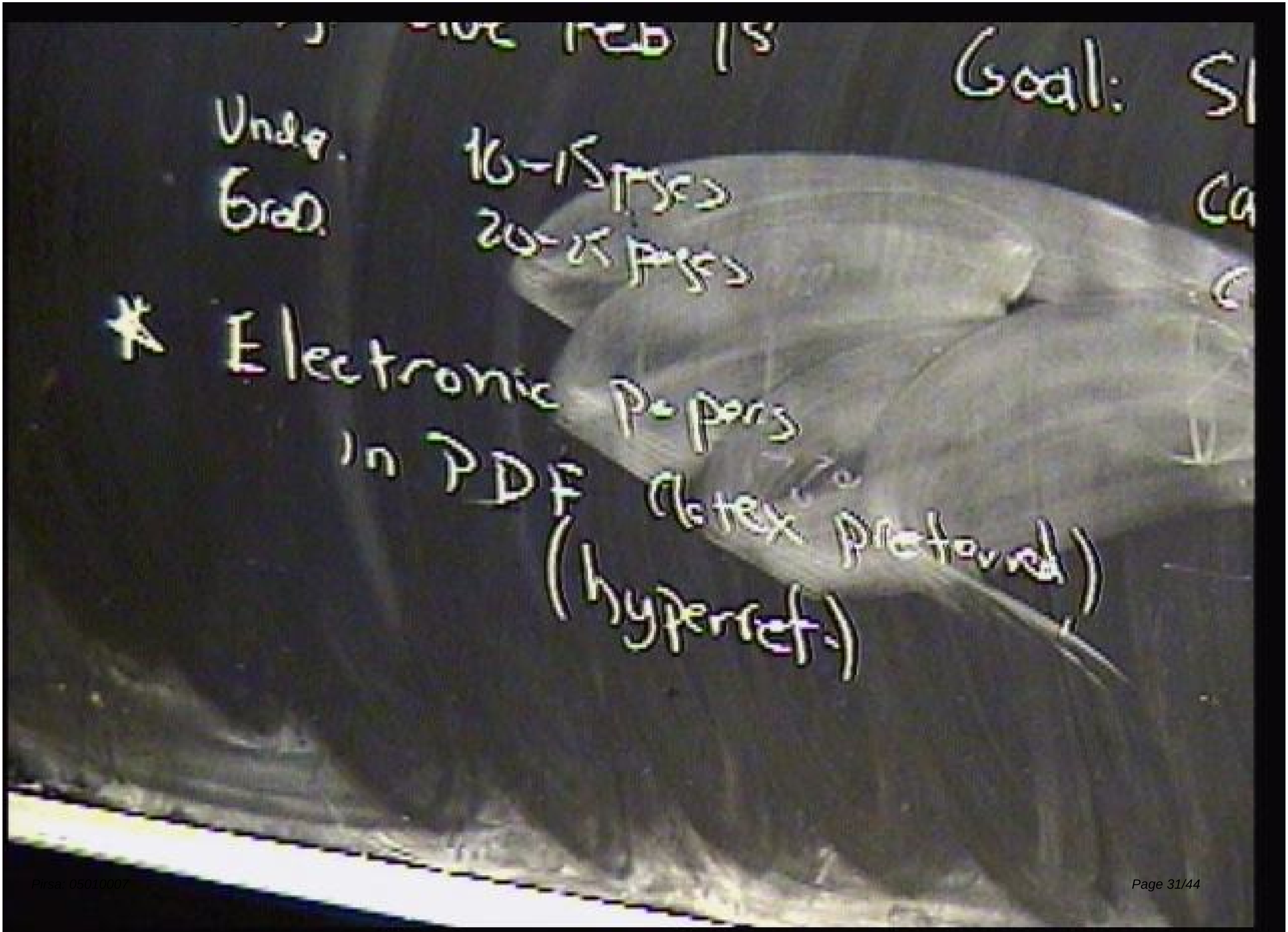
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