Title: Interpretation of Quantum Theory: Lecture 3

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Abstract:

Generalized states, measurements, and transformations.

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A) Classical indeterminacy (ignorance)

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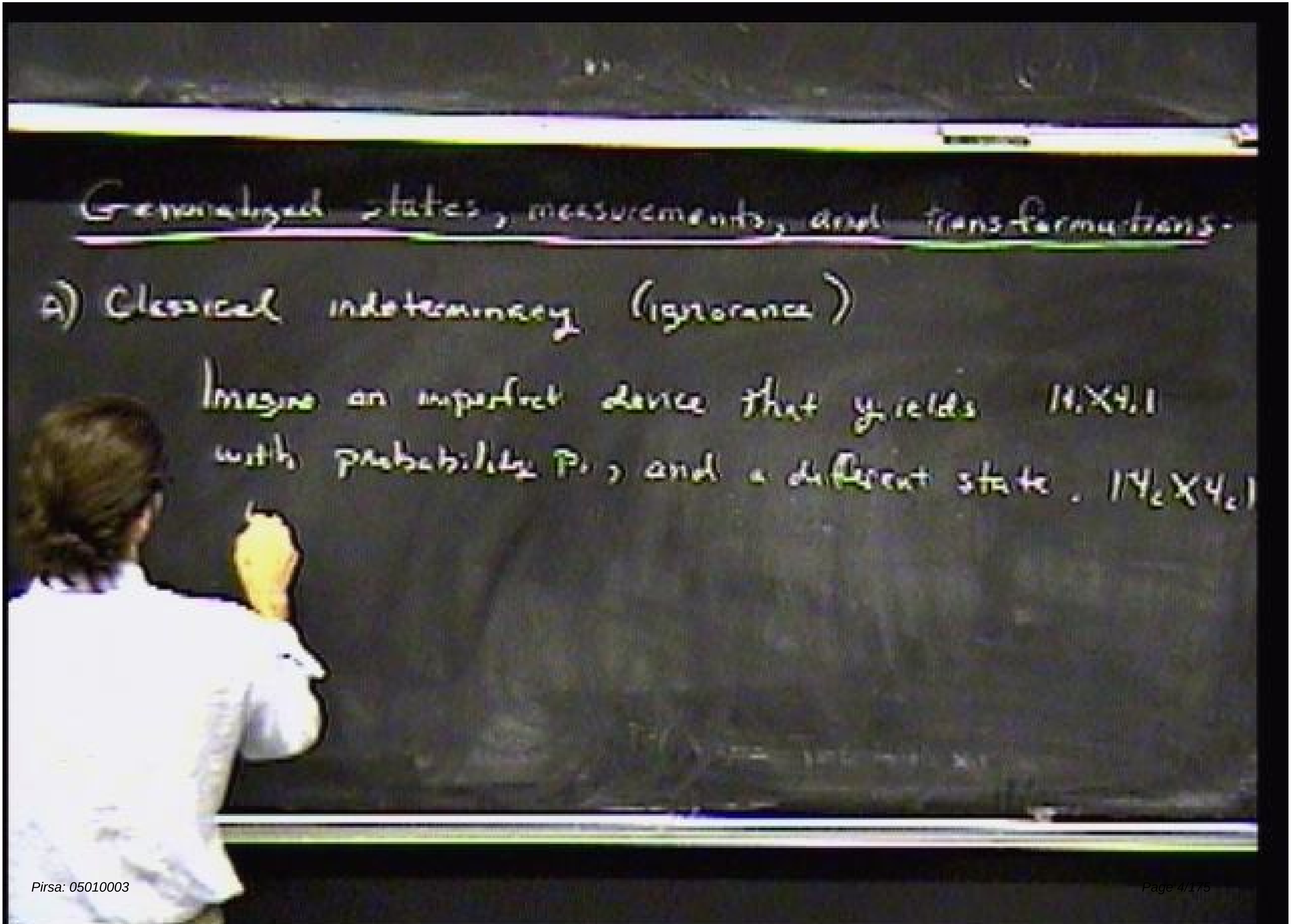
Generalized states, measurements, and transformations.
A) Classical indeterminacy (ignorance)
Imagine an imperfect device that yields $|\psi_1\rangle\langle\psi_1|$
with probability p_1, and a different state, $|\psi_2\rangle\langle\psi_2|$

Generalized states, measurements, and transformations:

a) Classical indeterminacy (ignorance)

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Generalized states, measurements, and transformations.

A) Classical indeterminacy (ignorance)

Imagine an imperfect device that yields $|\psi_1\rangle\langle\psi_1|$ with probability p_1, and a different state, $|\psi_2\rangle\langle\psi_2|$ with prob. $p_2 = 1-p_1$

$$\rho = p_1 |\psi_1\rangle\langle\psi_1| + (1-p_1)|\psi_2\rangle\langle\psi_2|$$

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prob. $p_2 = 1 - p_1$.

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Assuming $|\psi_1\rangle \in \mathcal{H}$, $|\psi_2\rangle \in \mathcal{H}$, s.t. $\langle\psi_1|\psi_2\rangle \neq 0$

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$$\rho = p_1 |\psi_1\rangle\langle\psi_1| + (1-p_1)|\psi_2\rangle\langle\psi_2|$$

Assuming $|\psi_1\rangle \in \mathcal{H}$, $|\psi_2\rangle \in \mathcal{H}$, s.t. $\langle\psi_1|\psi_2\rangle \neq 0$,

$$tr(|\psi_1\rangle\langle\psi_1|) = tr(|\psi_2\rangle\langle\psi_2|) = 1.$$

$\Rightarrow \mathrm{tr}(\rho) = 1$

$\Rightarrow \quad \mathrm{tr}(\rho) \leq 1.$

$\langle u|\rho|u\rangle \geq 0 \quad \forall\, |u\rangle \in \mathcal{H}$

$\mathrm{tr}(\rho^2) \leq 1$

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$\langle u|\rho|u\rangle \geq 0 \quad \forall\, |u\rangle \in \mathcal{H}.$

$\mathrm{tr}(\rho^2) \leq 1$ $\Leftarrow$ saturated only if $\rho = \{0,1\}$.

For d

$\Rightarrow \quad tr(\rho) = 1.$

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For $\dim(\mathcal{H}) = D$

$$\frac{1}{D} \leq tr(\rho^2) \leq 1$$

$\rightarrow \mathrm{tr}(\rho) = 1.$

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$\hat{\rho}$ is general state operator, mixed state

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Maximally mixed state $\hat{\rho} = \frac{\mathbb{1}}{D}$

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For $\dim(\mathcal{H}) = D$

$$\frac{1}{D} \leq \mathrm{tr}(\rho^2) \leq 1$$

$\hat{\rho}$ is general state operator (mixed state).

Maximally mixed state $\hat{\rho} = \frac{\mathbb{1}}{D}$ (total ignorance) for $\dim(\mathcal{H}) = D$

$$\frac{1}{D} \leq tr(\rho^2) \leq 1$$

$\hat{\rho}$ is general state operator (mixed stat

[max]imally mixed state $\hat{\rho} = I/D$ (total ignoran

$$\hat{\rho} = \int d\alpha\, p(\alpha)\, |\psi(\alpha)\rangle\langle\psi(\alpha)|$$

Consider a non-ideal device implementing with prob p_i
a unitary U_i

Consider a non-ideal device implementing with prob. p_1 a unitary U_1, and with prob $(1-p_1)$ implementing $U_2 \neq U_1$.

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Linear Map $\quad \mathcal{E}(\rho) = p_1 U_1 \rho U_1^\dagger + (1-p_1) U_2 \rho U_2^\dagger$

Consider a non-ideal device implementing with prob p_1 a unitary U_1, and with prob $(1-p_1)$ implementing U_2

Linear Map $\mathcal{E}(\rho) = p_1 U_1 \rho U_1^\dagger + (1-p_1) U_2 \rho U_2^\dagger$

$\hookrightarrow$ not unitary.

Positive: takes state operators to states operators.

Consider a non-ideal device implementing with prob. p_1 a unitary U_1, and with prob $(1-p_1)$ implementing $U_2 \neq U_1$.

Linear Map $\quad \mathcal{E}(\hat{\rho}) = p_1 U_1 \hat{\rho} U_1^\dagger + (1-p_1) U_2 \hat{\rho} U_2^\dagger = \hat{\rho}_f$

not unitary

Positive: takes state operators to state operators.

Linear Map $\quad \mathcal{E}(\hat{\rho}) = p_1 U_1 \hat{\rho}_0 U_1^{\dagger} + (1-p_1) U_2$

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In general, $\quad tr(\rho_{\mathcal{E}}^2) \le tr(\rho_0^2)$

Consider a non-ideal device implementing with prob. p_1 a unitary $\mathcal{U}_1$, and with prob $(1-p_1)$ implementing $\mathcal{U}_2 \neq \mathcal{U}$

Linear Map $\quad \mathcal{E}(\hat{\rho}) = p_1 \, \mathcal{U}_1 \hat{\rho}_i \mathcal{U}_1^\dagger + (1-p_1) \, \mathcal{U}_2 \hat{\rho}_i \mathcal{U}_2^\dagger \equiv \hat{\rho}_e$

↳ not unitary.

Positive: takes state operators to state operators.

In general, $\mathrm{tr}(\rho_e^2) \leq \mathrm{tr}(\rho_i^2)$ $\Rightarrow$ equality is saturated iff $p_1 = \{0,1\}$.

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: takes state operators to states operators.

$\mathrm{tr}(\rho_f^2) \leq \mathrm{tr}(\rho_i^2)$ $\Rightarrow$ equality is saturated if $\rho_i = \{0, 1\}$.

Linear Map $\mathcal{E}(\hat{\rho}) = p_i \mathcal{U}_i \hat{\rho}_i \mathcal{U}_i^{+} + (1-p_i)$

$\hookrightarrow$ not unitary.

Positive : takes state operators to states o

In general, $\mathrm{tr}(\rho_f^2) \leq \mathrm{tr}(\rho_i^2)$ $\Rightarrow$ equality

$\mathcal{E}(\hat{\rho}) = \int d\alpha \, p(\alpha) \, \mathcal{U}(\alpha) \hat{\rho} \, \mathcal{U}^{\dagger}(\alpha)$ i

$\hat{\rho} = \int d\alpha \; p(\alpha) \; |\psi(\alpha)\rangle\langle\psi(\alpha$

Transformations

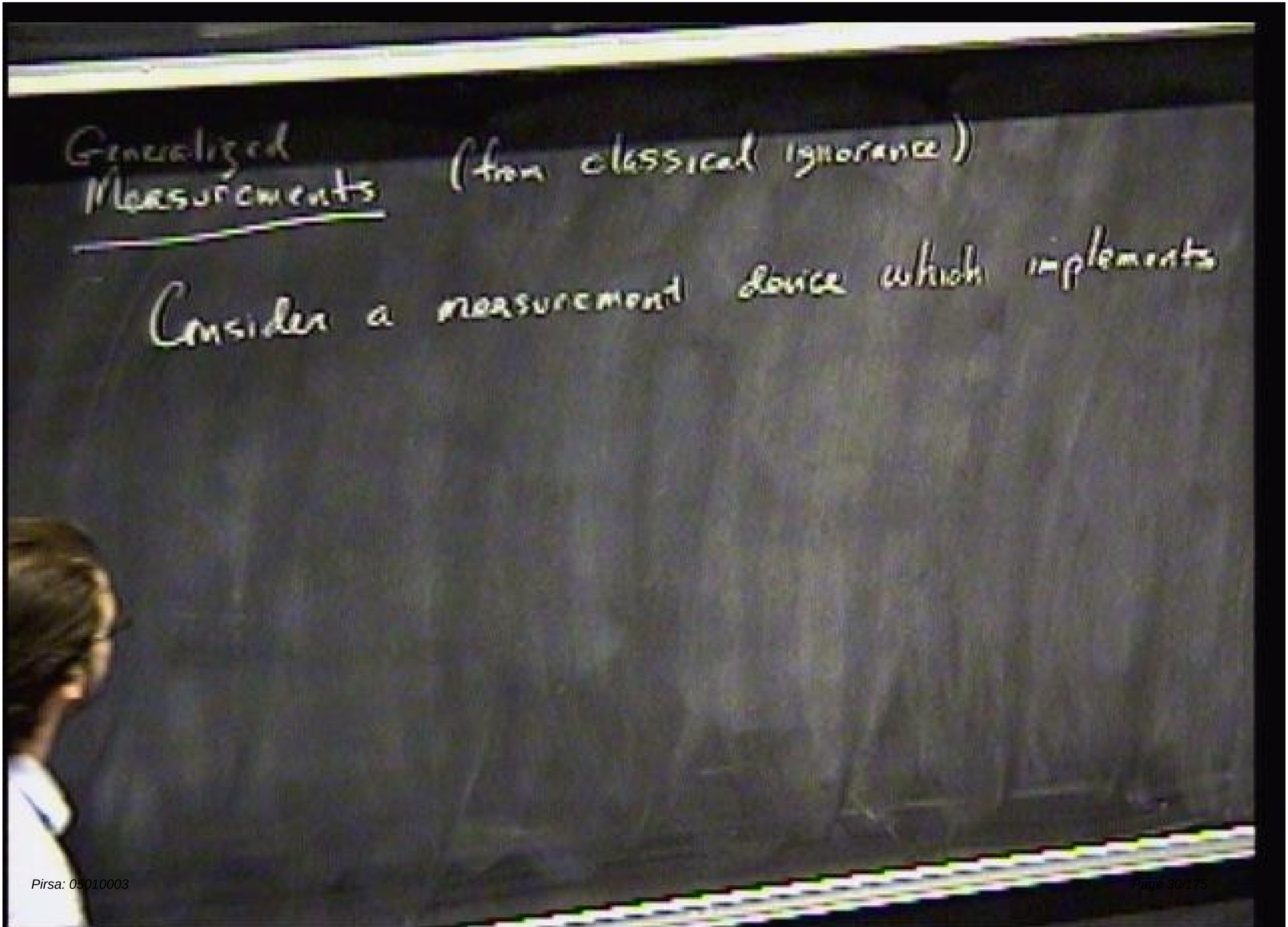
Consider a non-ideal device

a unitary U_1 , and with

Linear Map. $\mathcal{E}(\hat{\rho}) = p_1$

not uni

Generalized
Measurements (from classical ignorance)


Generalized
Measurements (from classical ignorance))
Consider a measurement device which implements

Aside on measurement

$$\hat{O} = \hat{O}^{+}$$

$$O = \sum_{k} \lambda_k |\lambda_k\rangle\langle\lambda_k|$$

Aside on measurement

$$\hat{O} = \hat{O}^{\dagger}$$

$$\hat{O} = \sum_k \lambda_k \underbrace{|\lambda_k\rangle\langle\lambda_k|}$$

Aside on measurement

$$\hat{O} = \hat{O}^{\dagger}$$

$$\hat{O} = \sum_k \lambda_k \, |\lambda_k\rangle\langle\lambda_k|$$

$$\hat{\rho} \rightsquigarrow \mathrm{Prob}_k = \mathrm{tr}\left(\hat{\rho}\, |\lambda_k\rangle\langle\lambda_k|\right)$$

...de on measurement

$\hat{O} = \hat{O}^{\dagger}$

$$\hat{O} = \sum_k \lambda_k \underbrace{|\lambda_k\rangle\langle\lambda_k|}$$

$$\hat{\rho} \Rightarrow \mathrm{Prob}_k = \mathrm{tr}(\hat{\rho}\,|\lambda_k\rangle\langle\lambda_k|) = p_k$$

$|\lambda_k\rangle\langle\lambda_k|$

$\text{prob}_k = \text{tr}(\hat{\rho}\,|\lambda_k\rangle\langle\lambda_k|) = p_k$

$p_k =$ relative freq $= \dfrac{\#\text{ of outcomes } k}{\text{total }\#\text{ of tests}}$

measurement

$\hat{O} = \hat{O}^{+}$

$$\hat{O} = \sum_k \lambda_k \underbrace{|\lambda_k\rangle\langle\lambda_k|}$$

$$\hat{\rho} \rightarrow \text{prob}_k = \text{tr}(\hat{\rho}\,|\lambda_k\rangle\langle\lambda_k|) = p_k$$

$$\text{tr}(\hat{O}\hat{\rho}) = \sum_k \lambda_k p_k$$

p_k = relative freq = $\frac{\#\text{ of outcomes}}{\text{total}\ \#\text{ of t}}$

measurement

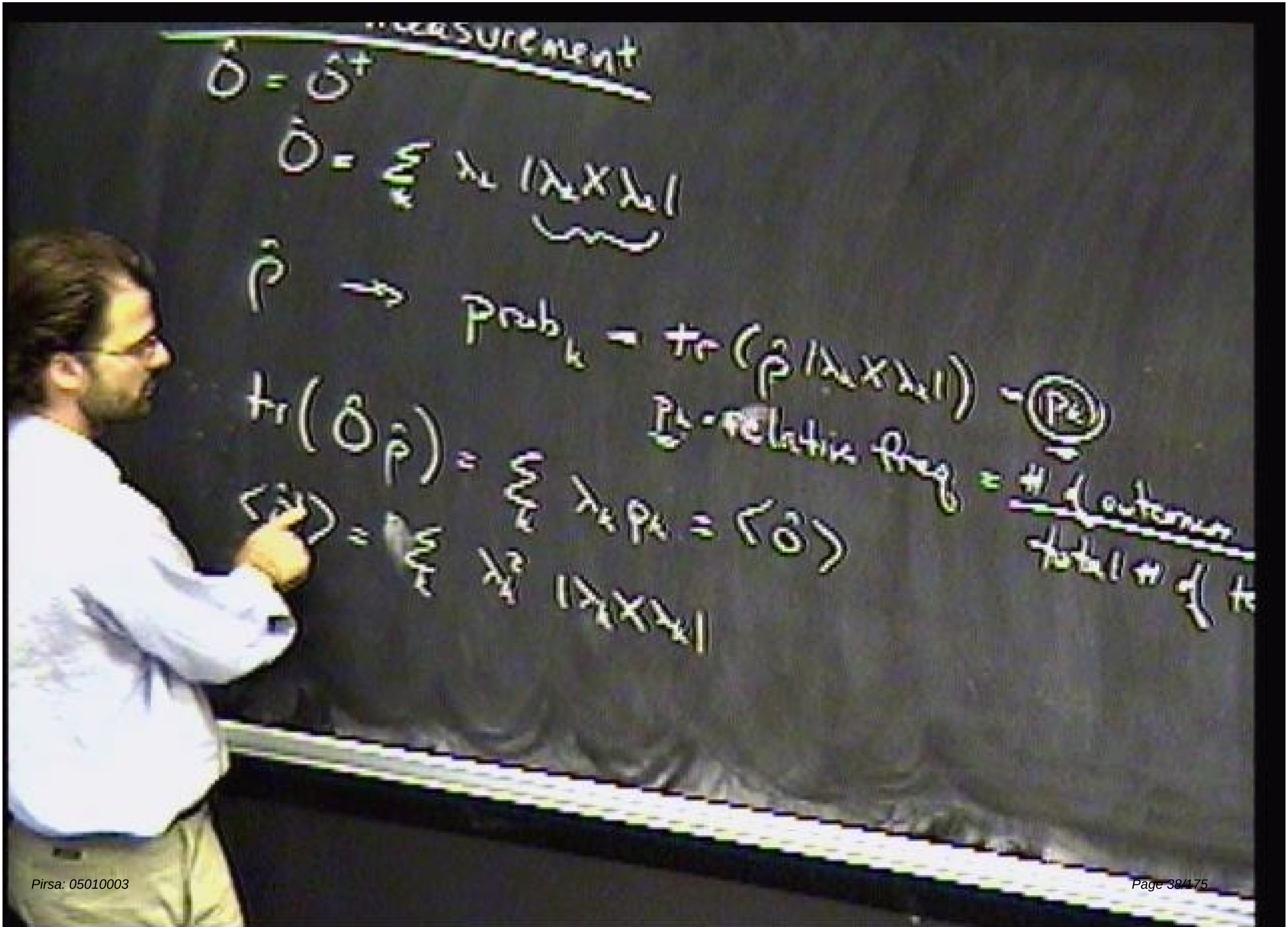
$\hat{O} = \hat{O}^\dagger$

$$\hat{O} = \sum_k \lambda_k \underbrace{|\lambda_k\rangle\langle\lambda_k|}$$

$$\hat{\rho} \rightsquigarrow \text{prob}_k = tr(\hat{\rho}\,|\lambda_k\rangle\langle\lambda_k|) = p_k$$

p_k = relative freq = $\frac{\#\text{ of outcomes}}{\text{total \# of tr}}$

$$tr(\hat{O}\hat{\rho}) = \sum_k \lambda_k p_k = \langle \hat{O} \rangle$$


measurement
$\hat{O} = \hat{O}^{\dagger}$
$\hat{O} = \sum_k \lambda_k |\lambda_k\rangle\langle\lambda_k|$
$\hat{\rho} \rightarrow \text{prob}_k = \text{tr}(\hat{\rho}|\lambda_k\rangle\langle\lambda_k|) = p_k$
p_k = relative freq = # of outcomes / total # of
$\text{tr}(\hat{O}\hat{\rho}) = \sum_k \lambda_k p_k = \langle\hat{O}\rangle$
$\langle\hat{O}^2\rangle = \sum_k \lambda_k^2 |\lambda_k\rangle\langle\lambda_k|$

$$\hat{\rho} \longrightarrow \text{Prob}_k = \text{tr}(\hat{\rho}\,|\lambda_k\rangle\langle\lambda_k|) = p_k$$

p_k = relative freq = $\frac{\# \{o\ldots}{\text{total}}$

$$\text{tr}(\hat{O}\hat{\rho}) = \sum_k \lambda_k p_k = \langle \hat{O} \rangle$$

$$\langle \hat{O}^2 \rangle = \text{tr}\Big(\sum_k \lambda_k^2\, |\lambda_k\rangle\langle\lambda_k|\,\hat{\rho}\Big) = \sum_k \lambda_k^2 p_k$$

$$\mathrm{tr}(\hat{O}\hat{\rho}) = \sum_k \lambda_k \rho$$

$$\langle \hat{O}^2 \rangle = \mathrm{tr}\Big(\sum_k \lambda_k^2 \, |\lambda_k\rangle$$

$$f(\hat{O}) = \sum_k f(\lambda_k)\, |\lambda_k\rangle\langle\lambda_k|$$

$$P_k = |\lambda_k\rangle\langle\lambda_k|$$

$$P_k^2 = P_k$$

$$(\hat{\rho}\, |\lambda_k\rangle\langle\lambda_k|) = P_k$$

elative fre

$$P_k = |\lambda_k\rangle\langle\lambda_k|$$

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$\Rightarrow$ need not be rank-one.

