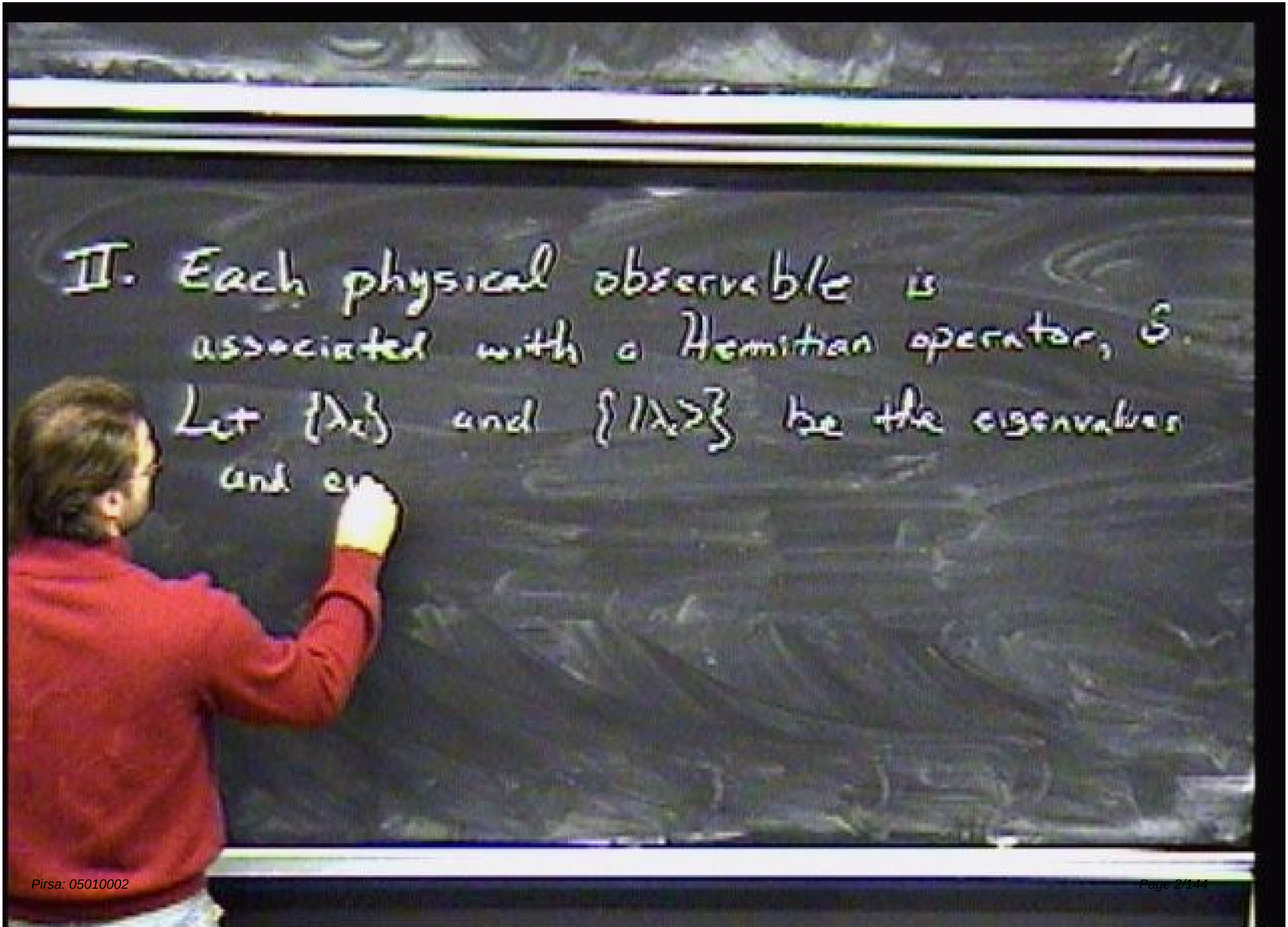
Title: Interpretation of Quantum Theory: Lecture 2

Date: Jan 06, 2005 02:20 PM

URL: http://pirsa.org/05010002

Abstract:


II. Each physical observable is
associated with a Hermitian operator, Ô.
Let {λᵢ} and {|λᵢ>} be the eigenvalues
and e

Each physical observable is associated with a Hermitian operator, $\hat{O}$

Let $\{\lambda_i\}$ and $\{|\lambda_i\rangle\}$ be the eigenvalues and eigenvectors, respectively.

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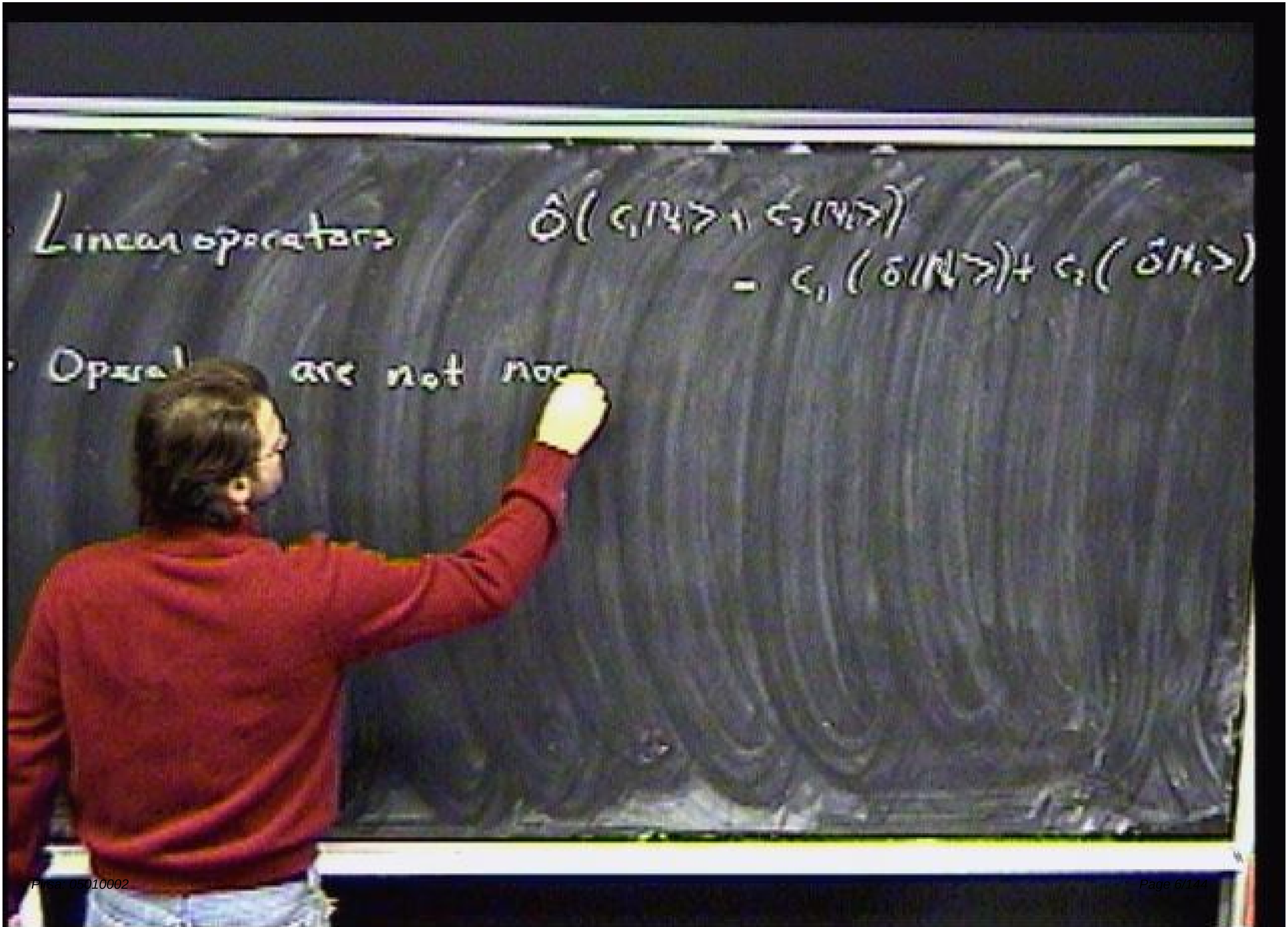
Let $\{\lambda_i\}$ and $\{|\lambda_i\rangle\}$ be the eigenvalues and eigenvectors, respectively.

a) The set of allowed observable outcomes is determined from $\{\lambda_i\}$.

b) Given some state $\hat{\rho}$, the probability of observing outcome λ_i, is $P(\lambda_i) = \mathrm{Tr}(\hat{\rho}\,|\lambda_i\rangle\langle\lambda_i|)$.


Linear operators
$\hat{O}(c_1|\psi_1\rangle + c_2|\psi_2\rangle)$
$= c_1(\hat{O}|\psi_1\rangle) + c_2(\hat{O}|\psi_2\rangle)$
are not

- Linear operators $\hat{O}(c_1|\psi_1\rangle + c_2|\psi_2\rangle) = c_1(\hat{O}|\psi_1\rangle) + c_2(\hat{O}|\psi_2\rangle)$

- Operators are not necessarily commutative

$$[\hat{A}, \hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A} \neq 0$$

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- Adjoint $(A^\dagger \varphi, \psi) = (\varphi, A\psi) \quad \forall \varphi, \psi \in \mathcal{H}$

$$\langle \phi | A^\dagger | \psi \rangle = \langle \psi | A | \phi \rangle^*$$

- Linear operators $\hat{O}(c_1|\psi_1\rangle + c_2|\psi_2\rangle)$
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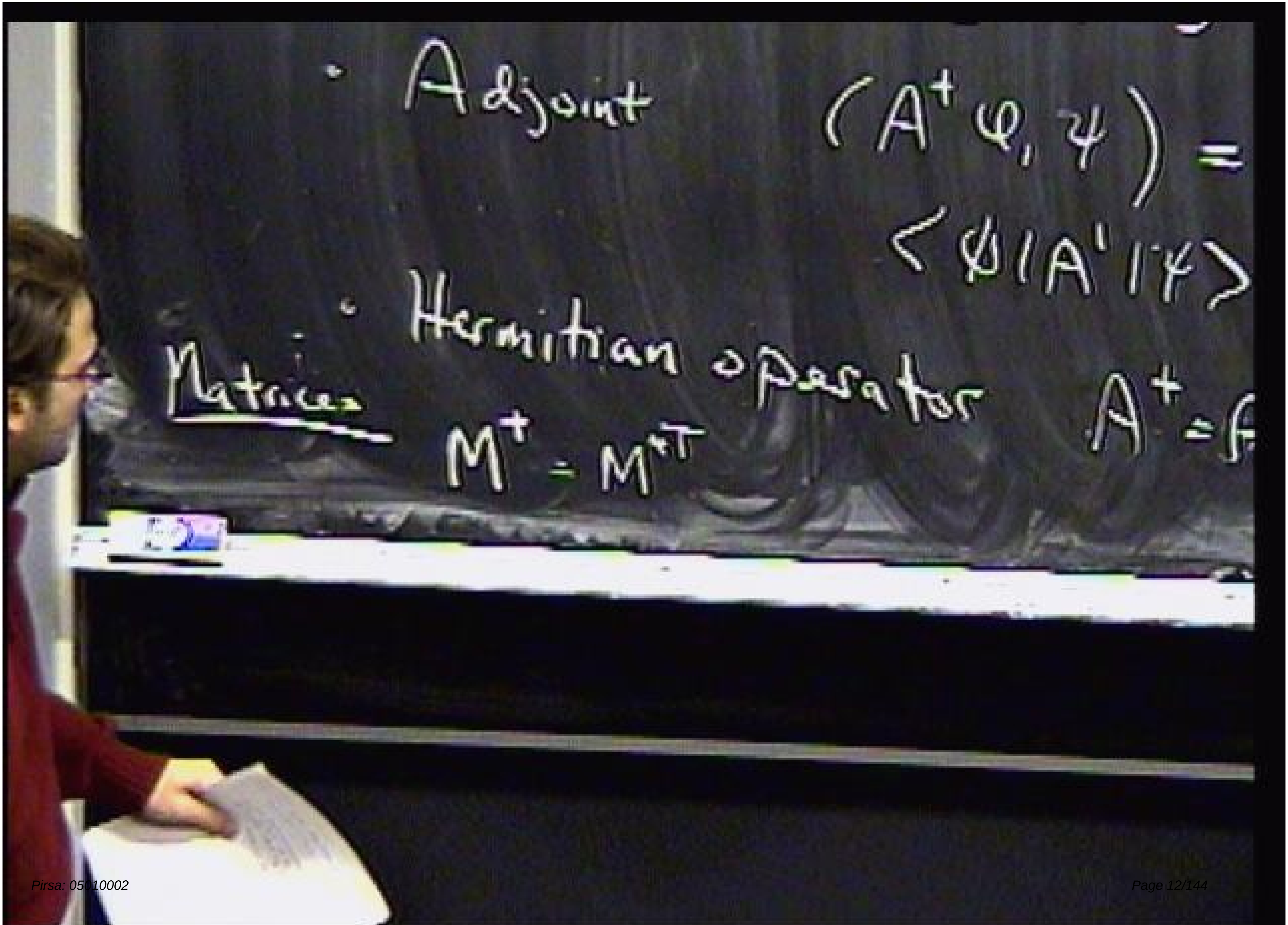
- Adjoint $(A^\dagger \varphi, \psi) = (\varphi, A\psi) \quad \forall \varphi, \psi \in \mathcal{H}$
$$\langle \phi | A^\dagger | \psi \rangle = \langle \psi | A | \phi \rangle^*$$
- Hermitian operator $A^\dagger = A$

$$(\varphi, A\psi) \quad \forall\, \varphi, \psi \in \mathcal{H}$$

$$= \langle \psi | A | \phi \rangle^*$$

$$(AB)^\dagger = B^\dagger A^\dagger$$

$$(cA)^\dagger = c^* A^\dagger$$


Adjoint
$(A^\dagger \varphi, \psi) =$
$\langle \phi | A^\dagger | \psi \rangle$
Hermitian operator
$A^\dagger =$
Matrices
$M^\dagger = M^{*T}$

Adjoint $(A^\dagger \varphi, \psi) = (\varphi, A\psi)$

$\langle \phi | A^\dagger | \psi \rangle = \langle \psi | A | \phi \rangle^*$

Hermitian operator $A^\dagger = A$

$M^\dagger = M^{*T}$

$\hookrightarrow$ has real eigenvalues

Hermitian Operator admits a spectral decomposition

(i) Discrete Spectrum

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Operator admits a spectral decomposit

crete Spectrum

$$\hat{O} = \sum_k \lambda_k \underbrace{|\lambda_k\rangle\langle\lambda_k|}_{P_k}$$

P_k = projector $\Rightarrow P_k^2 = P_k$

$M^\dagger = M^{*T}$ ↳ has real eigenvalues $(cA)^\dagger = c^* A^\dagger$

Hermitian Operator admits a spectral decomposition

(i) Discrete Spectrum

$$\hat{O} = \sum_{\lambda} \lambda_\ell \underbrace{|\lambda_\ell\rangle\langle\lambda_\ell|}_{P_\ell}$$

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(ii) Continuous Spectrum

$M^{\dagger} = M^{*T}$ ↳ has real eigenvalues | $(cA)^{\dagger} = c^{*}A^{\dagger}$

Hermitian Operator admits a spectral decomposition

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$$\hat{O} = \sum_i \lambda_i \underbrace{|\lambda_i\rangle\langle\lambda_i|}_{P_i}$$

P_i = projector $\Rightarrow P_i^2 = P_i$

(ii) Continuous Spectrum

$$\hat{O} = \int d\lambda \, \lambda \, |\lambda\rangle\langle\lambda|$$

P_k = projector $\Rightarrow P_k^2 = P_k$

Dirac

$|\lambda\rangle\lambda|$

$|\lambda\rangle \in$ rigged Hilbert spaces

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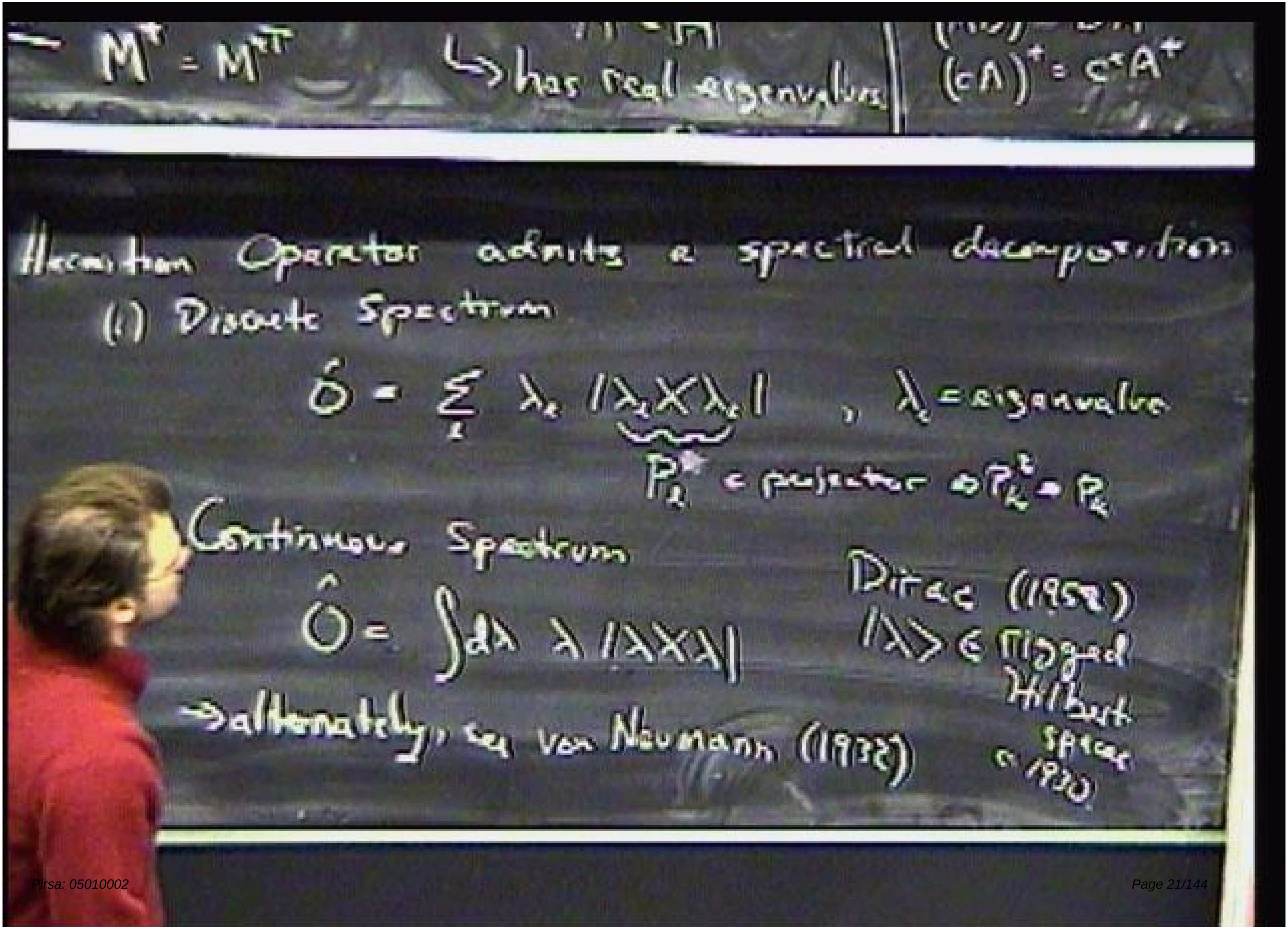
$$\hat{O} = \int d\lambda \, \lambda \, |\lambda\rangle\langle\lambda|$$

Dirac (1958)

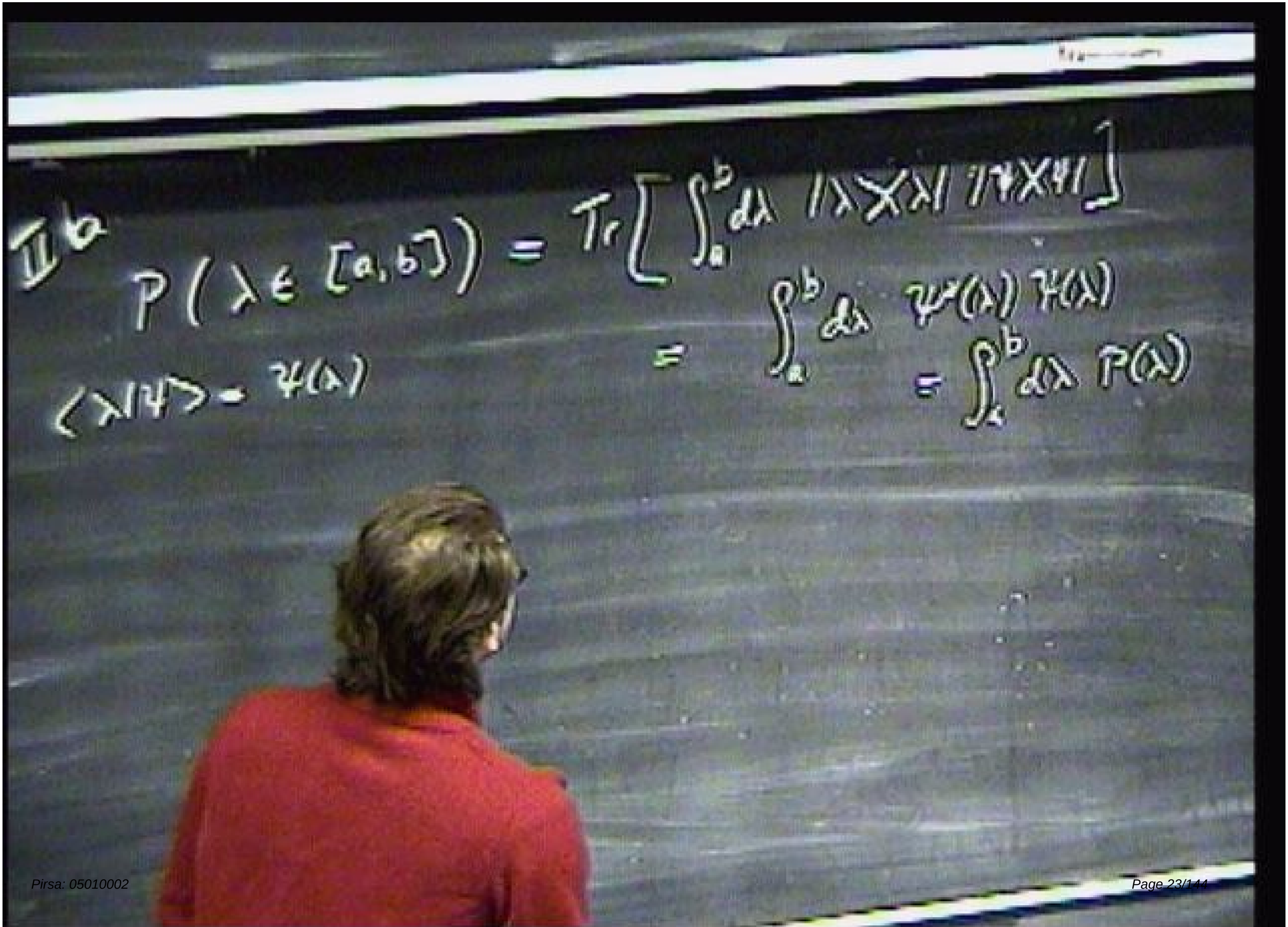
$|\lambda\rangle \in$ rigged Hilbert spaces c. 1970

→ alternately, see von Neumann (1932)


$M^\dagger = M^{*T}$
↳ has real eigenvalues
$(cA)^\dagger = c^* A^\dagger$
Hermitian Operator admits a spectral decomposition
(i) Discrete Spectrum
$\hat{O} = \sum_i \lambda_i \underbrace{|\lambda_i\rangle\langle\lambda_i|}_{P_i}$, λ_i = eigenvalue
P_i = projector ⇔ $P_i^2 = P_i$
Continuous Spectrum
$\hat{O} = \int d\lambda \, \lambda \, |\lambda\rangle\langle\lambda|$
Dirac (1958)
$|\lambda\rangle \in$ rigged Hilbert space
~1930
→ alternately, see von Neumann (1932)

IIb

$$P(\lambda \in [a,b]) = \mathrm{Tr}\left[\int_a^b d\lambda \, |\lambda\rangle\langle\lambda| \, |\psi\rangle\langle\psi| \right]$$



II b
$P(\lambda \in [a,b]) = \mathrm{Tr}\left[\int_a^b d\lambda \, |\lambda\rangle\langle\lambda| \, |\psi\rangle\langle\psi|\right]$
$= \int_a^b d\lambda \, \psi^*(\lambda)\, \psi(\lambda)$
$= \int_a^b d\lambda \, P(\lambda)$
$\langle\lambda|\psi\rangle = \psi(\lambda)$

II b

$$P(\lambda \in [a,b]) = Tr\left[\int_a^b d\lambda \; |\lambda\rangle\langle\lambda| \; |\psi\rangle\langle\psi|\right]$$

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$$\langle\lambda|\psi\rangle = \psi(\lambda)$$

$$f(\mathcal{O}) = \sum_\lambda f(\lambda)\,|\lambda\rangle\langle\lambda|$$

II b

$$P(\lambda \in [a,b]) = \mathrm{Tr}\left[\int_a^b d\lambda\, |\lambda\rangle\langle\lambda|\, |\psi\rangle\langle\psi|\right]$$