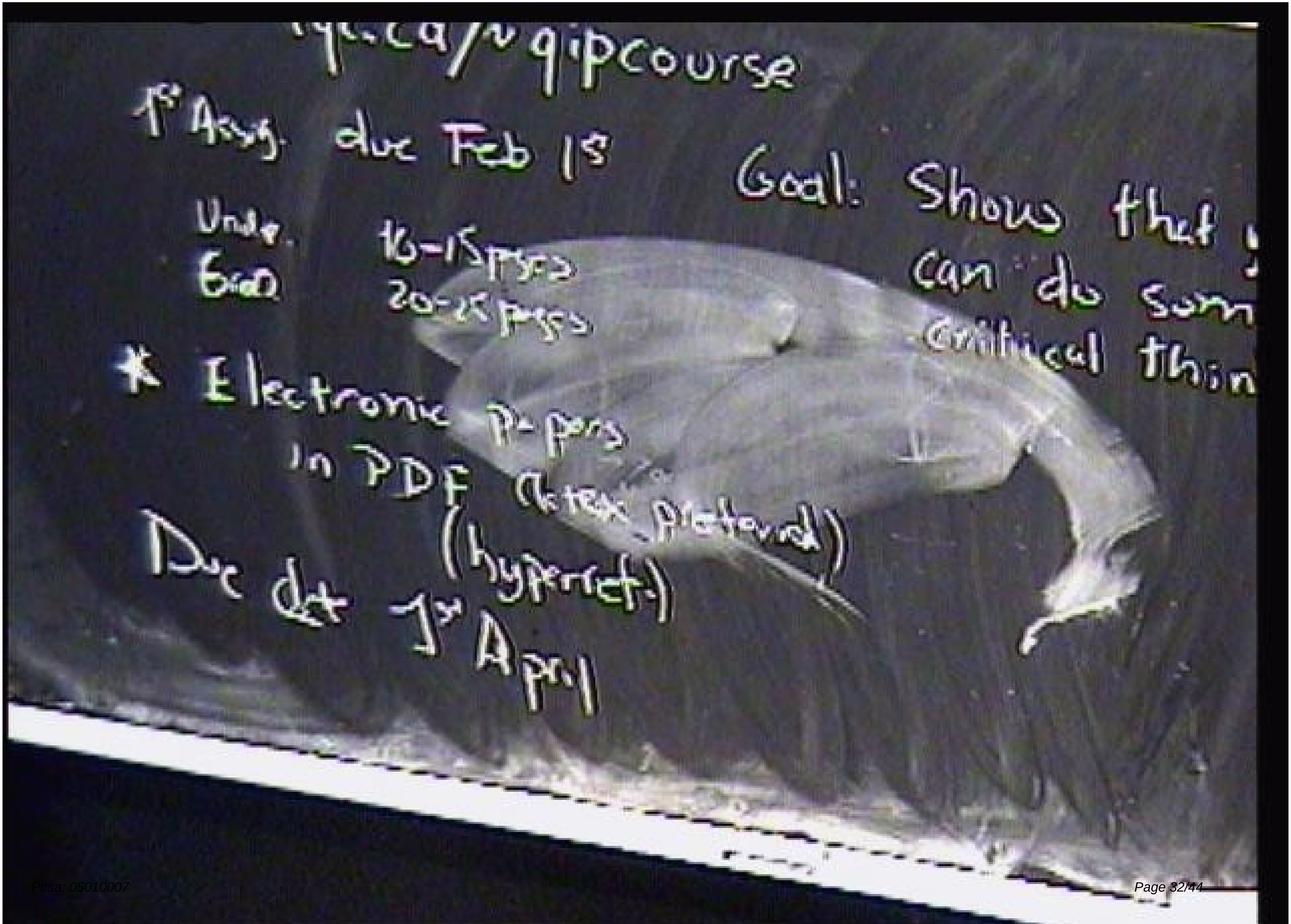
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