$\lambda_k|) = P_k$

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$\lambda_k|) = P_k$

$$\hat{P}_k = |\lambda_k\rangle\langle\lambda_k|$$

$$\hat{P}_k^2 = \hat{P}_k \Longleftrightarrow \text{projector}$$

$\Longrightarrow$ need not be rank-one.

$$|\lambda_k\rangle\langle\lambda_k|) = P_k$$

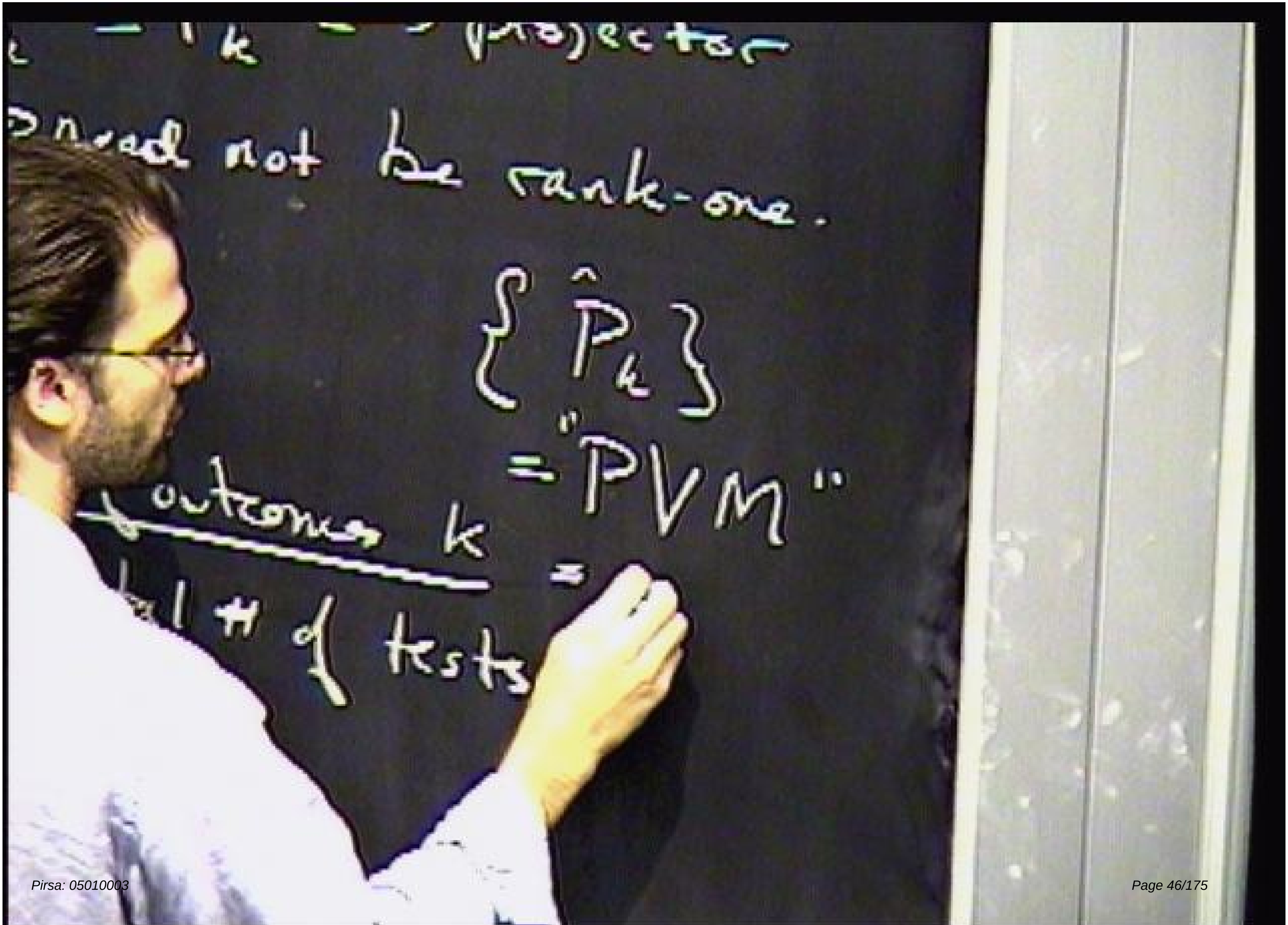
$\quad = P_k \quad$ projector

$\Rightarrow$ need not be rank-one.

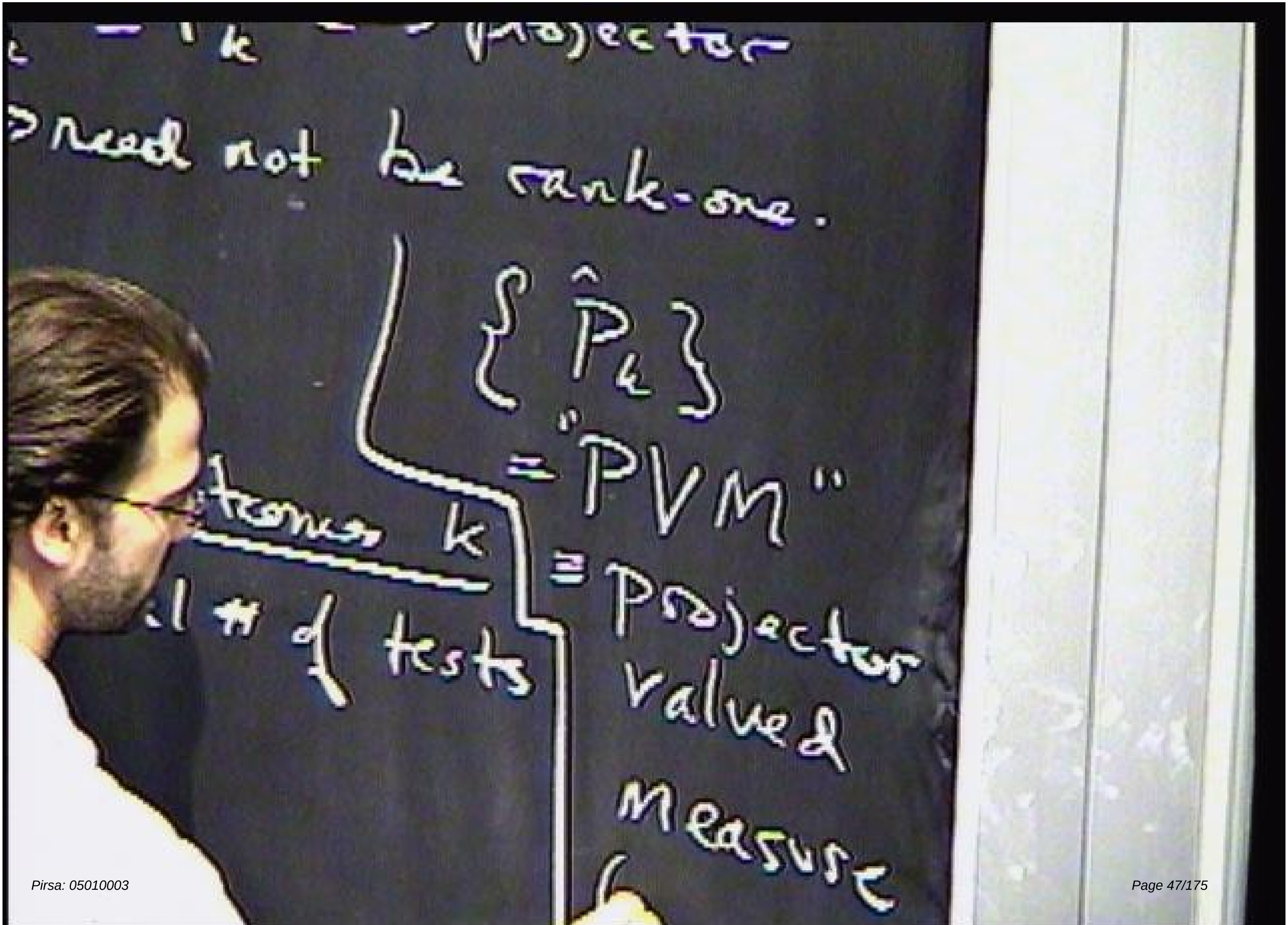
P_k $\qquad \{ P_k \}$

$$\frac{\#\text{ of outcomes } k}{\text{total }\#\text{ of tests}}$$

k



projector
need not be rank-one.
{ P̂k }
= "PVM"
outcomes k
# of tests


k
projector
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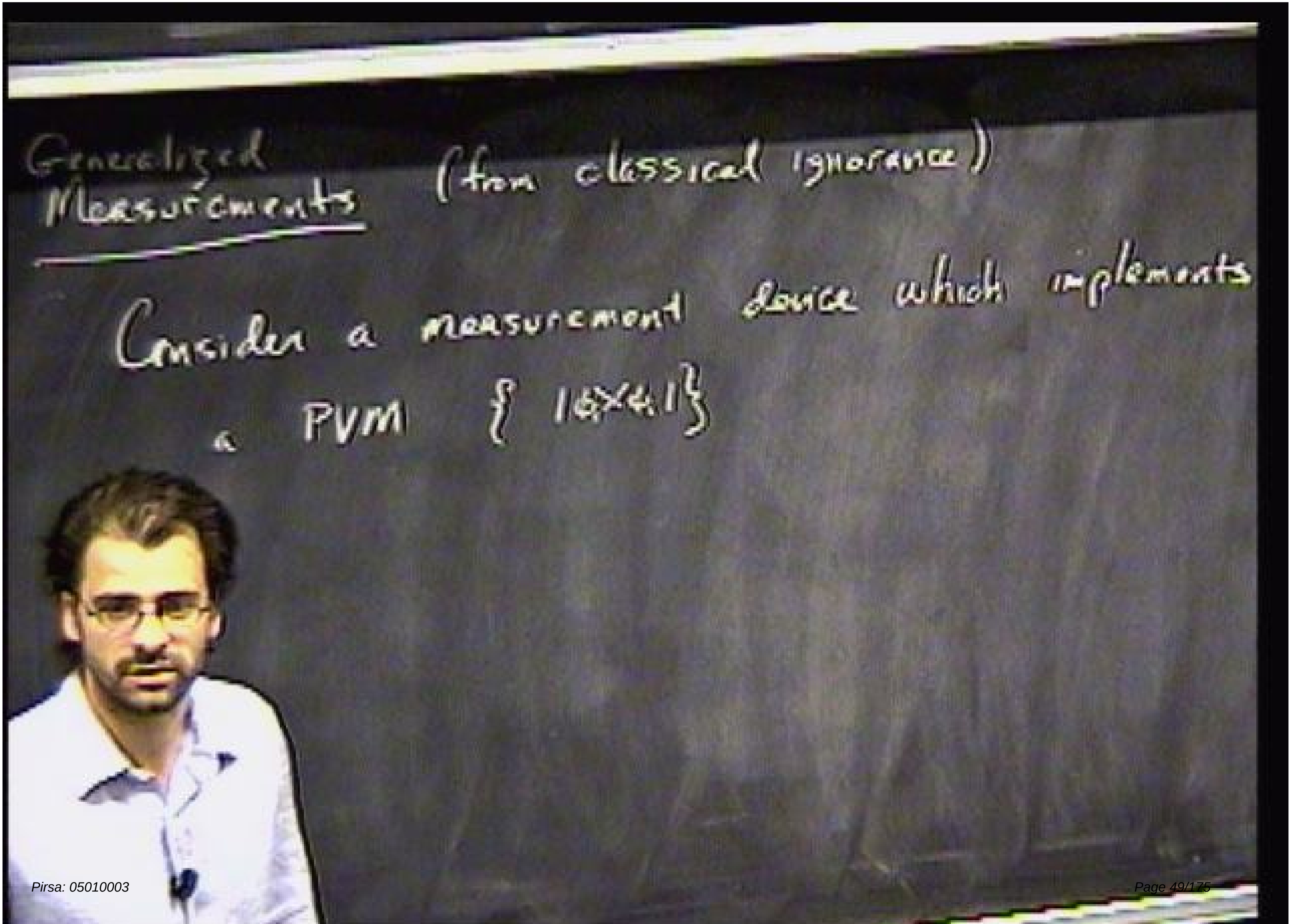
{ P̂k }
= "PVM"
k
= Projector
valued
Measure
# of tests

$\frac{\text{outcomes } k}{\text{total \# of tests}}$

$\{\hat{P}_k\}$

= "PVM"

= Projector valued measure (von Neumann, 1932)


Generalized Measurements (from classical ignorance)
Consider a measurement device which implements
a PVM $\{ |a\rangle\langle a| \}_a$

neralized
easurements (from classical ignorance)

Consider a measurement device which implements

a PVM $\{|\phi_i\rangle\langle\phi_i|\}$ with prob. p,

and the PVM $\{|\psi_i\rangle\langle\psi_i|\}$ with prob. $(1-p)$.

$\hat{P}_k = |\lambda_k\rangle\langle\lambda_k|$, $P_k P_j = P_j \delta_{kj}$

$\hat{P}_k^2 = \hat{P}_k \iff$ projector

$\Rightarrow$ need not be rank-one.

$\lambda_k\rangle\langle\lambda_k|) \sim P_k$

$\{\hat{P}_k\}$

= "PVM"

k freq = # of out

$\hat{P}_k = |\lambda_k\rangle\langle\lambda_k|$, $P_k P_j = \delta_{kj}$

$\hat{P}_k^2 = \hat{P}_k \Longleftrightarrow$ projector

$\Longrightarrow$ need not be rank-one.

$\lambda_k\rangle\langle\lambda_k|) = P_k$

$\{\hat{P}_k\}$

$\hat{ }$k freq $= \#$ of out

= "PVM"

$$\hat{P}_k = |\lambda_k\rangle\langle\lambda_k| \,, \quad \hat{P}_k \hat{P}_j = \hat{P}_k \,\delta_{kj}$$

$$\hat{P}_k^2 = \hat{P}_k \iff \text{projector}$$

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$\langle\lambda_k|) = P_k$

$\{\hat{P}_k\}$

Generalized Measurements (from classical ignorance)

Consider a measurement device which implements

a PVM $\{|a_i\rangle\langle a_i|\}$ with prob. p,

and the PVM $\{|\psi_i\rangle\langle\psi_i|\}$ with prob. $(1-p)$

$$\{E_i\} = \{p\,|a_i\rangle\langle a_i| + (1-p)\,|\psi_i\rangle\langle\psi_i|\}$$

PVM $\{ |\phi_i\rangle\langle\phi_i| \}$ with prob p_1,

and the PVM $\{ |\psi_i\rangle\langle\psi_i| \}$ with p

$$\{ E_i \} = \{ p\, |\phi_i\rangle\langle\phi_i| + (1-p)\, |\psi_i\rangle\langle\psi_i| \}$$

$$p_i = p_1 \operatorname{tr}(\hat{\rho}\, |\phi_i\rangle\langle\phi_i|) + (1-p_1) \operatorname{tr}(\hat{\rho}\, |\psi_i\rangle\langle\psi_i|)$$

PVM $\{ |\phi_i\rangle\langle\phi_i| \}$ with prob. p,

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$$\{E_i\} = \{ p\,|\phi_i\rangle\langle\phi_i| + (1-p)\,|\psi_i\rangle\langle\psi_i| \}$$

$$p_i = p_1\,\mathrm{tr}(\hat{\rho}\,|\phi_i\rangle\langle\phi_i|) + (1-p_1)\,\mathrm{tr}(\hat{\rho}\,|\psi_i\rangle\langle\psi_i|)$$

$$= \mathrm{tr}\left[\hat{\rho}\,(p_1|\phi_i\rangle\langle\phi_i| + (1-p_1)|\psi_i\rangle\langle\psi_i|)\right]$$

$\langle u | E_i | u \rangle \geq 0 \quad \forall \ |u\rangle \in \mathcal{H}$

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$\hookrightarrow$ positive operators

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↳ positive operators

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$\hookrightarrow$ positive operators

$\sum_i p_i = 1 \quad \Rightarrow \quad \sum_i E_i = \mathbb{1}$

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↳ positive operators.

$\sum_i p_i = 1 \iff \boxed{\sum_i E_i = \mathbb{1}}$

Positive operator valued measures.