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$$= \int_a^b d\lambda\, P(\lambda)$$

$$f(O) = \sum_\lambda f(\lambda)\, |\lambda\rangle\langle\lambda|$$

$$\langle x\rangle = \int dx\; x\, \psi^*(x)\,\psi(x)$$

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II b

$$P(\lambda \in [a,b]) = Tr\left[\int_a^b d\lambda \, |\lambda\rangle\langle\lambda| \, |\psi\rangle\langle\psi| \right]$$

$$\left\{ = \int_a^b d\lambda \, \psi^*(\lambda)\, \psi(\lambda) \right.$$

$$= \int_a^b d\lambda \, P(\lambda) \}$$

$$\langle \lambda | \psi \rangle = \psi(\lambda)$$

$$f(\hat{O}) = \sum_\lambda f(\lambda) \, |\lambda\rangle\langle\lambda|$$

$$\langle x \rangle = \int dx \; x \, \psi^*(x)\, \psi(x) = Tr(x \, |\psi\rangle\langle\psi|)$$

$$= Tr\left[\int dx \; x \, |x\rangle\langle x| \, |\psi\rangle\langle\psi| \right]$$

$$=$$

II b

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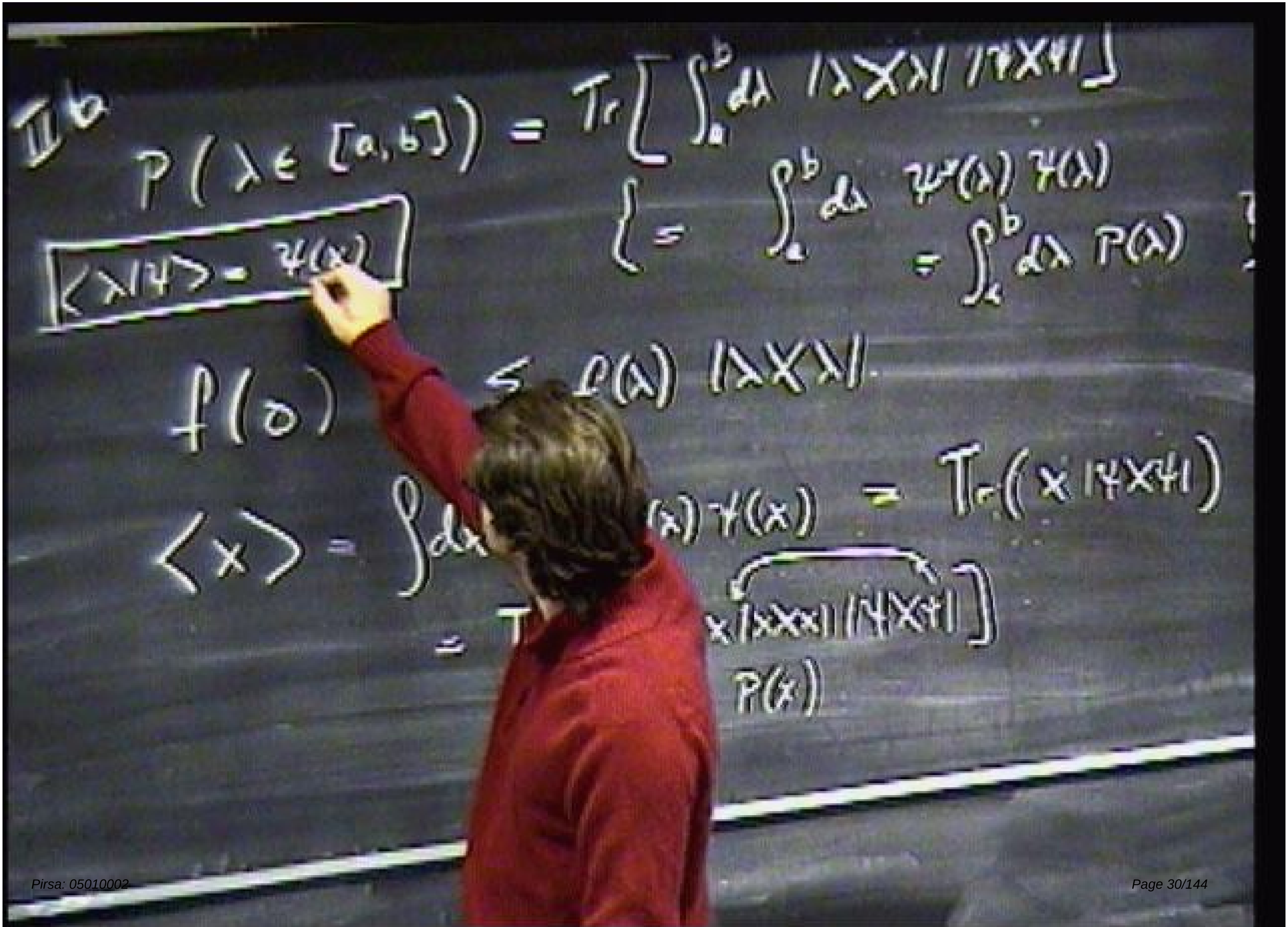
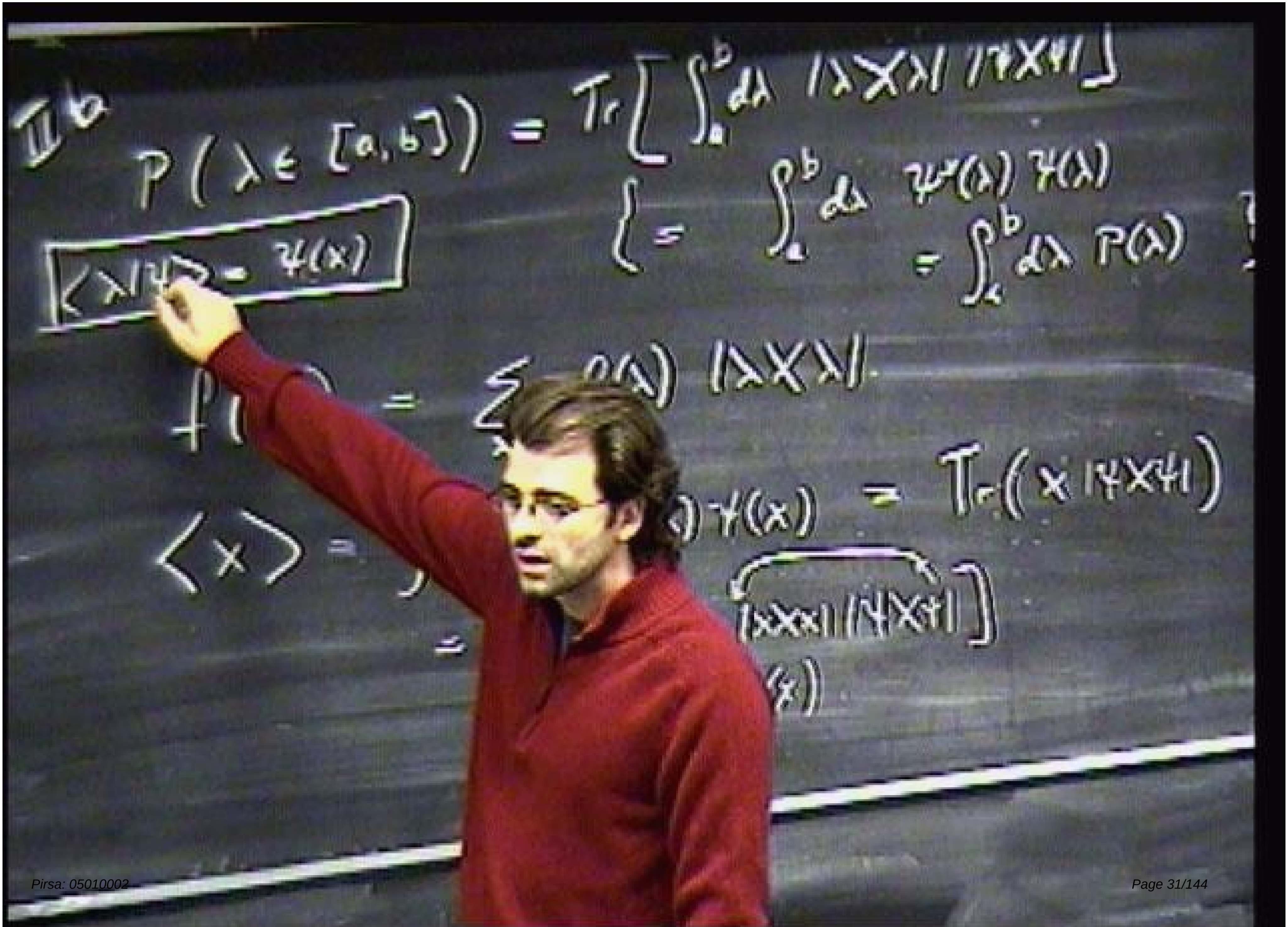
$$= \mathrm{Tr}\left[\int dx\ x\ |x\rangle\langle x|\ |\psi\rangle\langle\psi|\right]$$

$$= \int dx\ x\ P(x)$$



II b
$P(\lambda \in [a,b]) = \mathrm{Tr}\left[\int_a^b d\lambda\, |\lambda\rangle\langle\lambda|\, |\psi\rangle\langle\psi|\right]$
$= \int_a^b d\lambda\, \psi^*(\lambda)\,\psi(\lambda)$
$= \int_a^b d\lambda\, P(\lambda)$
$\boxed{\langle\lambda|\psi\rangle = \psi(\lambda)}$
$f(O)$
$f(\lambda)\, |\lambda\rangle\langle\lambda|$
$\langle x\rangle = \int dx$
$(x)\,\psi(x) = \mathrm{Tr}(x\, |\psi\rangle\langle\psi|)$
$= T$
$x\, |x\rangle\langle x|\, |\psi\rangle\langle\psi|]$
$P(x)$



IIb
P (λ ∈ [a,b]) = Tr [∫ₐᵇ dλ |λ⟩⟨λ| |ψ⟩⟨ψ|]
= ∫ₐᵇ dλ ψ*(λ) ψ(λ)
= ∫ₐᵇ dλ P(λ)
⟨λ|ψ⟩ = ψ(x)
|λ⟩⟨λ|
Tr(x |ψ⟩⟨ψ|)
⟨x⟩ =
|x⟩⟨x| |ψ⟩⟨ψ|

$$P(\lambda \in [a,b]) = \ldots$$

$$\boxed{\langle \lambda | \psi \rangle = \psi(x)}$$

$$= \int_a^b d\lambda\, \psi^*(\lambda) \ldots$$

$$= \int_a \ldots$$

$$f(\hat{o}) = \sum_\lambda f(\lambda)\, |\lambda\rangle\langle\lambda|$$

$$\langle x \rangle = \int dx\; x\; \psi^*(x)\,\psi(x) = \mathrm{Tr}(\ldots$$

$$= \mathrm{Tr}\Big[\int dx\; x\, |x\rangle\langle x|\, |\psi\rangle\langle\psi|\Big]$$

$$= \int dx\; x\; P(x)$$

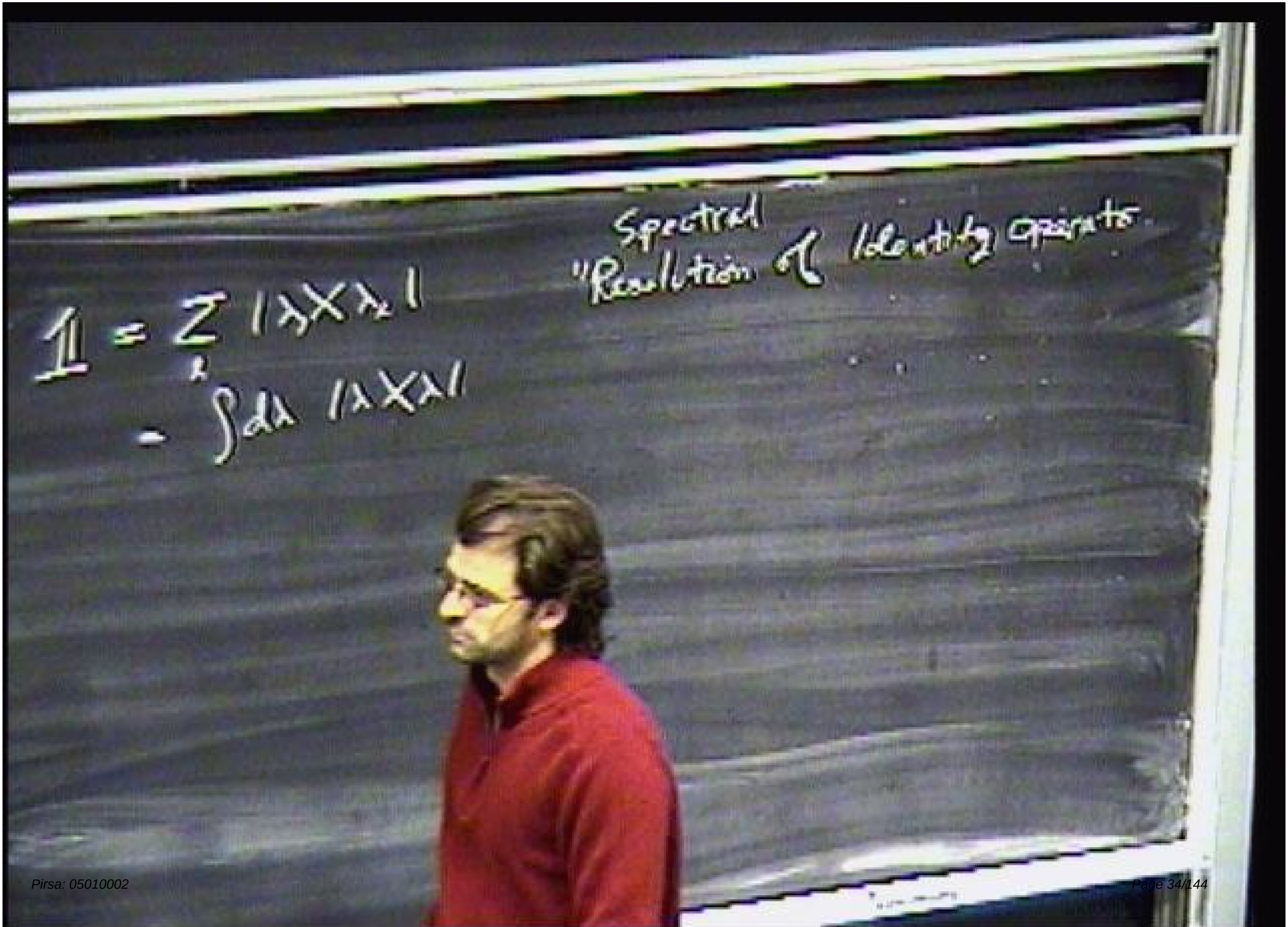
($|\psi\rangle$ is the state)

$d\lambda \quad \psi^*(\lambda)\, \psi(\lambda)$

$= \int_a^b d\lambda \; P(\lambda) \quad \}$

$\langle \hat{p} \rangle = \int dx\, \psi^*(x) \left(-i\hbar \frac{d}{dx}\right) \psi(x)$

$= \mathrm{Tr}\left(X\, |\psi\rangle\langle\psi| \right)$


Spectral
"Resolution of Identity operator
$\mathbb{1} = \sum_k |\lambda_k\rangle\langle\lambda_k|$
$= \int d\lambda \, |\lambda\rangle\langle\lambda|$

Spectra

"Resolution

$$\mathbb{1} = \sum_{i} |\lambda_i\rangle\langle\lambda_i|$$

$$= \int d\lambda \; |\lambda\rangle\langle\lambda|$$

where

$$\mathbb{1}|\psi\rangle = |\psi\rangle$$

"Resolution of Identity"

$$\mathbb{1} = \sum_k |\lambda_k\rangle\langle\lambda_k|$$

$$= \int d\lambda \, |\lambda\rangle\langle\lambda|$$

where

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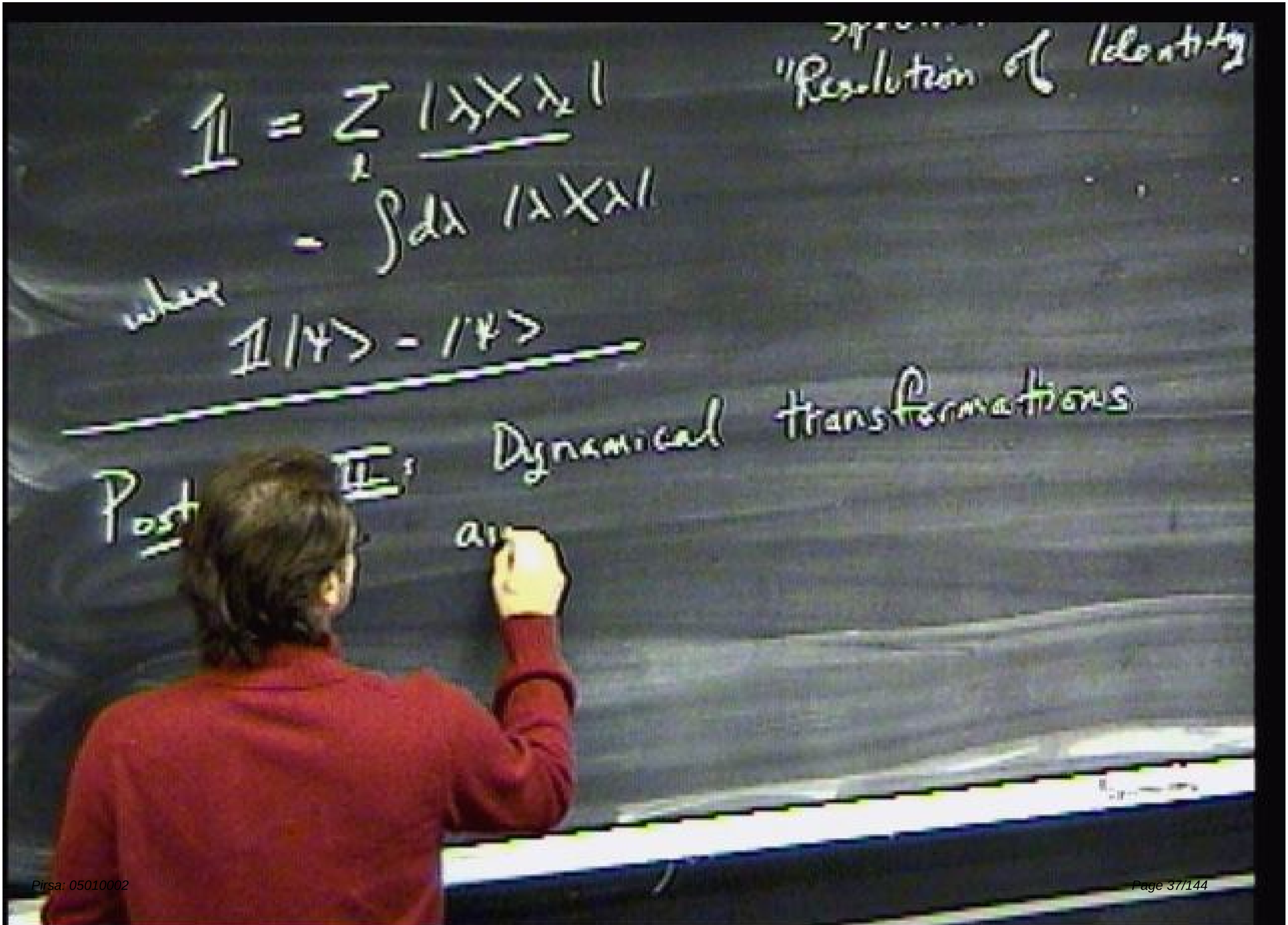
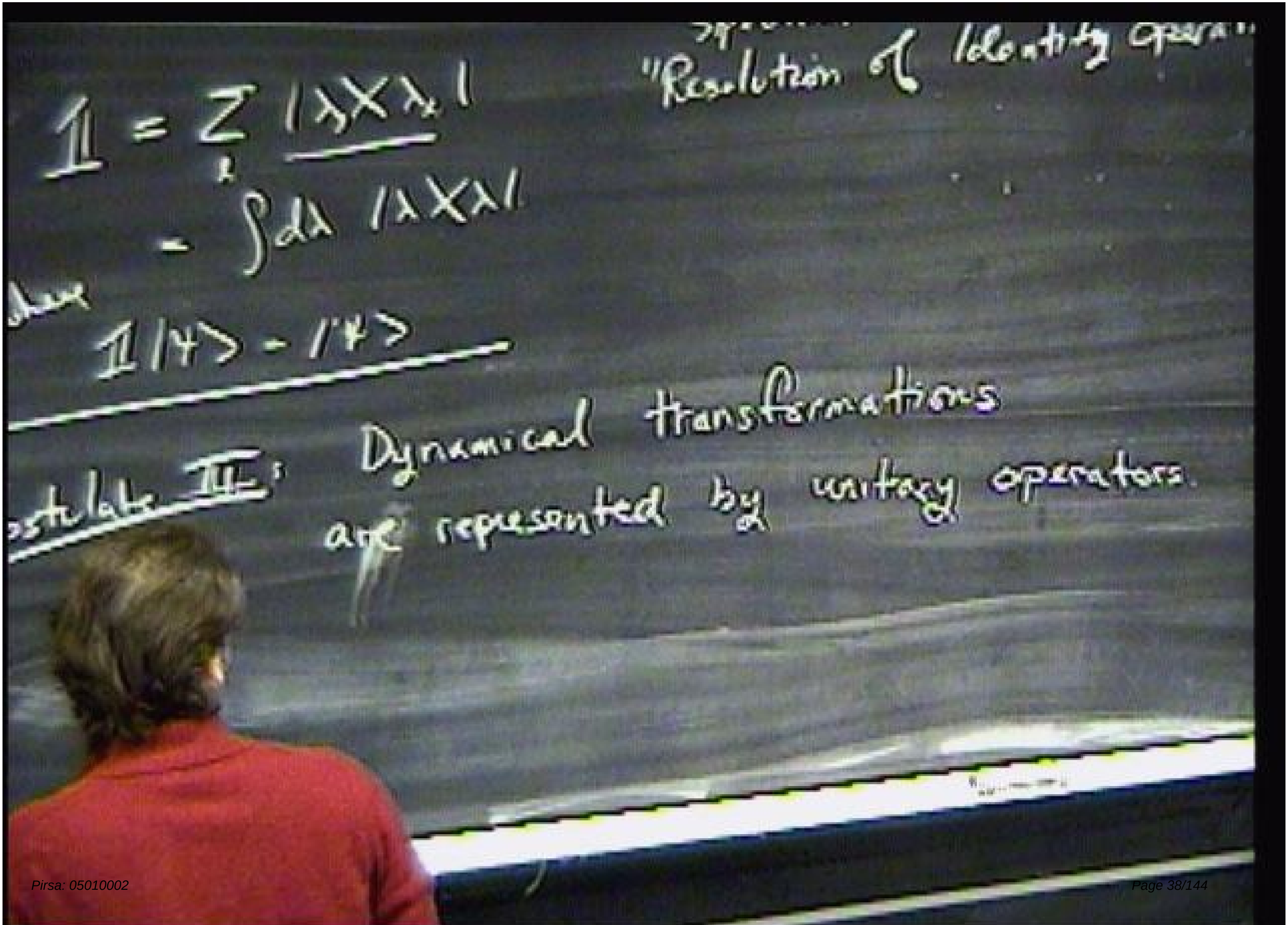
Postulate III:


"Resolution of Identity
$\mathbb{1} = \sum_i |\lambda_i\rangle\langle\lambda_i|$
$= \int d\lambda\, |\lambda\rangle\langle\lambda|$
where
$\mathbb{1}|\psi\rangle = |\psi\rangle$
Post
II: Dynamical transformations
a


"Resolution of Identity
$\mathbb{1} = \sum_k |\lambda_k\rangle\langle\lambda_k|$
$= \int d\lambda\, |\lambda\rangle\langle\lambda|$
$\mathbb{1}|\psi\rangle = |\psi\rangle$
III: Dynamical transformations
are represented by unitary operators.

mical transformations

epresented by unitary operators

$$|\psi(s_2)\rangle = U(s_2, s_1)\,|\psi(s_1)\rangle$$

-mical transformations

-epresented by unitary operators

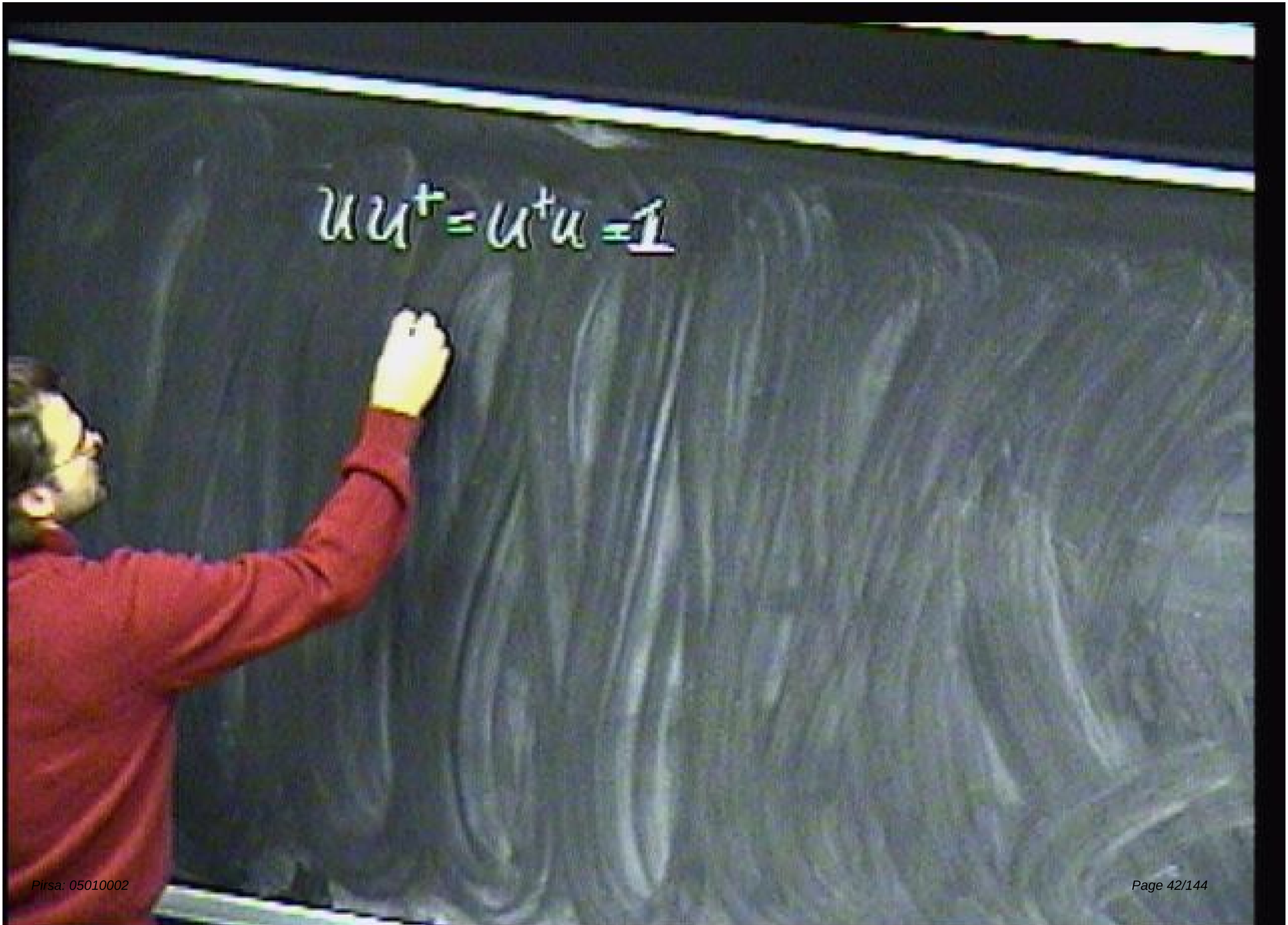
$$|\psi(s_2)\rangle = \underline{\underline{U(s_2, s_1)}}\, |\psi(s_1)\rangle$$

namical transformations

represented by unitary operators.

$$|\psi(s_2)\rangle = \underline{U(s_2, s_1)}\, |\psi(s_1)\rangle$$

$$\rho(s_2) = U(s_2, s_1)\, \rho(s_1)\, U^{\dagger}(s_2, s_1)$$


$UU^\dagger = U^\dagger U = 1$

$$UU^{+} = U^{+}U = \underline{1}$$

$$U^{+} = U^{-1}$$

$$UU^{\dagger} = U^{\dagger}U = \mathbb{1}$$

$$U^{\dagger} = U^{-1}$$

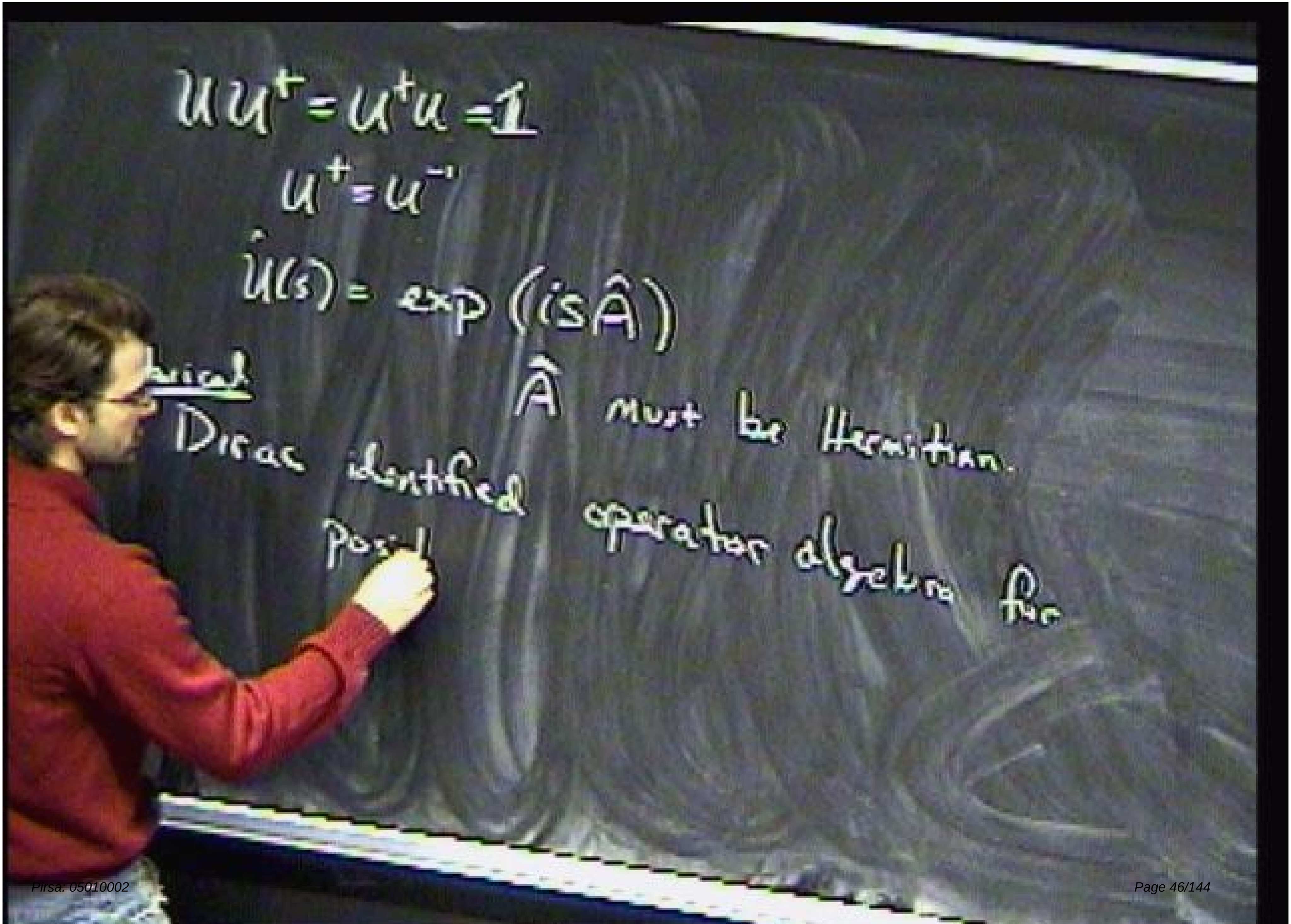
$$U(s) = \exp(is\hat{A})$$

$$UU^{+} = U^{+}U = \underline{1}$$

$$U^{+} = U^{-1}$$

$$\hat{U}(s) = \exp(is\hat{A})$$

$\hat{A}$ must be Hermitian.