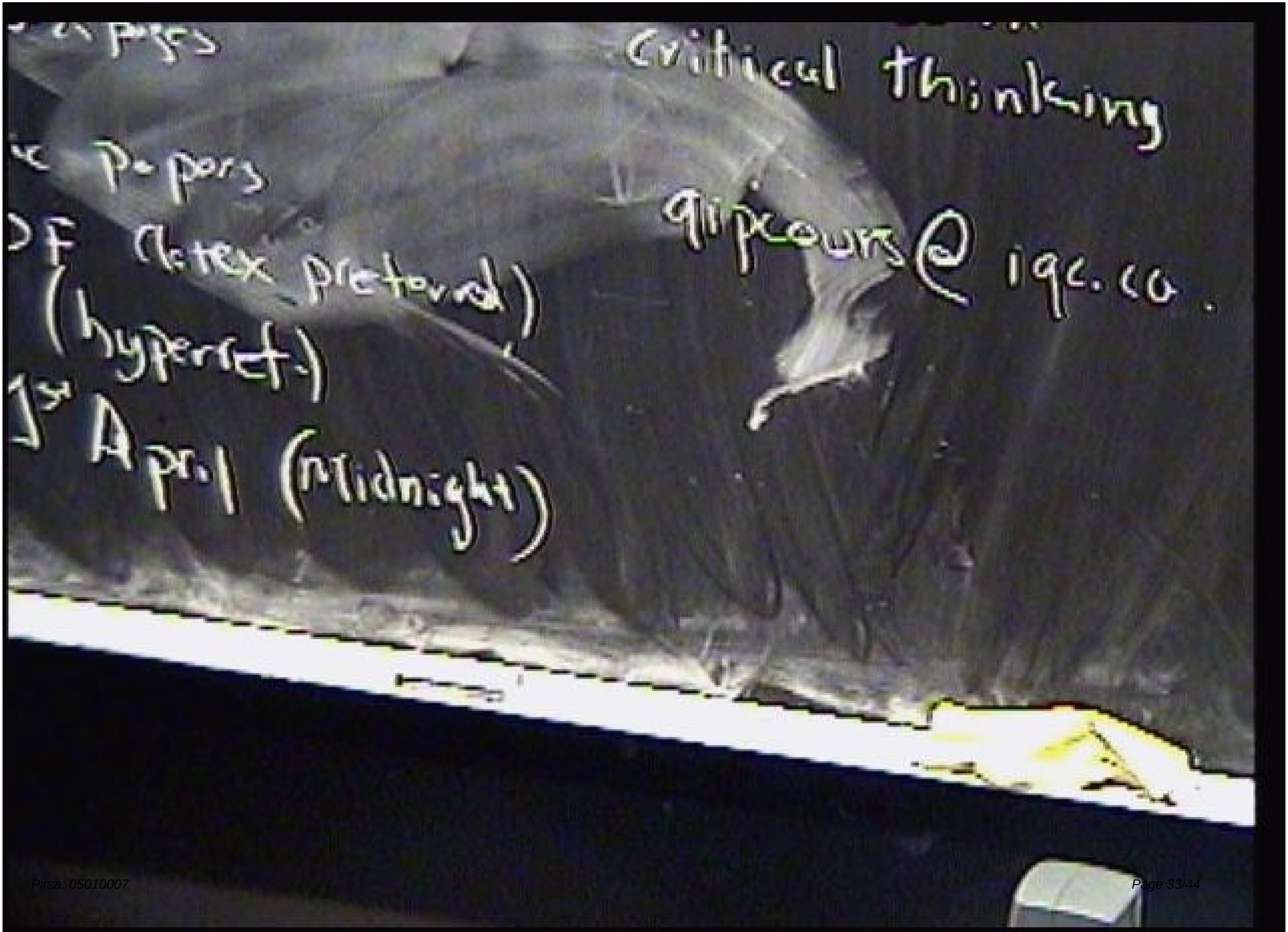
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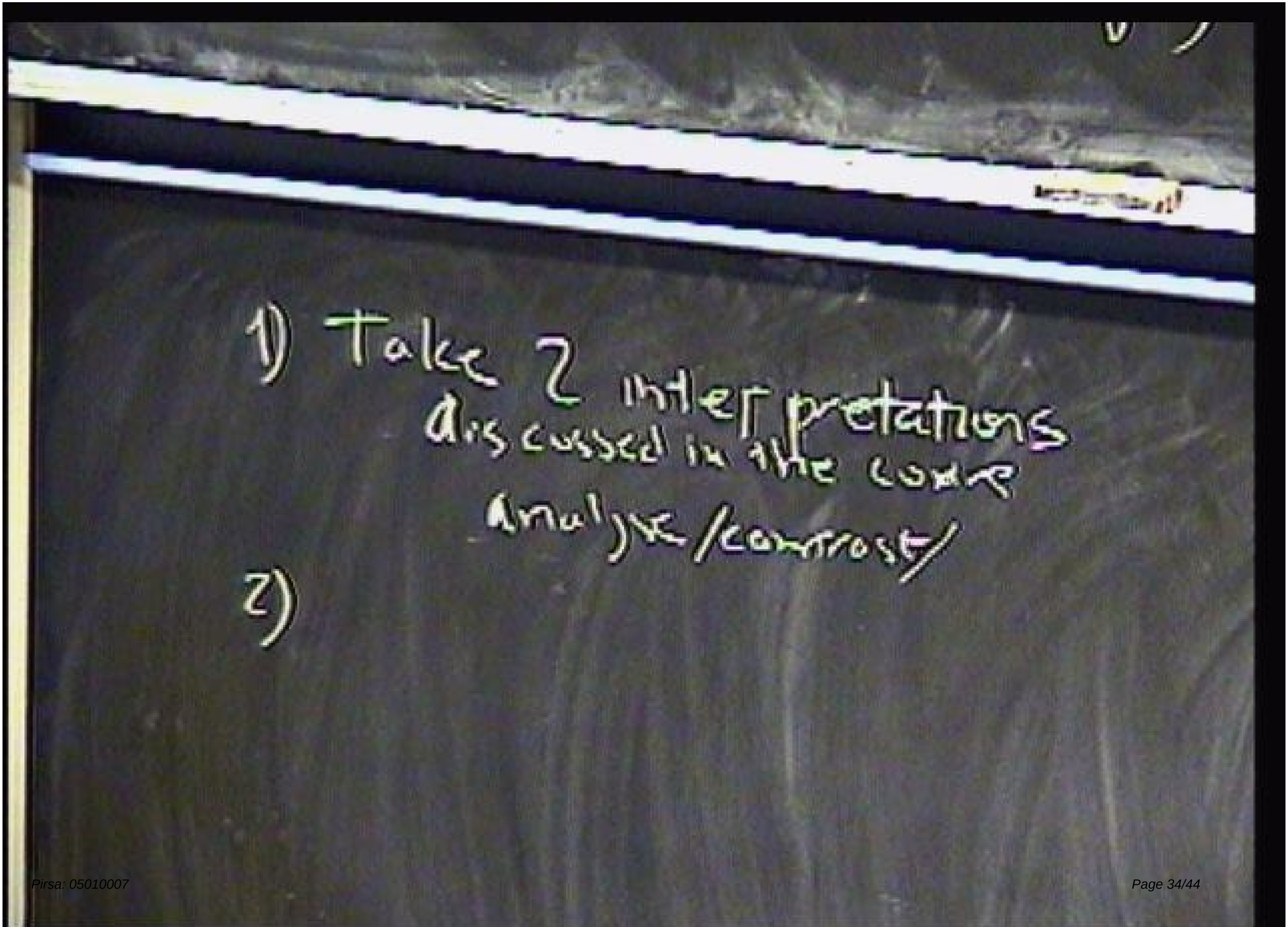