$\langle u | E_i | u \rangle \geq 0 \quad \forall\ |u\rangle \in \mathcal{H}$

$\hookrightarrow$ positive operators

$\sum_i p_i = 1 \iff \boxed{\sum_i E_i = \mathbb{1}}$

Positive operator valued measures

$E_i^2 \neq E_i$

$\langle u|E_i|u\rangle \geq 0 \quad \forall\, |u\rangle \in \mathcal{H}$

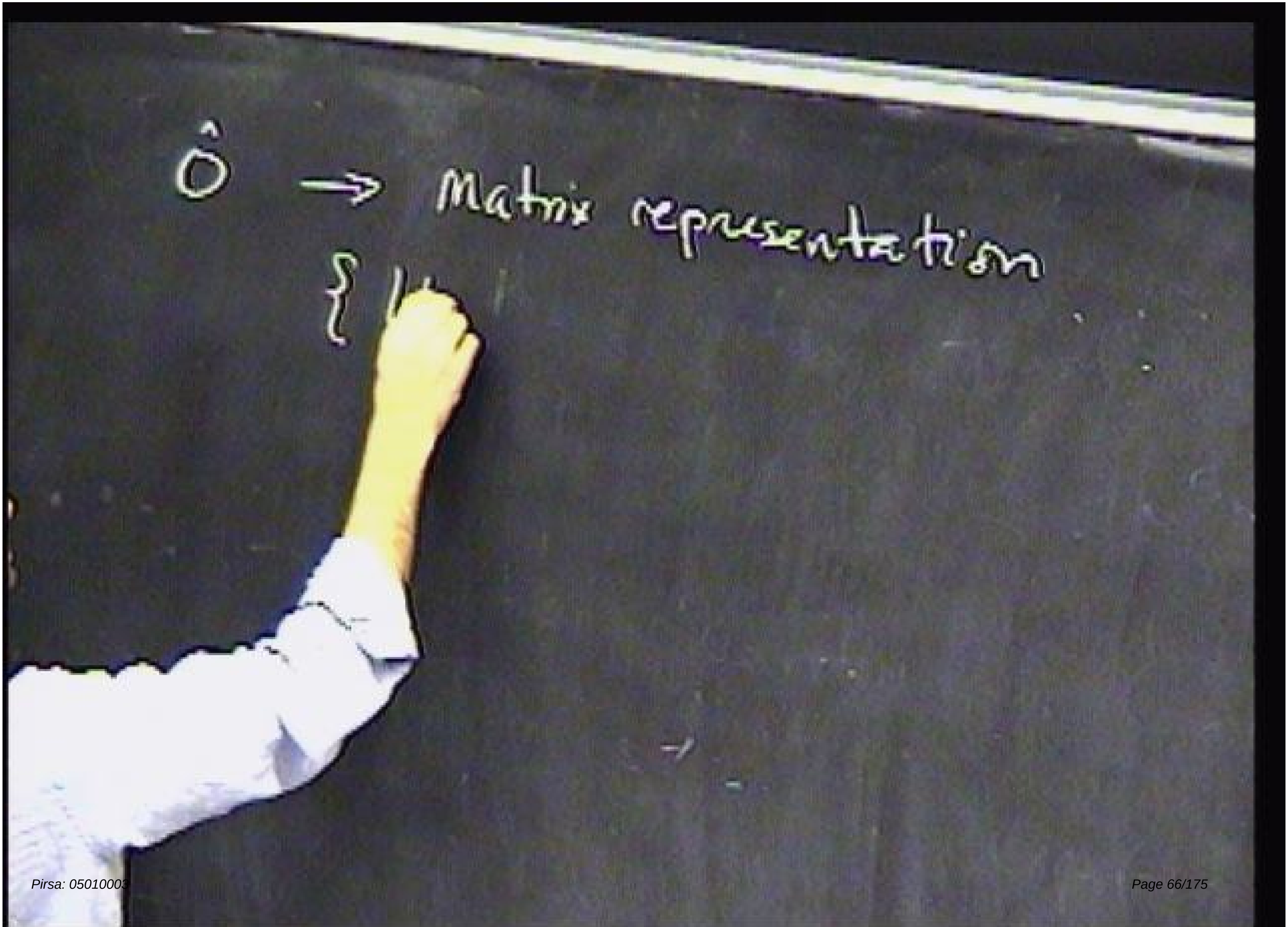
$\hookrightarrow$ positive operators

$\sum_i p_i = 1 \iff \boxed{\sum_i E_i = \mathbb{1}}$

Positive operator valued measures.

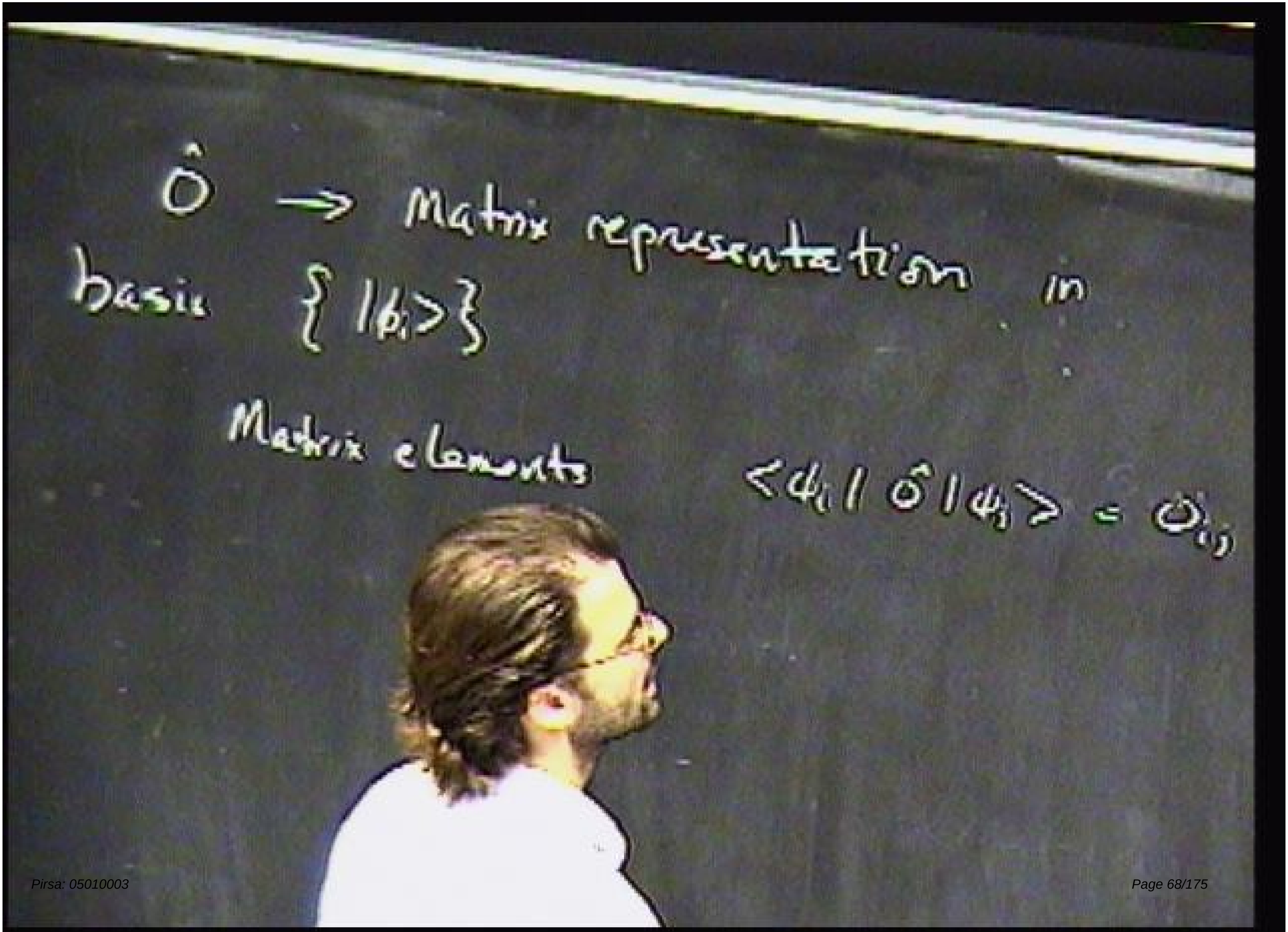
$E_i^2 \neq E_i$

PVM are special case $E_i^2 = E_i$.


$\hat{O} \rightarrow$ Matrix representation
$\{ |$

$\hat{O} \longrightarrow$ Matrix representation

basis $\{ |u_i\rangle \}$


$\hat{O}$ → Matrix representation in
basis $\{ |\phi_i\rangle \}$
Matrix elements $\langle \phi_i | \hat{O} | \phi_j \rangle = O_{ij}$

$\hat{O}$ → Matrix representation in basis $\{|\phi_i\rangle\}$

Matrix elements $\langle\phi_i|\hat{O}|\phi_j\rangle = O_{ij}$ i = row n, j = col n

(= row #

) = col. #

$\hat{O} \rightarrow$ Matrix representation in basis $\{|\phi_i\rangle\}$

Matrix elements $\langle\phi_i|\hat{O}|\phi_j\rangle = O_{ij}$

$$\mathrm{tr}(\hat{O}) = \sum_i \langle\phi_i|\hat{O}|\phi_i\rangle.$$

...ation in

basis $\{|\phi_i\rangle\}$

Matrix elements $\langle\phi_i|\hat{O}|\phi_j\rangle = O_{ij}$

$$\mathrm{tr}(\hat{O}) = \sum_i \langle\phi_i|\hat{O}|\phi_i\rangle .$$

ssignment problem:

...ents $\langle \phi_i | \hat{O} | \phi_j \rangle = O_{ij}$ i = row #
j = col. #

$\sum_i \langle \phi_i | \hat{O} | \phi_i \rangle$.

$\{|0\rangle, |1\rangle\}$, $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

coherent superposition

...rix elements $\langle \phi_i | \hat{O} | \phi_j \rangle = O_{ij}$ i = row # , j = col #

...r $(\hat{O}) = \sum_i \langle \phi_i | \hat{O} | \phi_i \rangle$.

...oblem: $\{|0\rangle, |1\rangle\}$, $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ ← coherent superposit...

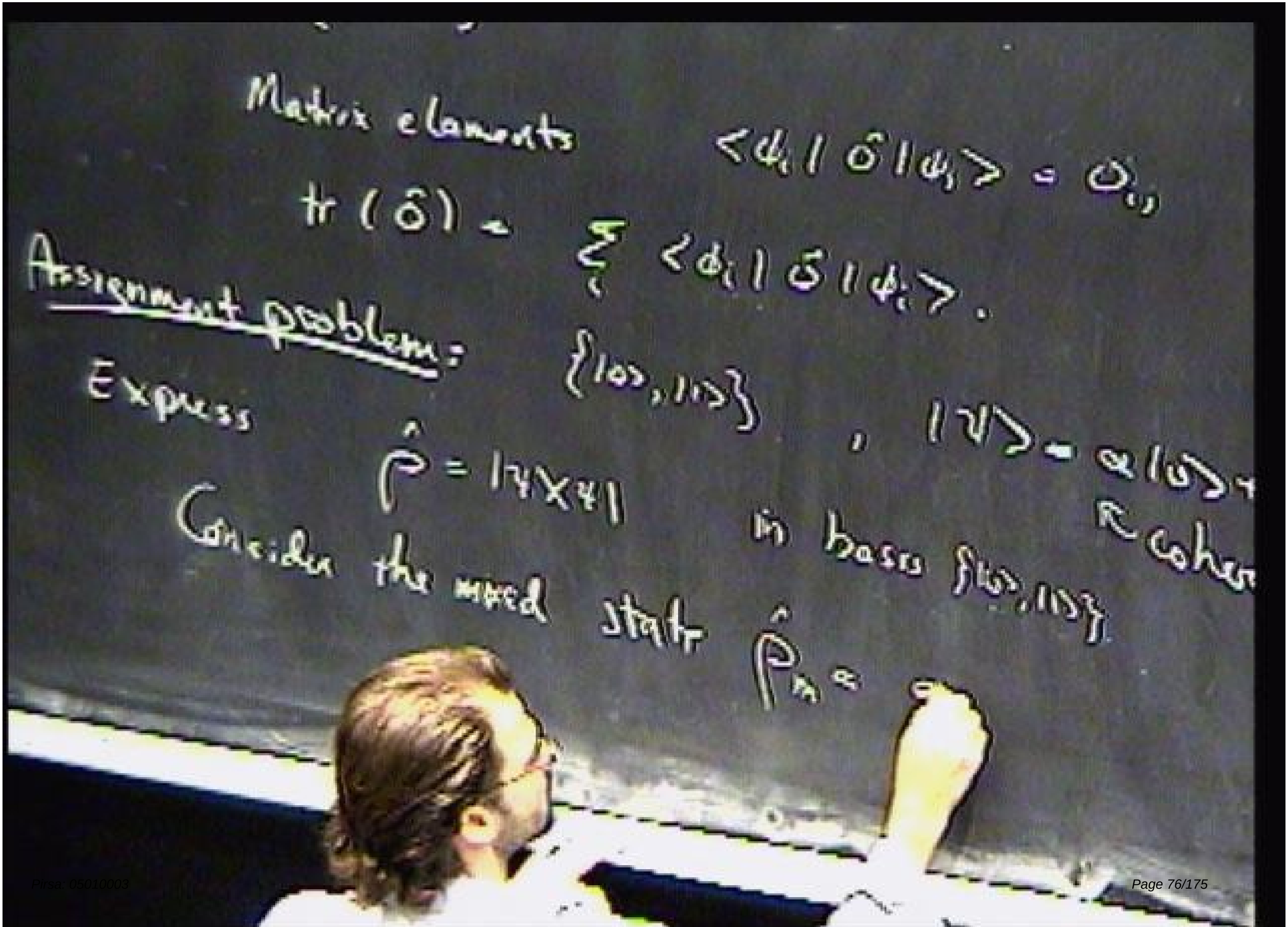
$\hat{\rho} = |\psi\rangle\langle\psi|$

$\langle \phi_i | \hat{O} | \phi_j \rangle = O_{ij}$

$$\mathrm{tr}(\hat{O}) = \sum_i \langle \phi_i | \hat{O} | \phi_i \rangle .$$

Lem: $\{|0\rangle, |1\rangle\}$, $|\psi\rangle = \alpha|0\rangle + \ldots$ ← coher

$\hat{\rho} = |\psi\rangle\langle\psi|$ in basis $\{|0\rangle, |1\rangle\}$


Matrix elements $\langle \phi_i | \hat{O} | \phi_j \rangle = O_{ij}$
$\mathrm{tr}(\hat{O}) = \sum_i \langle \phi_i | \hat{O} | \phi_i \rangle$.
Assignment problem: $\{|0\rangle, |1\rangle\}$
Express $\hat{\rho} = |\psi\rangle\langle\psi|$ in basis $\{|0\rangle, |1\rangle\}$
$|\psi\rangle = \alpha|0\rangle +$
Consider the mixed state $\hat{\rho}_m =$

$\}$, $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

$\leftarrow$ coherent superposition

in basis $\{|0\rangle, |1\rangle\}$

$\hat{\rho}_M = |\alpha|^2 |0\rangle\langle 0| + |\beta|^2 |1\rangle\langle 1|$

...stem: $\{|0\rangle, |1\rangle\}$, $|\psi\rangle = \alpha|0\rangle + \beta$

$\hat{\rho} = |\psi\rangle\langle\psi|$ in basis $\{|0\rangle, |1\rangle\}$ ⇖ coherent

...en the mixed state $\hat{\rho}_m = |\alpha|^2 |0\rangle\langle 0| + |\beta|^2$

express in same basis.

$$|+\rangle = |0\rangle + |1\rangle \qquad \{|+\rangle, |-\rangle\}$$
$$|-\rangle = |0\rangle - |1\rangle$$

m

$\rangle = O_{ij}$

i = row #
j = col. #

$|+\rangle = |0\rangle + |1\rangle$

$|-\rangle = |0\rangle - |1\rangle$

$\{|+\rangle, |-\rangle\}$

Express both ρ_T & ρ_m in this basis.

O_{ij}

i = row #

j = col #.

$|+\rangle = |0\rangle + |1\rangle$

$|-\rangle = |0\rangle - |1\rangle$

$\{|+\rangle, |-\rangle\}$

Express both ρ_T & ρ_m in this basis. Determine a cond$^{\underline{n}}$ on $\{\alpha, \beta\}$ so that ρ_m is diagonal in every basis

O_{ij}

i = row #

j = col #

$= \alpha|0\rangle + \beta|1\rangle$

$\rho_A = |\alpha|^2 |0\rangle\langle 0| + |\beta|^2 |1\rangle\langle 1|$

express in same basis.

Tensor Product Structure $\rightarrow$ Composite Systems

Tensor Product Structure → Composite Systems

Two systems S_A & S_B, total system $S = S_A + S_B$

Tensor Product Structure → Composite Systems

Two systems S_A & S_B, total system $S = S_A + S_B$

$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$$

Tensor Product Structure → Composite Systems

Two systems S_A & S_B, total system $S = S_A + S_B$

$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$

$|\psi_A\rangle \in \mathcal{H}_A$, $|\psi_B\rangle \in \mathcal{H}_B$

systems S_A & S_B, total syst

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[illegible] posite systems

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Let $\{|u_{A,i}\rangle\}$ be a basis for $\mathcal{H}_A$, and $\{|u_{B,j}\rangle\}$ be a basis for $\mathcal{H}_B$.

Composite systems

Two systems S_A & S_B, total system $S = S_A + S_B$

$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$$

$|\psi_A\rangle \in \mathcal{H}_A$, $|\psi_B\rangle \in \mathcal{H}_B$

$|\psi\rangle \in \mathcal{H}$

Let $\{|\phi_i\rangle\}$ be a basis for $\mathcal{H}_A$, and $\{|\theta_j\rangle\}$ be a basis for $\mathcal{H}_B$.

$$|\psi\rangle = \sum_i a_i |\phi_i\rangle$$

$|\psi_A\rangle \in \mathcal{H}_A$, $|\psi_B\rangle$

$|\psi\rangle \in \mathcal{H}$

Let $\{|\phi_{A,i}\rangle\}$ be a basis for $\mathcal{H}_A$

$$|\psi\rangle_A = \sum_i a_i |\phi_i\rangle$$

S_A & S_B , total system S.

$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$

$|\psi_A\rangle \in \mathcal{H}_A$, $|\psi_B\rangle \in \mathcal{H}_B$

$|\psi\rangle \in \mathcal{H}$

Let $\{|\phi_{A,i}\rangle\}$ be a basis for $\mathcal{H}_A$, and $\{|\phi_{B,j}\rangle\}$ be a basis

$|\psi_A\rangle = \sum_i a_i |\phi_{A,i}\rangle$, $|\psi_B\rangle = \sum_j b_j |\phi_{B,j}\rangle$

$|\psi_A\rangle \in \mathcal{H}_A$, $|\psi_B\rangle \in \mathcal{H}_B$

$|\psi\rangle \in \mathcal{H}$

$\{|\phi_{A,i}\rangle\}$ be a basis for $\mathcal{H}_A$, and $\{|\phi_{B,j}\rangle\}$ be a ba

$|\psi_A\rangle = \sum_i a_i |\phi_{A,i}\rangle$, $|\psi_B\rangle = \sum_j b_j |\phi_{B,j}\rangle$

$a_i = \langle \phi_{A,i} | \psi_A \rangle$, $b_j = \langle \phi_{B,j} | \psi_B \rangle$

$|\psi_A\rangle \in \mathcal{H}_A$, $|\psi_B\rangle \in \mathcal{H}_B$

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$|\psi_A\rangle = \sum_i a_i |\phi_i\rangle$, $|\psi_B\rangle = \sum_j b_j |\phi_{B,j}\rangle$

$a_i = \langle \phi_{A,i} | \psi_A \rangle$, $b_j = \langle \phi_{B,j} | \psi_B \rangle$

otal system $S = S_A + S_B$

$$|\psi\rangle = \sum_{ij} c_{ij} \, |\phi_{A,i}\rangle \otimes |\phi_{B,j}\rangle$$

$\mathcal{H}_B$

nd $\{|\phi_{B,i}\rangle\}$ be a basis for $\mathcal{H}_B$.

$= \sum_j b_j \, |\phi_{B,j}\rangle$

$= \langle\phi \quad |\psi\rangle$

otal system $S = S_A + S_B$

$|\psi\rangle = \sum_{ij} c_{ij} \underbrace{|\phi_{A,i}\rangle \otimes |\phi_{B,j}\rangle}$

$\mathcal{H}_B$ $\rightarrow$ form a basis for $\mathcal{H}$.

nd $\{|\phi_{B,j}\rangle\}$ be a basis for $\mathcal{H}_B$.