$UU^{\dagger} = U^{\dagger}U = \mathbb{1}$
$U^{\dagger} = U^{-1}$
$U(s) = \exp(is\hat{A})$
$\hat{A}$ must be Hermitian.
Dirac identified operator algebra for

$$\mathcal{U}\mathcal{U}^{+} = \mathcal{U}^{+}\mathcal{U} = \mathbb{1}$$

$$\mathcal{U}^{+} = \mathcal{U}^{-1}$$

$$\hat{\mathcal{U}}(s) = \exp(is\hat{A})$$

Historical

$\hat{A}$ must be Hermitian.

Dirac identified operator algebra for position, momentum and angular momentum

$$UU^{\dagger} = U^{\dagger}U = \mathbb{1}$$

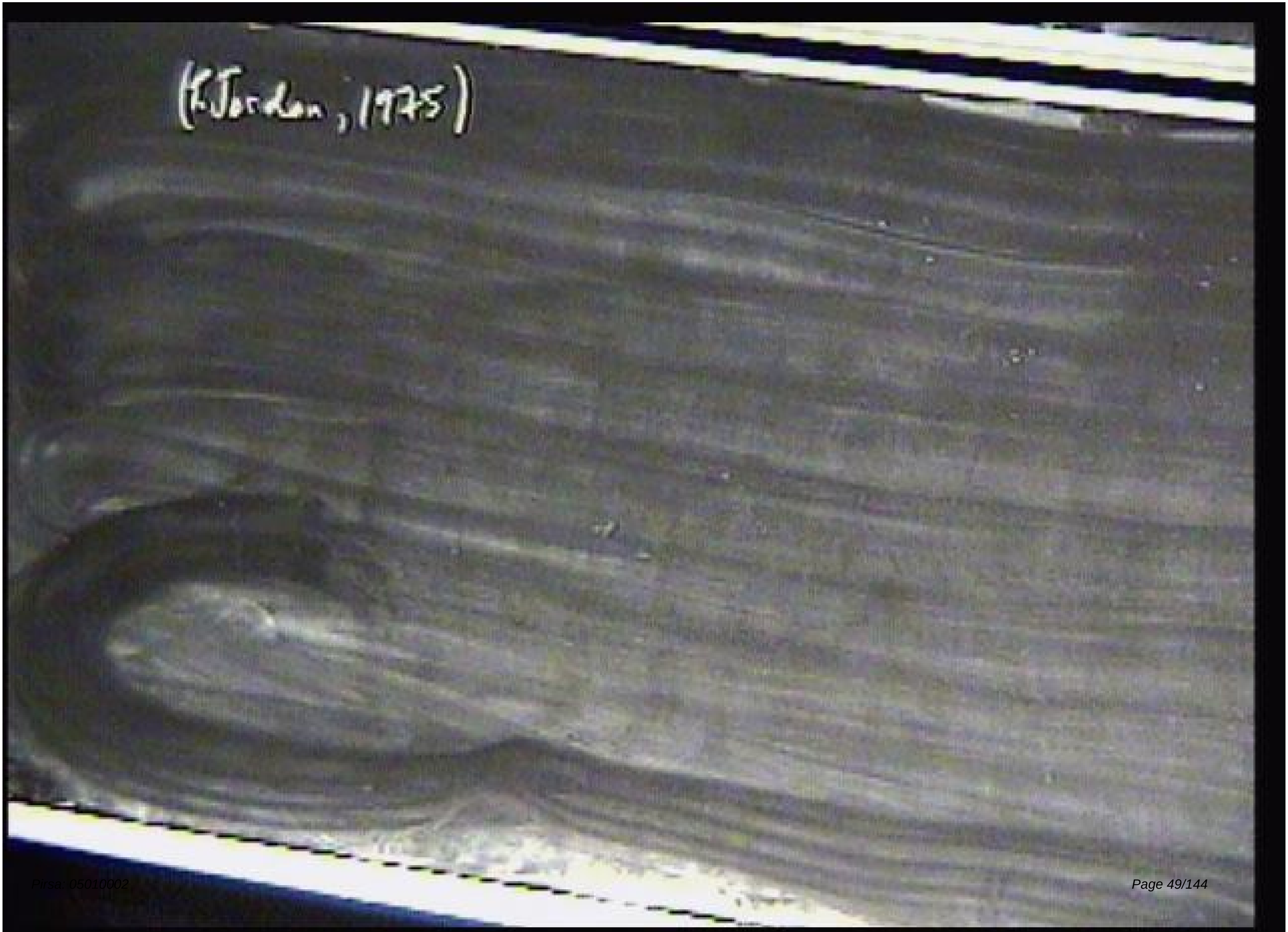
$$U^{\dagger} = U^{-1}$$

$$\dot{U}(s) = \exp((is\hat{A}))$$

<u>Historical</u>

$\hat{A}$ must be Hermitian.

Dirac identified operator algebra for position, momentum and angular momentum by quantizing the classical Poisson bracket.

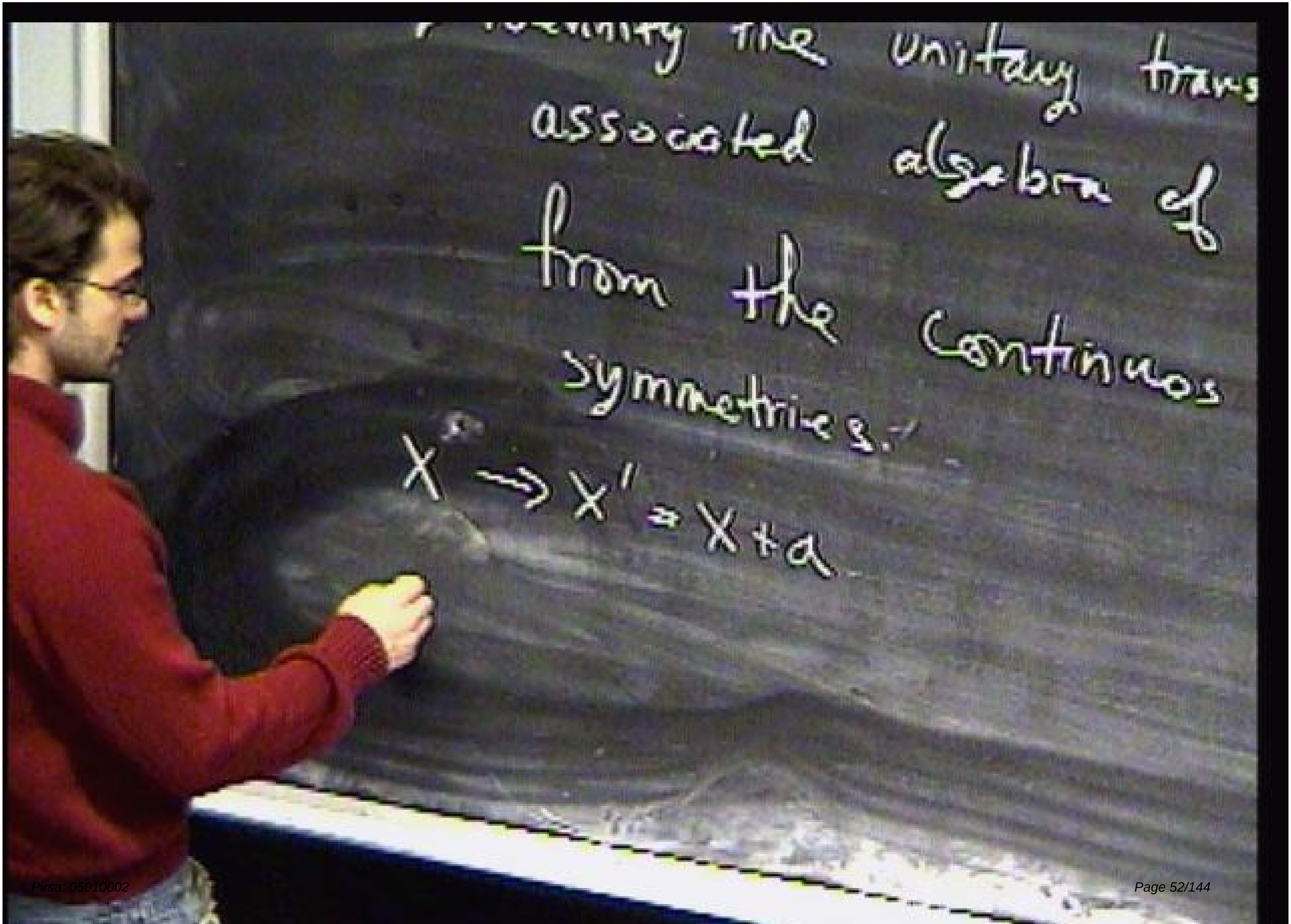

(F. Jordan, 1975)

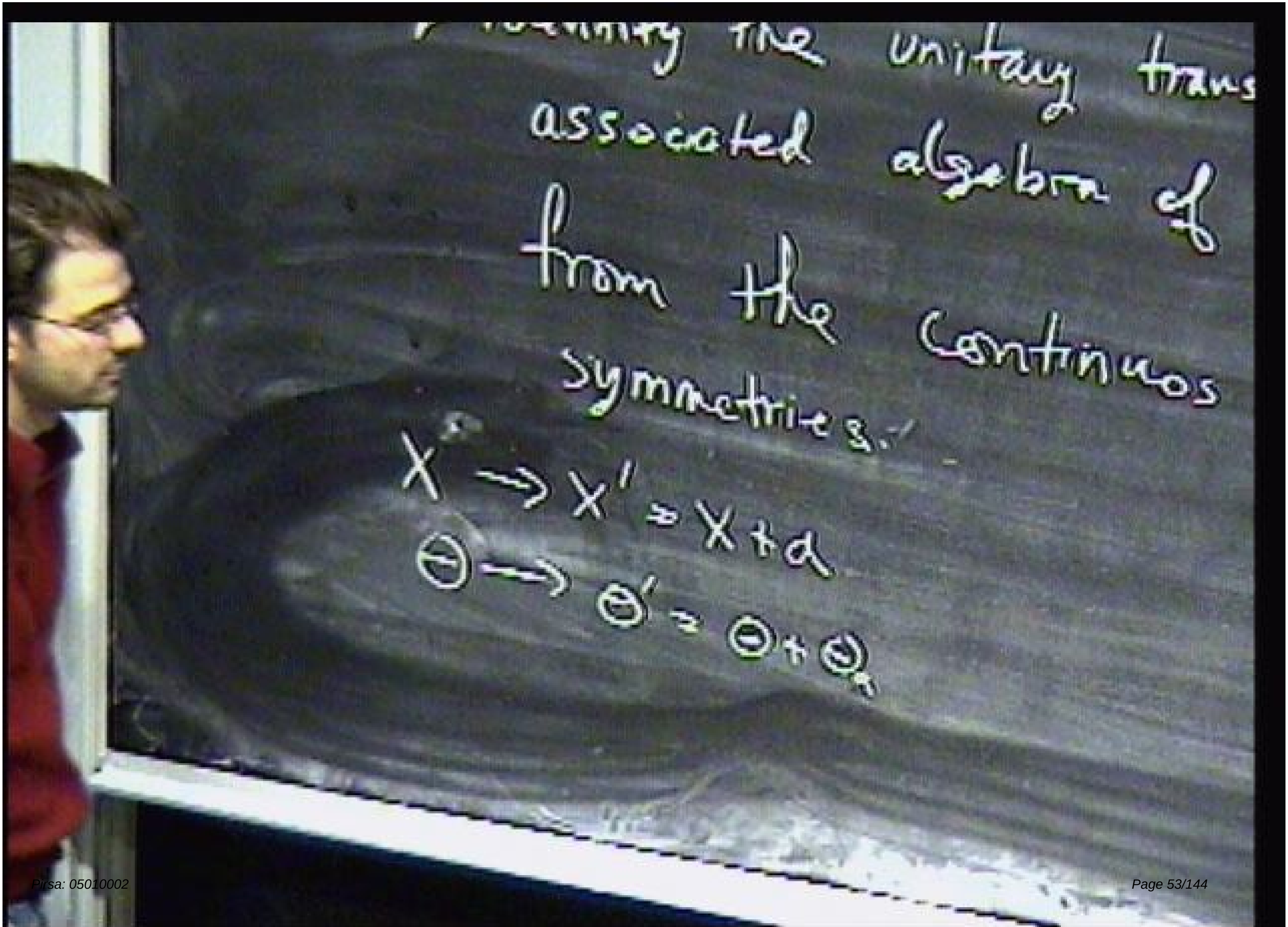
...rdan, 1975)

→ Identify the unitary transformation, and the associated algebra of the generators $\{\hat{A}\}$

...rdan, 1975)

→ Identify the unitary transformation, and the associated algebra of the generators $\{\hat{A}\}$ from the continuous Galilean space-time symmetries.


the unitary trans
associated algebra of
from the Continuos
symmetries:
X → X' = X + a


the unitary trans
associated algebra of
from the Continuos
symmetries
X → X' = X + a
Θ → Θ' = Θ + Θ

⇒ Identify the unitary transformation, and associated algebra of the generators from the continuous Galilean spa[ce] symmetries.

$$X \rightarrow X' = X + a$$

$$\Theta \rightarrow \Theta' = \Theta + \Theta_a$$

$$\Theta' \rightarrow \Theta'' = \Theta' + \Theta_b$$

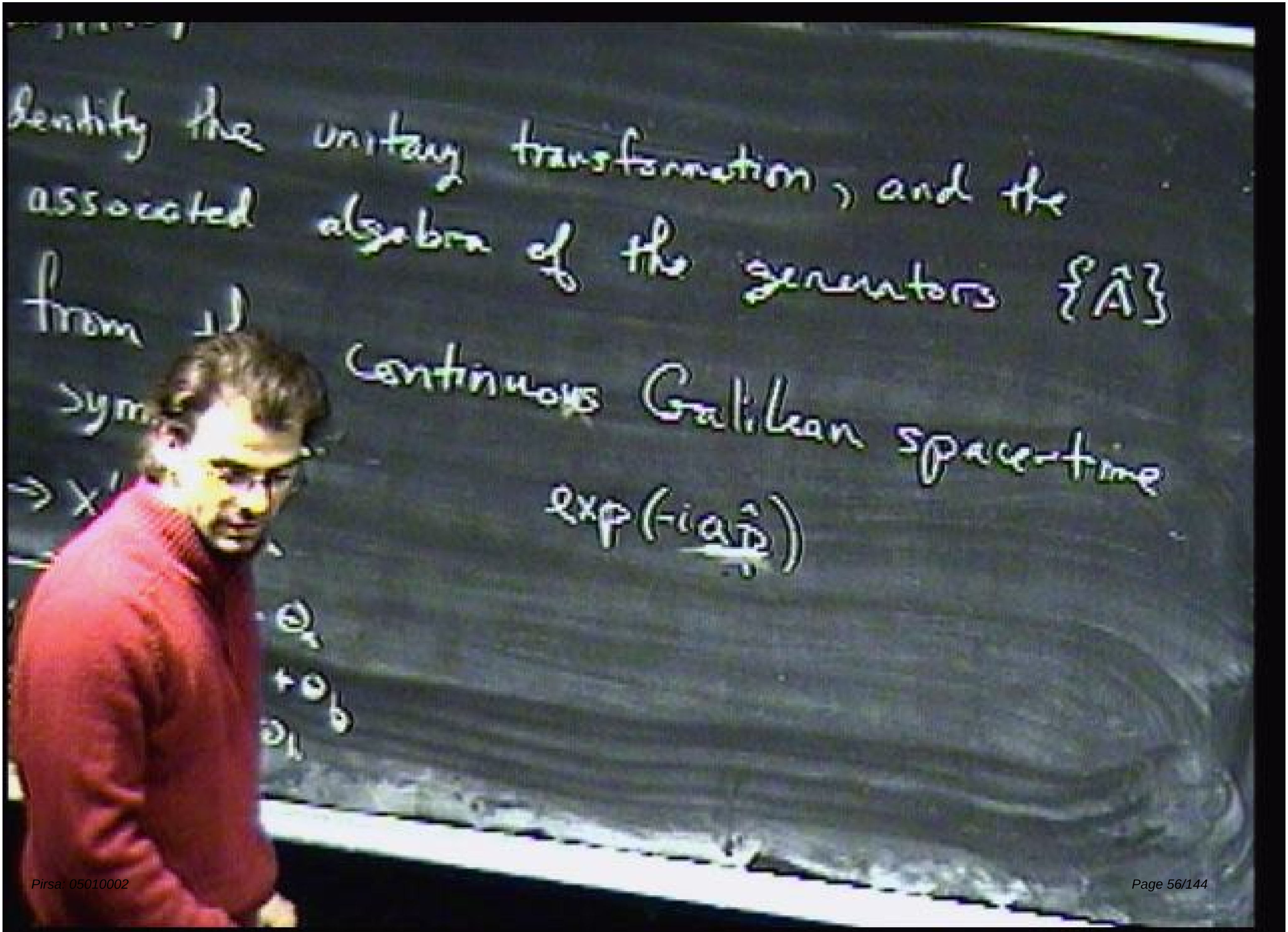
→ Identify the unitary transformation, and associated algebra of the generators from the continuous Galilean spa[ce] symmetries.

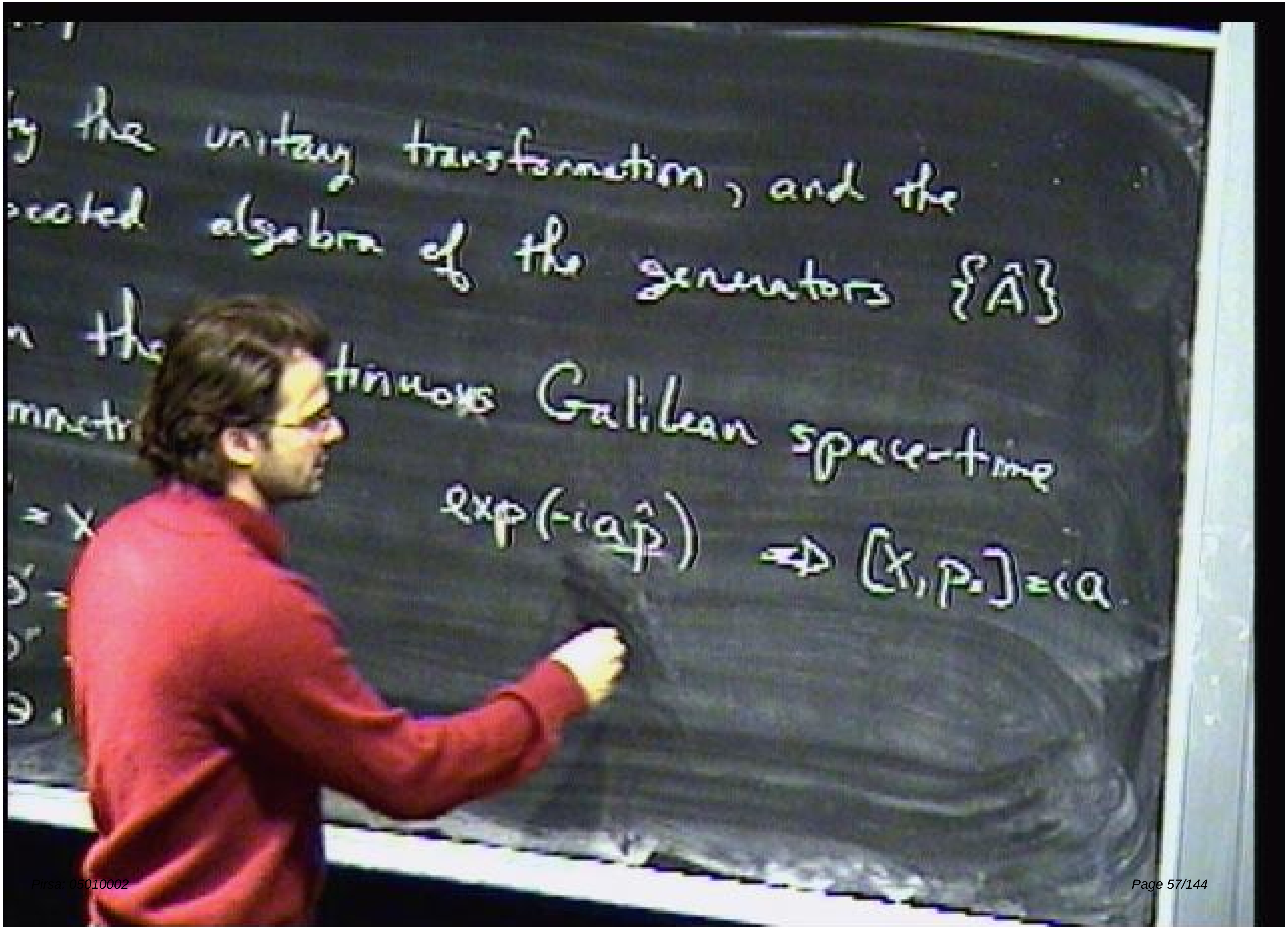
$X \rightarrow X' = X + a$

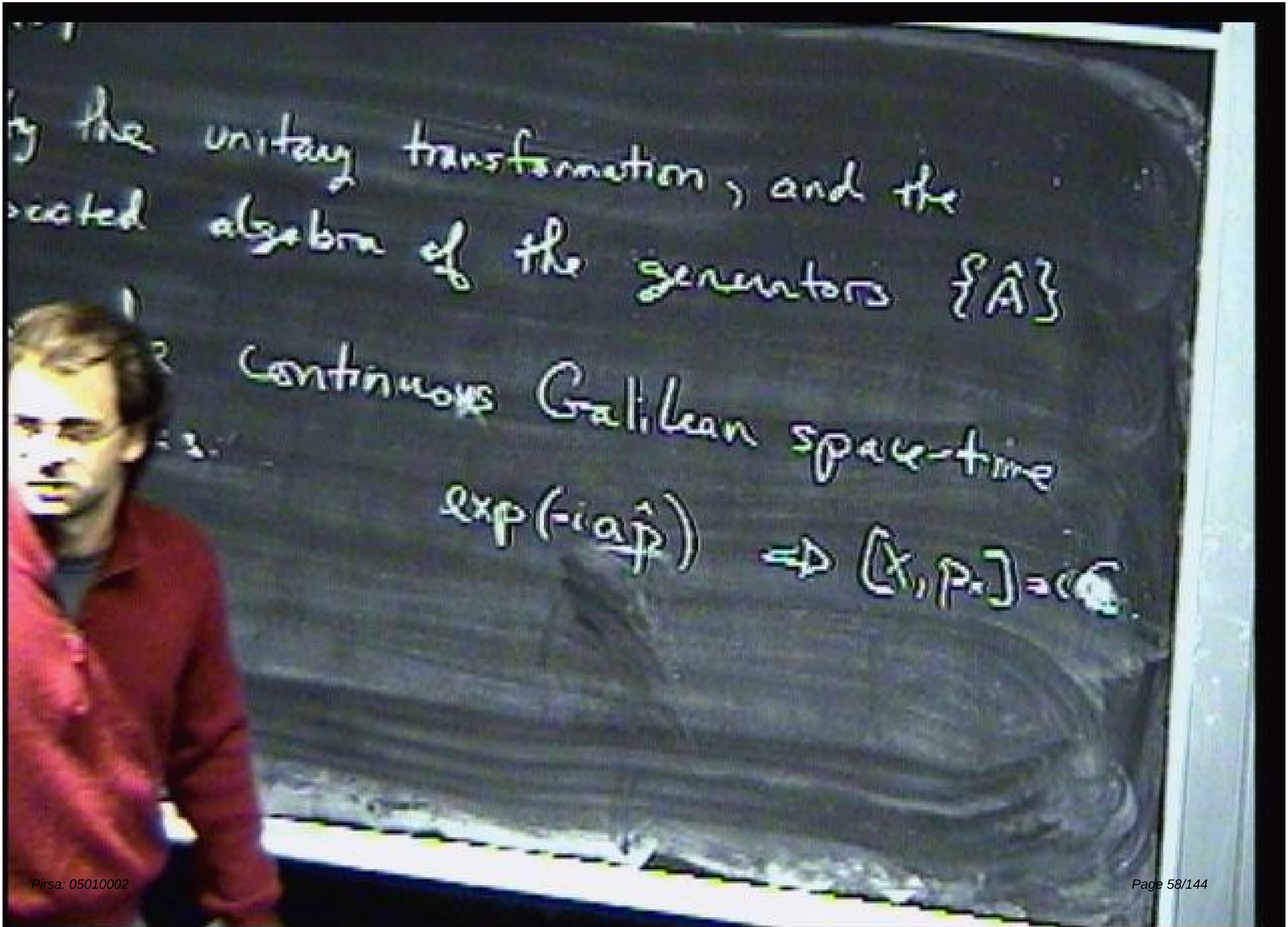
$\Theta \rightarrow \Theta' = \Theta + \Theta_a$