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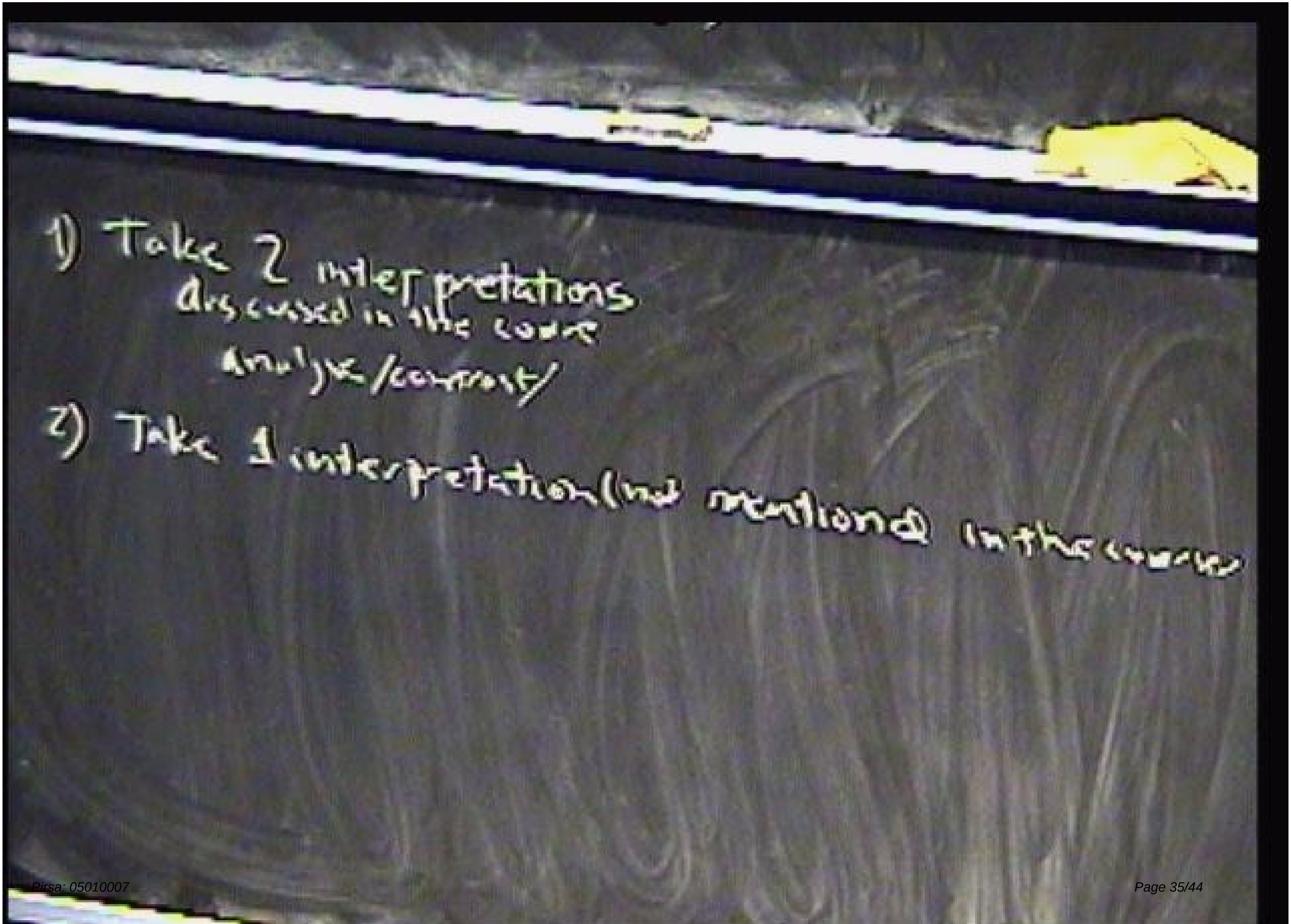
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- decoherence
- modal interp.
- macro realism