$= \sum_j b_j |\phi_{B,j}\rangle$

$= \langle\phi \quad |\psi\rangle$

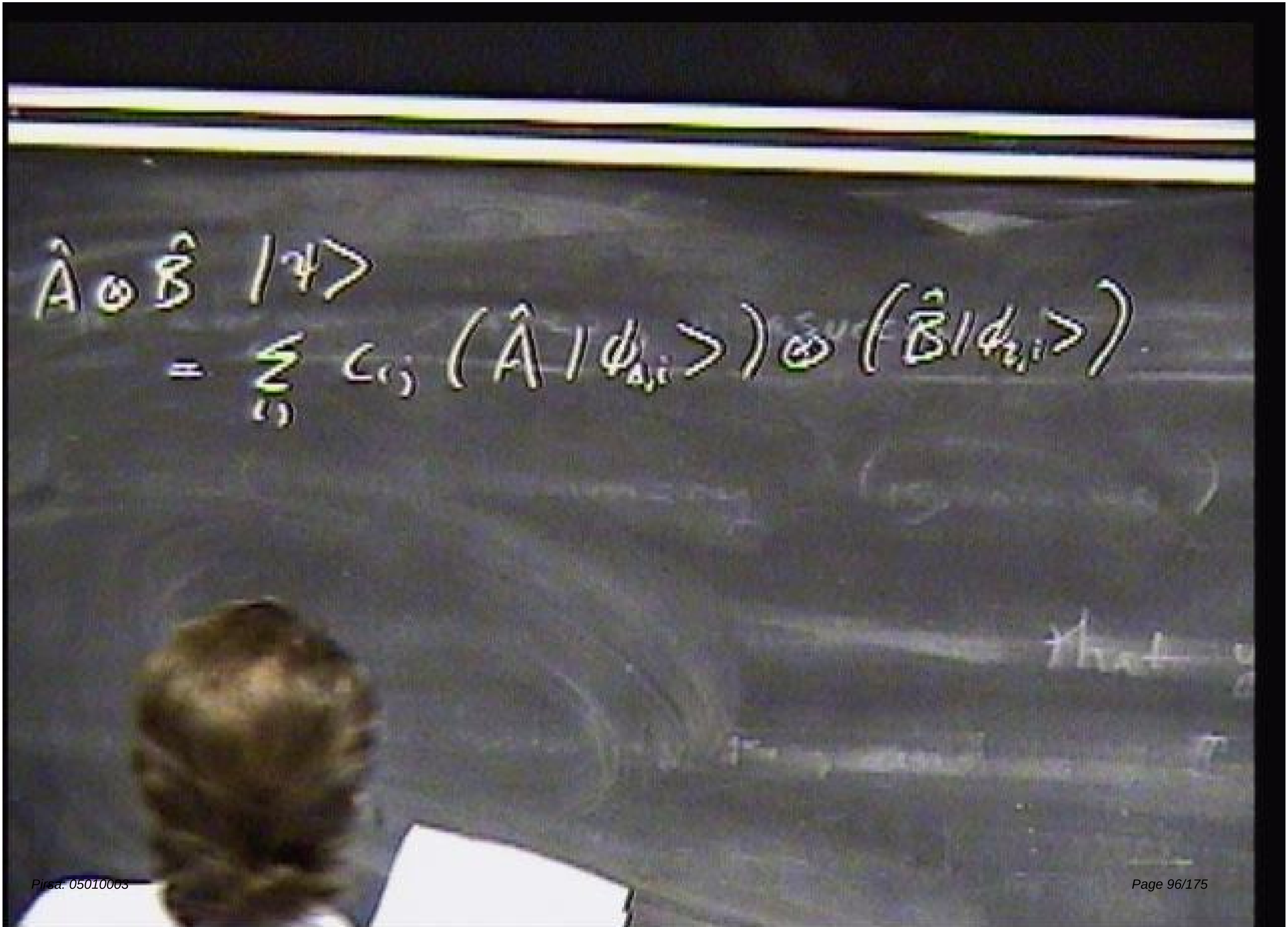
in

$$|+\rangle = (|0\rangle + |1\rangle)\frac{1}{\sqrt{2}}$$

$$|-\rangle = (|0\rangle - |1\rangle)\frac{1}{\sqrt{2}}$$

Express both ρ_P & ρ_m

basis. Determine a


$\hat{A}\otimes\hat{B}\,|\psi\rangle$
$= \sum_{ij} c_{ij}\,(\hat{A}\,|\phi_{a,i}\rangle)\otimes(\hat{B}\,|\phi_{b,j}\rangle)$
that

$$\hat{A}\otimes\hat{B}\,|\psi\rangle$$
$$= \sum_{i,j} c_{ij}\,(\hat{A}\,|\phi_{A,i}\rangle)\otimes(\hat{B}\,|\phi_{B,j}\rangle)$$

Any operator $\hat{O}$ acting on $\mathcal{H}$ can be expressed

$$\hat{O} = \sum_k A_k \otimes B_k$$

$$= \sum_{ij} c_{ij} \ldots$$

Any operator $\hat{O}$ acting on $\mathcal{H}$ can

$$\hat{O} = \sum_k \hat{A}_k \otimes \hat{B}_k$$

$$\hat{O} = \hat{A} \otimes \mathbb{1}$$

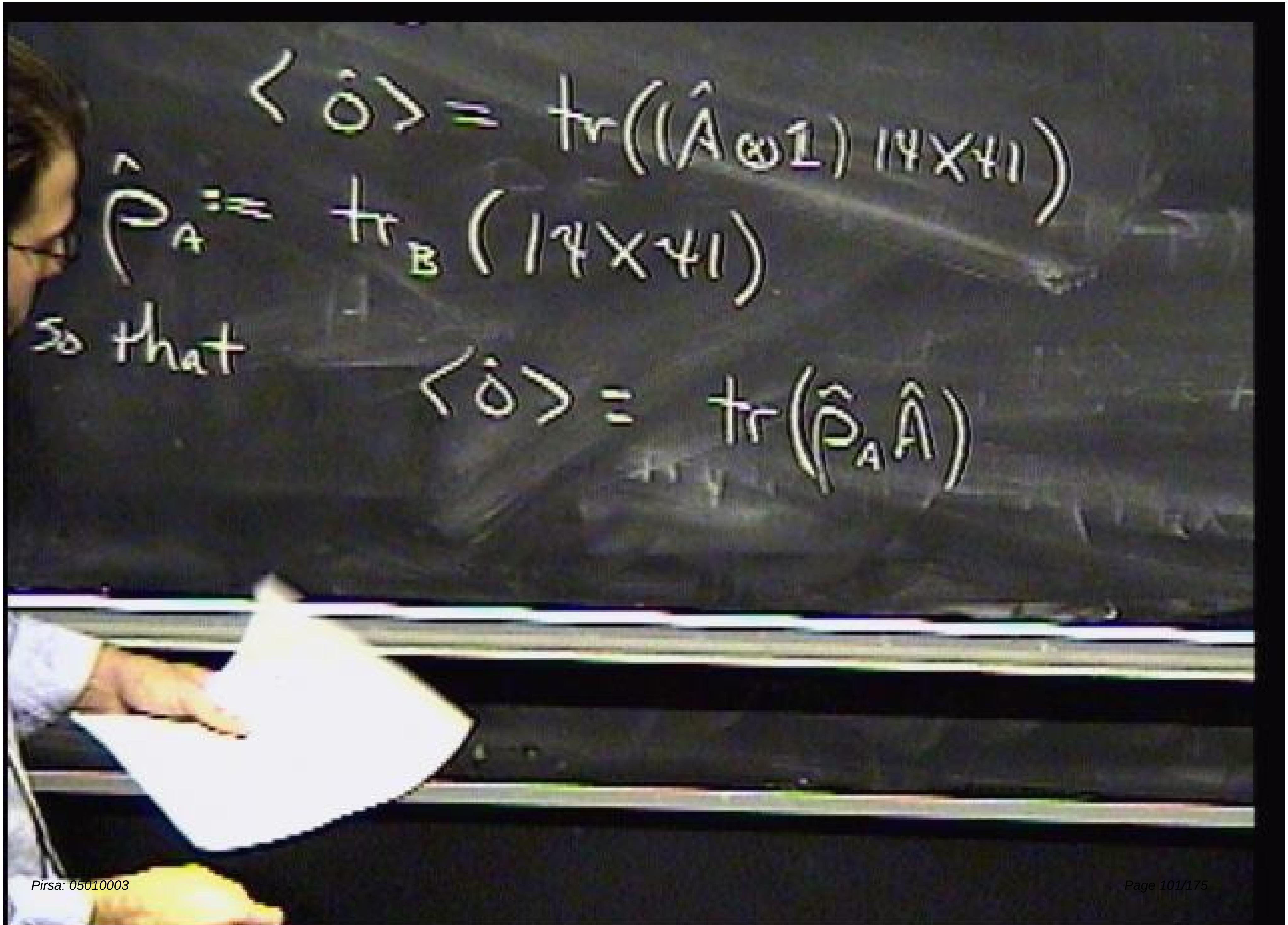
$$\hat{O} = \sum_k A_k \otimes B_k$$

$$\hat{O} = \hat{A} \otimes \mathbb{1}$$

$$\langle \hat{O} \rangle = \mathrm{tr}\left((\hat{A} \otimes \mathbb{1}) |\Psi\rangle\langle\Psi| \right)$$

$$\langle \hat{O} \rangle = \mathrm{tr}\left((\hat{A} \otimes \mathbb{1}) |\psi\rangle\langle\psi| \right)$$

$$\rho_A := \mathrm{tr}_B \left(|\psi\rangle\langle\psi| \right)$$

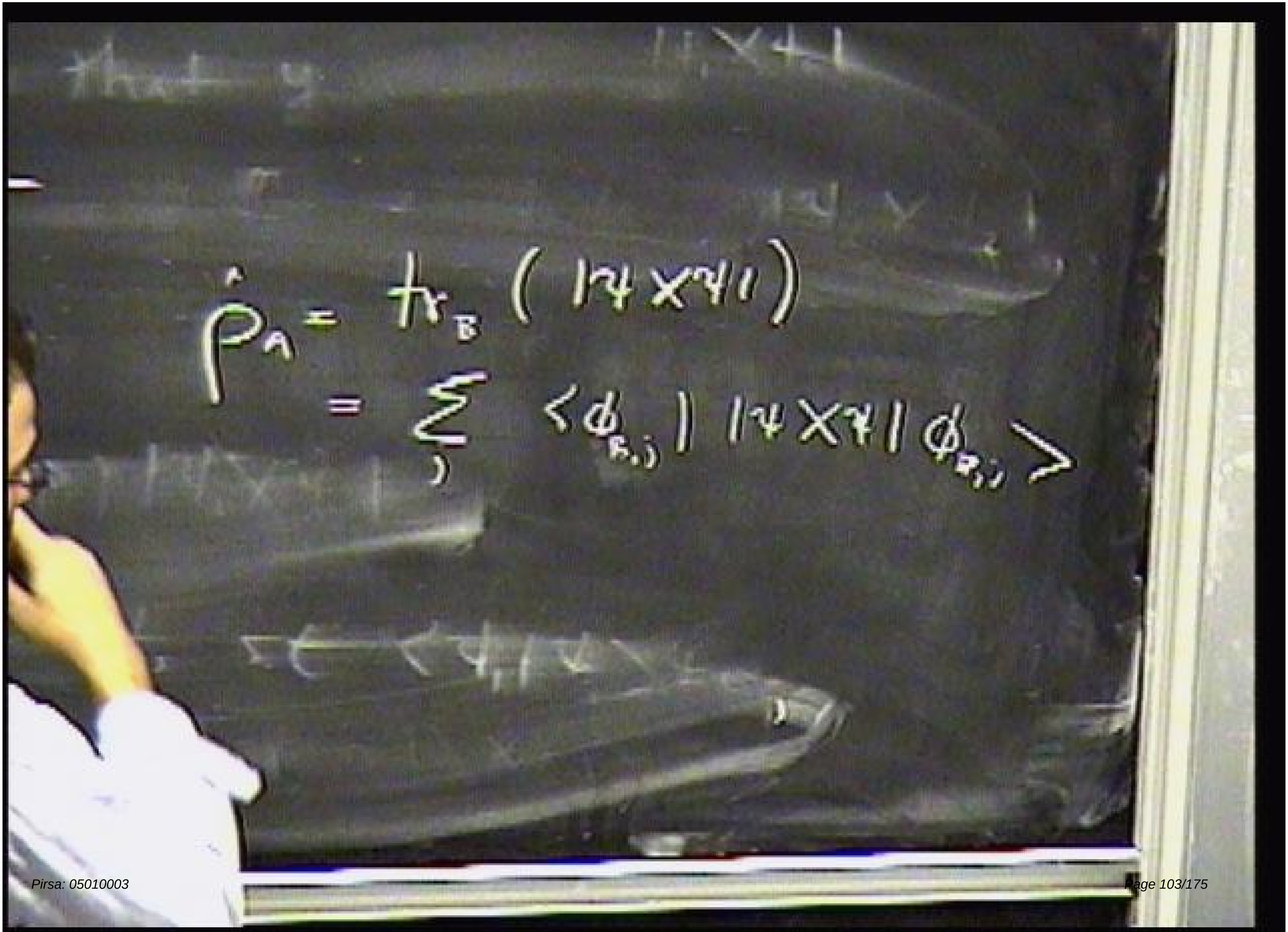

$\langle \hat{O} \rangle = \mathrm{tr}((\hat{A} \otimes \mathbb{1}) |\psi\rangle\langle\psi|)$
$\hat{\rho}_A := \mathrm{tr}_B(|\psi\rangle\langle\psi|)$
so that
$\langle \hat{O} \rangle = \mathrm{tr}(\hat{\rho}_A \hat{A})$

$$\langle \hat{O} \rangle = \mathrm{tr}\left((\hat{A} \otimes \mathbb{1})\, |\psi\rangle\langle\psi| \right)$$

$$\hat{\rho}_A := \mathrm{tr}_B \left(|\psi\rangle\langle\psi| \right)$$

so that

$$\langle \hat{O} \rangle = \mathrm{tr}_{(A)}\left(\hat{\rho}_A \hat{A} \right)$$


$\hat{\rho}_A = tr_B (|\psi\rangle\langle\psi|)$
$= \sum_j \langle \phi_{B,j} | \, |\psi\rangle\langle\psi| \, | \phi_{B,j} \rangle$

$$\hat{\rho}_A = \mathrm{tr}_B \left(|\psi\rangle\langle\psi| \right)$$

$$= \sum_j \langle \phi_{B,j} | \, |\psi\rangle\langle\psi| \, \phi_{B,j} \rangle$$

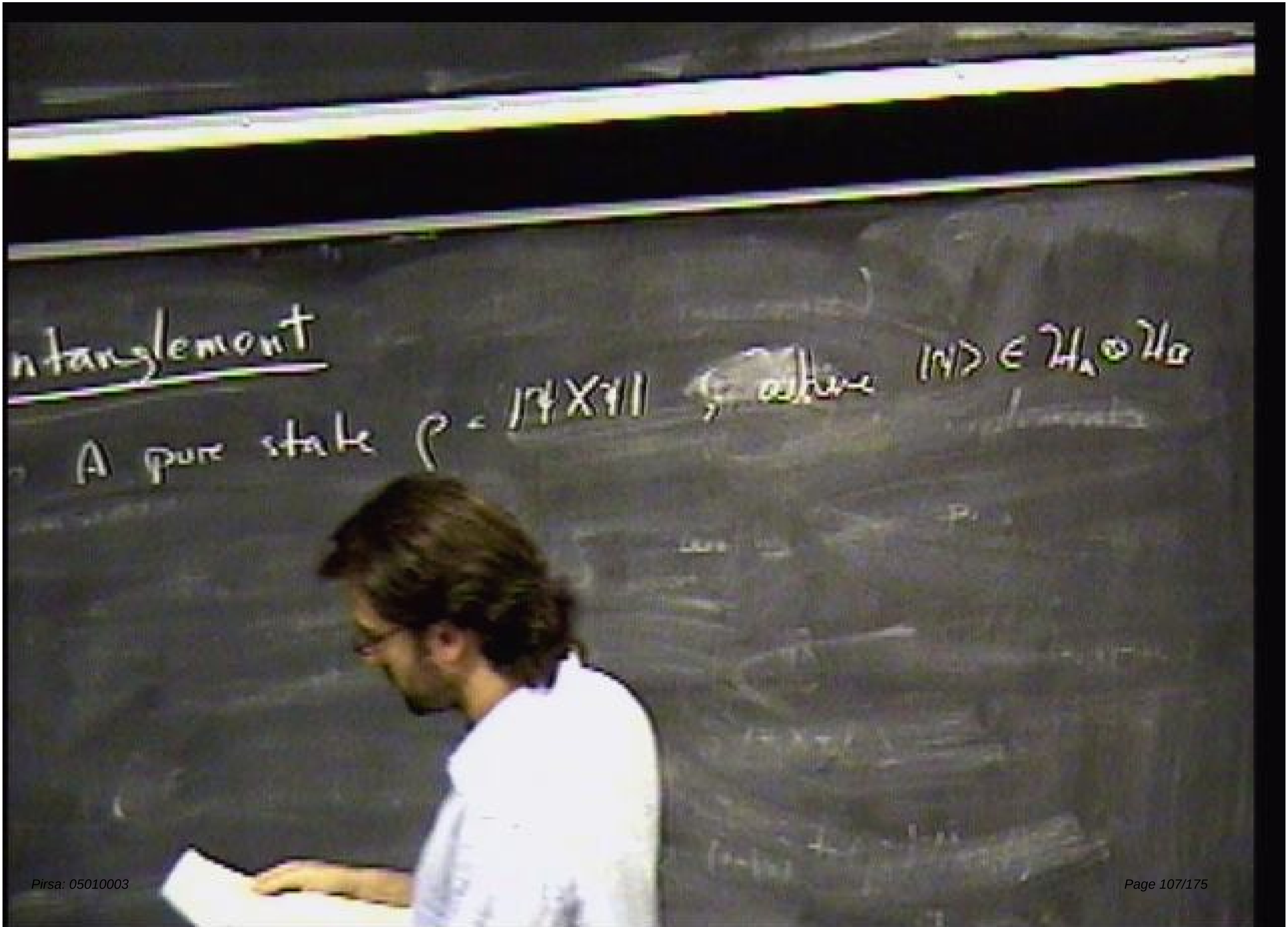
$\Rightarrow$ produces

$$\hat{\rho}_A = tr_B (|\psi\rangle\langle\psi|)$$

$$= \sum_j \langle \phi_{B,j} | \, |\psi\rangle\langle\psi| \, | \phi_{B,j} \rangle$$

$\Rightarrow$ uniquely produces a state operator


Entanglement


ntanglement
A pure state $\rho = |\psi\rangle\langle\psi|$; where $|\psi\rangle \in \mathcal{H}_A \otimes \mathcal{H}_B$

ntanglement

A pure state $\rho = |\psi\rangle\langle\psi|$; where $|\psi\rangle \in \mathcal{H}_A \otimes \mathcal{H}_B$

is entangled (factorable, separable) iff

$$|\psi\rangle = |\psi_A\rangle \otimes |\psi_B\rangle$$

ntanglement

A pure state $\rho = |\psi\rangle\langle\psi|$, where $|\psi\rangle \in \mathcal{H}_A \otimes \mathcal{H}_B$
is unentangled (factorable, separable) iff

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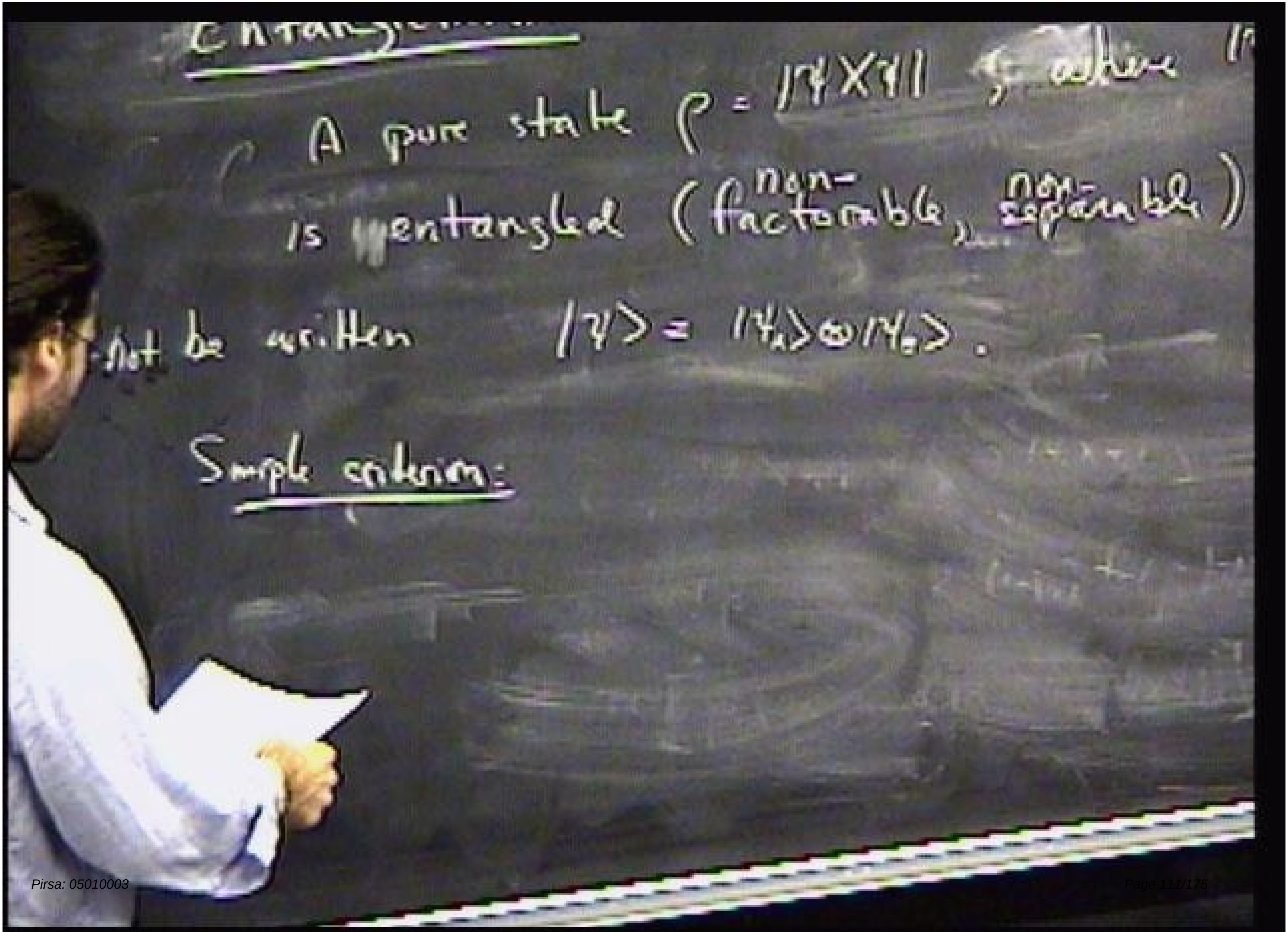
Entanglement