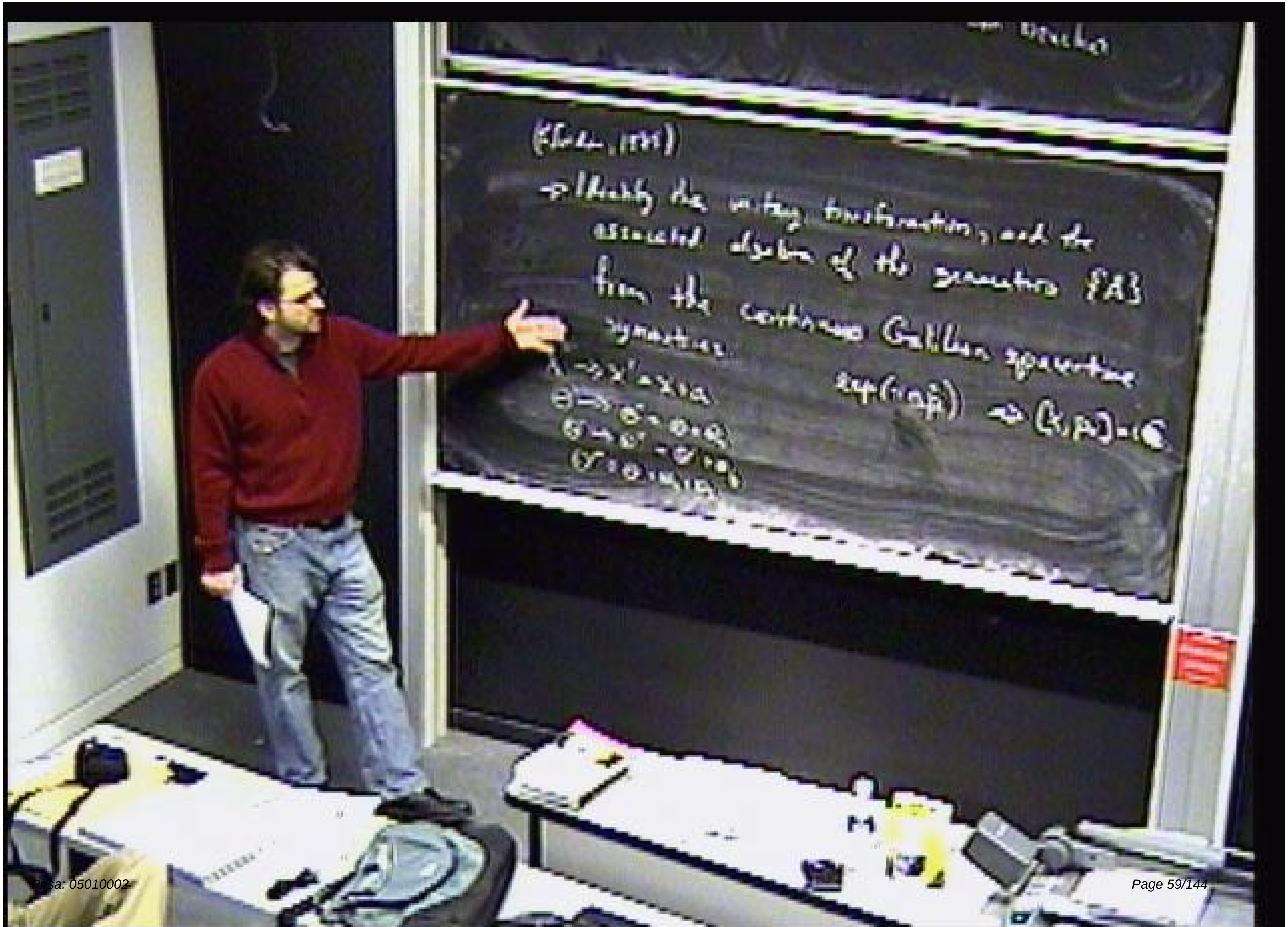
$\Theta' \rightarrow \Theta'' = \Theta' + \Theta_b$

$\Theta'' = \Theta + \Theta_a + \Theta_b$


dentify the unitary transformation, and the
associated algebra of the generators {Â}
from
continuous Galilean space-time
exp(-iap̂)


the unitary transformation, and the
algebra of the generators {Â}
the
Galilean space-time
exp(-iap̂) ⇒ [x, p] = ia


y the unitary transformation, and the
ociated algebra of the generators {Â}
continuous Galilean space-time
exp(-iap̂) ⇒ [x, p] =



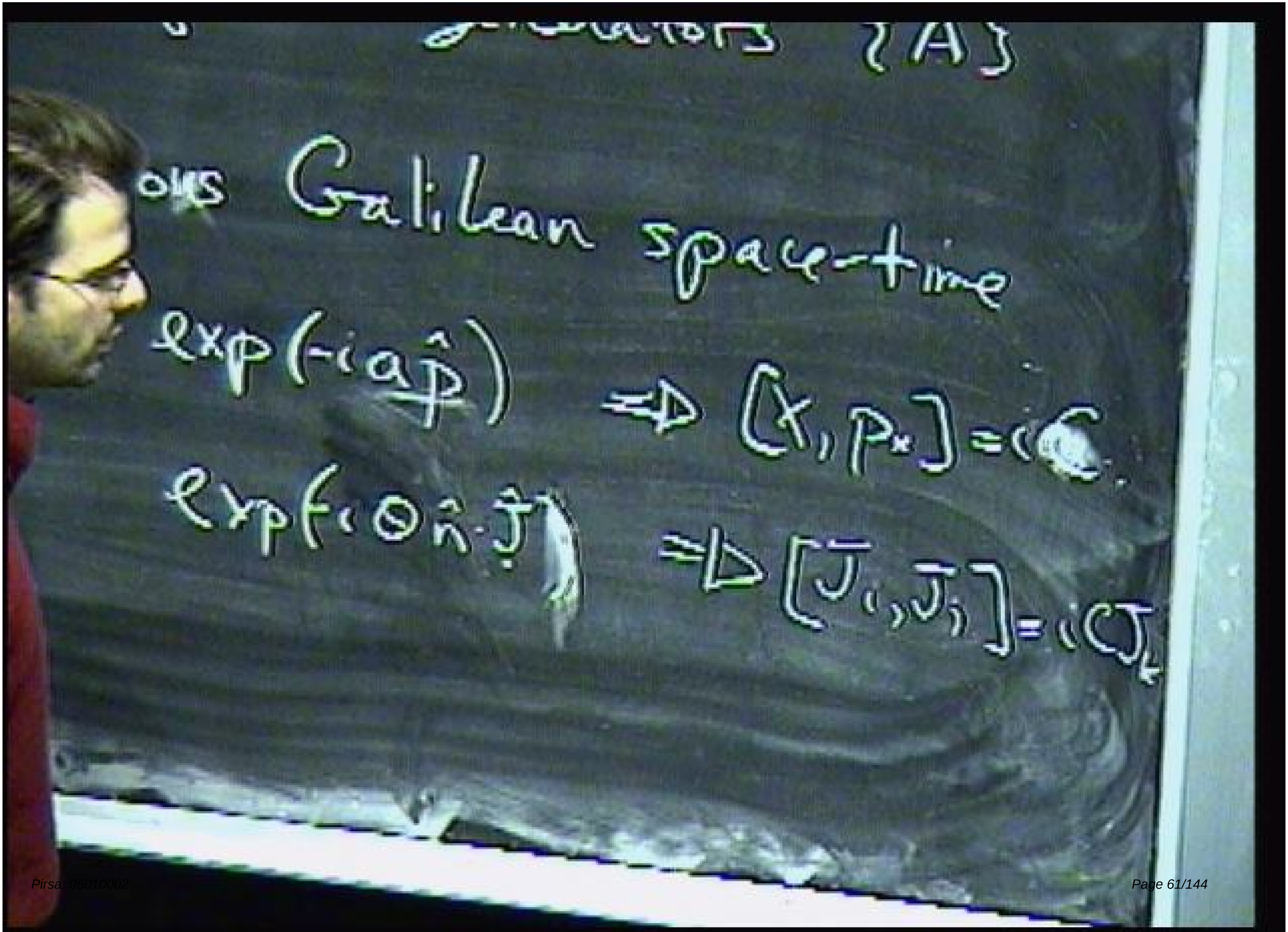
from the continuous Galilean

generators $\{A\}$

...inuous Galilean space-time

$\exp(-ia\hat{p}) \Rightarrow [x, p_x] = i\mathbb{C}$

$\exp(-i\theta\hat{n}\cdot\hat{J}) \Rightarrow [$


{A}
Galilean space-time
exp(-iap̂) ⇒ [x, pₓ] = iC
exp(iθn̂·Ĵ) ⇒ [Jᵢ, Jⱼ] = iCJₖ

generators $\{A\}$

...tinuous Galilean space-time

$\exp(-ia\hat{p}) \Rightarrow [x, p_x] = iC$

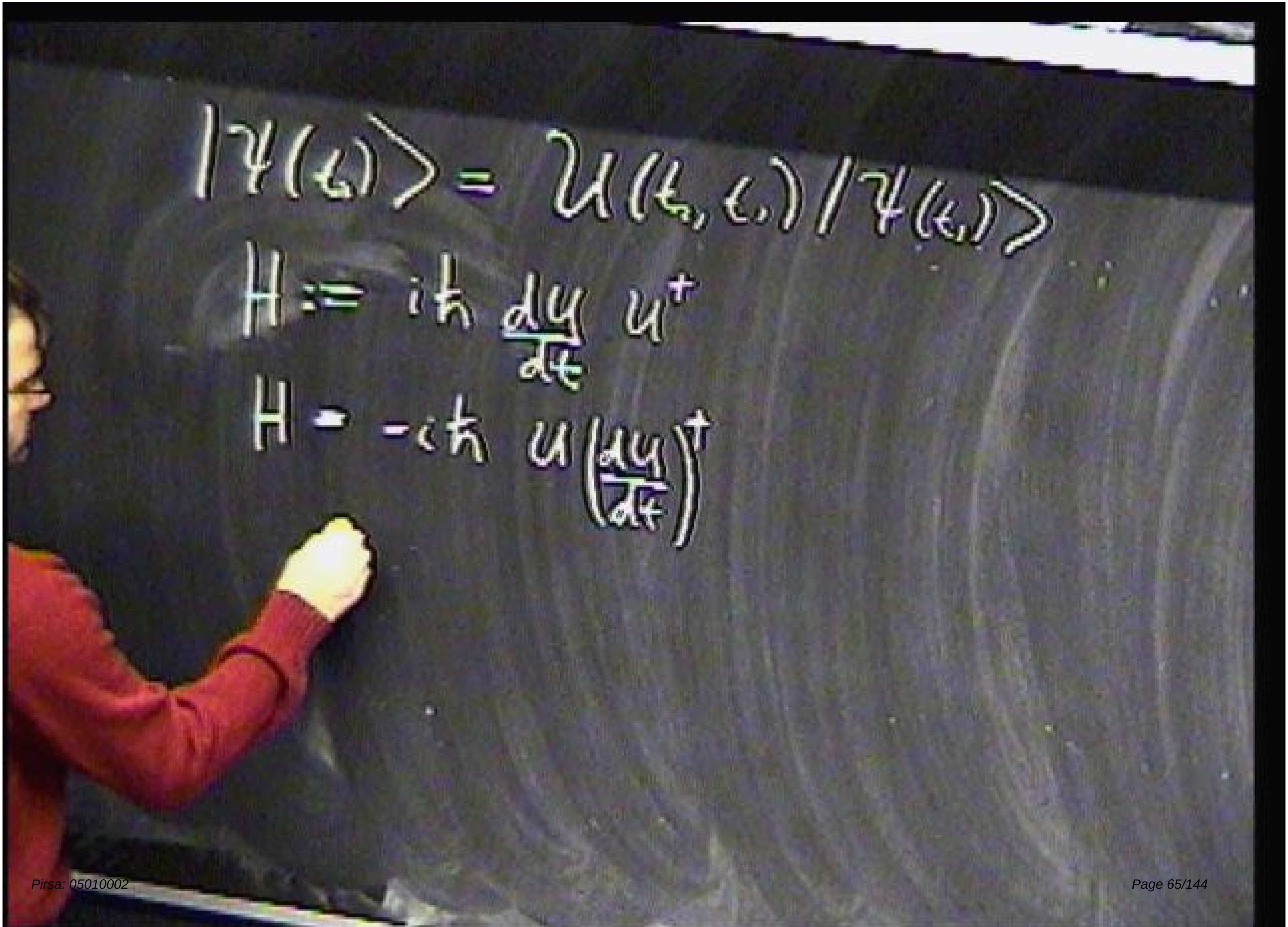
$\exp(i\theta\hat{n}\cdot\hat{J}) \Rightarrow [J_i, J_j] = iCJ_k$

$= -[J_j, J_i]$

$$|\psi(t_1)\rangle = \mathcal{U}(t_1, t_0)\,|\psi(t_0)\rangle$$

$$|\psi(t)\rangle = U(t,t_0)\,|\psi(t_0)\rangle$$

$$H := i\hbar \frac{dU}{dt} U^{\dagger}$$



$|\psi(t)\rangle = U(t,t_0)|\psi(t_0)\rangle$
$H := i\hbar \frac{dU}{dt} U^\dagger$
$H = -i\hbar\, U \left(\frac{dU}{dt}\right)^\dagger$

$$|\psi(t_1)\rangle = \mathcal{U}(t_1,t_0)\,|\psi(t_0)\rangle$$

$$H := i\hbar \frac{dU}{dt} U^\dagger$$

$$H = -i\hbar\, U \left(\frac{dU}{dt}\right)^\dagger$$

$$\Rightarrow H = H^\dagger$$

$$|\psi(t)\rangle = \mathcal{U}(t,t_0)\,|\psi(t_0)\rangle$$

$$H := i\hbar \frac{d\mathcal{U}}{dt}\,\mathcal{U}^{\dagger}$$

$$H = -i\hbar\, \mathcal{U}\left(\frac{d\mathcal{U}}{dt}\right)^{\dagger}$$

$$\Rightarrow H = H^{\dagger}$$

$$i\hbar \frac{d|\psi(t)\rangle}{dt} = \hat{H}\,|\psi(t)\rangle$$

$$|\psi(t_1)\rangle = \mathcal{U}(t_1, t_0)\,|\psi(t_0)\rangle$$

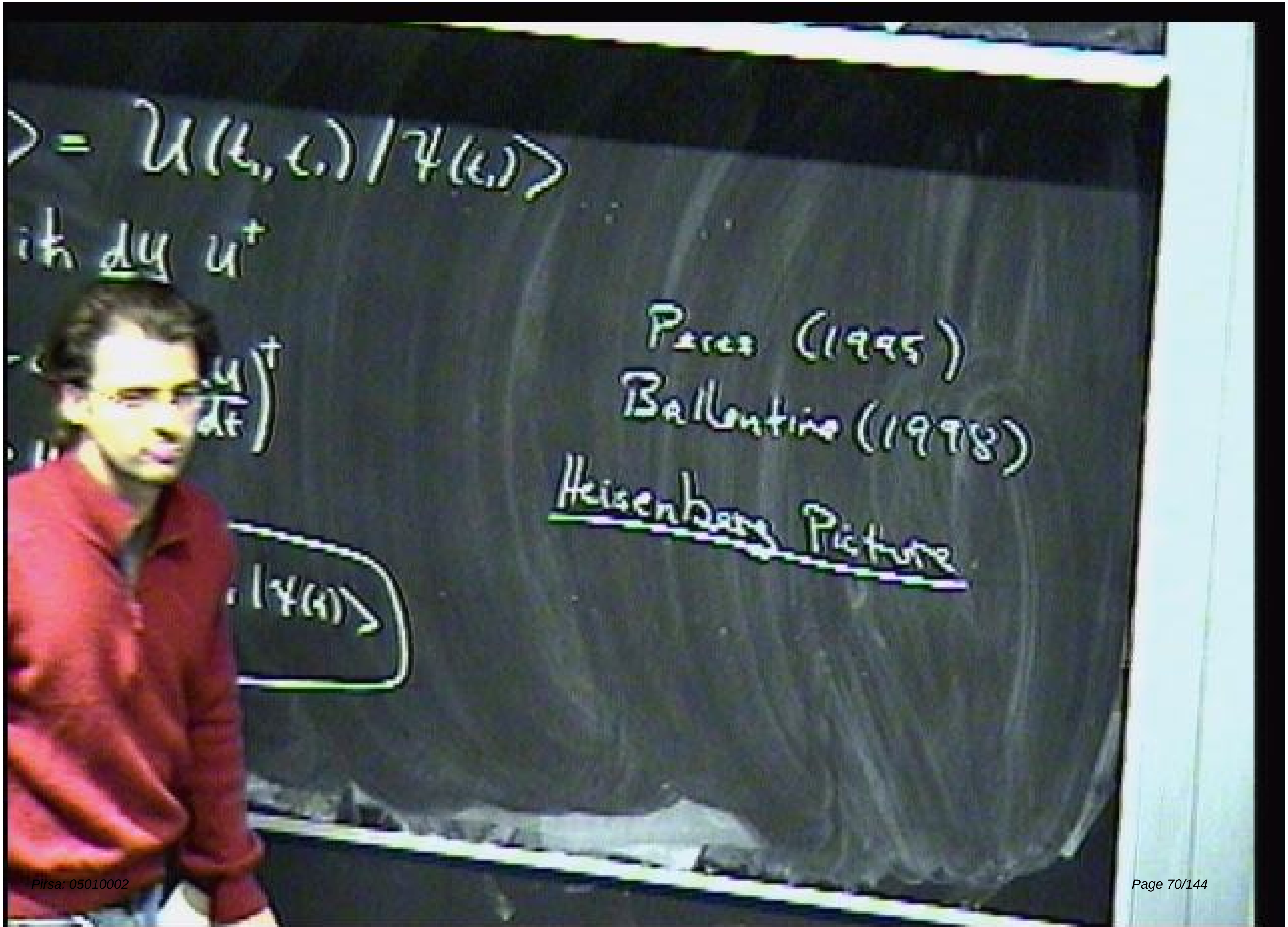
$$H := i\hbar \frac{d\mathcal{U}}{dt}\,\mathcal{U}^\dagger$$

$$H = -i\hbar\,\mathcal{U}\left(\frac{d\mathcal{U}}{dt}\right)^\dagger$$

$$\Rightarrow H = H^\dagger$$

$$\boxed{i\hbar \frac{d\,|\psi(t)\rangle}{dt} = \hat{H}\,|\psi(t)\rangle}$$

Peres (1995)
Ballentine (1998)


Peres (1995)
Ballentine (1998)
Heisenberg Picture

Peres (1995)
Ballentine (1998)

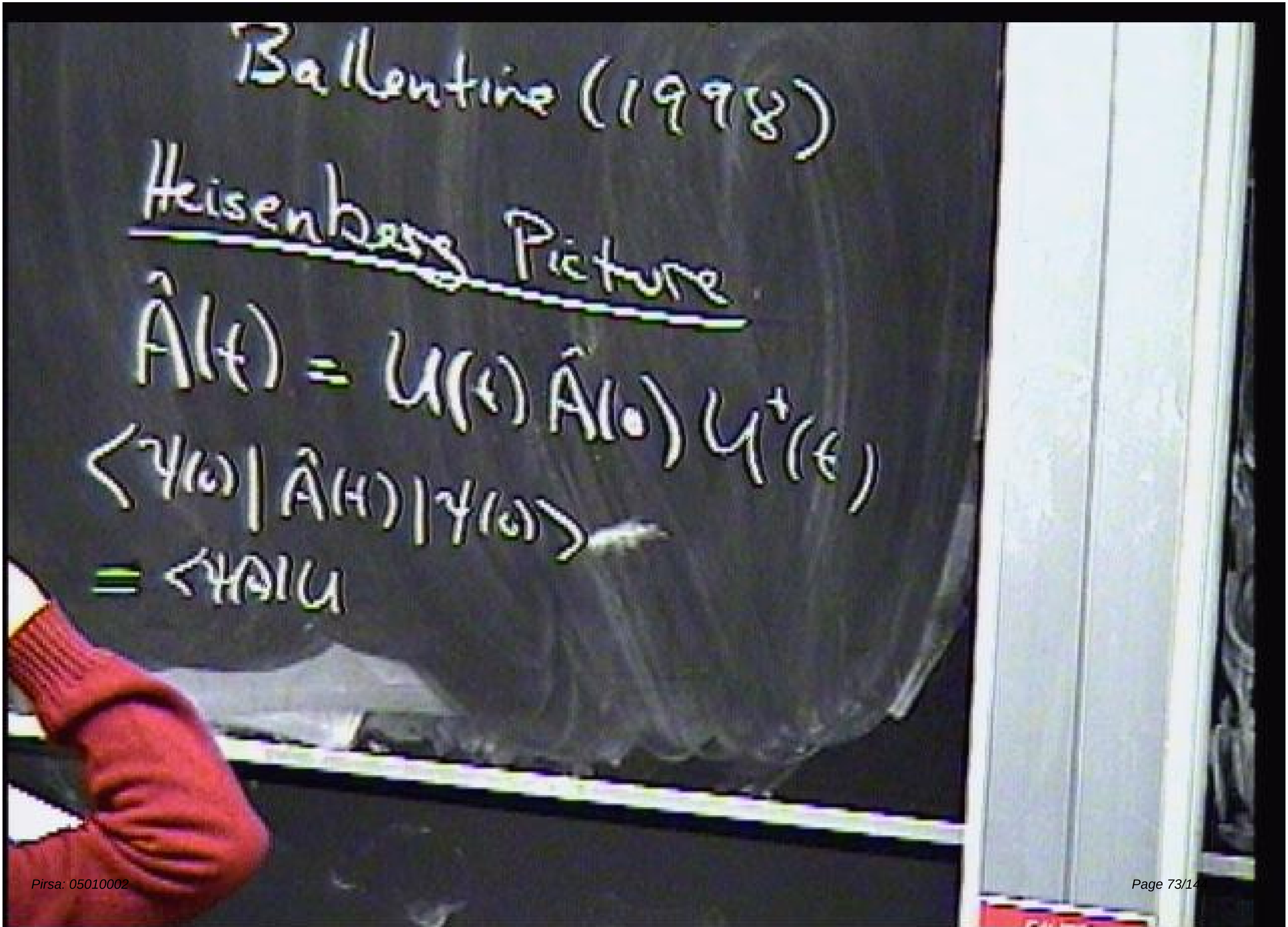
Heisenberg Picture

$$\hat{A}(t) = U(t)\,\hat{A}(0)\,U^{+}(t)$$

$$U = \exp(-i\hat{H}t)$$

$$H \neq H(t).$$

Peres (1995)



Ballentine (1998)
Heisenberg Picture
$\hat{A}(t) = U(t)\hat{A}(0)U^{+}(t)$
$\langle\psi(0)|\hat{A}(t)|\psi(0)\rangle$
$= \langle\psi(0)|U$

Ballentine (1998)

Heisenberg Picture

$$\hat{A}(t) = U^\dagger(t)\,\hat{A}(0)\,U(t)$$

$$\langle\psi(0)|\,\hat{A}(t)\,|\psi(0)\rangle$$

$$= \langle\psi(0)|\,U^\dagger A(0)\,U\,|\psi(0)\rangle$$

$$= \langle\psi$$

Ballentine (1998)