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- ....

(not ~~mentioned~~ covered in the course)

Zurek Review of 2003

Modern Phys. v75, p715-770

(not ~~mentioned~~ covered in the course)

Zurek Reviews of Modern Phys. v 75, p715-770 2003

Bub Interpreting the Q World

Leggett.

- Review an historical paper

J. A. Wheeler + W. H Zurek

(Q. Theory + Measurement.

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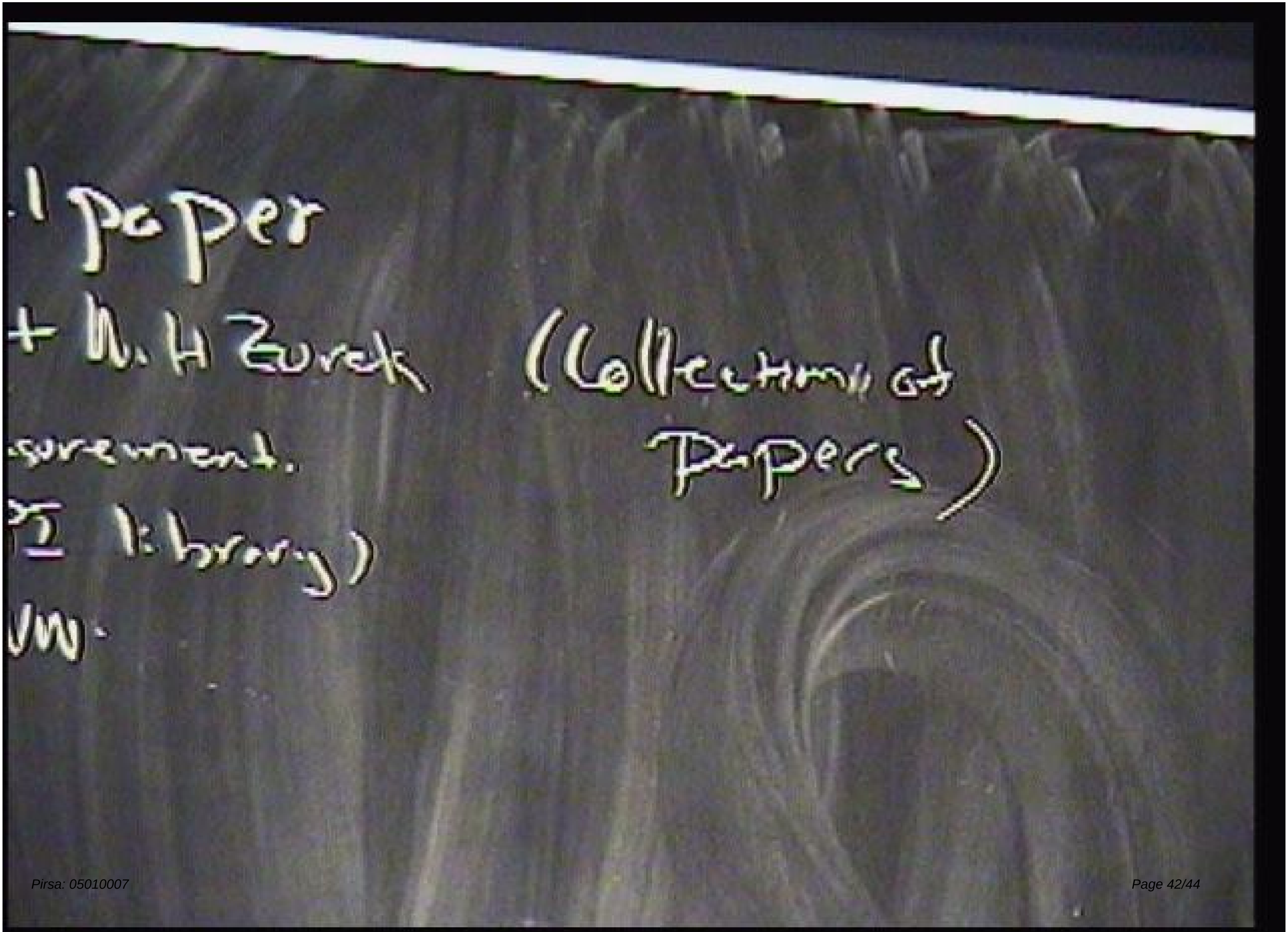