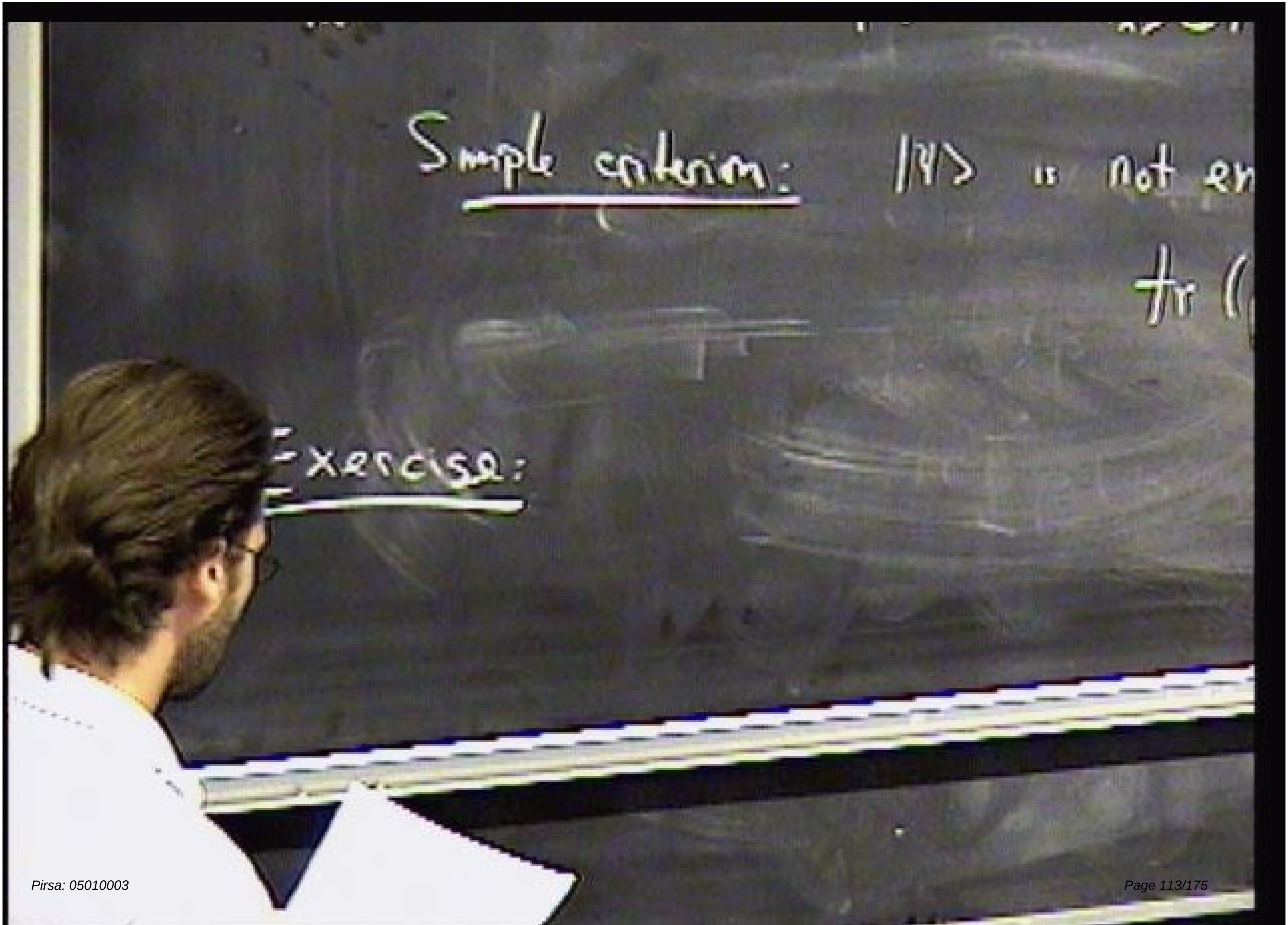
A pure state $\rho = |\Psi\rangle\langle\Psi|$ where $|\Psi\rangle \in \mathcal{H}_A \otimes \mathcal{H}_B$
is entangled (non-factorable, non-separable) iff it cannot be written $|\Psi\rangle = |\Psi_A\rangle \otimes |\Psi_B\rangle$


A pure state ρ = |ψ⟩⟨ψ| , where
is entangled (non-factorable, non-separable)
not be written |ψ⟩ = |ψ_A⟩⊗|ψ_B⟩ .
Simple criterion:

be written $|\Psi\rangle = |\Psi_A\rangle \otimes |\Psi_B\rangle$.

Simple criterion: $|\Psi\rangle$ is not entangled iff

$$\mathrm{tr}(\rho_A^2) = 1$$


Simple criterion: |Ψ⟩ is not en
tr (
Exercise:

hen $\quad |\psi\rangle = |\psi_A\rangle \otimes |\psi_B\rangle$.

criterion: $\quad |\psi\rangle$ is not entangled iff

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Use Schmidt decomposition to show that

$$\mathrm{tr}(\rho_A^2) = \mathrm{tr}(\rho_B^2) \quad \forall \; |\psi\rangle \in \mathcal{H}_A \otimes \mathcal{H}_B$$

then $\quad |\Psi\rangle = |\Psi_A\rangle \otimes |\Psi_B\rangle$.

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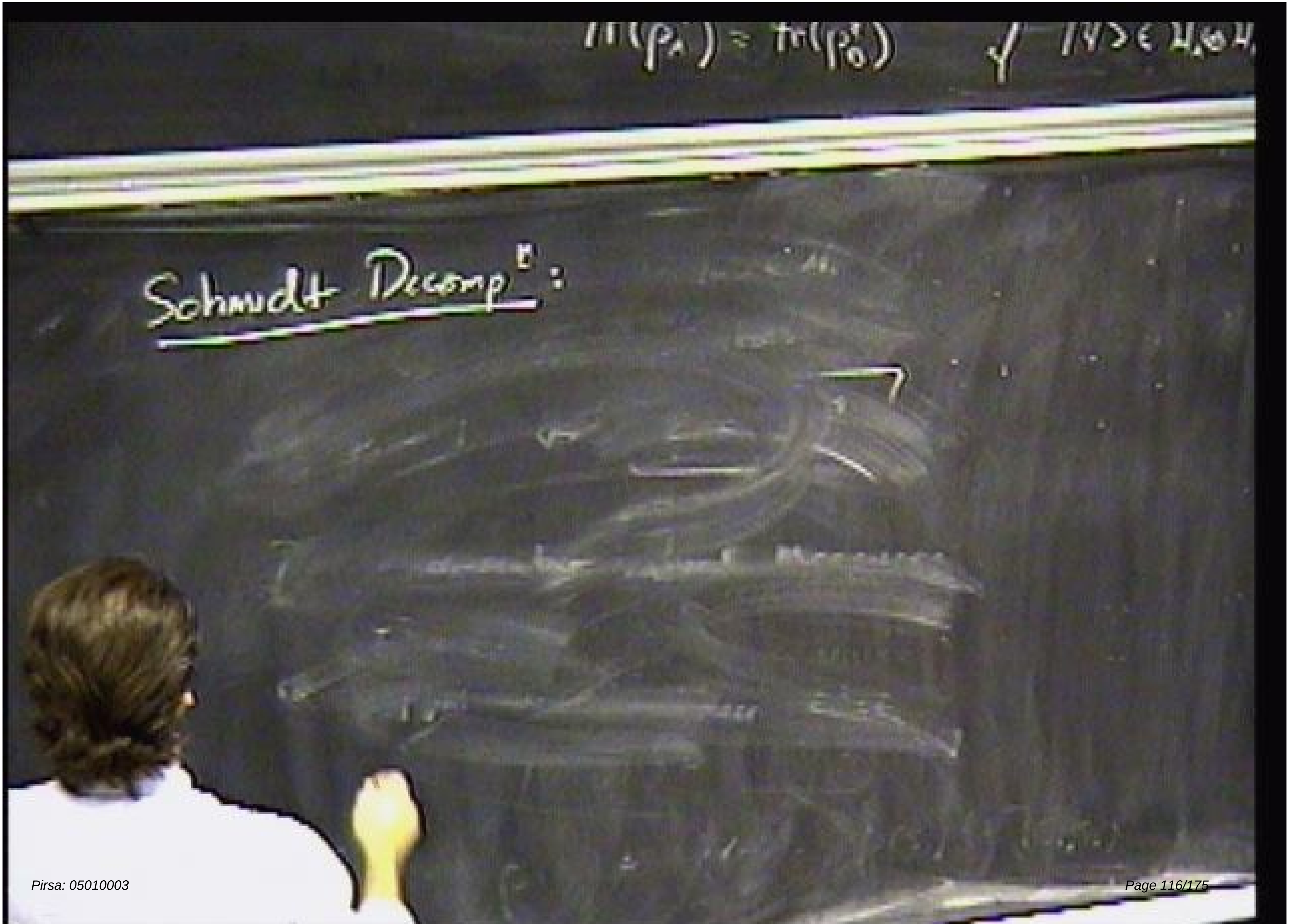
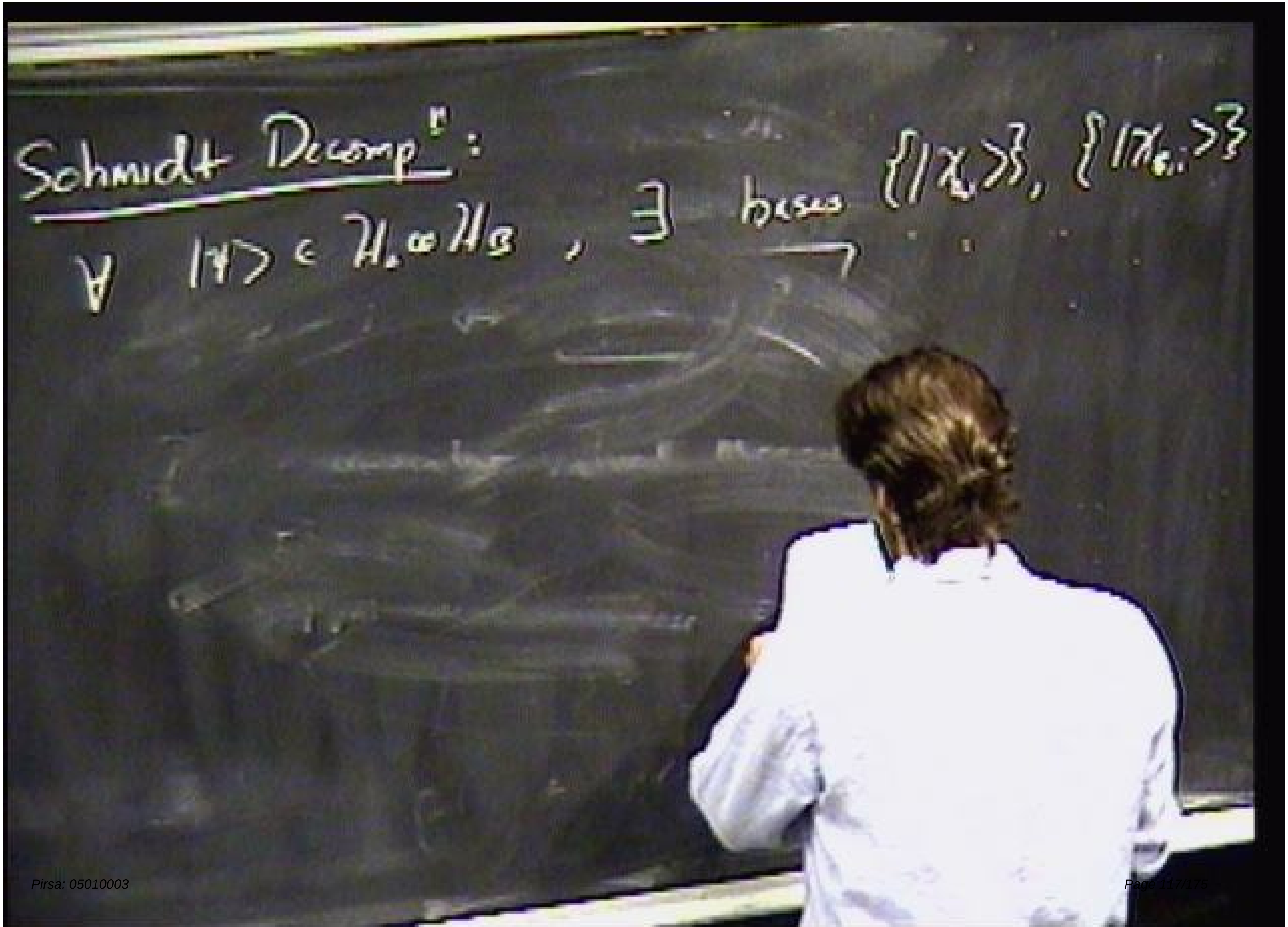

Schmidt Decomp^n:


Schmidt Decomp^n:
$\forall\ |\psi\rangle \in \mathcal{H}_A \otimes \mathcal{H}_B$, $\exists$ bases $\{|\chi_{A,i}\rangle\}$, $\{|\chi_{B,i}\rangle\}$

, $\exists$ bases $\{|\chi_{A,i}\rangle\}, \{|\chi_{B,i}\rangle\}$ for $\mathcal{H}_A, \mathcal{H}_B$ respectively

$\mathrm{Tr}(\rho_A) = \mathrm{Tr}(\rho_B) \quad \forall \ |\psi\rangle \in \mathcal{H}_A \otimes \mathcal{H}_B$

Schmidt Decomp^{n}:

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such that

$$|\psi\rangle = \sum_i \alpha_i \ |\chi_{A,i}\rangle \otimes |\chi_{B,i}\rangle$$

where $\alpha_i \geq 0$

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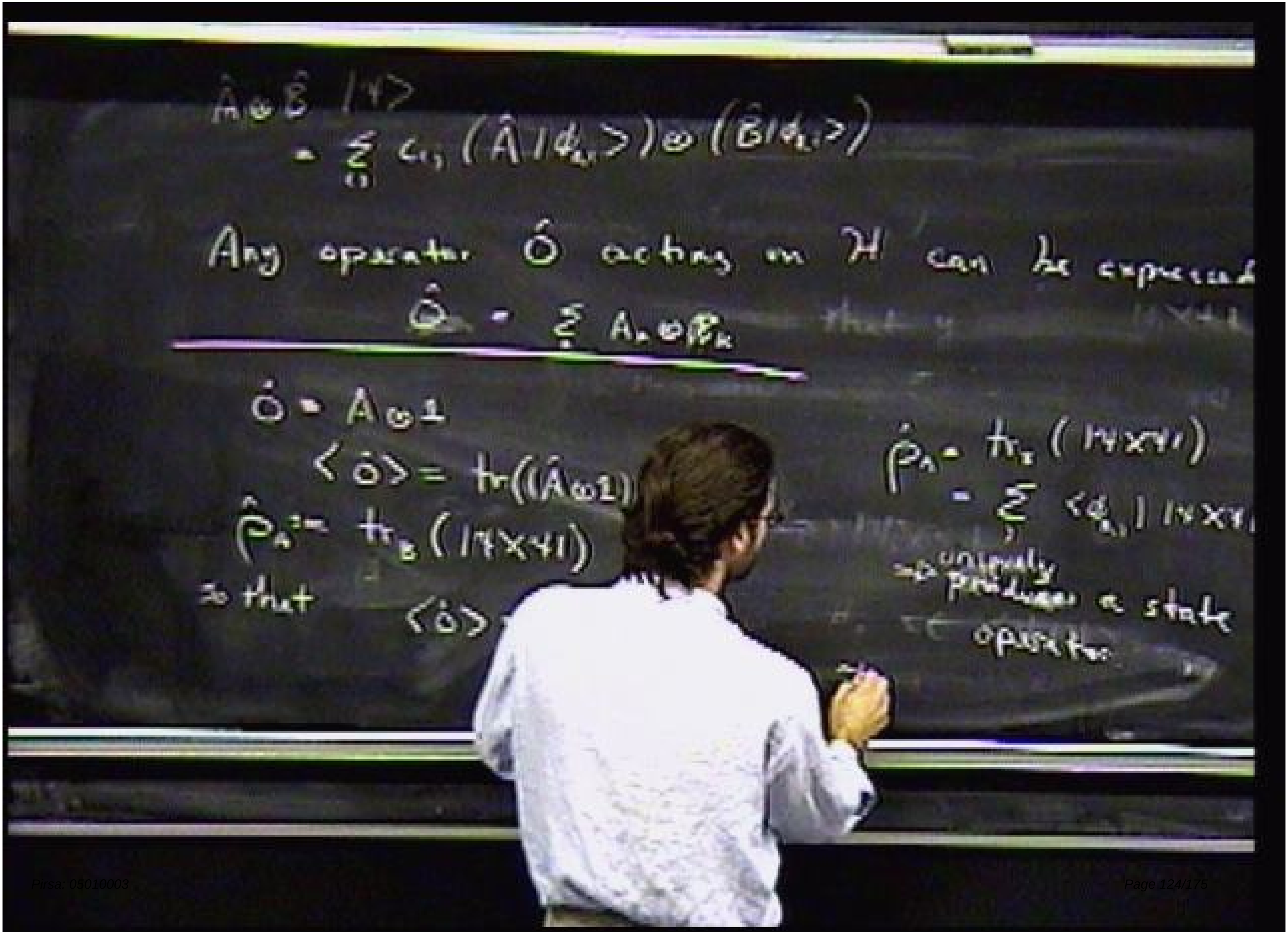
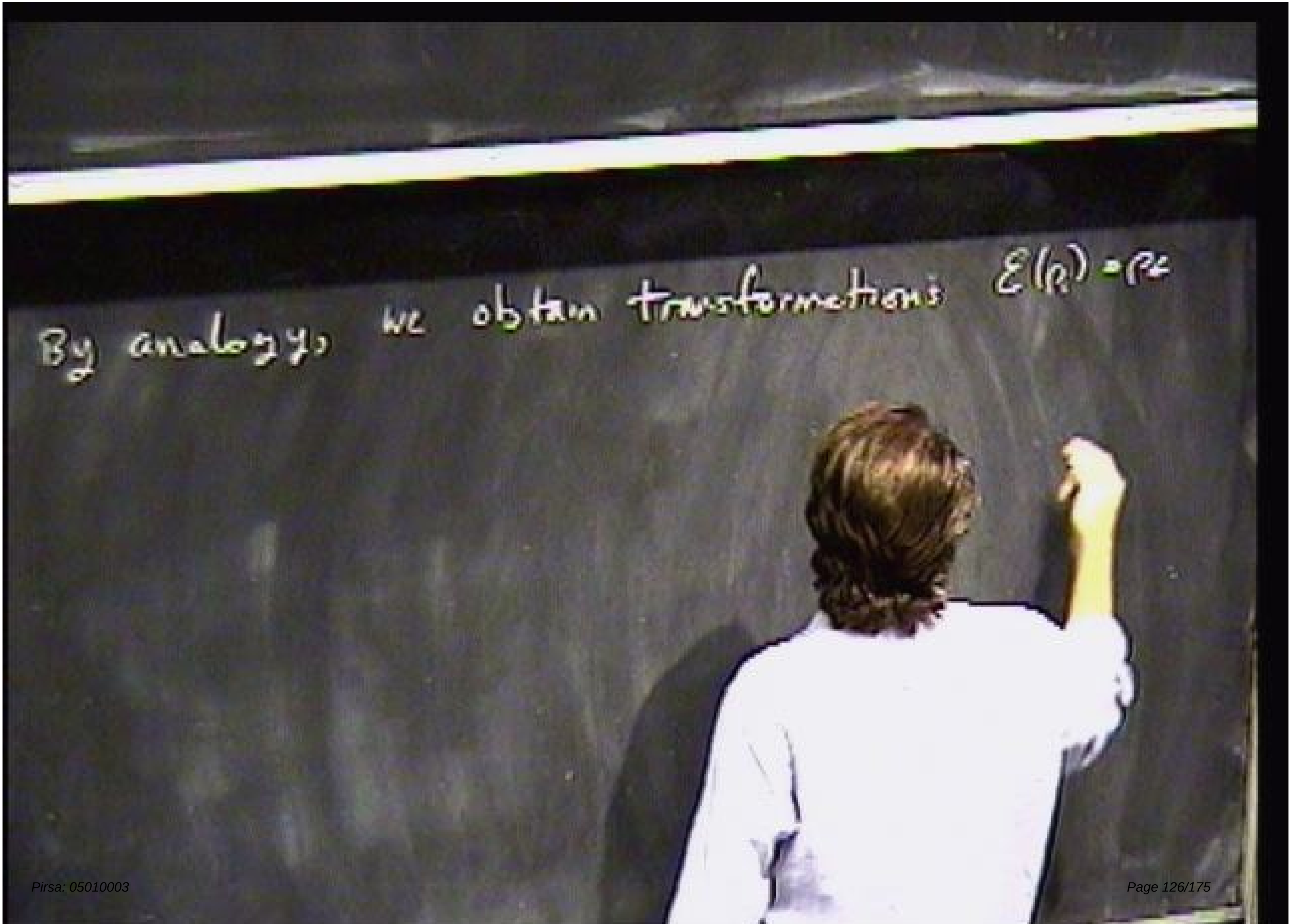