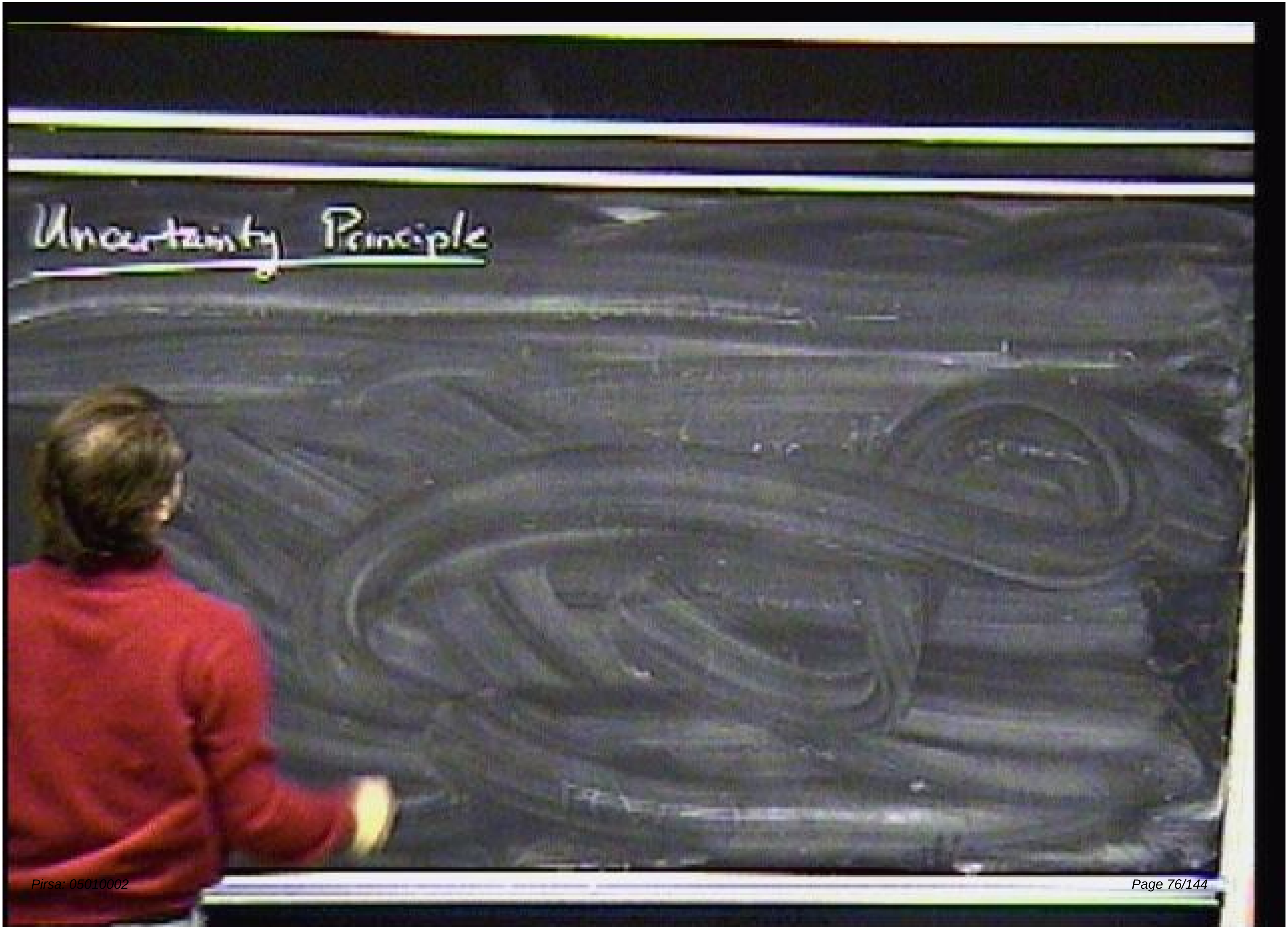
Heisenberg Picture

$$\hat{A}(t) = U^{\dagger}(t)\,\hat{A}(0)\,U(t)$$

$$\langle \psi(0)|\,\hat{A}(t)\,|\psi(0)\rangle$$

$$= \langle \psi(0)|\,U^{\dagger} A(0)\, U\,|\psi(0)\rangle$$

$$= \langle \psi(t)|\,A(0)\,|\psi(t)\rangle$$


Uncertainty Principle

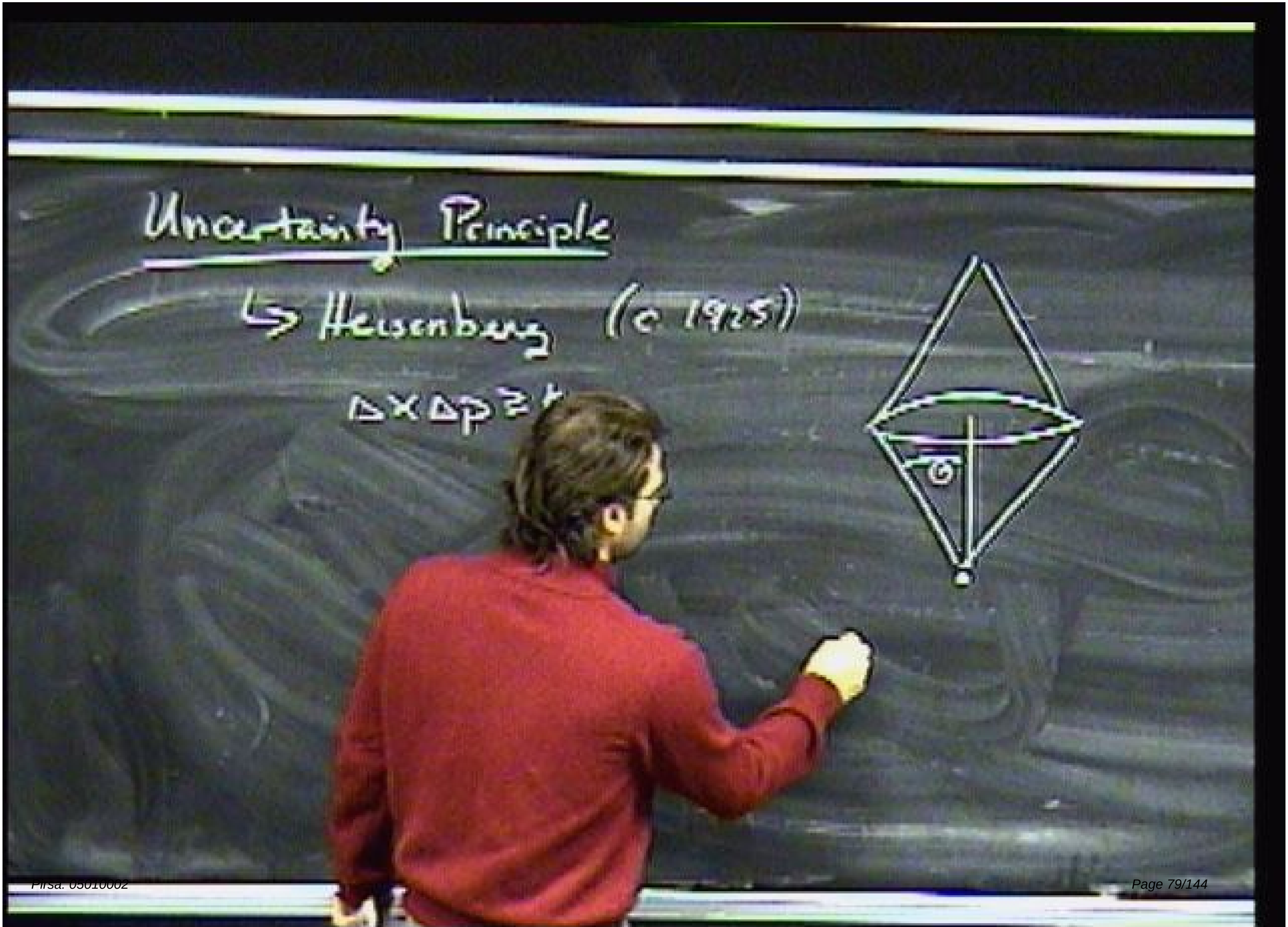
Uncertainty Principle

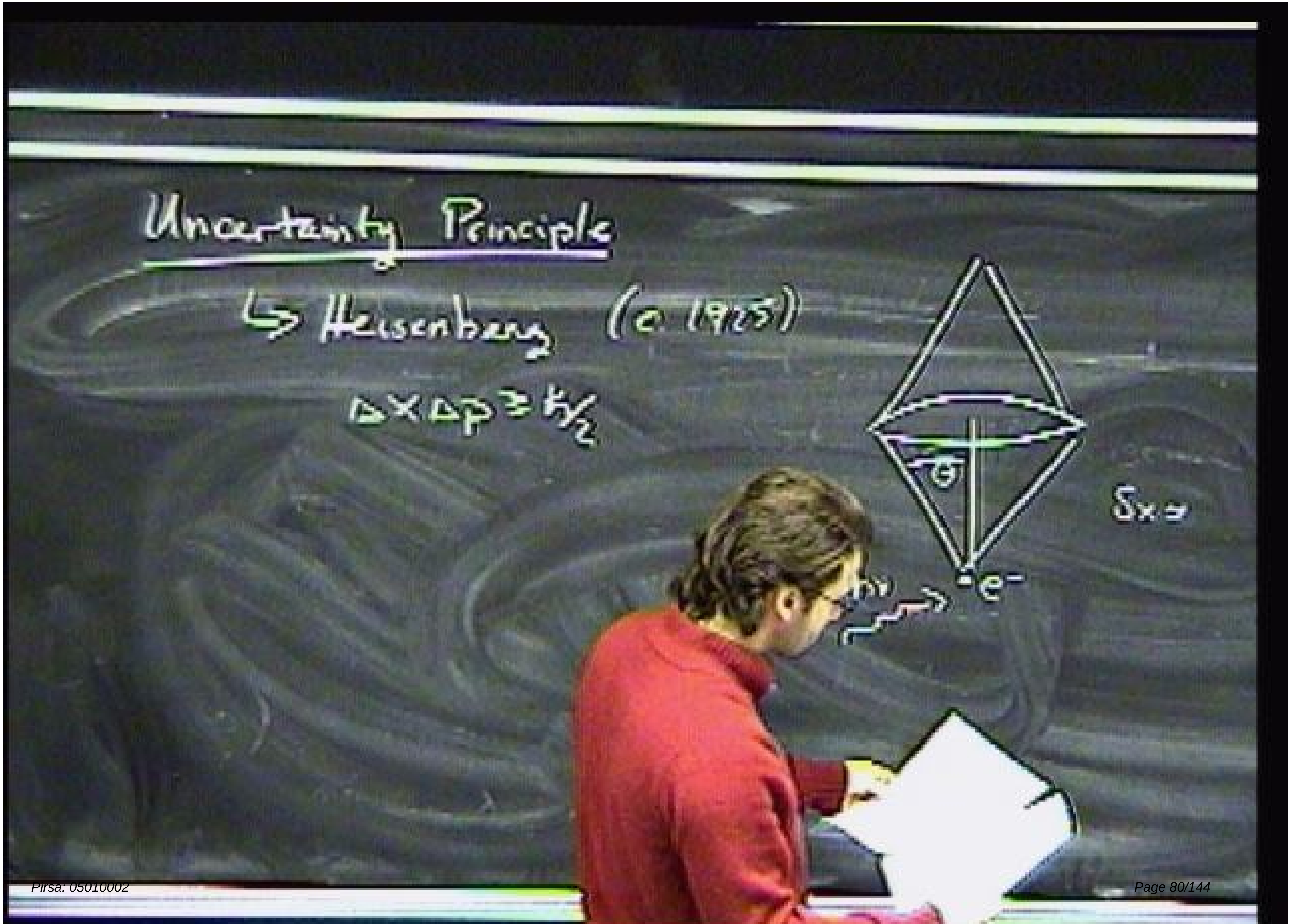
↳ Heisenberg (c. 1925)

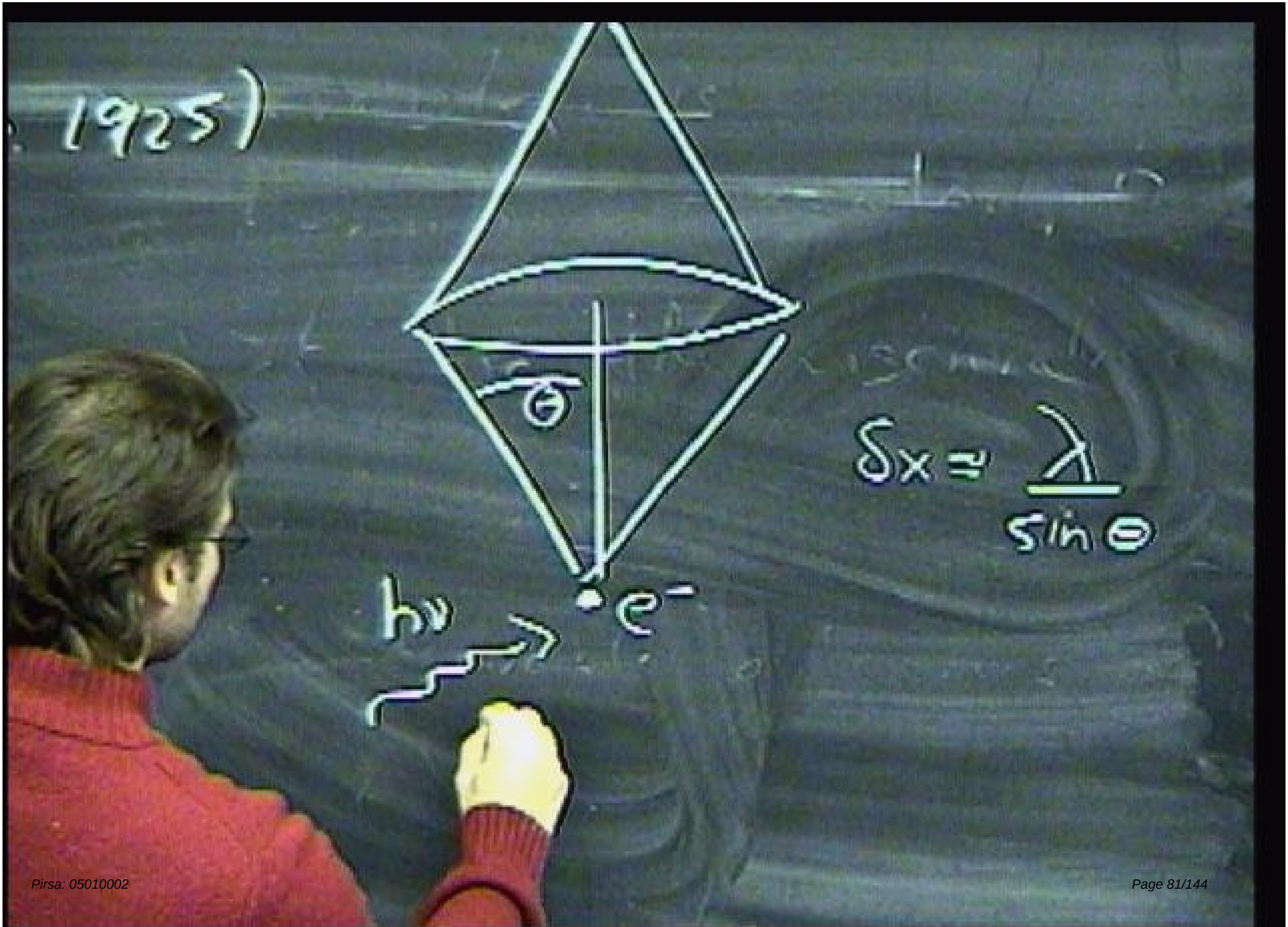
Uncertainty Principle

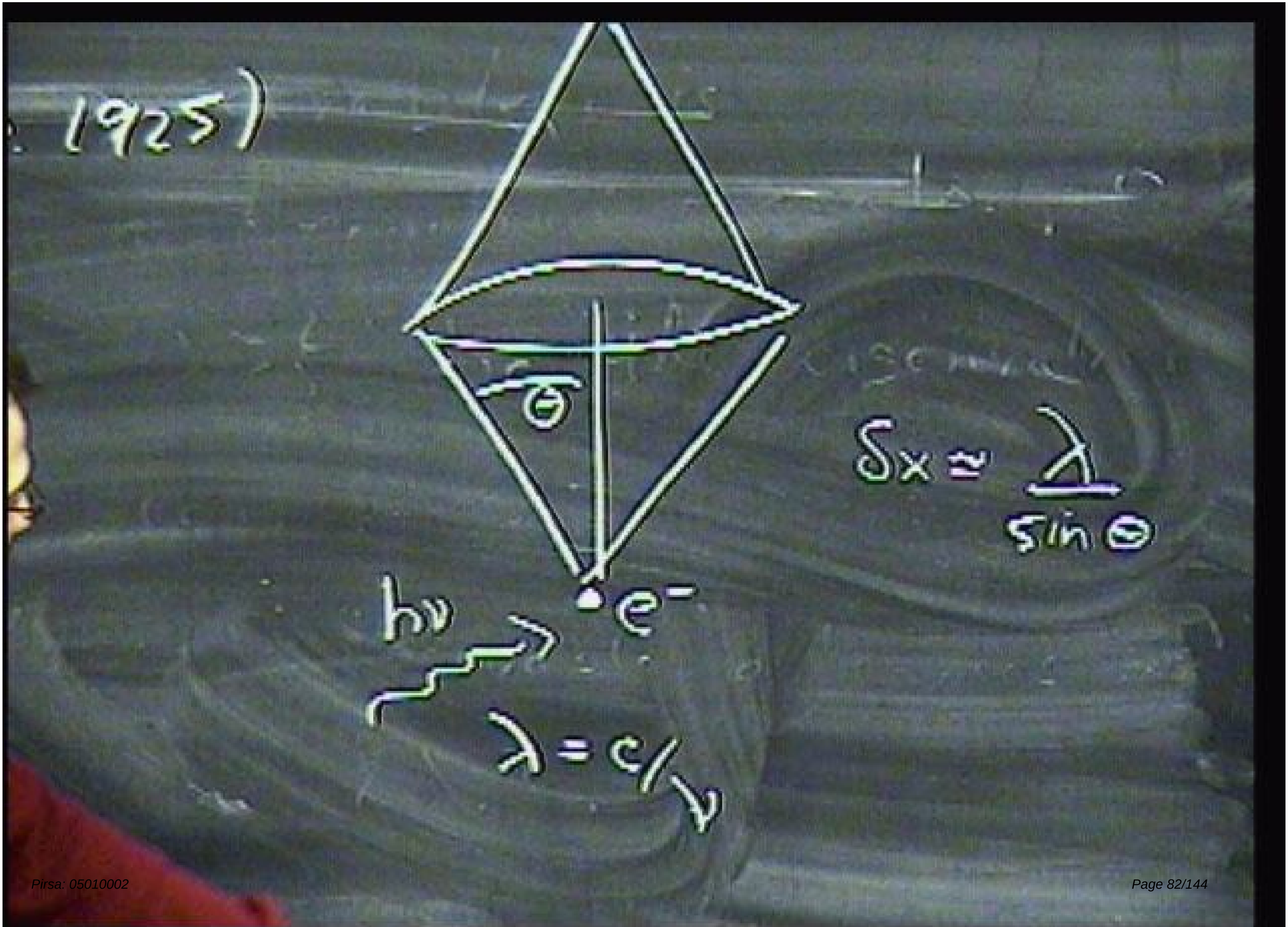
↳ Heisenberg (c. 1925)

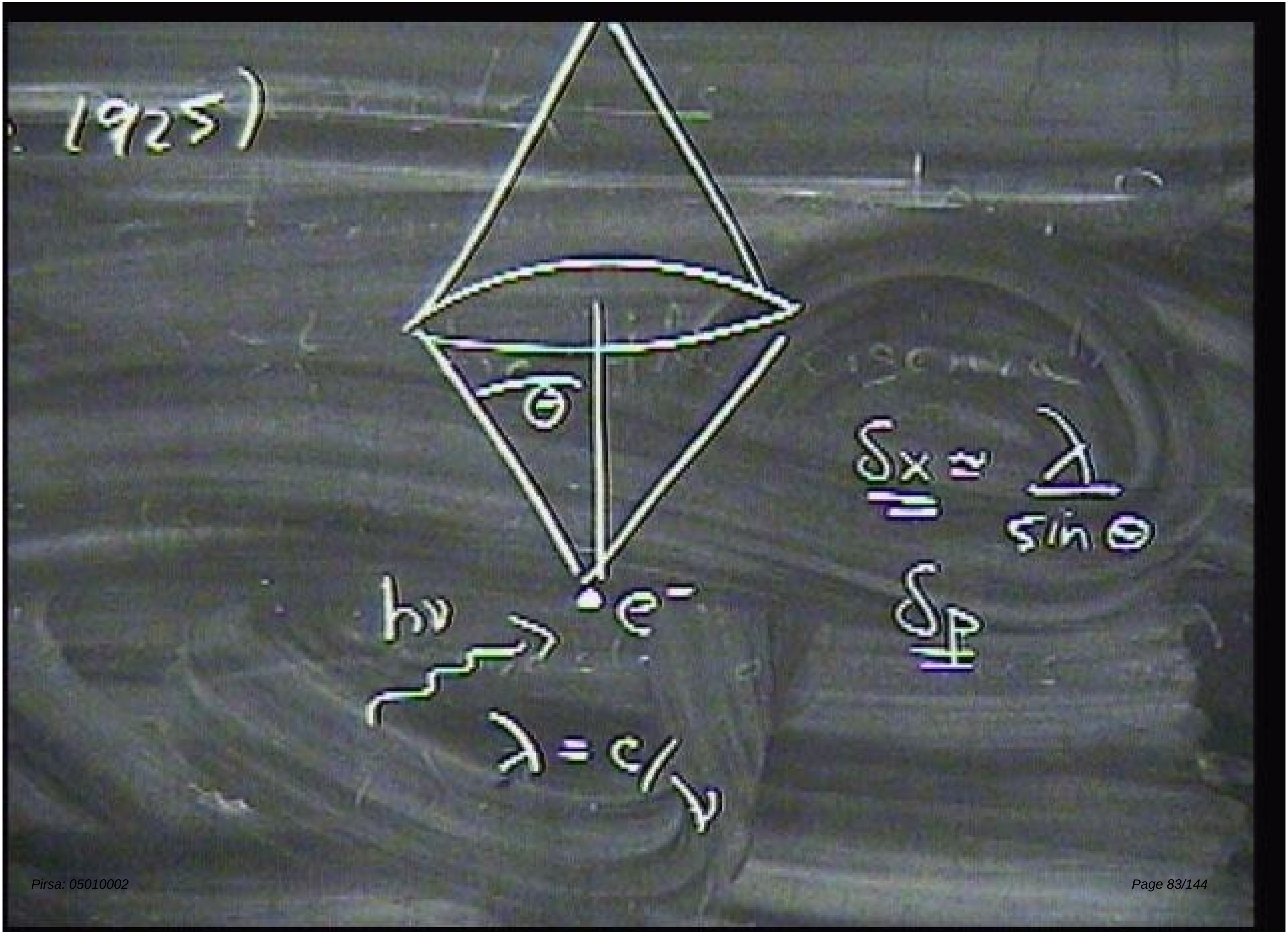
$$\Delta x \Delta p \geq \hbar/2$$


Uncertainty Principle
↳ Heisenberg (c 1925)
ΔxΔp ≥
θ


Uncertainty Principle
↳ Heisenberg (c. 1925)
$\Delta x \Delta p \geq \hbar/2$
θ
e^-
$\delta x \approx$


1925)
θ
δx ≃ λ / sin θ
hν
e⁻


1925)
θ
δx ≈ λ/sin θ
hν
e⁻
λ = c/ν

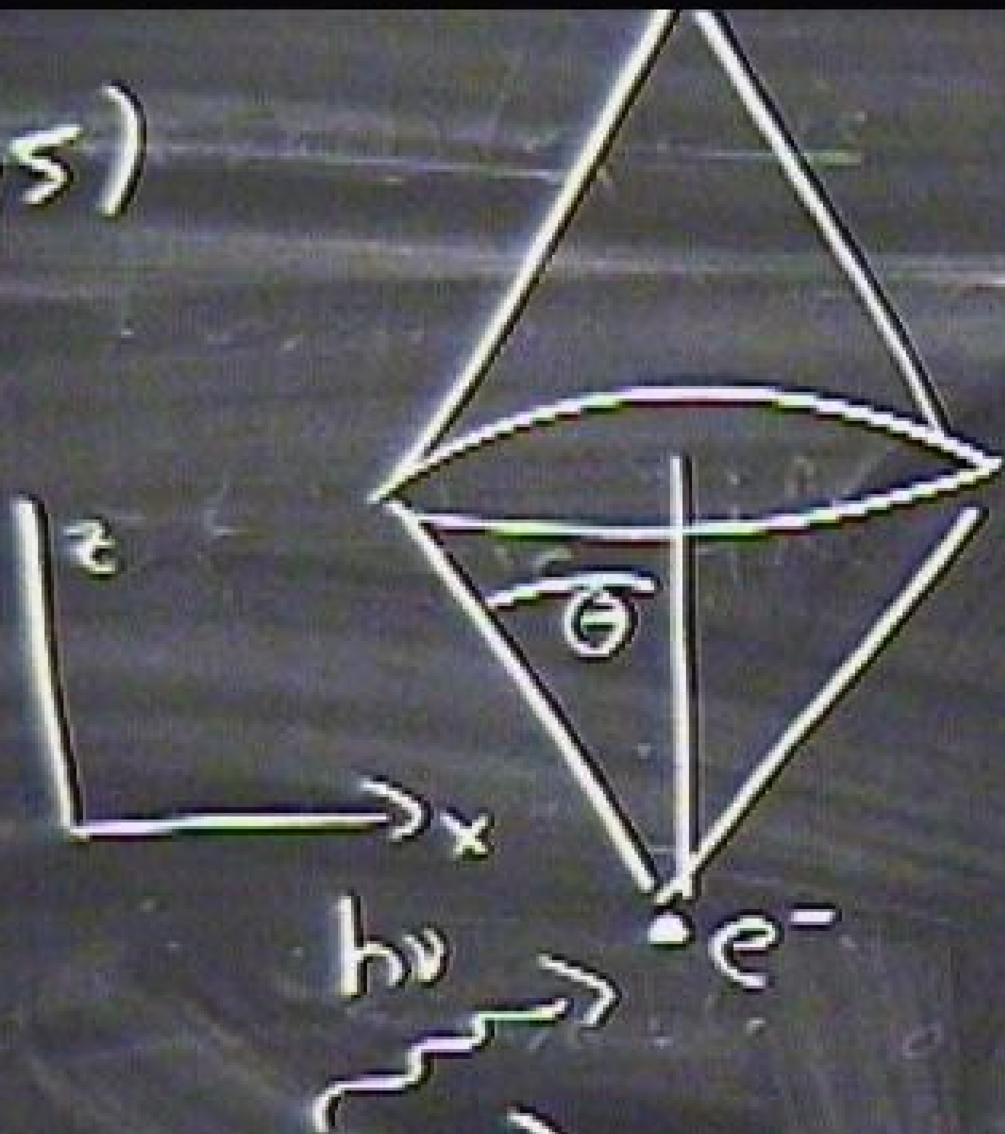

1925)
θ
δx ≈ λ / sin θ
δp
hν
e⁻
λ = c/ν

$$\delta x \simeq \frac{\lambda}{\sin\theta}$$

$$\delta p \simeq \frac{2h}{\lambda}\sin(\theta)$$

$$\lambda = c/\nu$$

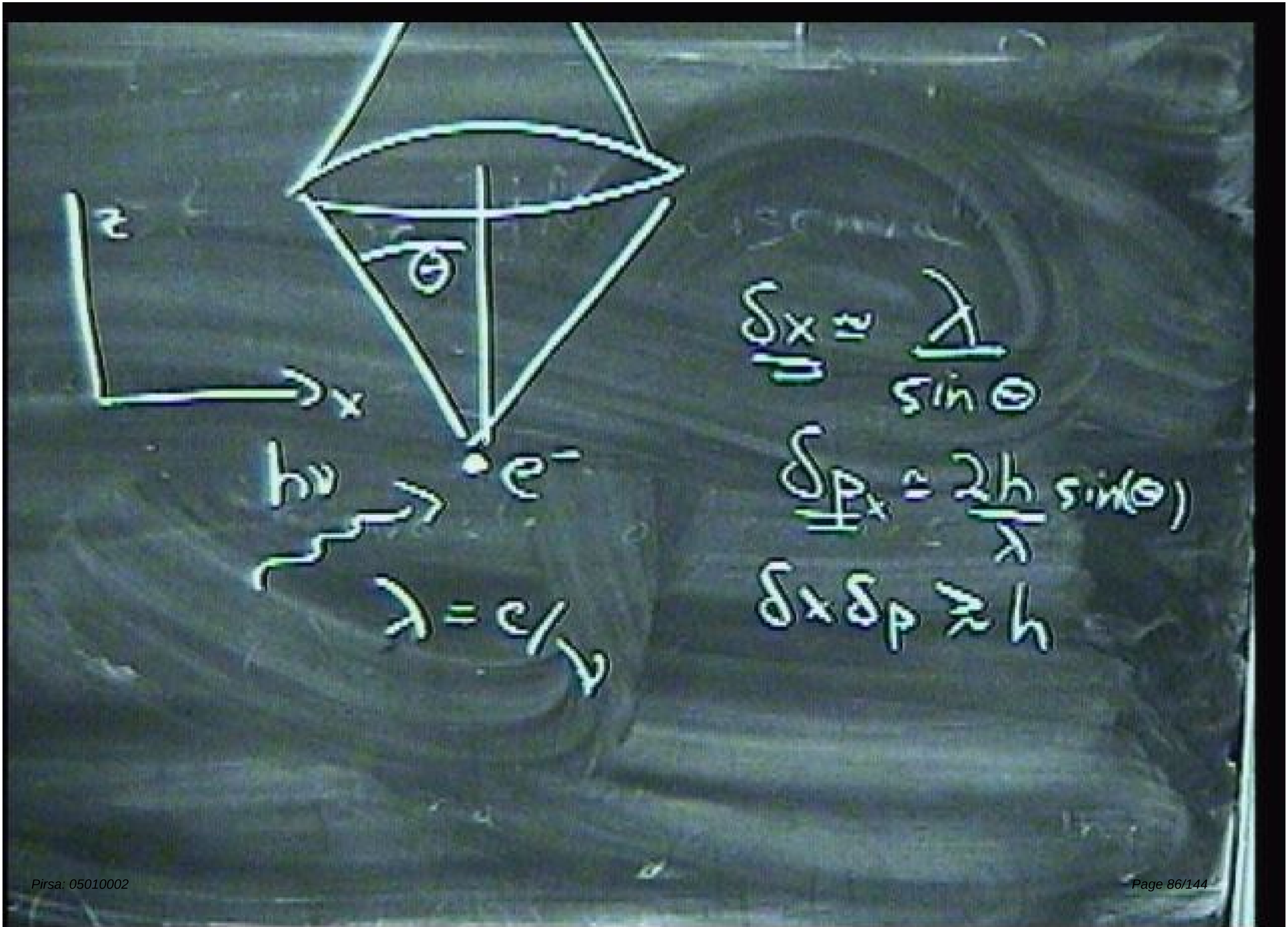
25)