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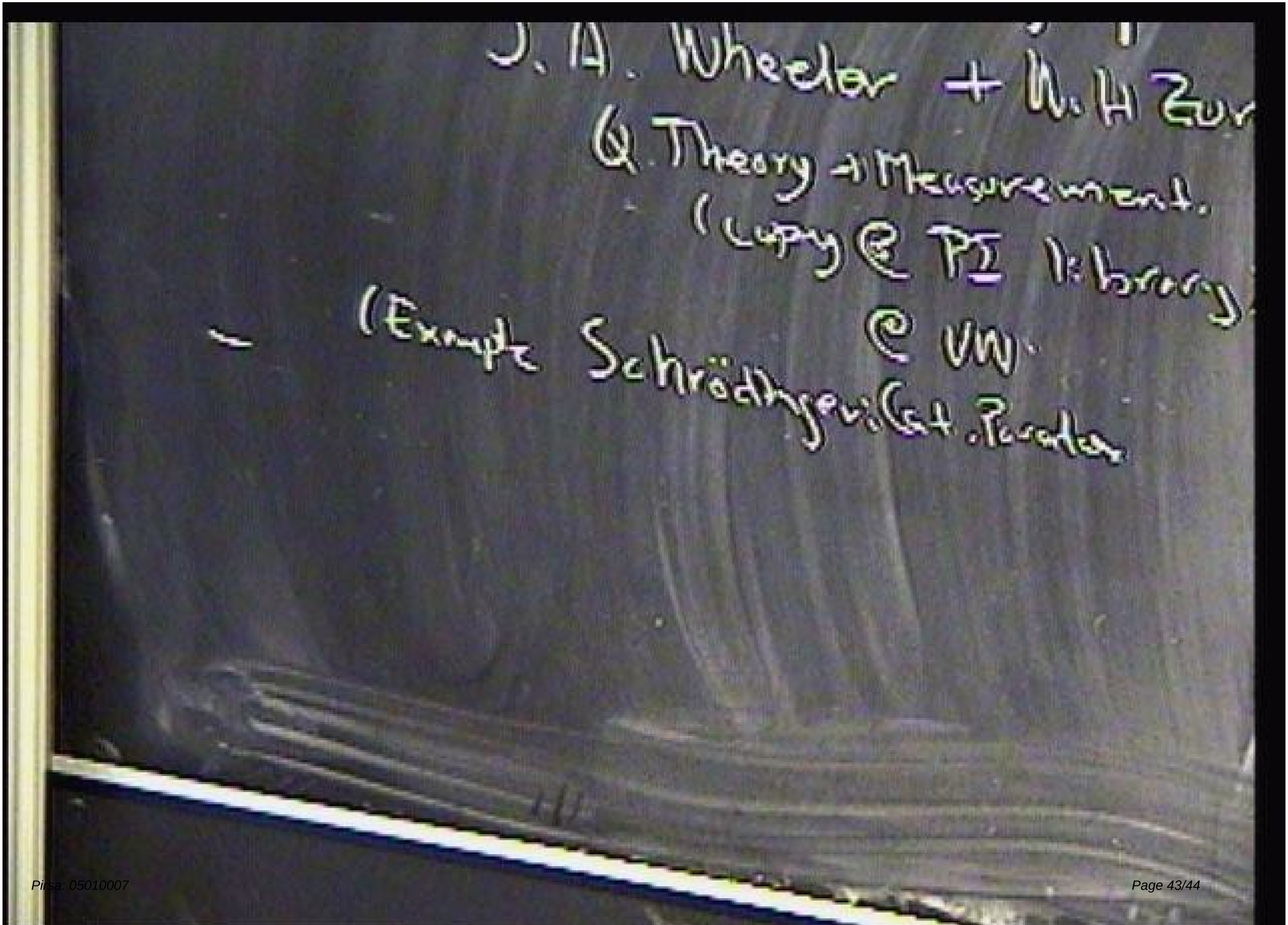
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9) Suggestion

+ Example Experimental realisation related F.QM