$\hat{A}\otimes\hat{B}\,|\Psi\rangle$
$= \sum_{i,j} c_{ij} (\hat{A}|\phi_{A_i}\rangle)\otimes(\hat{B}|\phi_{B_j}\rangle)$
Any operator $\hat{O}$ acting on $\mathcal{H}$ can be expressed
$\hat{O} = \sum_k A_k \otimes B_k$
$\hat{O} = \hat{A}\otimes 1$
$\langle\hat{O}\rangle = tr((\hat{A}\otimes 1)$
$\hat{\rho}_A := tr_B(|\Psi\rangle\langle\Psi|)$
so that
$\langle\hat{O}\rangle$
$\hat{\rho}_A = tr_B(|\Psi\rangle\langle\Psi|)$
$= \sum_j \langle\phi_{B_j}|\,|\Psi\rangle\langle\Psi|$
uniquely produces a state operator

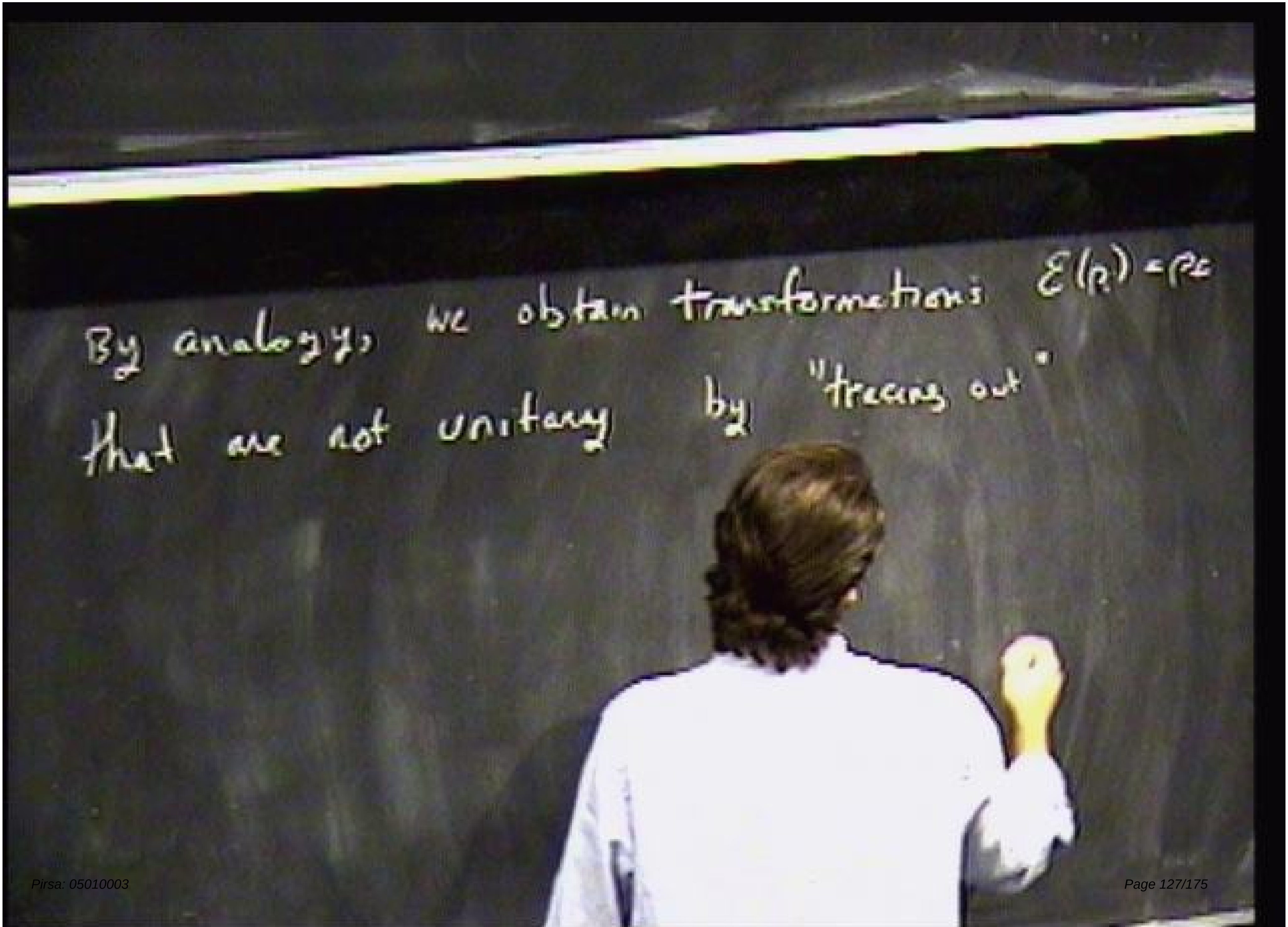
$$\hat{\rho}_A = tr_B (|\psi\rangle\langle\psi|)$$

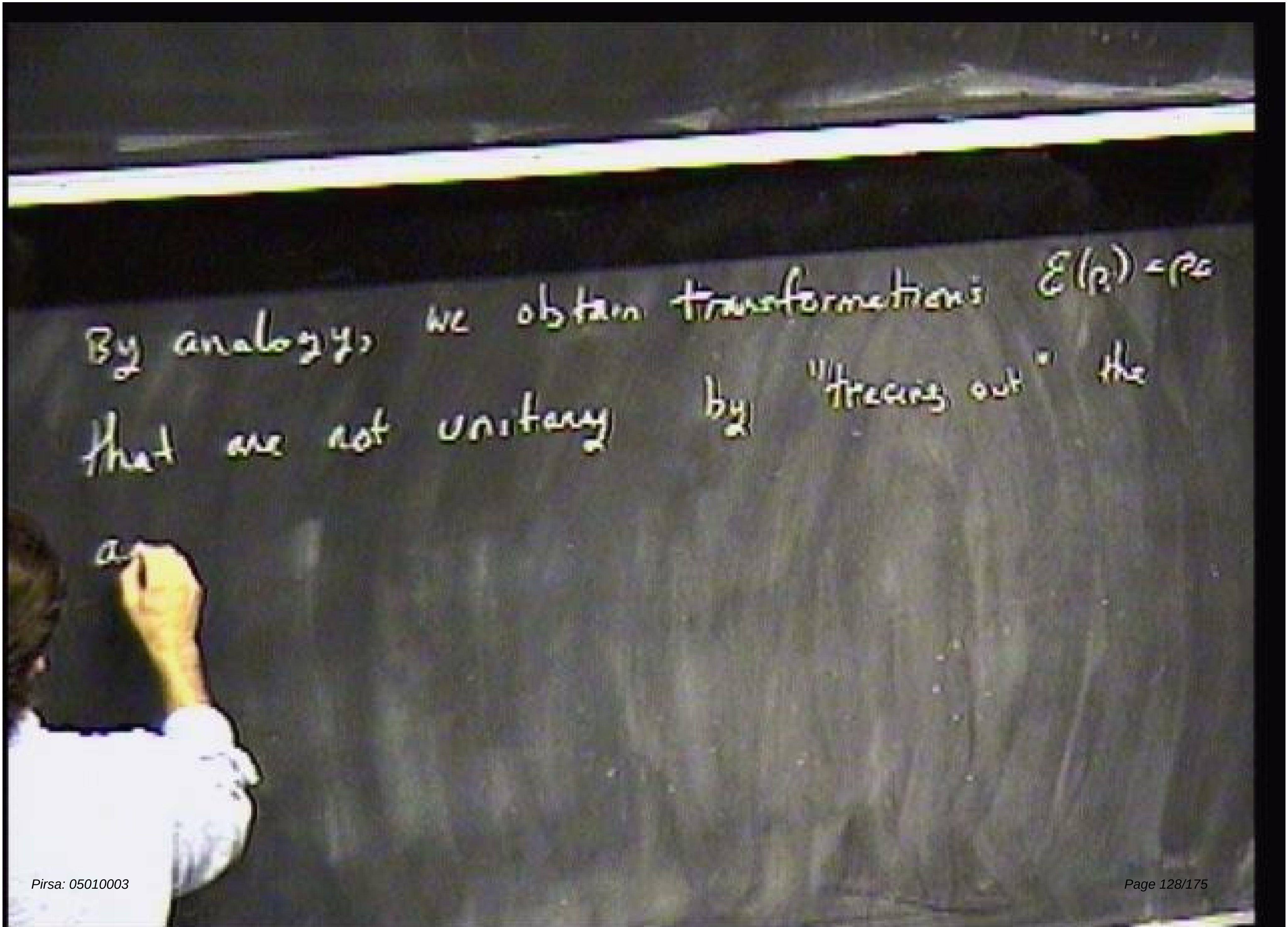
$$= \sum_j \langle \phi_{B,j} | \, |\psi\rangle\langle\psi| \, | \phi_{B,j} \rangle$$

$\Rightarrow$ uniquely produces a state operator

$\Rightarrow$ Generically $tr(\rho_A^2) < 1$


By analogy, we obtain transformations


By analogy, we obtain transformations $\mathcal{E}(\rho_S) = \rho_S$
that are not unitary by "tracing out".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a

By analogy, we obtain transformations $\mathcal{E}(\rho) = \rho_c$ that are not unitary by "tracing out" the action of a unitary $\mathcal{U} \in \mathcal{H}_A \otimes \mathcal{H}_B$.

By analogy, we obtain transformations $\mathcal{E}(\rho_S) \leftarrow \rho_S$

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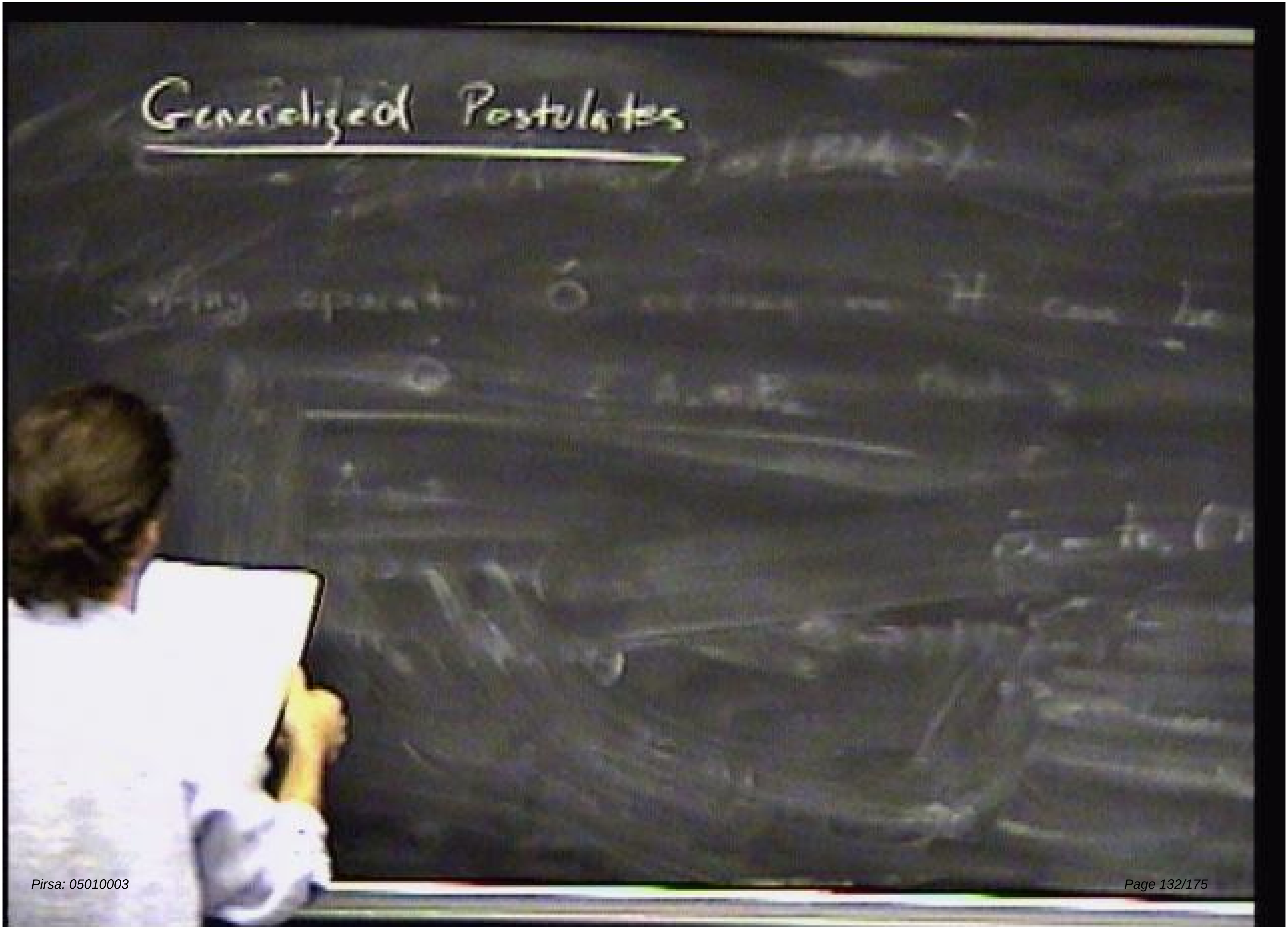
action of a unitary $\mathcal{U} \in \mathcal{H}_A \otimes \mathcal{H}_E$.

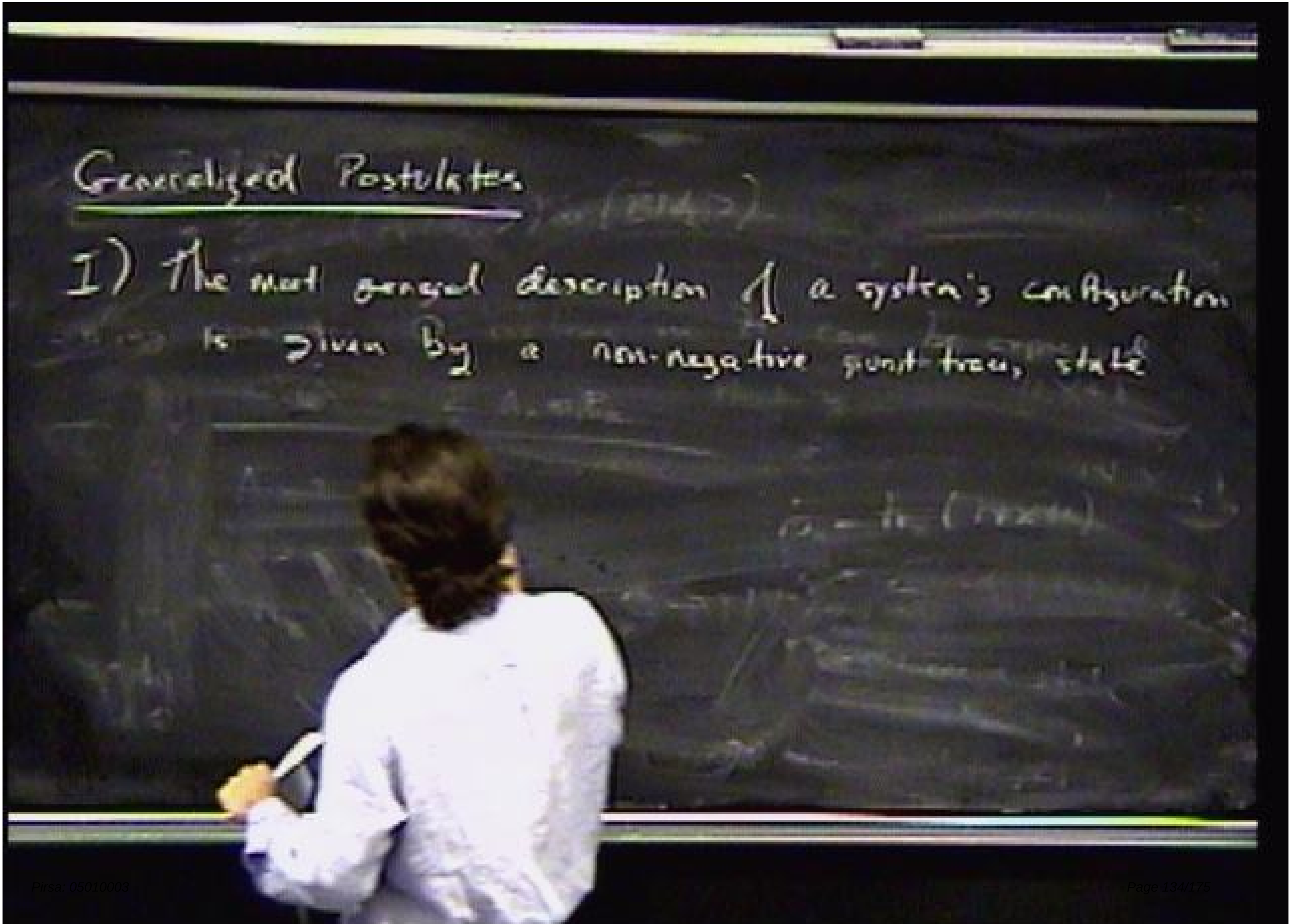
By analogy, "tracing out" the
that are not unitary by "tracing out" the
action of a unitary $\mathcal{U}$ on $\mathcal{H}_A \otimes \mathcal{H}_B$.

And similarly, we obtain POVM on $\mathcal{H}_A$
by "tracing out" the component of a PVM (on $\mathcal{H}_A \otimes \mathcal{H}_B$)
acting $\mathcal{H}_B$, leaving a POVM on $\mathcal{H}_A$.


Generalized Postulates

Generalized Postulates

1) The most general description of a system's configuration


Generalized Postulates
1) The most general description of a system's configuration
is given by a non-negative quantity, state

Generalized Postulates

I) The most general description of a system's configuration is given by a non-negative, unit-trace, state operator $\hat{\rho}$.