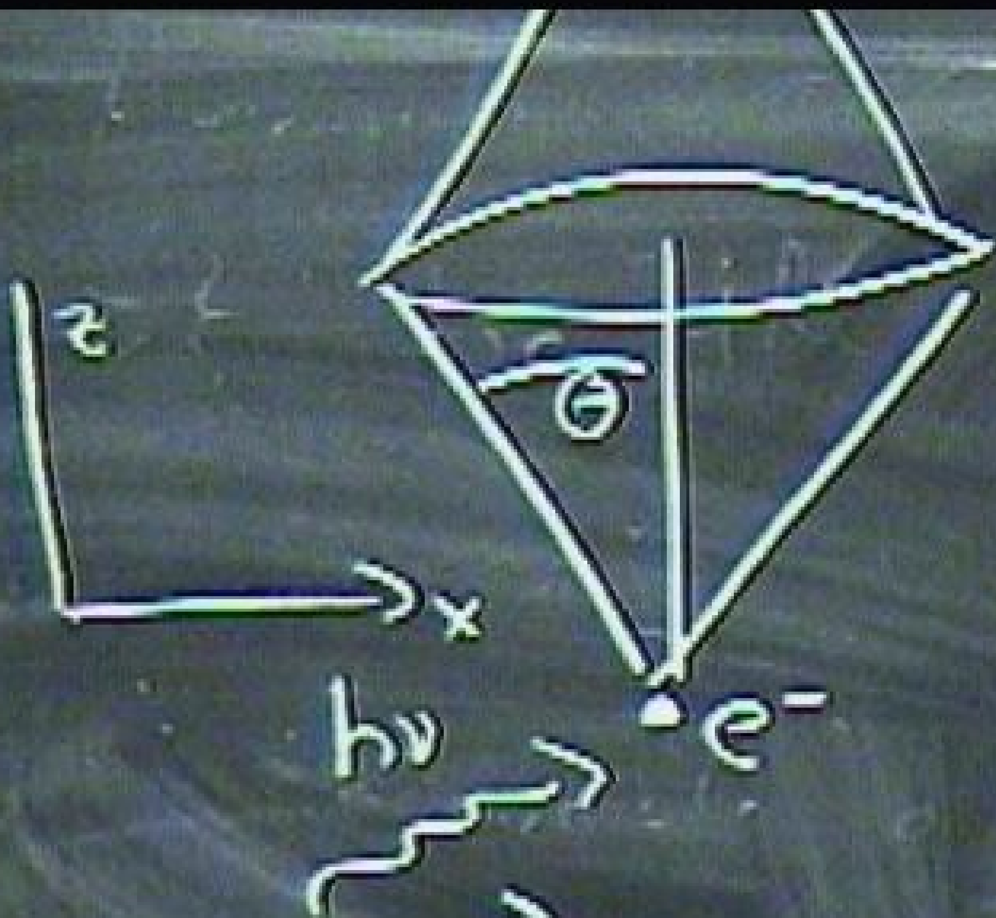


$$\delta x \simeq \frac{\lambda}{\sin\theta}$$

$$\delta p_x \simeq \frac{2h}{\lambda}\sin(\theta)$$

$$\lambda = c/\nu$$


z
x
θ
$h\nu$
e^-
$\lambda = c/\nu$
$\delta x \simeq \frac{\lambda}{\sin\theta}$
$\delta p_x \simeq \frac{2h}{\lambda}\sin(\theta)$
$\delta x \delta p \gtrsim h$



$$\lambda = c/\nu$$

$$\delta x \simeq \frac{\lambda}{\sin\theta}$$

$$\delta p_x \simeq \frac{2h}{\lambda}\sin(\theta)$$

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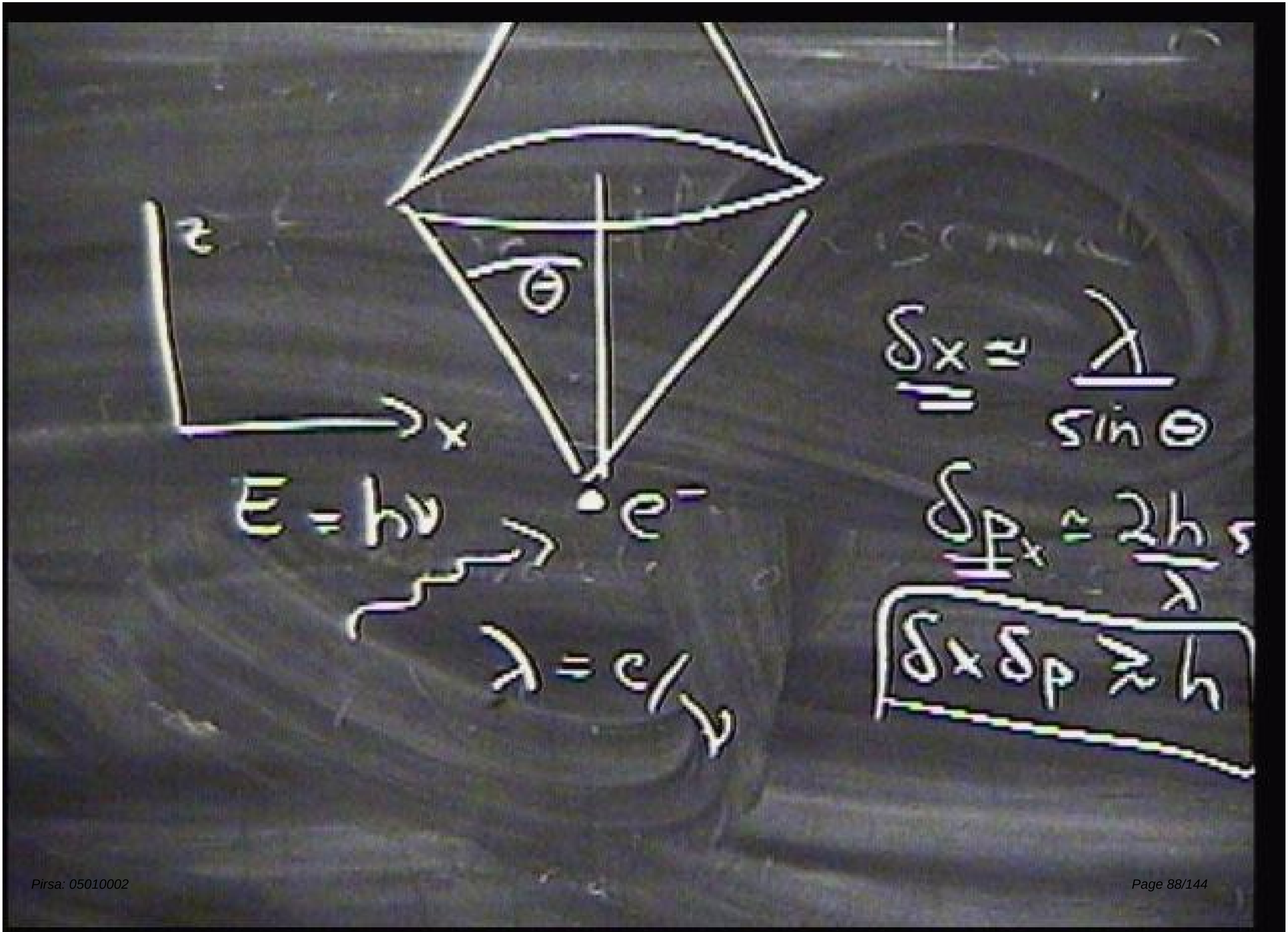
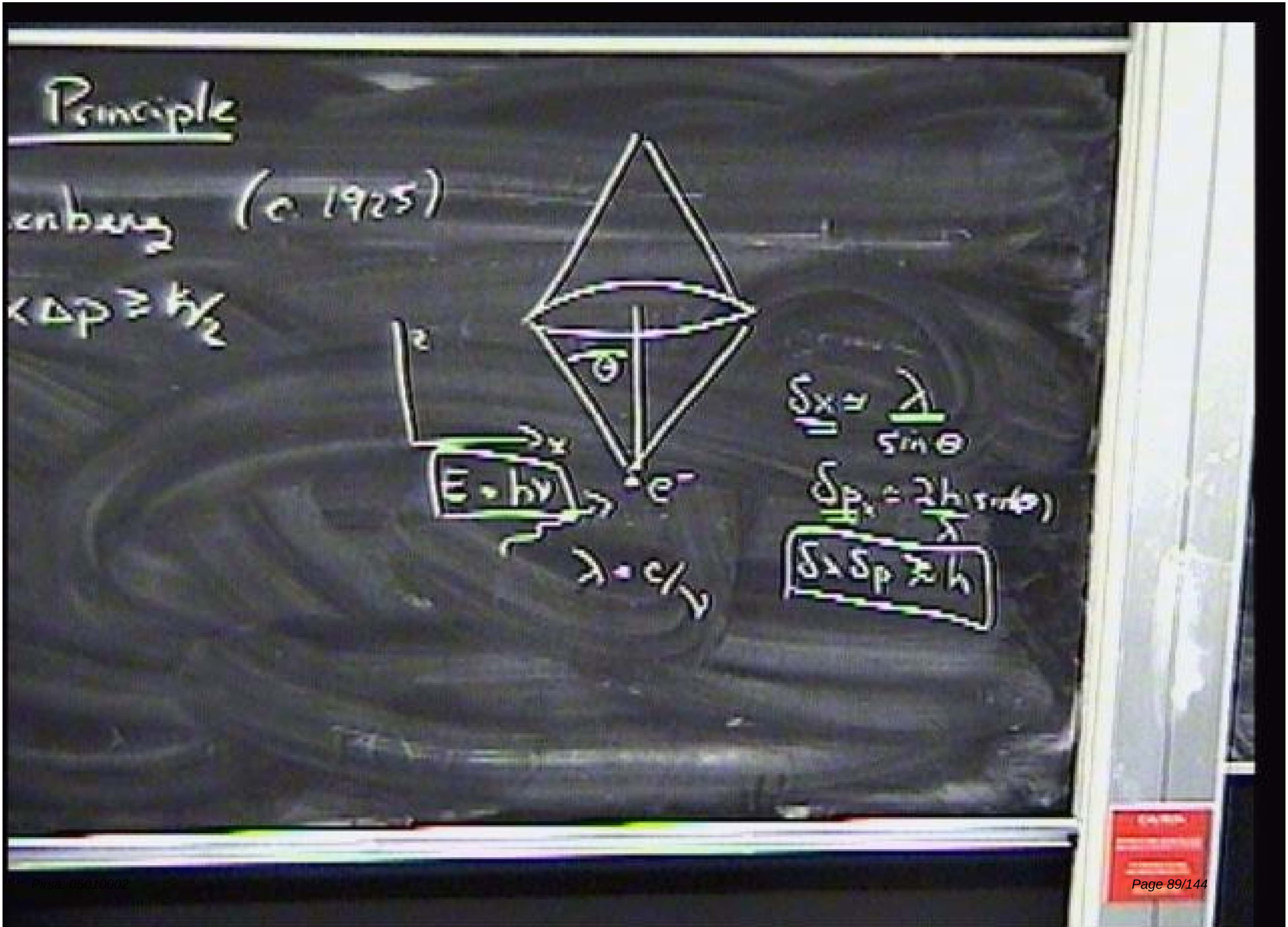
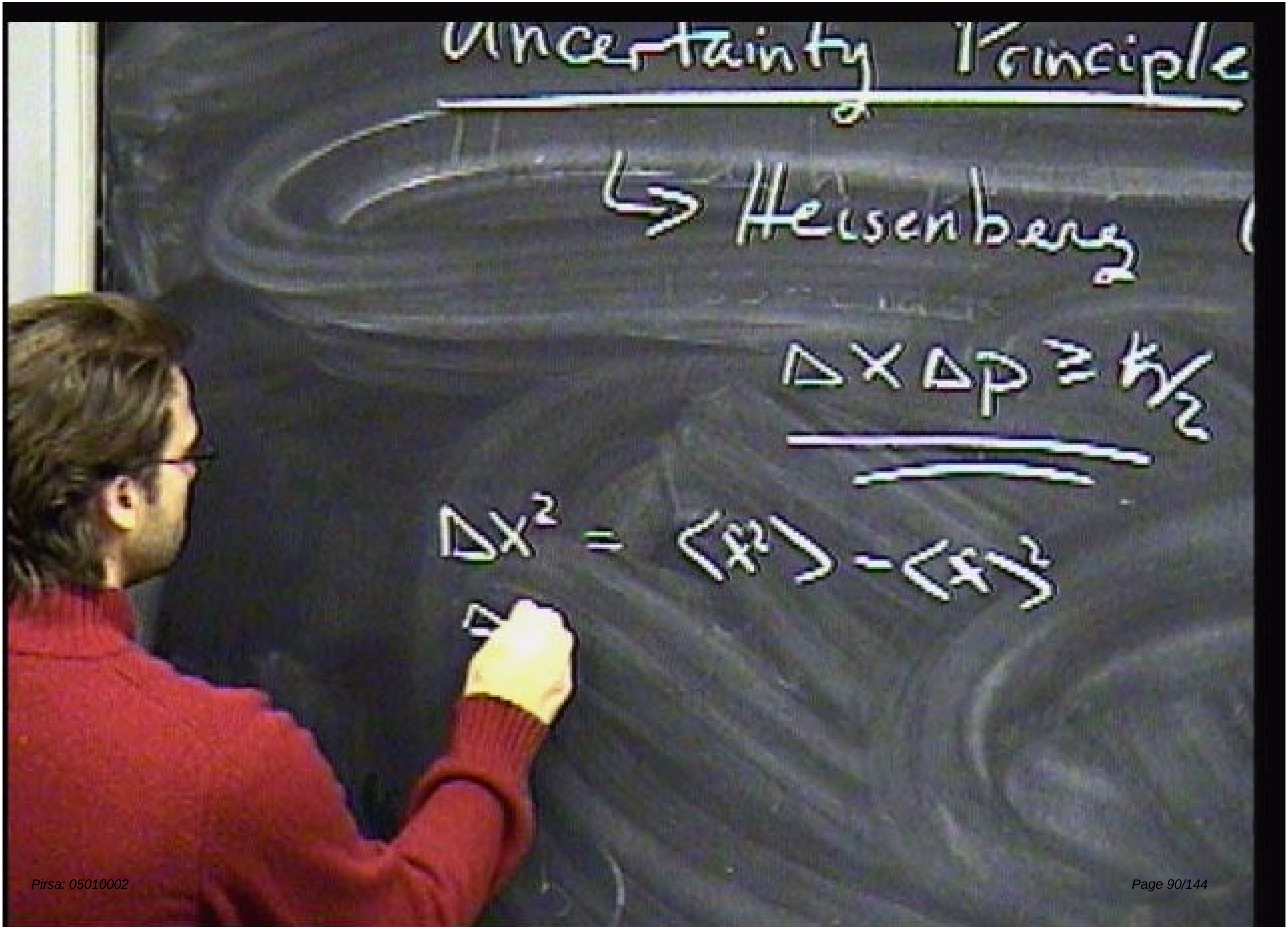
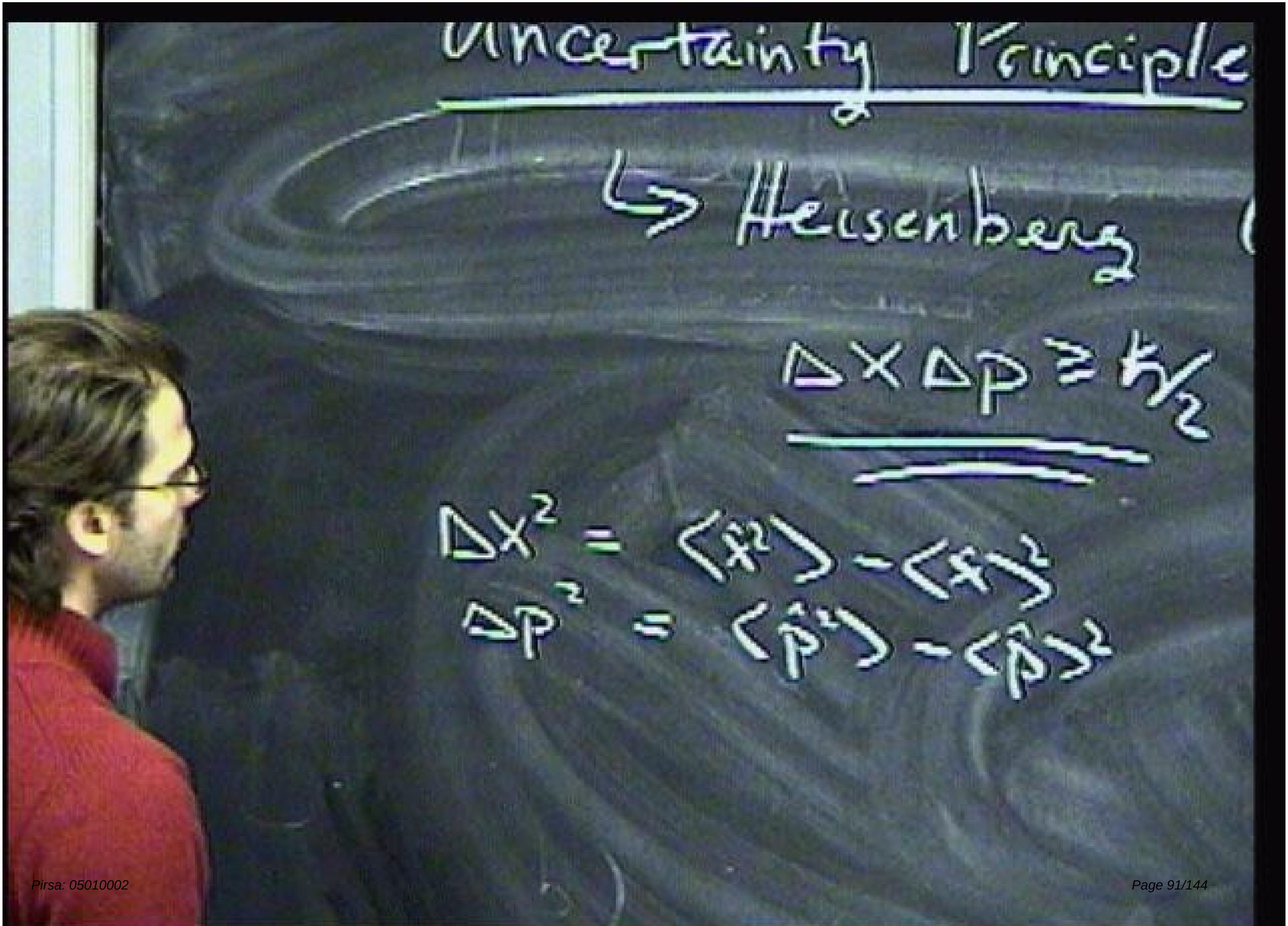
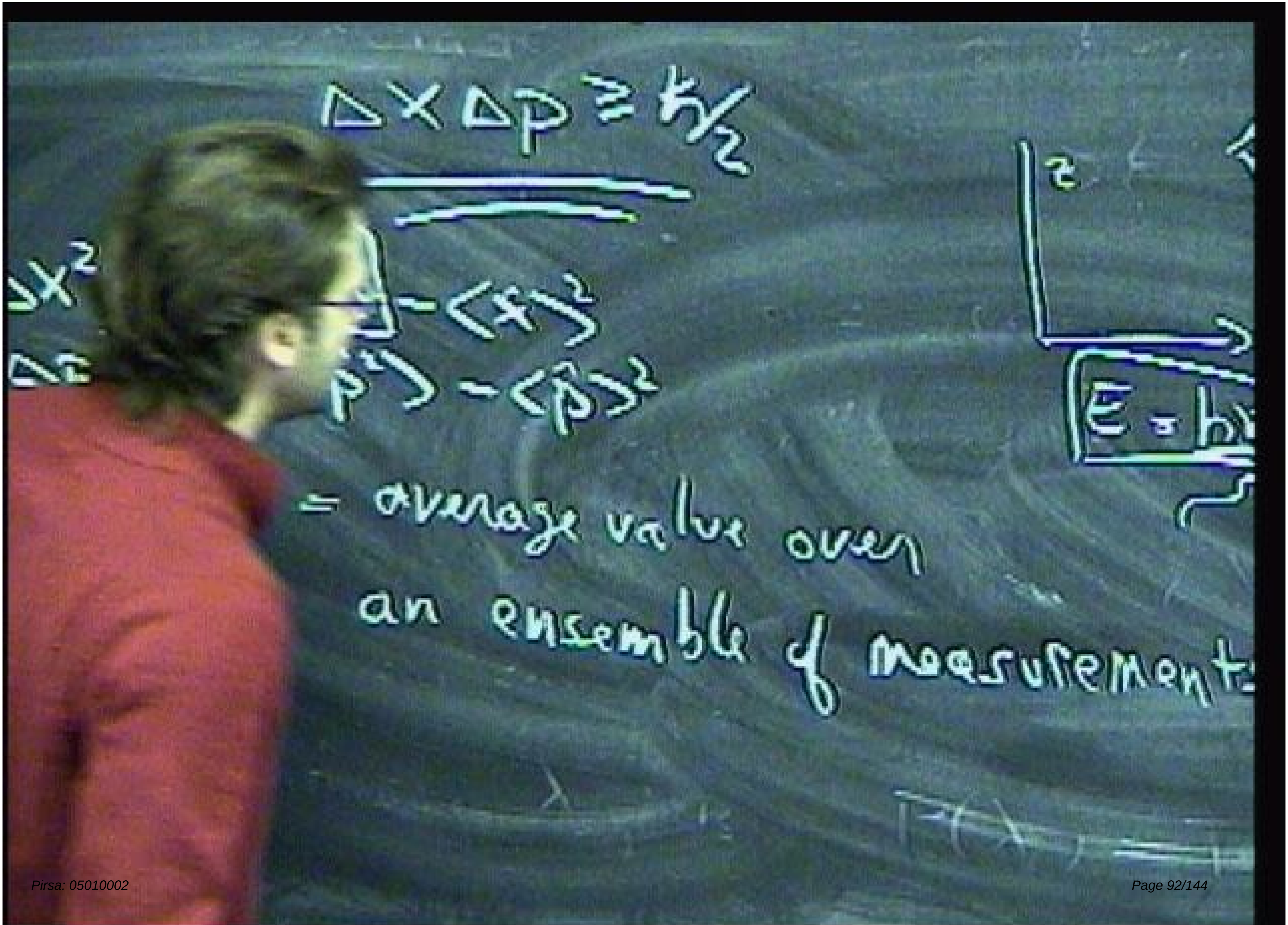

z
x
θ
E = hν
e⁻
λ = c/ν
δx ≃ λ / sin θ
δpₓ ≃ 2h s / λ
δx δp ≳ h


Principle
enberg (c. 1925)
xΔp ≥ ħ/2
z
x
θ
E = hν
e⁻
λ = c/ν
δx ≃ λ/sin θ
δpₓ ≃ 2h/λ sin(θ)
δx δp ≳ h


Uncertainty Principle
↳ Heisenberg (
$\Delta x \Delta p \geq \hbar/2$
$\Delta x^2 = \langle x^2 \rangle - \langle x \rangle^2$
Δ


Uncertainty Principle
↳ Heisenberg (
$\Delta x \Delta p \geq \hbar/2$
$\Delta x^2 = \langle \hat{x}^2 \rangle - \langle \hat{x} \rangle^2$
$\Delta p^2 = \langle \hat{p}^2 \rangle - \langle \hat{p} \rangle^2$


ΔxΔp ≥ ℏ/2
= average value over
an ensemble of measurement

$$\Delta x \Delta p \geq \hbar/2$$

$$\Delta x^2 = \boxed{\langle \hat{x}^2 \rangle} - \langle \hat{x} \rangle^2$$

$$\Delta p^2 = \langle \hat{p}^2 \rangle - \langle \hat{p} \rangle^2$$

$\langle \hat{f} \rangle$ = average value over an ensemble of measurements

$$\boxed{E = h\nu}$$

Uncertainty Principle

↳ Heisenberg (c 1925)

$$\Delta x \Delta p \geq \hbar/2$$

$$\Delta x^2 = \boxed{\langle \hat{x}^2 \rangle} - \langle \hat{x} \rangle^2$$

$$\Delta p^2 = \langle \hat{p}^2 \rangle - \langle \hat{p} \rangle^2$$

$\langle \hat{x} \rangle$ = average value over
$= \langle \psi | \hat{x} | \psi \rangle$ an ensemble of measurements

$\Delta p = \langle \hat{p}^2 \rangle - \langle \hat{p} \rangle^2$

$\langle \hat{f} \rangle$ = average value over

$= \langle \psi | \hat{f} | \psi \rangle$ an ensemble of mea.