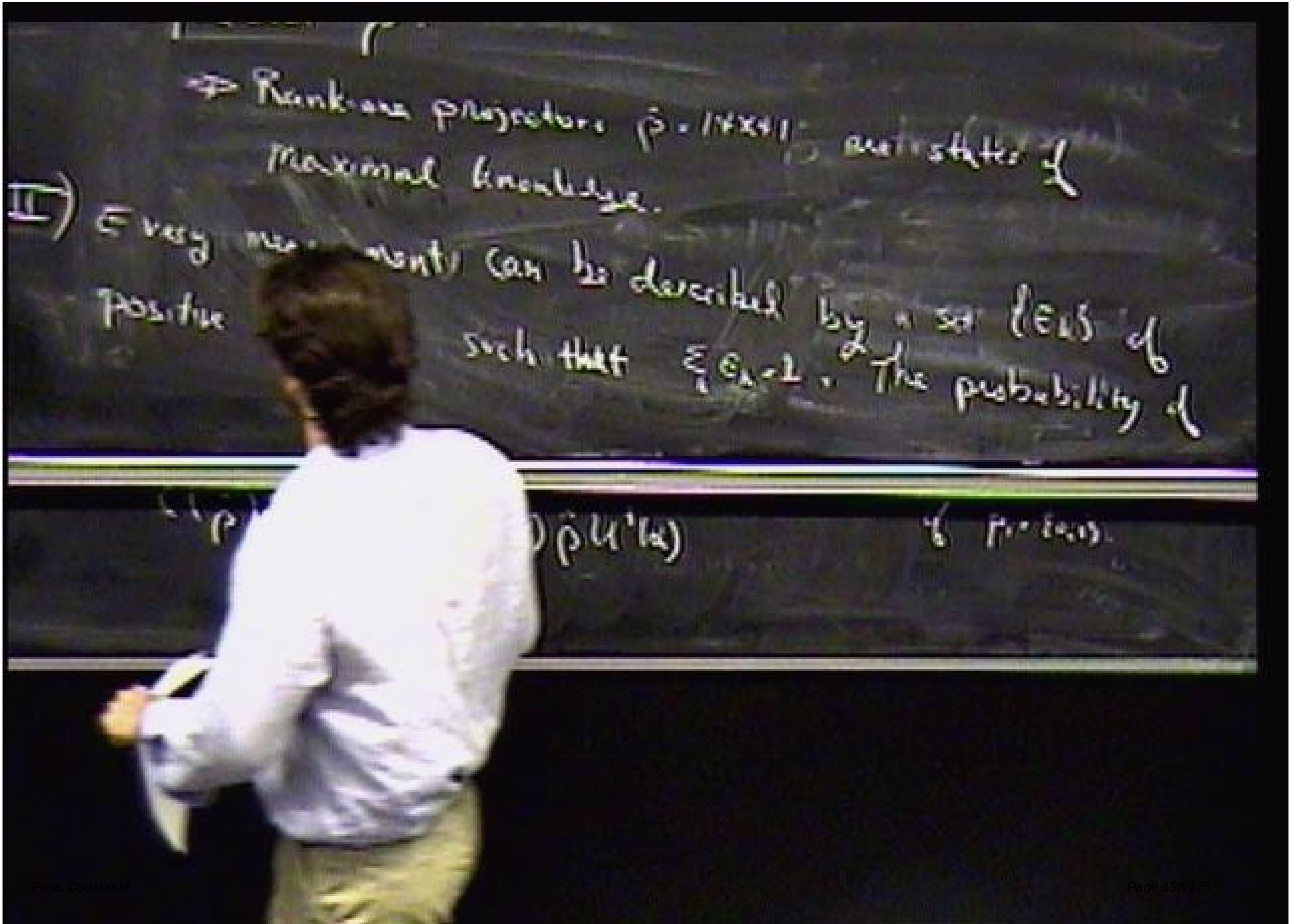
given by a non-negative unit trace, sta
erator $\hat{\rho}$.

Rank-one projectors $\hat{\rho} = |\psi\rangle\langle\psi|$ are states of
maximal knowledge.

I) The most general description of a system's configuration is given by a non-negative unit-trace, state operator $\hat{\rho}$.

$\Rightarrow$ Rank-one projectors $\hat{\rho} = |\psi\rangle\langle\psi|$ are states of maximal knowledge.

II) Every measurement can be described by a set $\{E_b\}$ of


⇒ Rank-one projectors $\hat{\rho} = |\psi\rangle\langle\psi|$ are states of maximal knowledge.
II) Every measurement can be described by a set $\{E_k\}$ of
positive
such that $\sum_k E_k = 1$. The probability of
of process.

⇒ Rank-one projectors $\hat{\rho} = |\psi\rangle\langle\psi|$ are states of maximal knowledge.

II) Every measurement can be described by a set $\{E_\alpha\}$ of positive operators such that $\sum_\alpha E_\alpha = 1$. The probability of out[illegible]

$$(\hat{\rho}) = \int d\alpha \, p(\alpha)\, U(\alpha)\, \hat{\rho}\, U^\dagger(\alpha)$$

of ρ [illegible]

⇒ Rank-one projectors $\hat{\rho} = |\psi\rangle\langle\psi|$ are states of maximal knowledge.

II) Every measurement can be described by a set $\{E_k\}$ of positive operators such that $\sum_k E_k = 1$. The probability of outcome "k"

$(\hat{\rho}) = \int d\alpha \, p(\alpha) \, U(\alpha) \hat{\rho} U^{\dagger}(\alpha)$ $\quad p \geq 0$

maximal knowledge.

II) Every measurement, can be described by positive operators such that $\sum_k E_k = \mathbb{1}$. outcome "k", for state $\hat{\rho}$, $p_k = \mathrm{tr}(\hat{\rho} E_k)$.

$$\mathcal{C}(\hat{\rho}) = \int d\alpha \, p(\alpha) \, U(\alpha) \, \hat{\rho} \, U^{\dagger}(\alpha)$$

ansformations

$\Rightarrow$ Rank-one PVM correspond to maximal tests.

Transformations

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III) The most general description of a transformation

$p_k = \mathrm{Tr}(\rho E_k)$.

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Transformations

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III) The most general description of a transformation is a completely positive linear map $\mathcal{E}(\rho)$ which takes the set of positive operators to itself.

The most general description of a transfor[mation]
is a completely positive linear maps $\mathcal{E}$
which takes the set of positive operator[s]

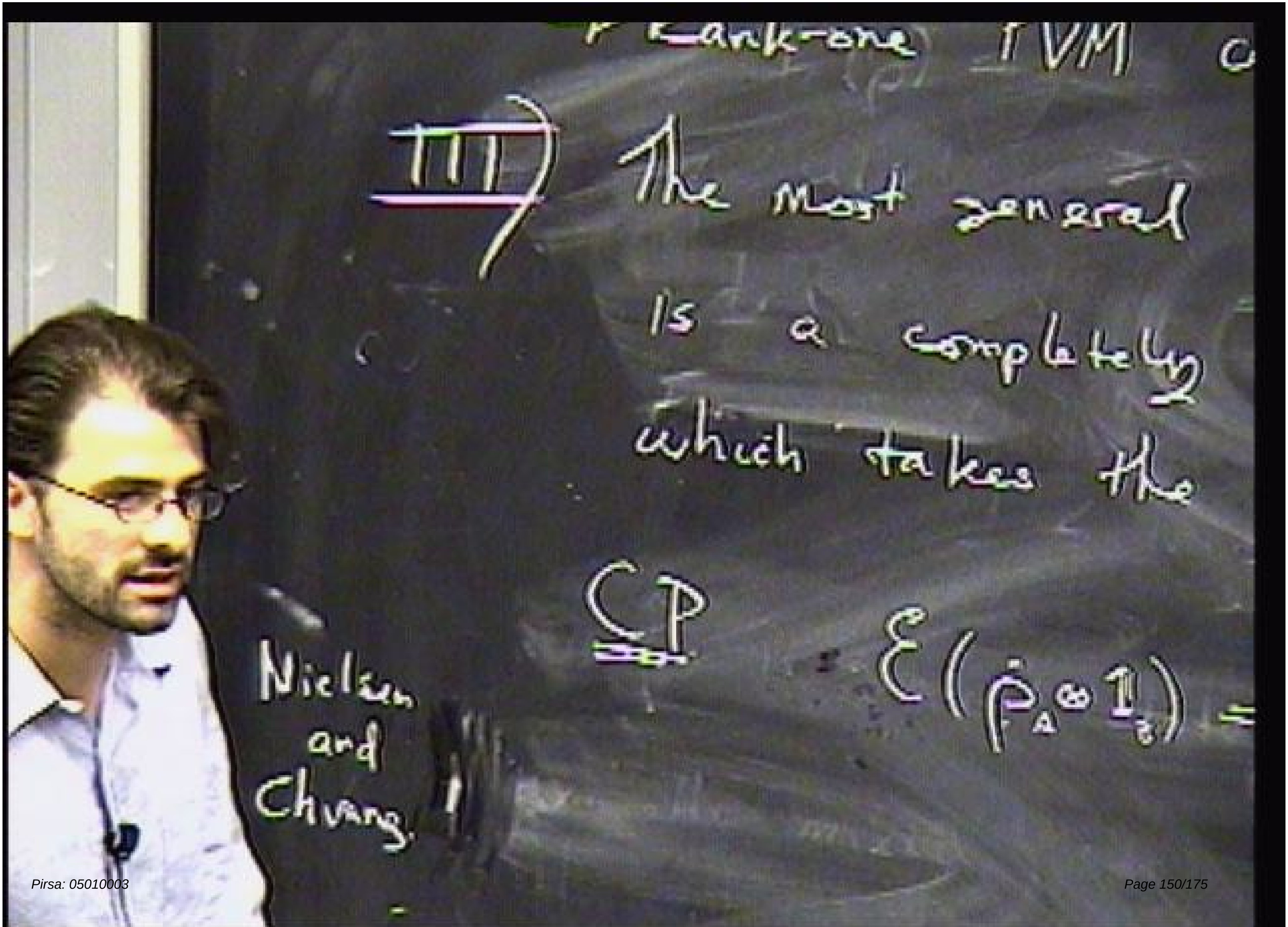
CP

$$\mathcal{E}(\rho_A \otimes \mathbb{1}_B) = \rho_{AB} \text{ that is nonnegative.}$$

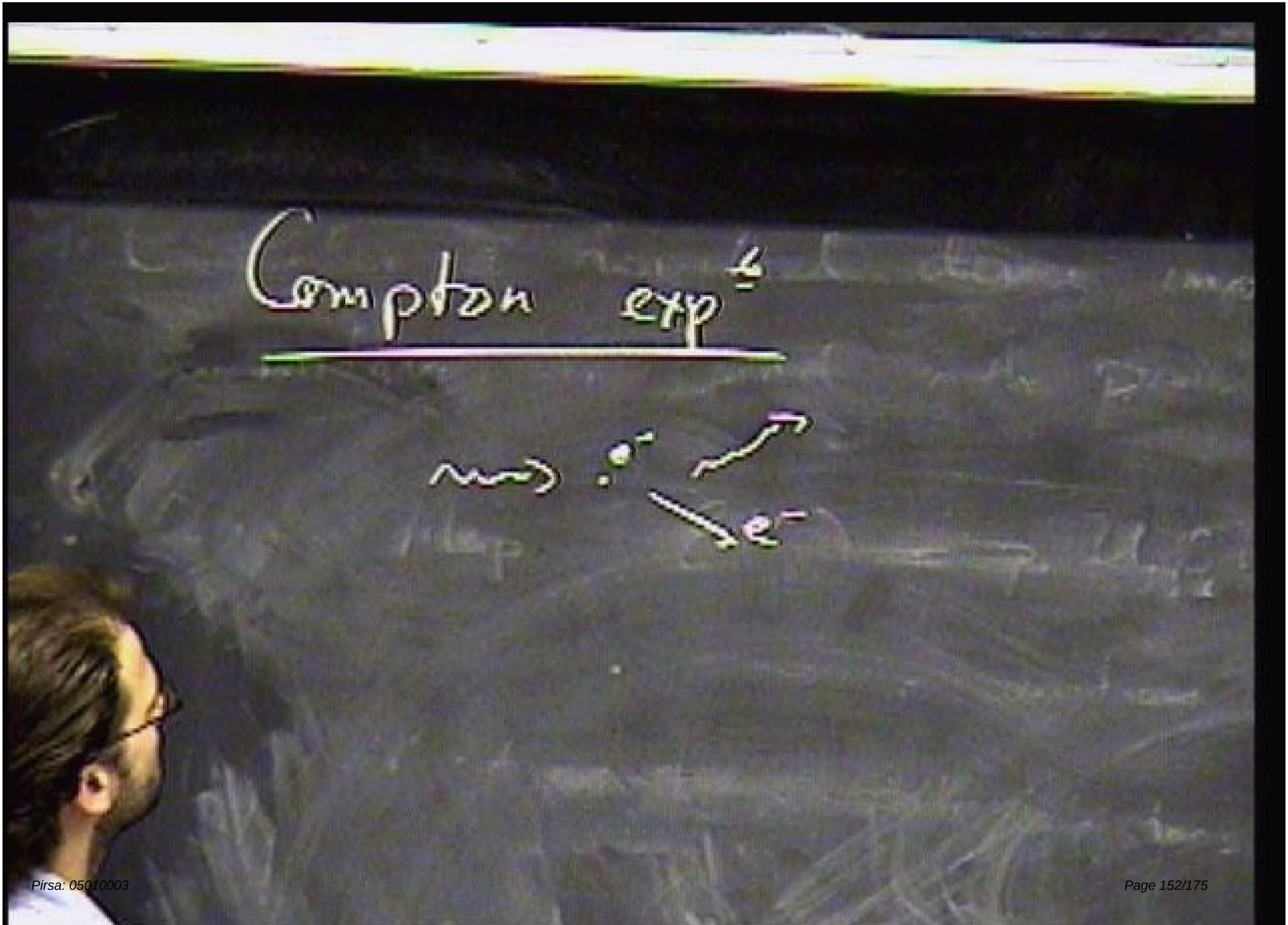
The most general description of a transfor[mation]
is a completely positive linear map $\mathcal{E}$
which takes the set of positive operator[s]

CP

$$\mathcal{E}(\hat{\rho}_1 \otimes \mathbb{1}_2) = \rho_{12} \text{ that is non-negative:}$$


Rank-one POVM
III) The most general
is a completely
which takes the
CP
Nielsen
and
Chuang.

Compton exp^t


Compton exp$^{\underline{t}}$
e^-
e^-

Compton exp$^{\underline{t}}$

$E = h\nu$ $\rightsquigarrow$ $\cdot e^-$ $\rightsquigarrow$ $h\nu'$

$\searrow e^-$

Compton exp$^{\underline{t}}$

$E = h\nu$ ⟿ e^- ⟿ $h\nu'$

$p = \frac{E}{c} = h/\lambda$ ⟶ e^-

Compton exp$^{\underline{t}}$

$E = h\nu$ $\rightsquigarrow$ $\cdot\, e^-$ $\rightsquigarrow$ $h\nu'$

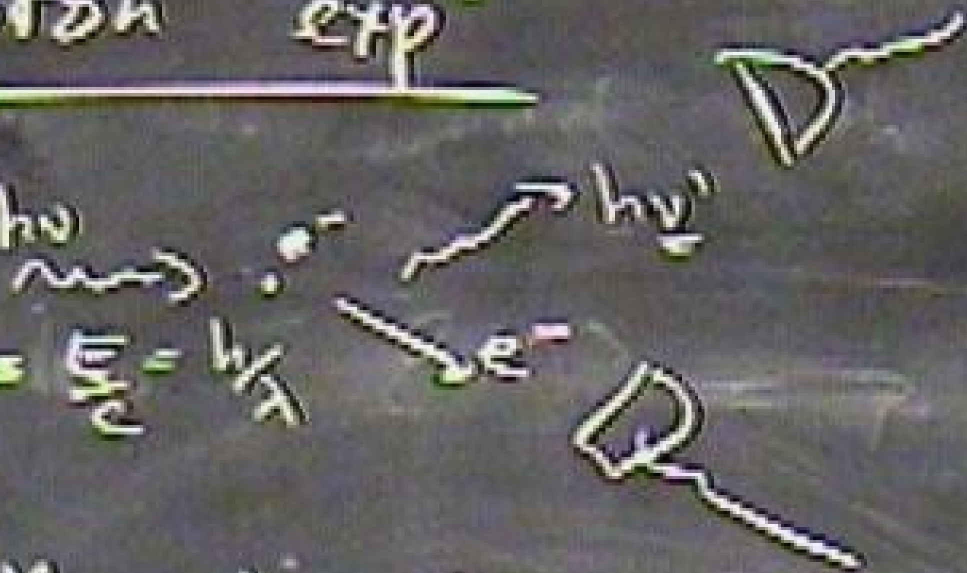
$\searrow e^-$

$p = \frac{E}{c} = h/\lambda$

Compton exp^{t}

$E = h\nu$

$p = \frac{E}{c} = h/\lambda$



1) Momentum of

Δt between interception

of e^-, photon.

Compton exp^t

$E = h\nu \rightsquigarrow e^- \rightsquigarrow h\nu'$

$p = \frac{E}{c} = h/\lambda \quad \searrow e^-$

Δt between int

$\gamma \quad e^-$

1) Momentum of e^- has a dispersion

Compton exp^{t}

$E = h\nu$, mass $= 0$ $\quad h\nu'$ $\quad e^-$

$p = \frac{E}{c} = \frac{h}{\lambda}$

Δt between interception of e^-, photon

1) Momentum of e^- has a dispersion and the momentum of photon has uncorrelated

Compton exp^s

$E = h\nu$

$p = \frac{E}{c} = \frac{h}{\lambda}$

$h\nu'$

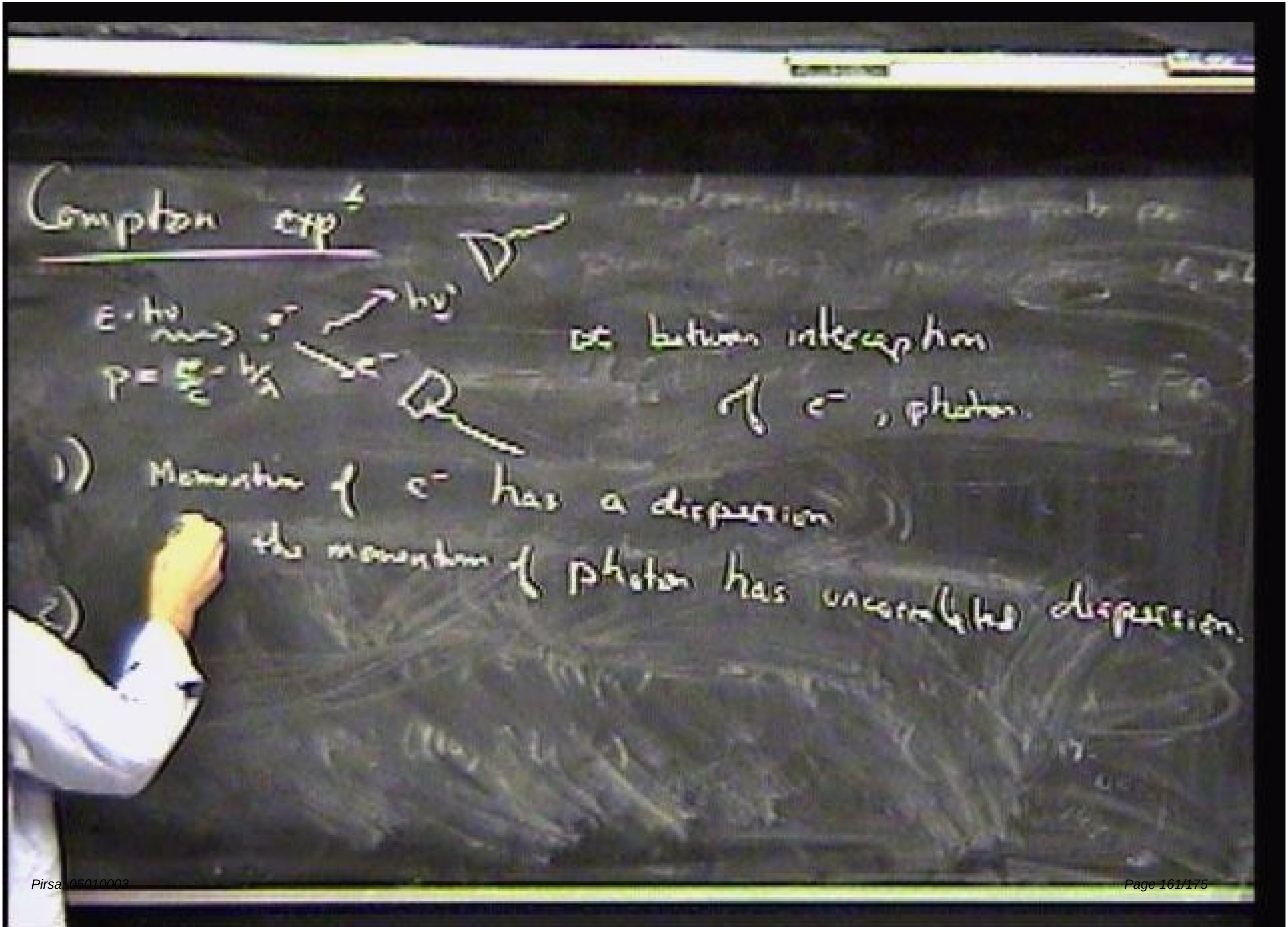
e^-

corr between interception

of e^-, photon

Von Neumann

1) Momentum of e^- has a dispersion and the momentum of photon has uncorrelated


Compton exp^t
E = hν
p = E/c = h/λ
hν'
e⁻
between interception
of e⁻, photon.
1) Momentum of e⁻ has a dispersion
the momentum of photon has uncorrelated dispersion.
2)

Compton exp^t

$E = h\nu$ $\rightsquigarrow$ e^- $\rightsquigarrow h\nu'$

$p = \frac{E}{c} = h/\lambda$ e^-

$\propto$ between interception

of e^-, photon.

a) Momentum of e^- has a dispersion

and b) the momentum of photon has uncorrelated d

2) 1a) holds but 1b) does not because the momentum of photon is fixed.

ompton exp^t

$E = h\nu$

$p = \frac{E}{c} = h/\lambda$

$h\nu'$

e^-

$\propto$ between interception of e^-, photon.

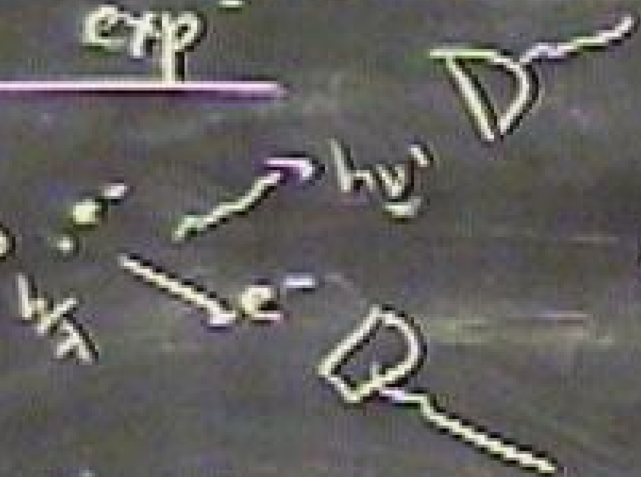
1) a) Momentum of e^- has a dispersion
and b) the momentum of photon has uncorrelated d

2) 1a) holds but 1b) does not, because the momentum of photon is fixed.

Compton exp^{t}

$E = h\nu$

$p = \frac{E}{c} = \frac{h}{\lambda}$



pc between interception of e^-, photon.

1) a) Momentum of e^- has a dispersion
and b) the momentum of photon has uncorrelated d

2) 1a) holds but 1b) does not because the momentum of photon is fixed.

3) Both

$E = h\nu$ $\rightsquigarrow e^-$ $\rightsquigarrow h\nu'$

$p = \frac{E}{c} = \frac{h}{\lambda}$ $\searrow e^-$

Δt between interception of e^-, photon.

1) a) Momentum of e^- has a dispersion
and b) the momentum of photon has uncorrelated d

2) 1a) holds but 1b) does not because the momentum of photon is fixed.

3) Both particle's momenta can be predicted certainty (no dispersion).

Compton exp^t

$E = h\nu$

$p = \frac{E}{c} = \frac{h}{\lambda}$

e^- $h\nu'$ e^-

Von Neumann

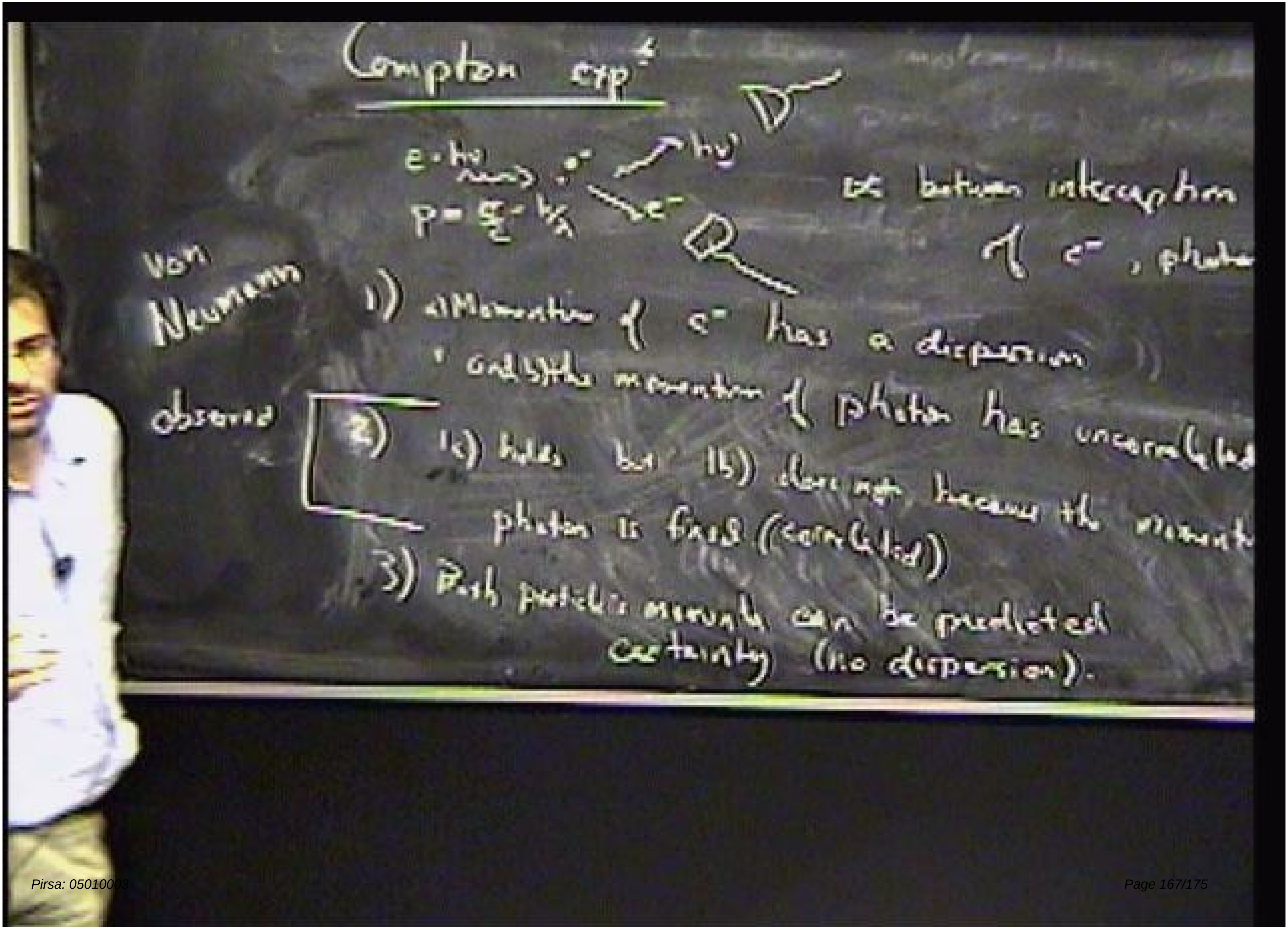
∝ between interception of e^-, photon

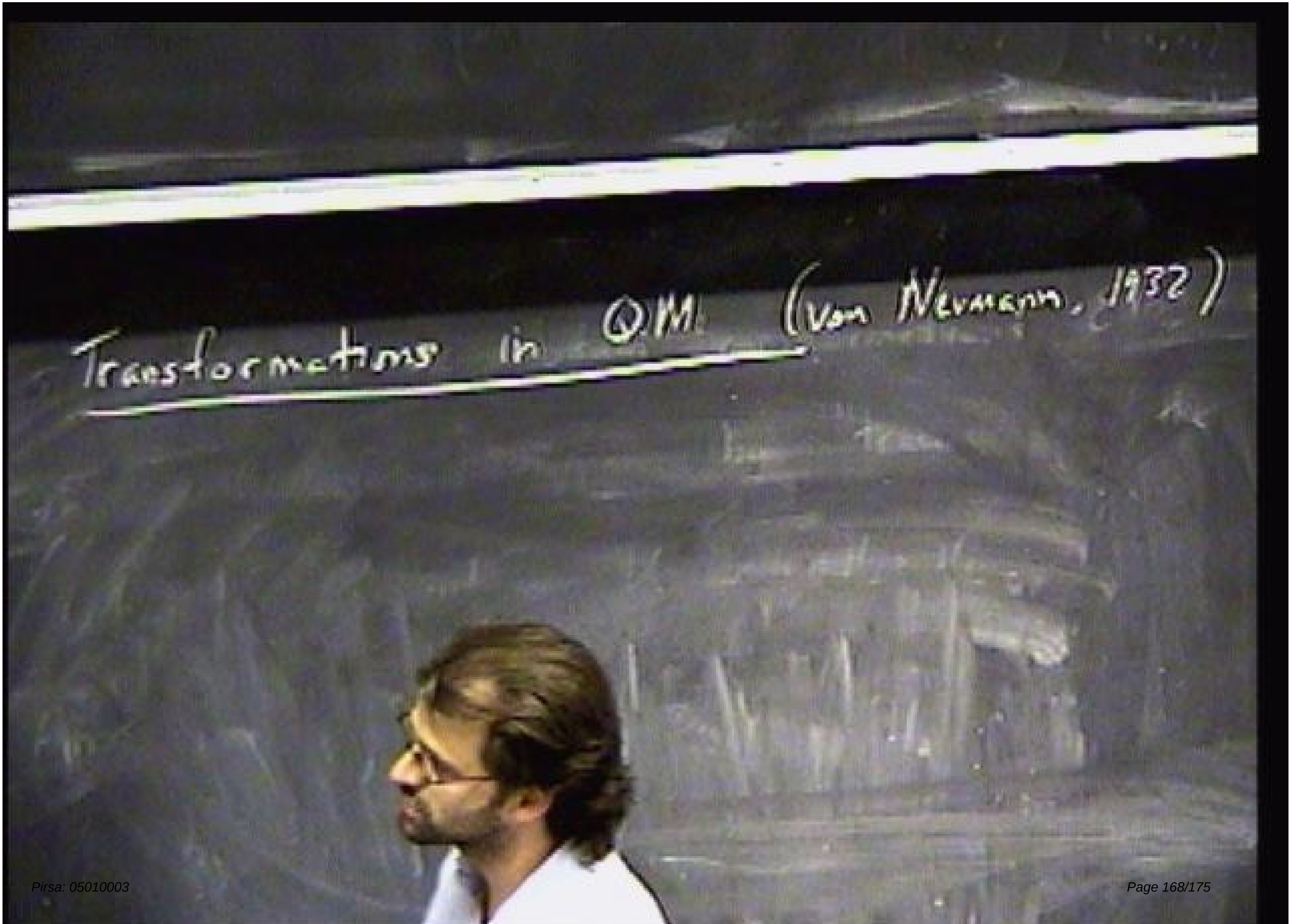
1) a) Momentum of e^- has a dispersion

b) and so the momentum of photon has uncorrelated

observed 2) 1a) holds but 1b) does not because the momentum photon is fixed

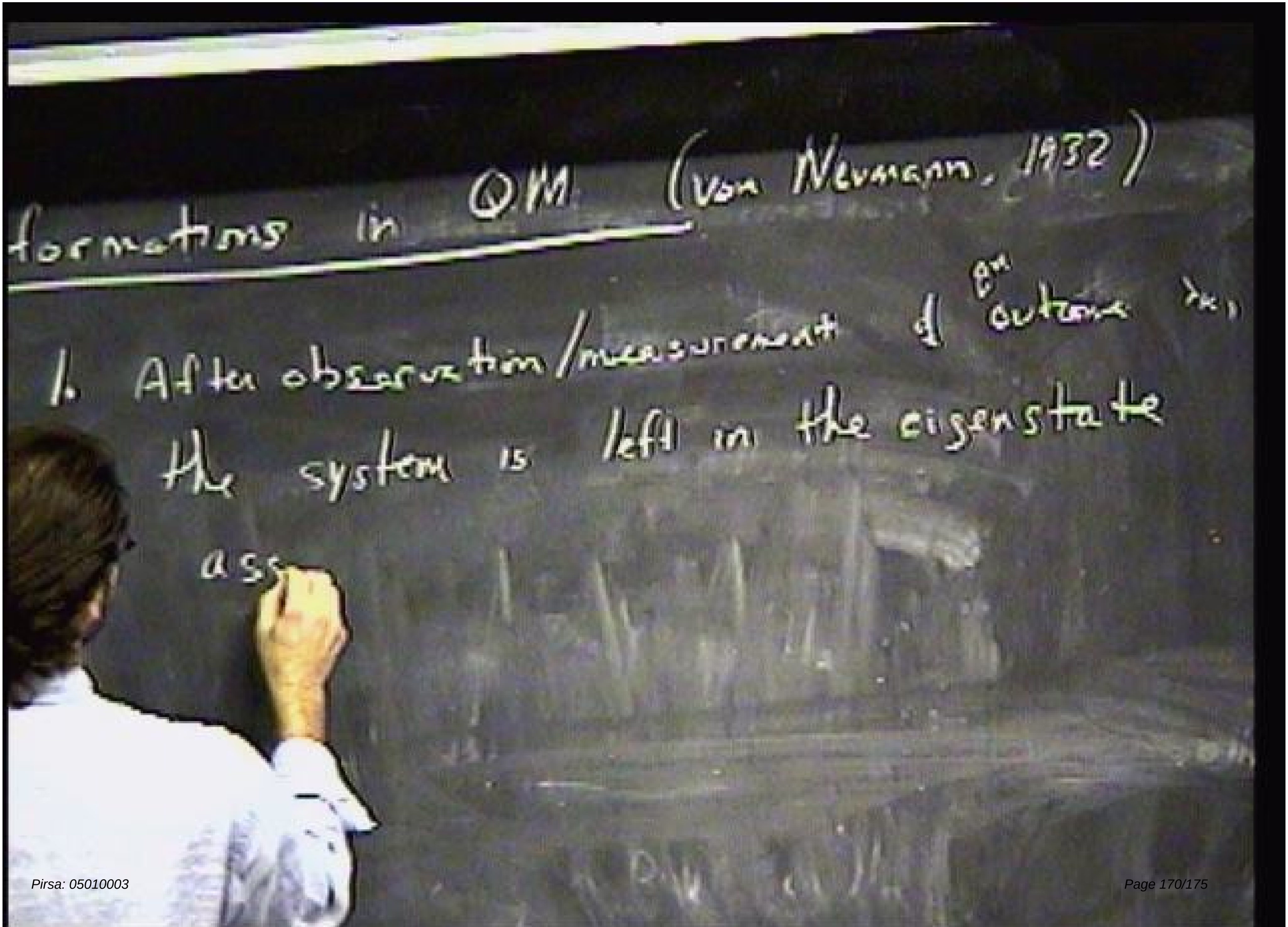
3) Both particles momenta can be predicted certainty (no dispersion).


Compton expt
E = hν
p = E/c = h/λ
hν'
e⁻
∝ between interception
of e⁻, photon
von Neumann
1) a) Momentum of e⁻ has a dispersion
b) the momentum of photon has uncorrelated
observed
2) 1a) holds but 1b) does not because the moment
photon is fixed (correlated)
3) Both particles momenta can be predicted
certainty (no dispersion).


Transformations in QM (von Neumann, 1932)

ansformations in QM (von Neumann, 1932)

1. After observation/measurement of $\overset{\text{an}}{\text{outcome}}$ x_1


formations in QM (von Neumann, 1932)
1. After observation/measurement of an outcome
the system is left in the eigenstate
ass

ormations in QM (von Neumann, 1932)

1. After observation/measurement of outcome λ_k (given), the system is left in the eigenstate $|\lambda_k\rangle$ associated with λ_k.

ormations in QM (von Neumann, 1932)

1. After observation/measurement of an outcome λ_i, the system is left in the eigenstate $|\lambda_i\rangle$ associated with λ_i.

2. Schrodinger evolution.

Transformations in QM (von

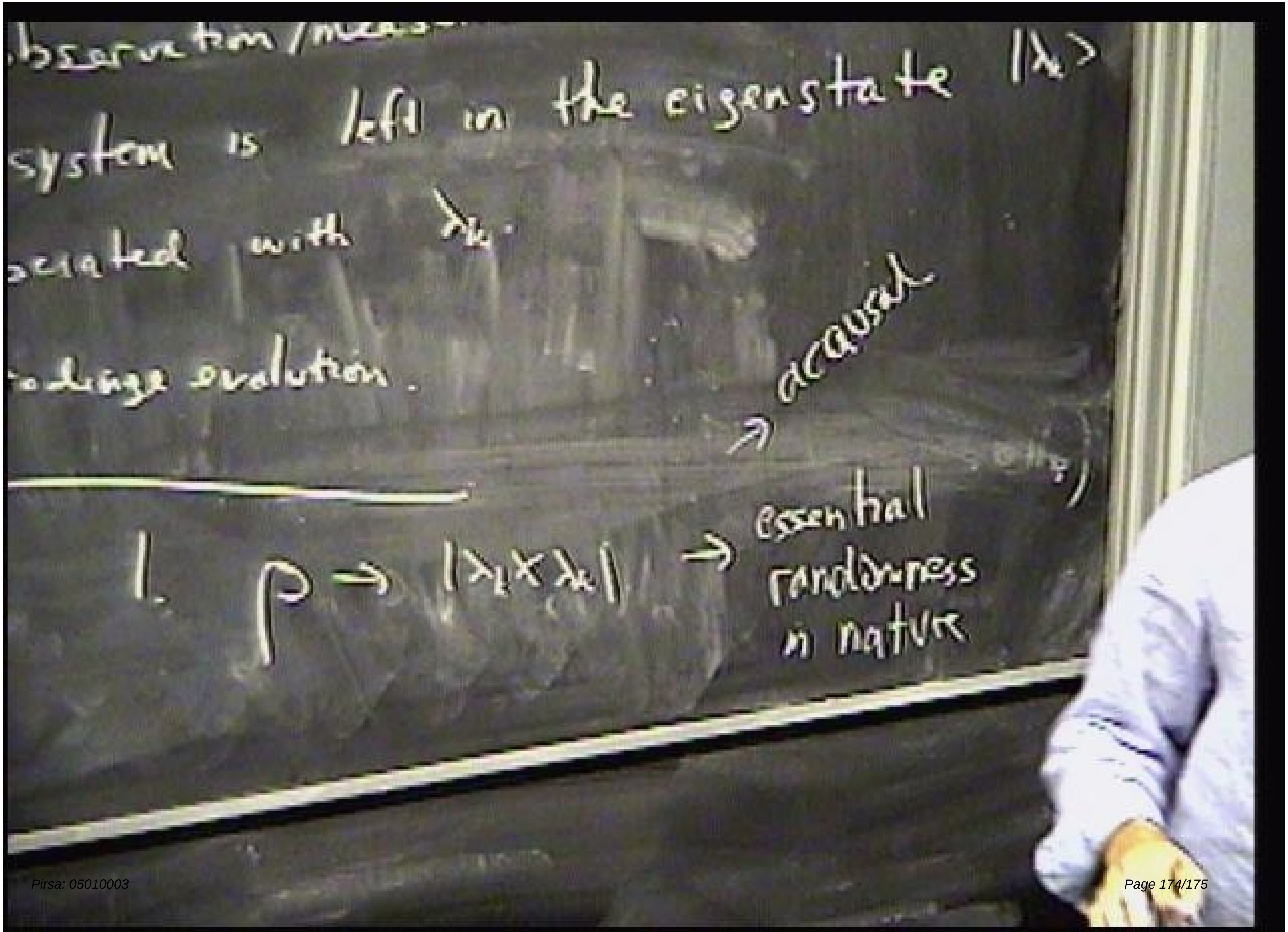
Process 1.

1. After observation/measurement the system is left in ... associated with the ...

Process 2

2. Schrodinger evolution.

$$\hat{O} = \sum_i \lambda_i |\lambda_i\rangle\langle\lambda_i|$$



left in the eigenstate |λ_k⟩
system is
with λ_k.
evolution.
acausal
1. ρ → |λ_k⟩⟨λ_k| → essential randomness in nature

bservation / measur

system is left in the eigenstate $|\lambda_k\rangle$

sciated with λ_k.

rodinger evolution.

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1. $\rho \rightarrow |\lambda_k\rangle\langle\lambda_k|$ → essential randomness in nature

→ acausal