$\Delta f > 0 \Rightarrow$ a variance in set of outcomes

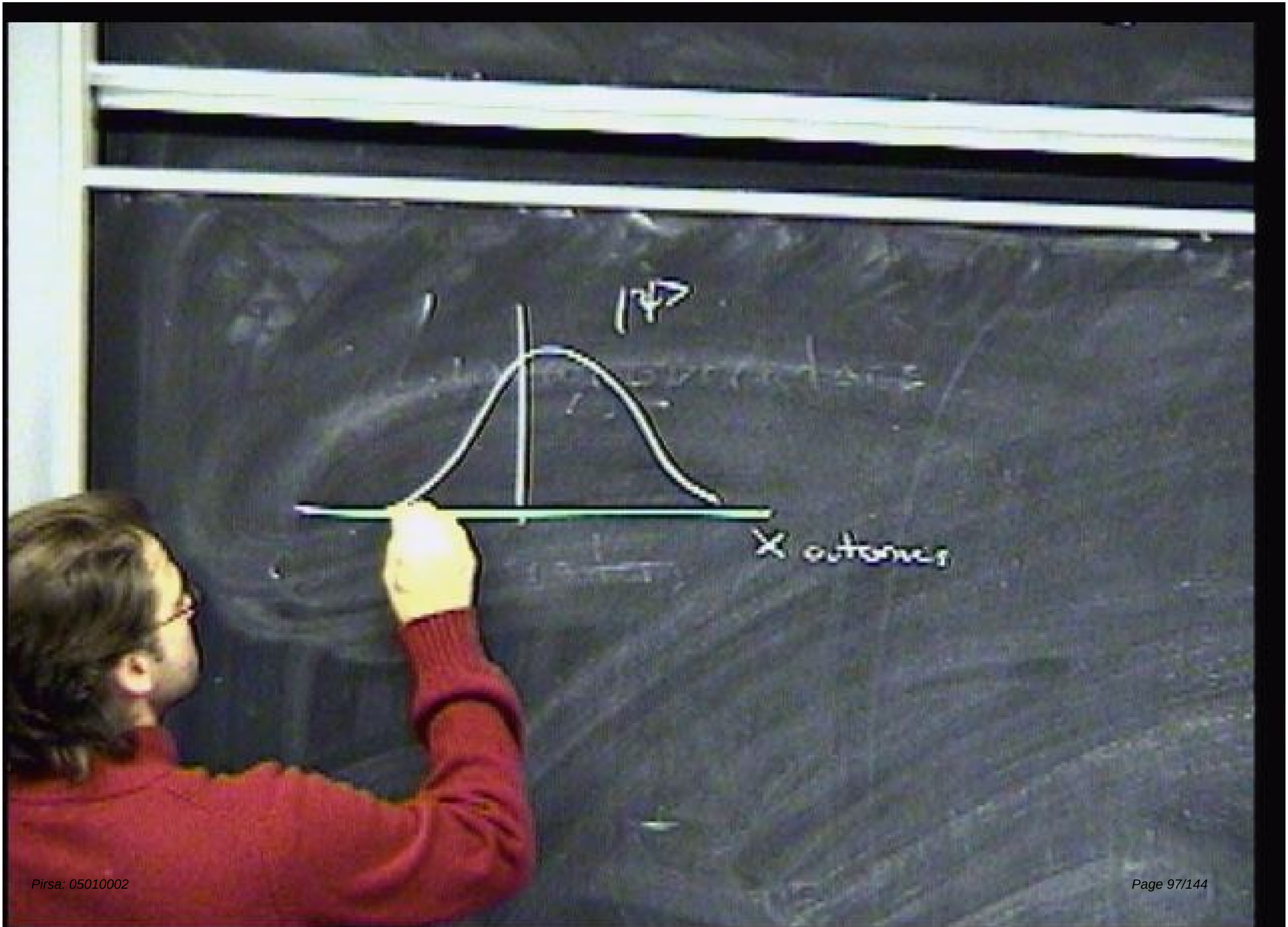
$\Delta p = \langle \hat{p}^2 \rangle - \langle \hat{p} \rangle^2$

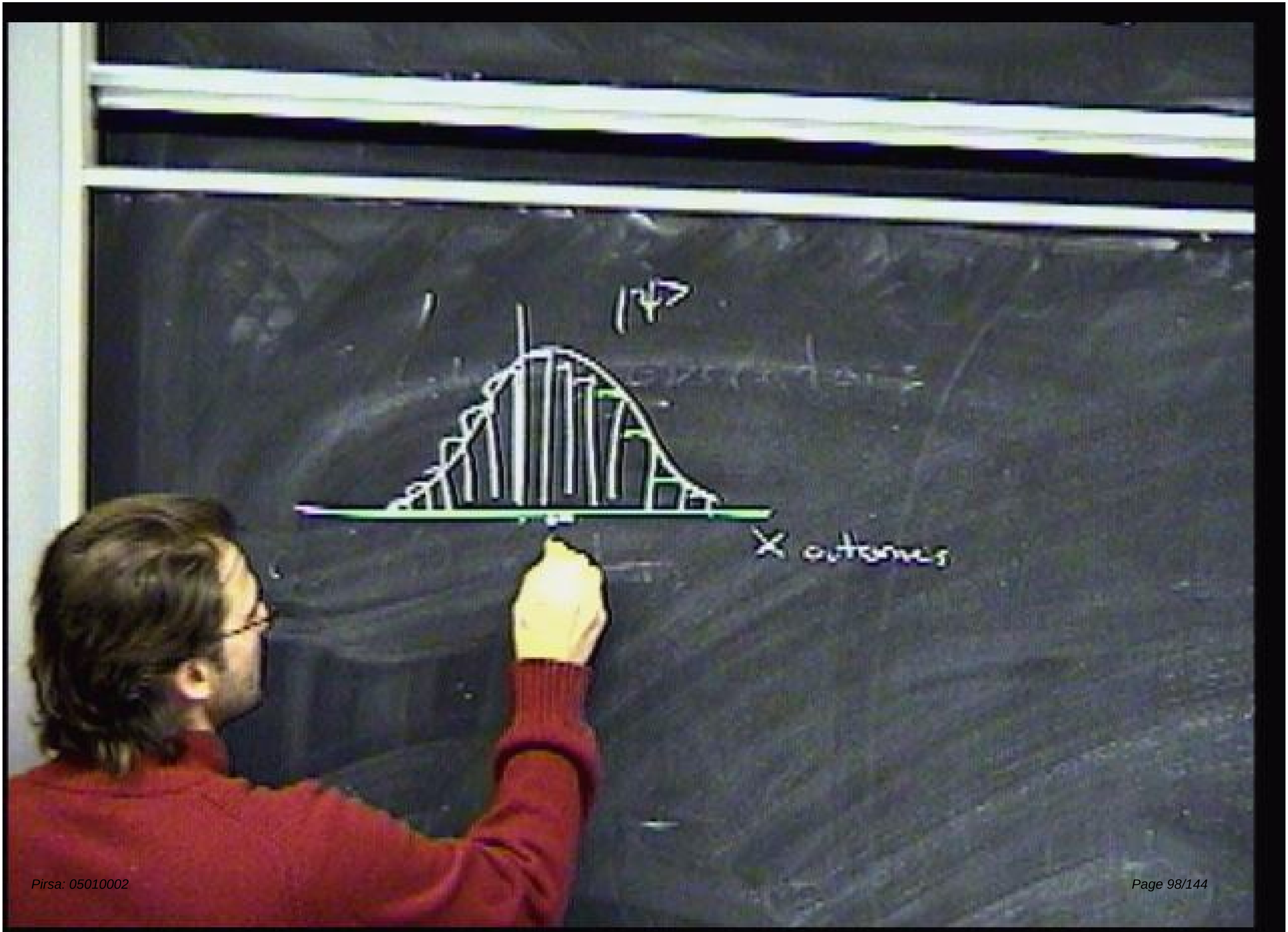
$\langle \hat{f} \rangle$ = average value over

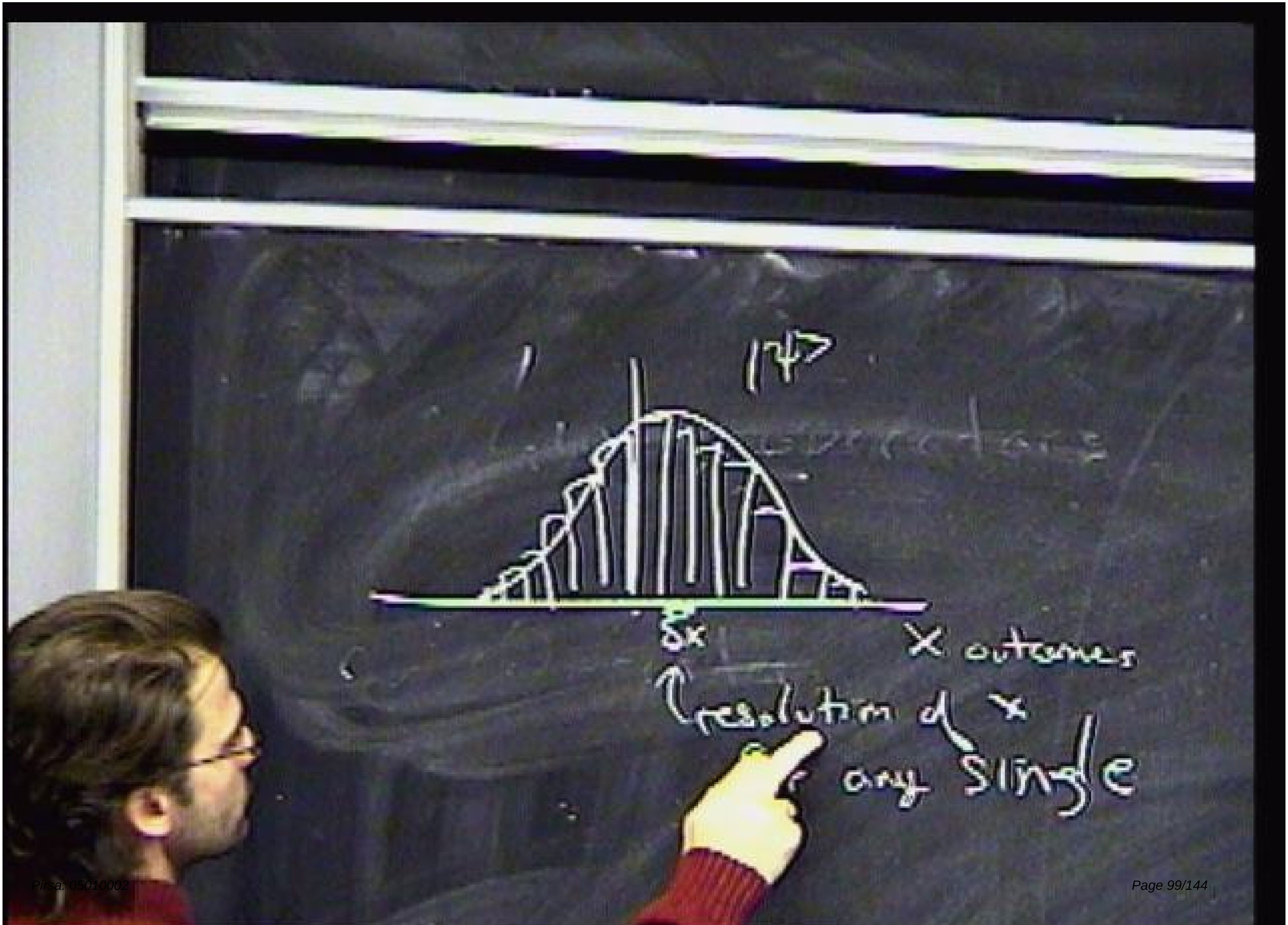
$= \langle \psi | \hat{f} | \psi \rangle$ an ensemble of mea.

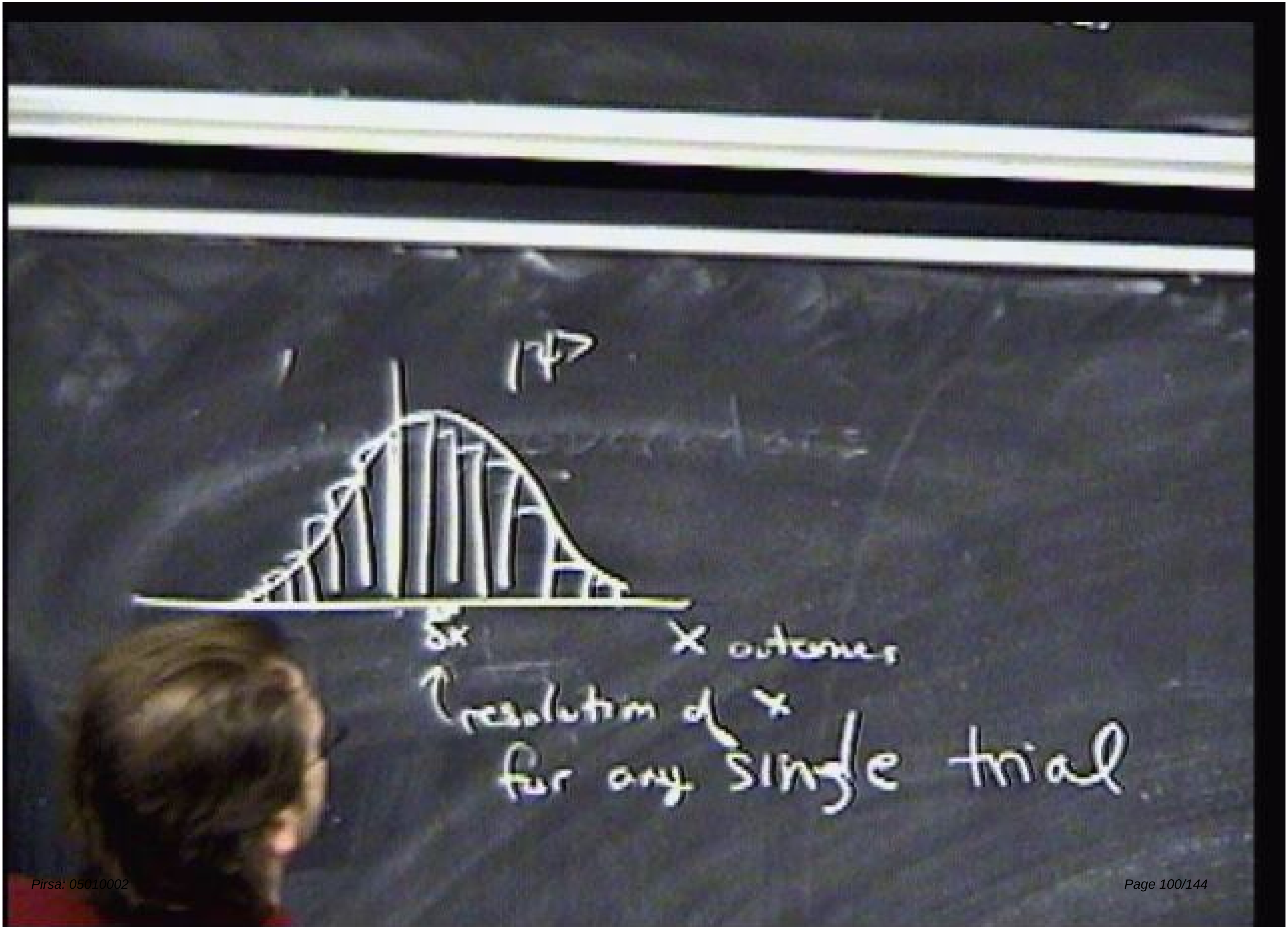
$\Delta x > 0 \Rightarrow$ a variance in

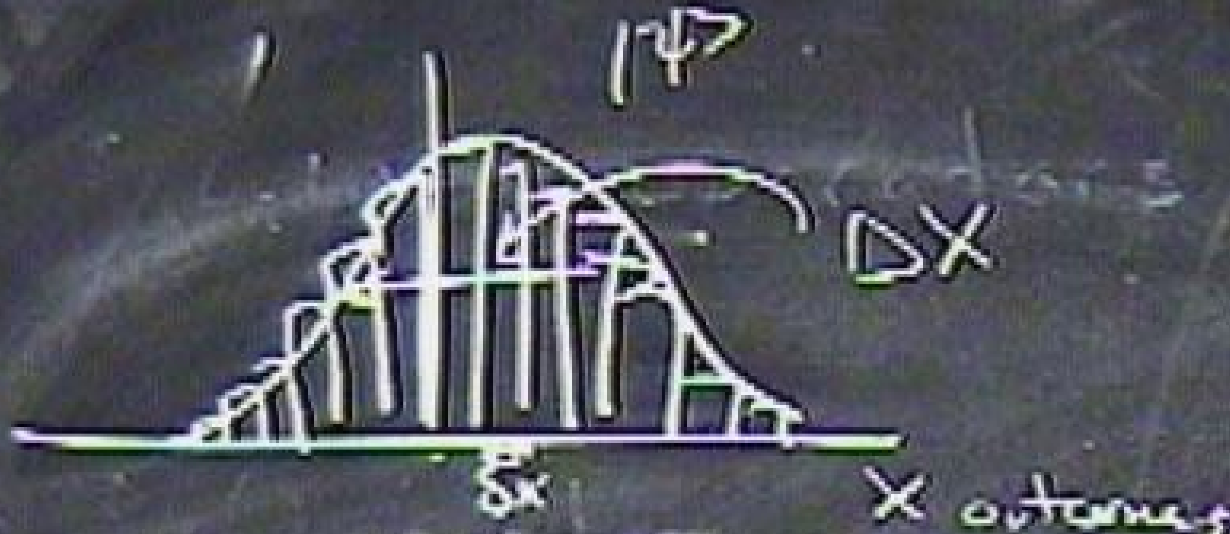
$\Delta p > 0$ set of outcomes


X outcomes


X outcomes


δx
x outcomes
resolution of x
any single


δx
x outcomes
resolution of x
for any single trial



resolution of x
for any single trial

δx $\qquad$ x outcomes

↑ resolution of x for any single trial

$\exists$ preparations, i.e. $\exists\ |\psi\rangle$ for which $\Delta X = 0$

outcomes

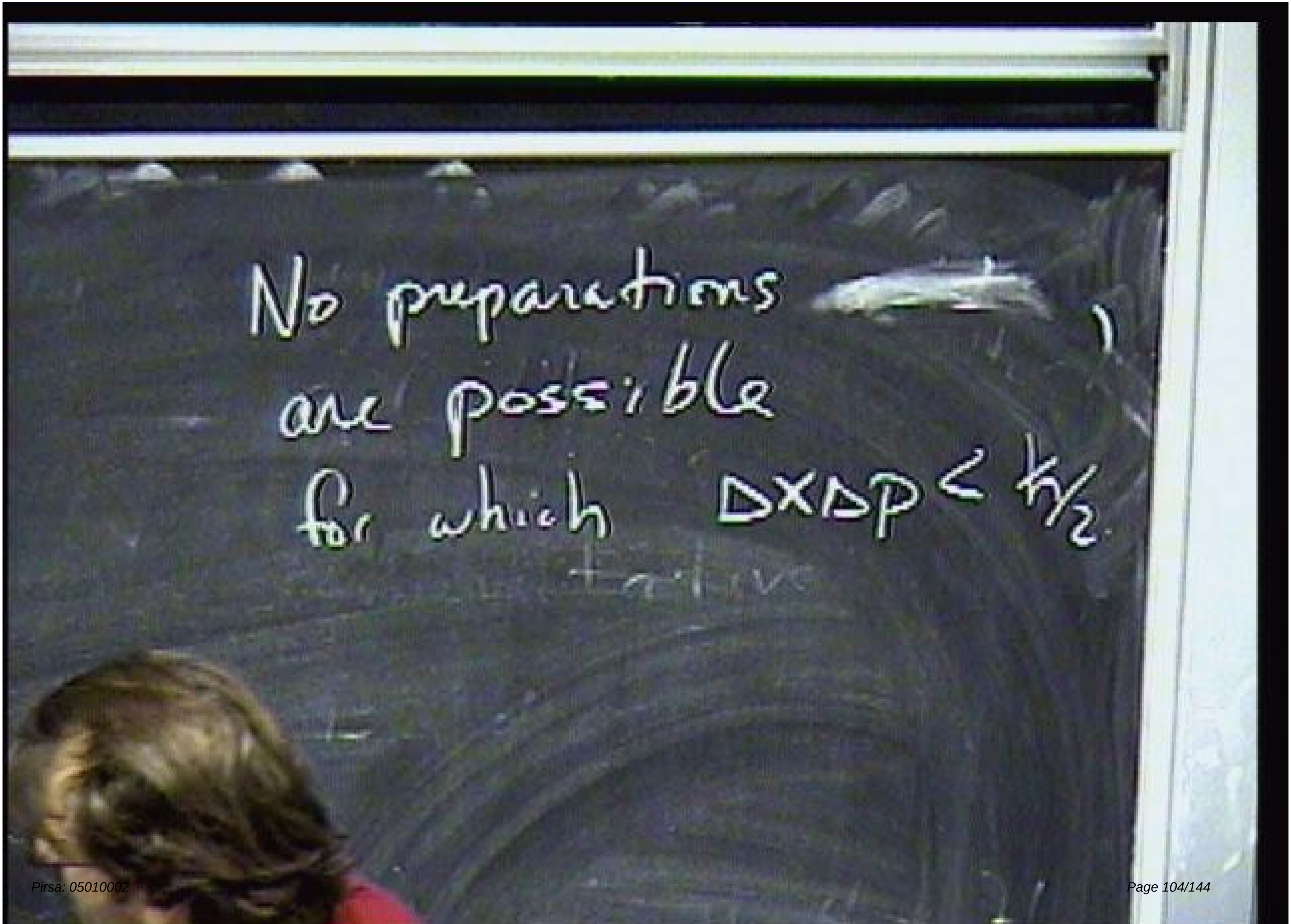
resolution of x

for any single trial

$\exists$ preparations, i.e. $\exists\ |\psi\rangle$

for which $\Delta x = 0$

$\exists$ preparations for which $\Delta p = 0$


No preparations
are possible
for which $\Delta x \Delta p < \hbar/2$.

No preparations
are possible
for which $\Delta x \Delta p <$

...mes

...gle trial

... $\exists\ |\psi\rangle$

... $\Delta x = \delta x$ where δx can in principle
be arbitrarily small

for which $\Delta p = 0$

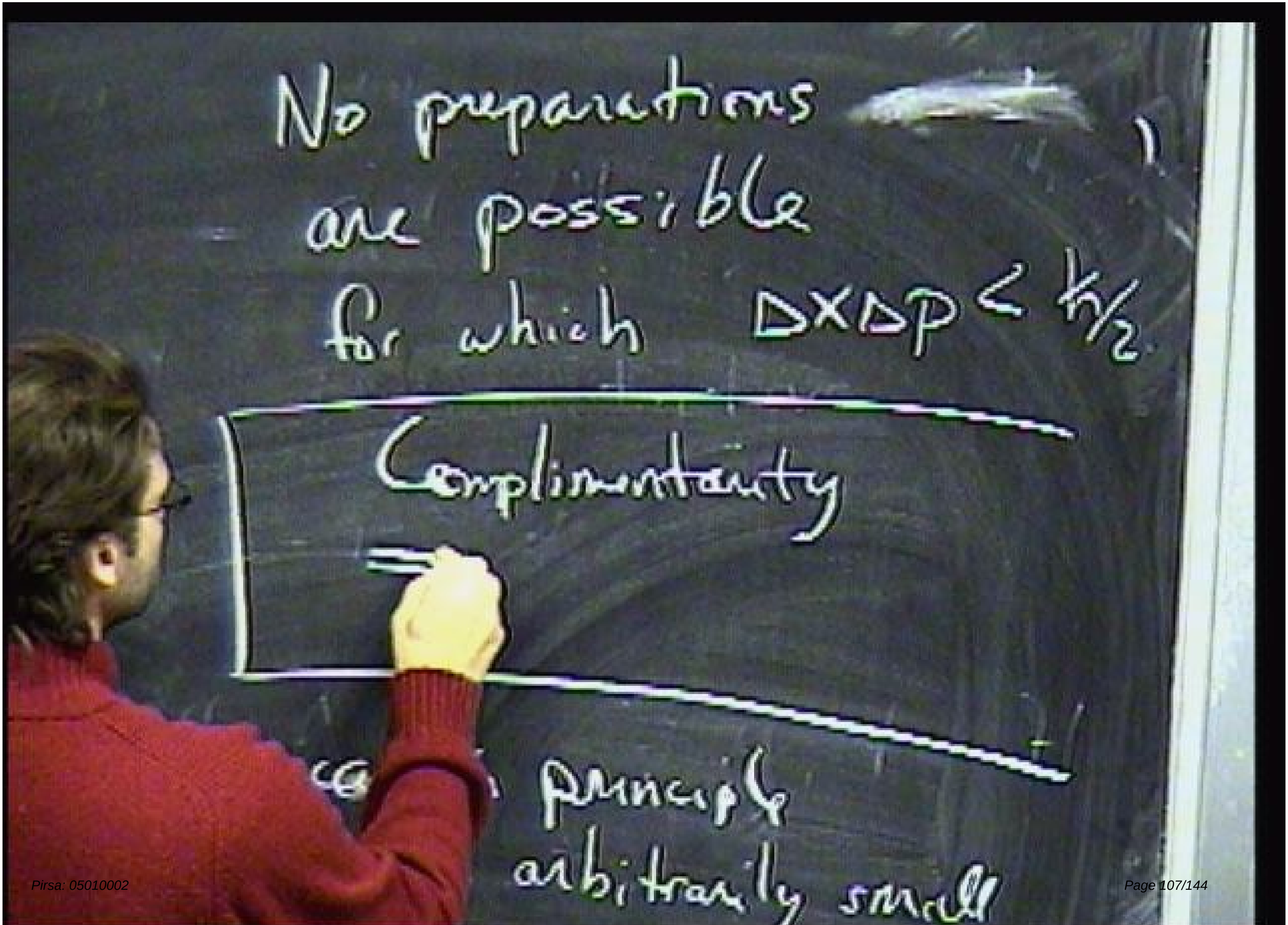
of outcomes

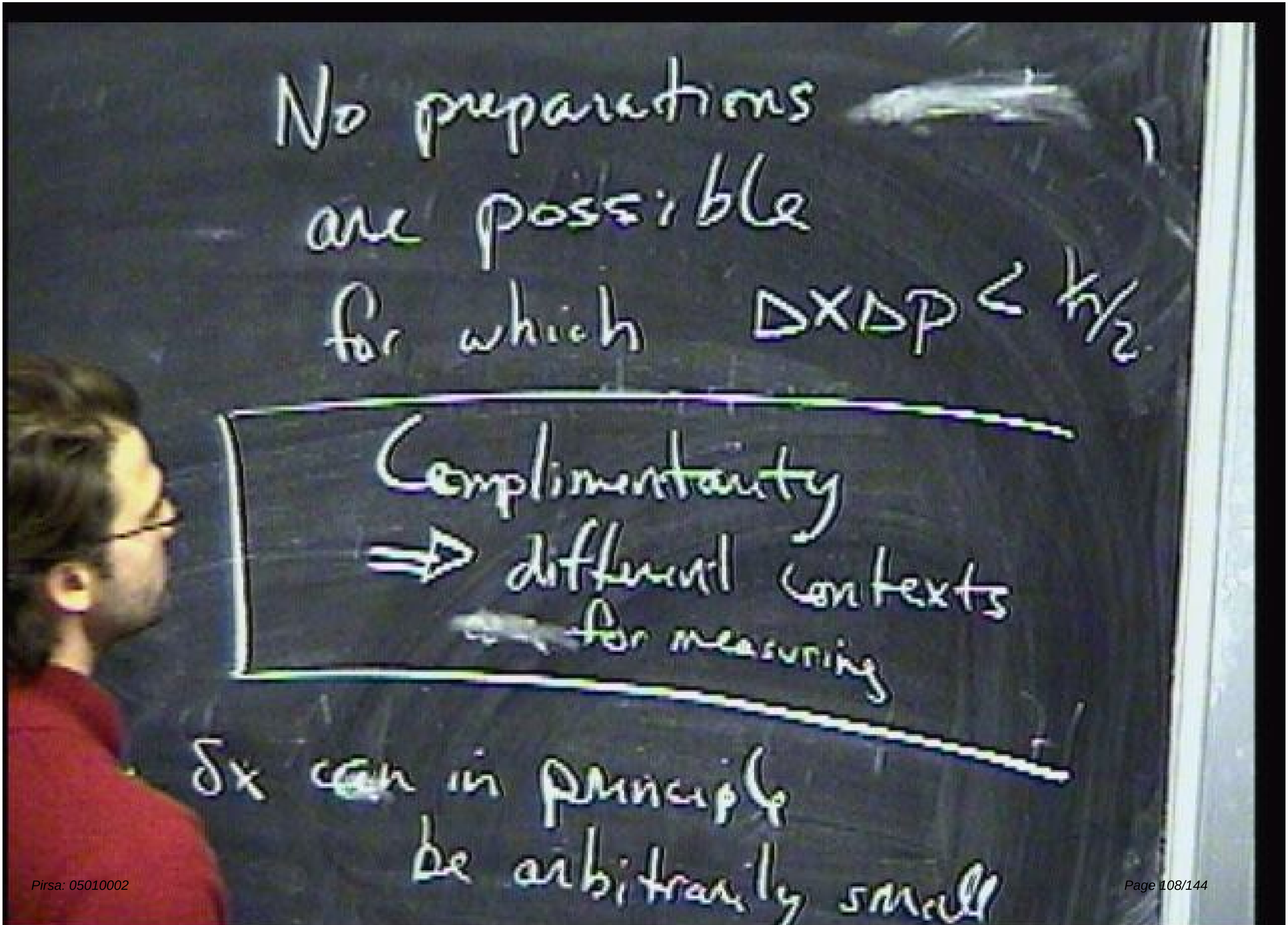
prediction of x for any single trial

No preparations are possible for which $\Delta x \Delta p < \hbar/2$

$\exists$ preparations, i.e. $\exists$ $|\psi\rangle$ for which $\Delta x = \delta x$ where δx can in principle be arbitrarily small

$\exists$ preparations for which $\Delta p = \delta p$


No preparations
are possible
for which ΔxΔp < ħ/2
Complimentarity
principle
arbitrarily small

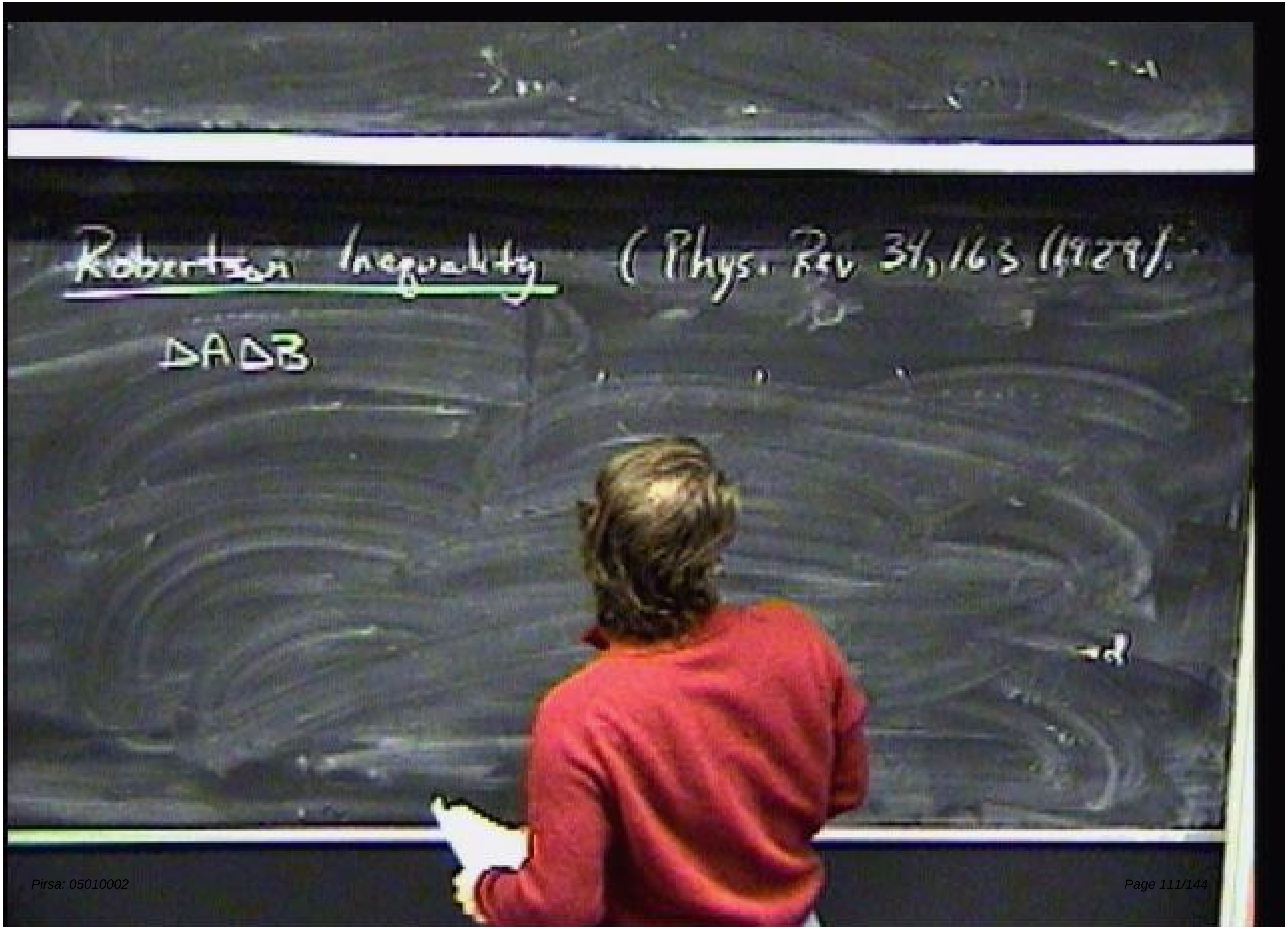

No preparations
are possible
for which $\Delta x \Delta p < \hbar/2$
Complimentarity
$\Rightarrow$ different contexts
for measuring
δx can in principle
be arbitrarily small

No preparations are possible for which $\Delta x \Delta p < \hbar/2$

Complimentarity
$\Rightarrow$ different contexts are required for measuring complimentary observables

where δx can in principle be arbitrarily small

Robertson Inequality


Robertson Inequality (Phys. Rev 34, 163 (1929).
ΔAΔB

Robertson Inequality (Phys. Rev 34, 163 (1929).)

$$\Delta A \Delta B \geq \frac{1}{2} |\langle [A,B] \rangle|$$

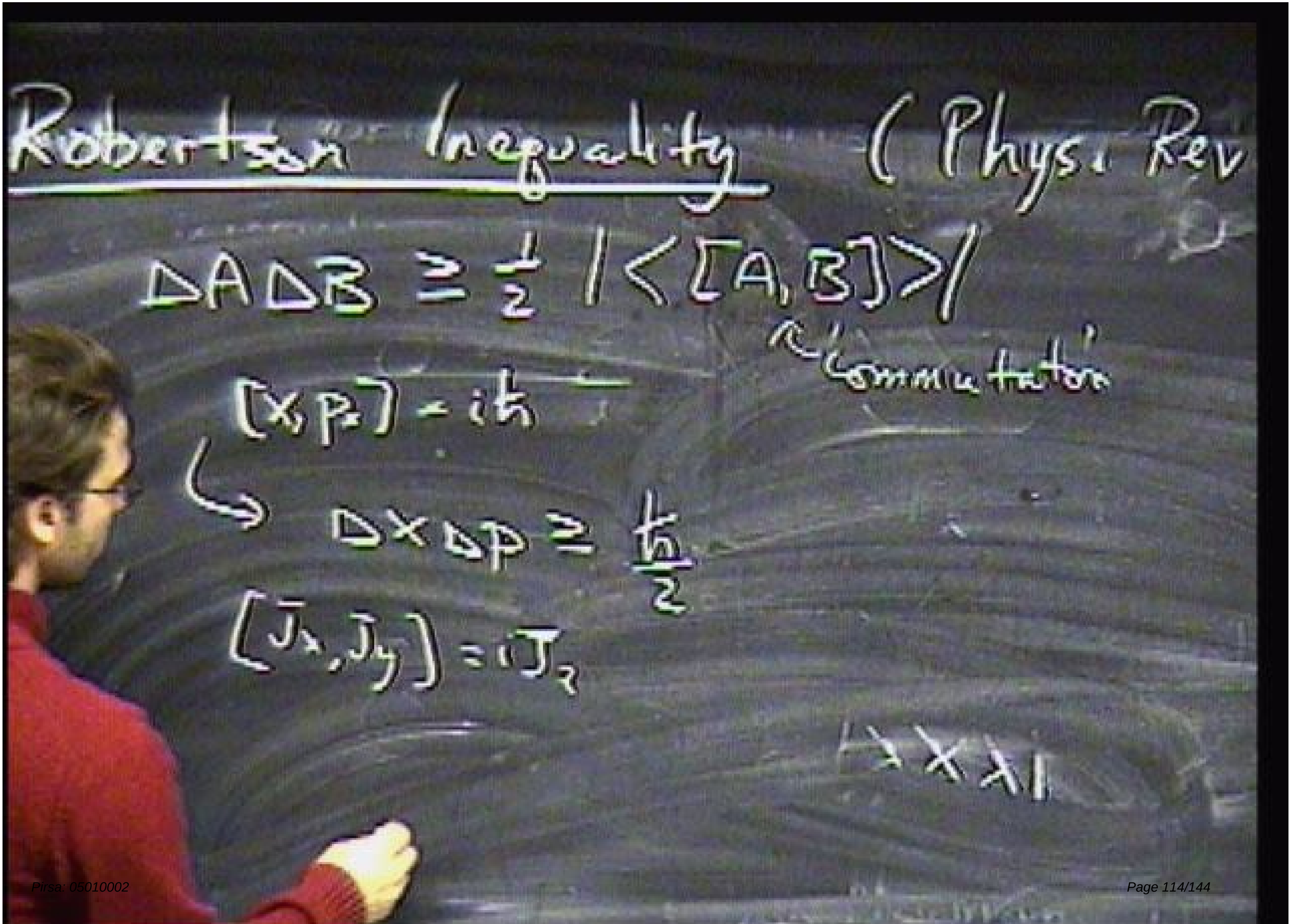
Robertson Inequality (Phys. Rev 34, 165 (1929).)

$$\Delta A \Delta B \geq \frac{1}{2} |\langle [A,B] \rangle|$$

commutator

$[x,p] = i\hbar$

$\hookrightarrow \Delta x \Delta p \geq \frac{\hbar}{2}$


Robertson Inequality (Phys. Rev
$\Delta A \Delta B \geq \frac{1}{2} |\langle [A,B] \rangle|$
commutator
$[x,p] = i\hbar$
$\Delta x \Delta p \geq \frac{\hbar}{2}$
$[J_x, J_y] = iJ_z$

Robertson Inequality (Phys. Rev

$$\Delta A \Delta B \geq \frac{1}{2} |\langle [A, B] \rangle|$$

'commutator'

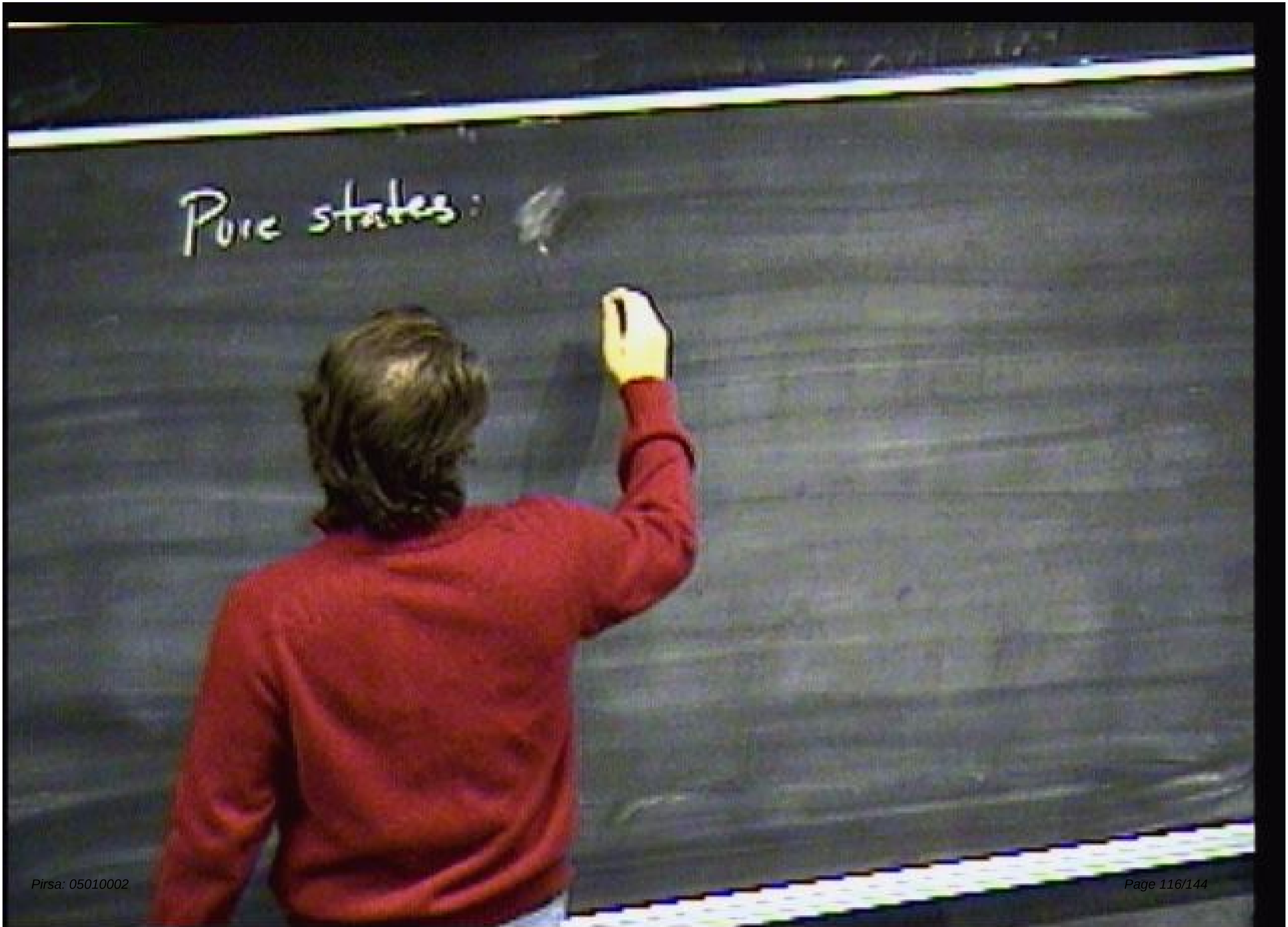
$[x, p_x] = i\hbar$

$\hookrightarrow \quad \Delta x \Delta p \geq \frac{\hbar}{2}$

$[J_x, J_y] = i J_z \hbar$

$\{J_z, \varphi\}$

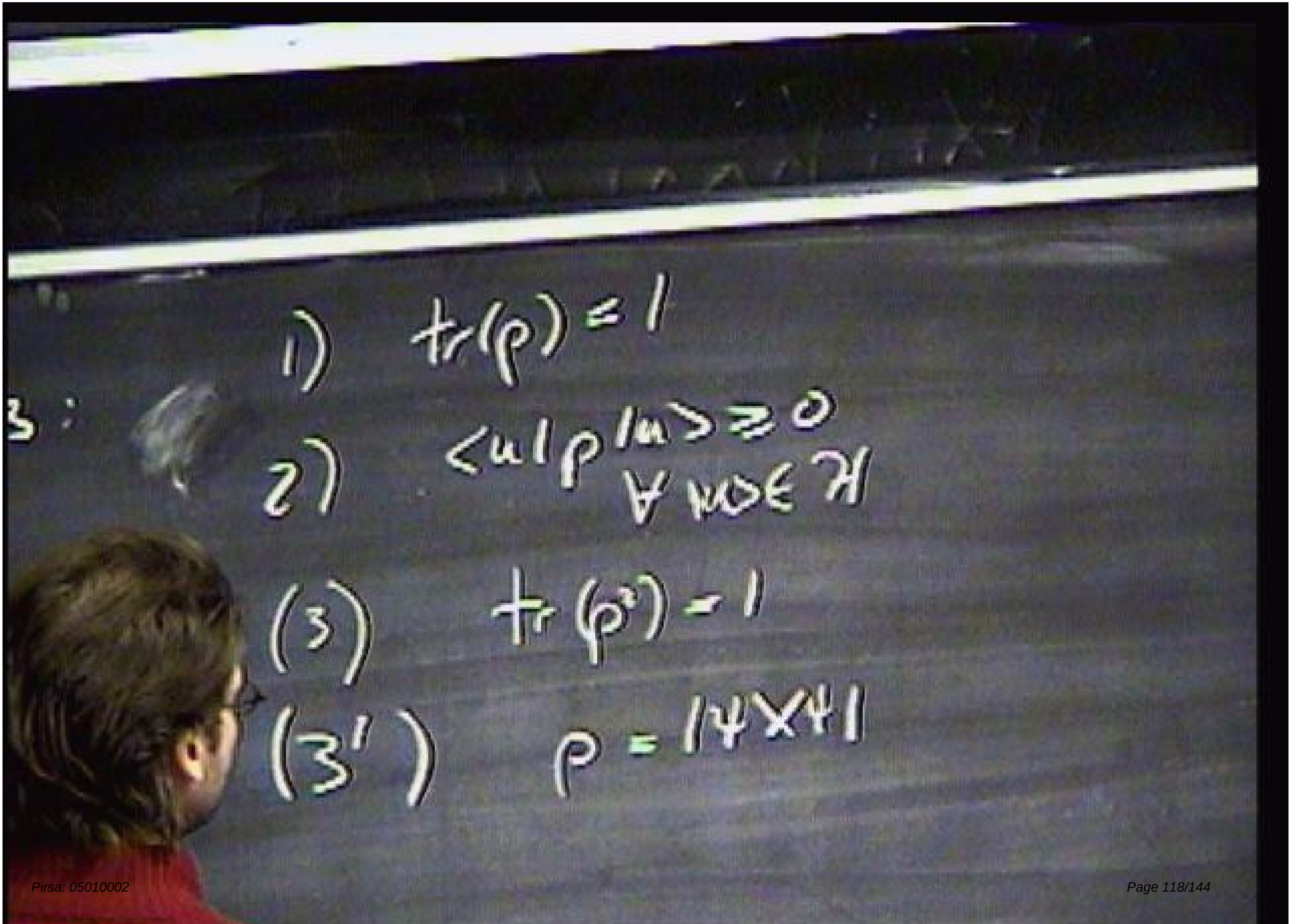
$\rightarrow \times \lambda 1$


Pure states:

3 :

1) $tr(\rho) = 1$

2) $\langle u|\rho|u\rangle \geq 0$
$\forall |u\rangle \in \mathcal{H}$


1) tr(ρ) = 1
2) ⟨u|ρ|u⟩ ≥ 0 ∀ |u⟩ ∈ H
(3) tr(ρ²) = 1
(3') ρ = |ψ⟩⟨ψ|

3 :

1) $\mathrm{tr}(\rho) = 1$

2) $\langle u|\rho|u\rangle \geq 0$ $\quad \forall\, |u\rangle \in \mathcal{H}$

(3) $\mathrm{tr}(\rho^2) = 1$

(3') $\rho = |\psi\rangle\langle\psi|$

(3'') $\rho^2 = \rho$

3 :

1) $\mathrm{tr}(\rho) = 1$

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ites:

1) $tr(\rho) = 1$

2) $\langle u|\rho|u\rangle \geq 0$ $\forall |u\rangle \in \mathcal{H}$

drop

(3) $tr(\rho^2) = 1$

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(3'') $\rho^2 = \rho$

ites :

$$\begin{cases} 1) & tr(\rho) = 1 \\ 2) & \langle u|\rho|u\rangle \geq 0 \quad \forall |u\rangle \in \mathcal{H} \end{cases}$$

drop

$$\left[\begin{array}{ll} (3) & tr(\rho^2) = 1 \\ (3') & \rho = |\psi\rangle\langle\psi| \\ (3'') & \rho^2 = \rho \end{array} \right.$$

Pure states: $\begin{cases} 1) & tr(\rho) = 1 \\ 2) & \langle u|\rho|u\rangle \geq 0 \\ & \forall u \in \mathcal{H} \end{cases}$

drop

$(3)\quad tr(\rho^2) = 1$

$(3')\quad \rho = |\psi\rangle\langle\psi|$

$(3'')\quad \rho^2 = \rho$

$|\psi_1\rangle\langle\psi_1|$

Pure states:

1) $\mathrm{tr}(\rho) = 1$

2) $\langle u|\rho|u\rangle \geq 0 \quad \forall u \in \mathcal{H}$

drop

(3) $\mathrm{tr}(\rho^2) = 1$

(3') $\rho = |\psi\rangle\langle\psi|$

(3'') $\rho^2 = \rho$

$\rho \cdot |\psi_1\rangle\langle\psi_1|$

Pure states:
$$\begin{cases} 1) & tr(\rho) = 1 \\ 2) & \langle u|\rho|u\rangle \geq 0 \\ & \forall u \in \mathcal{H} \end{cases}$$

drop
$$\left[\begin{array}{ll} (3) & tr(\rho^2) = 1 \\ (3') & \rho = |\psi\rangle\langle\psi| \\ (3'') & \rho^2 = \rho \end{array}\right.$$

$$\rho = p_1 |\psi_1\rangle\langle\psi_1| + p_2 |\psi_2\rangle\langle\psi_2|$$

$$p_2 = 1 - p_1$$

Pure states: $\left\{ \begin{array}{l} 1) \quad tr(\rho) = 1 \\ 2) \quad \langle u | \rho | u \rangle \geq 0 \\ \qquad \forall u \in \mathcal{H} \end{array} \right.$

drop $\left[\begin{array}{ll} (3) & tr(\rho^2) = 1 \\ (3') & \rho = |\psi\rangle\langle\psi| \\ (3'') & \rho^2 = \rho \end{array} \right.$

$\rho = p_1 |\psi_1\rangle\langle\psi_1| + p_2 |\psi_2\rangle\langle\psi_2|$

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Pure states: $\begin{cases} 1) & tr(\rho) = 1 \\ 2) & \langle u|\rho|u\rangle \geq 0 \quad \forall |u\rangle \in \mathcal{H} \end{cases}$

drop $\left[\begin{array}{ll} (3) & tr(\rho^2) = 1 \\ (3') & \rho = |\psi\rangle\langle\psi| \\ (3'') & \rho^2 = \rho \end{array}\right.$

$$\rho = p_1 |\psi_1\rangle\langle\psi_1| + p_2 |\psi_2\rangle\langle\psi_2|$$

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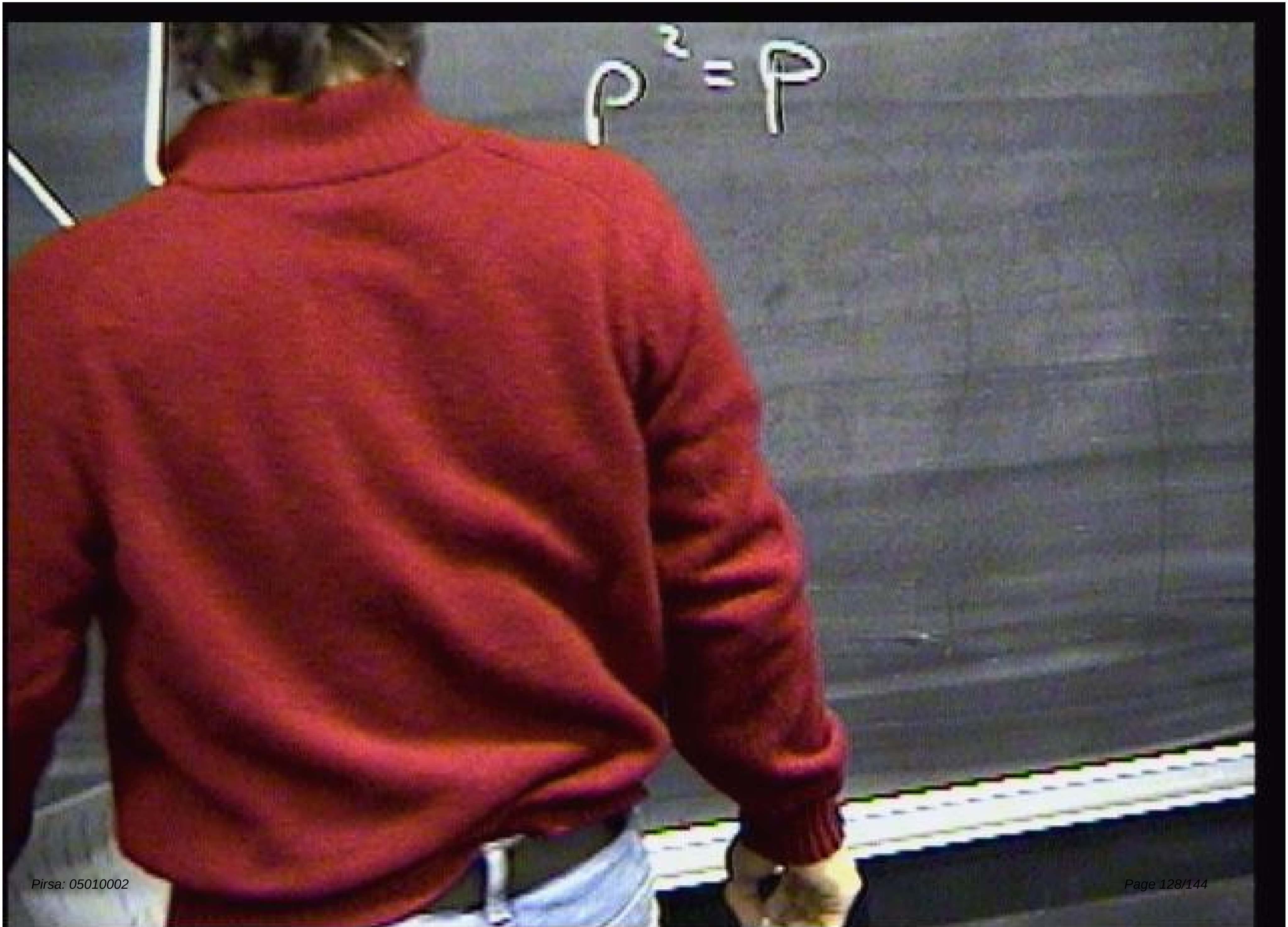
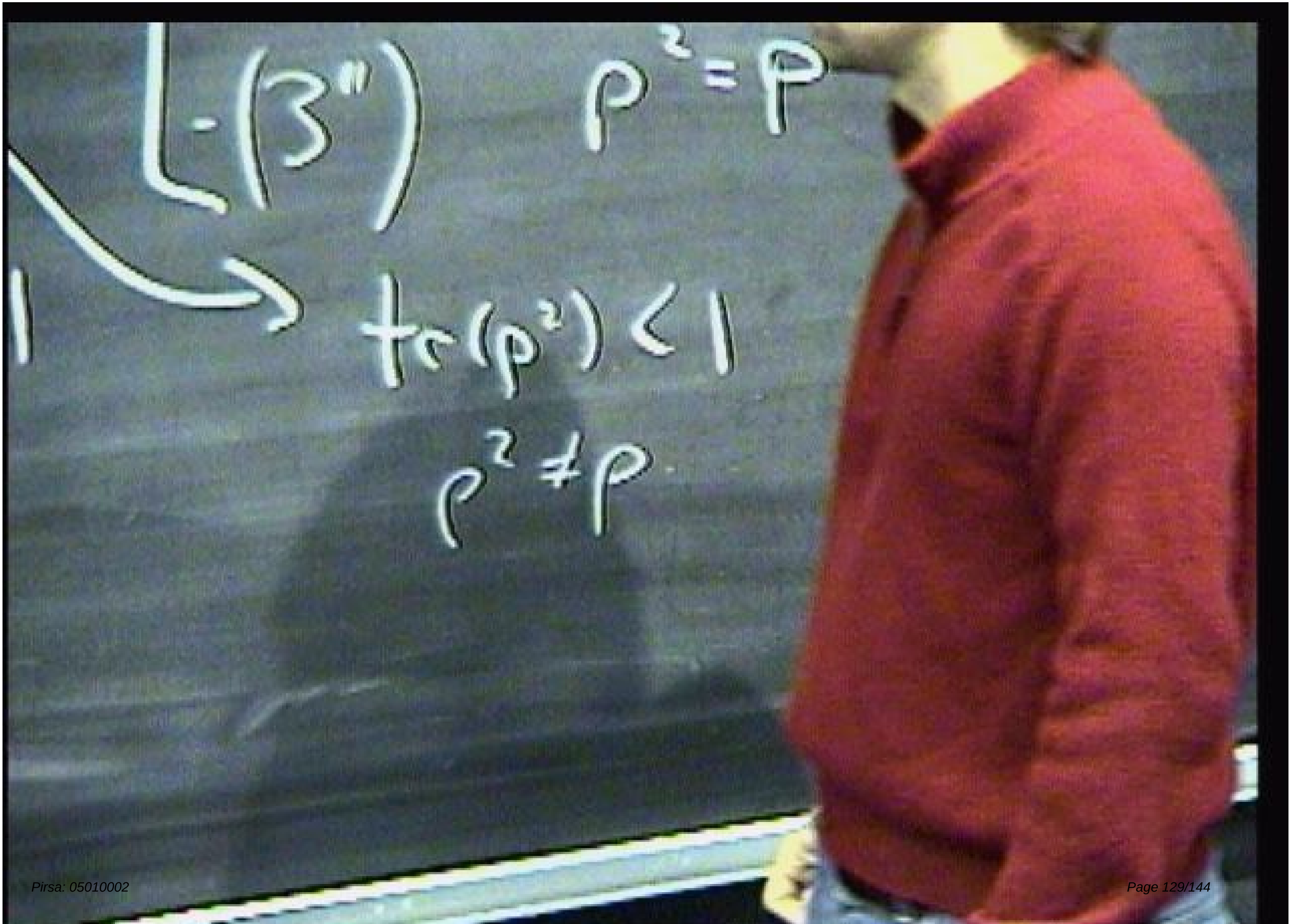

$\rho^2 = \rho$

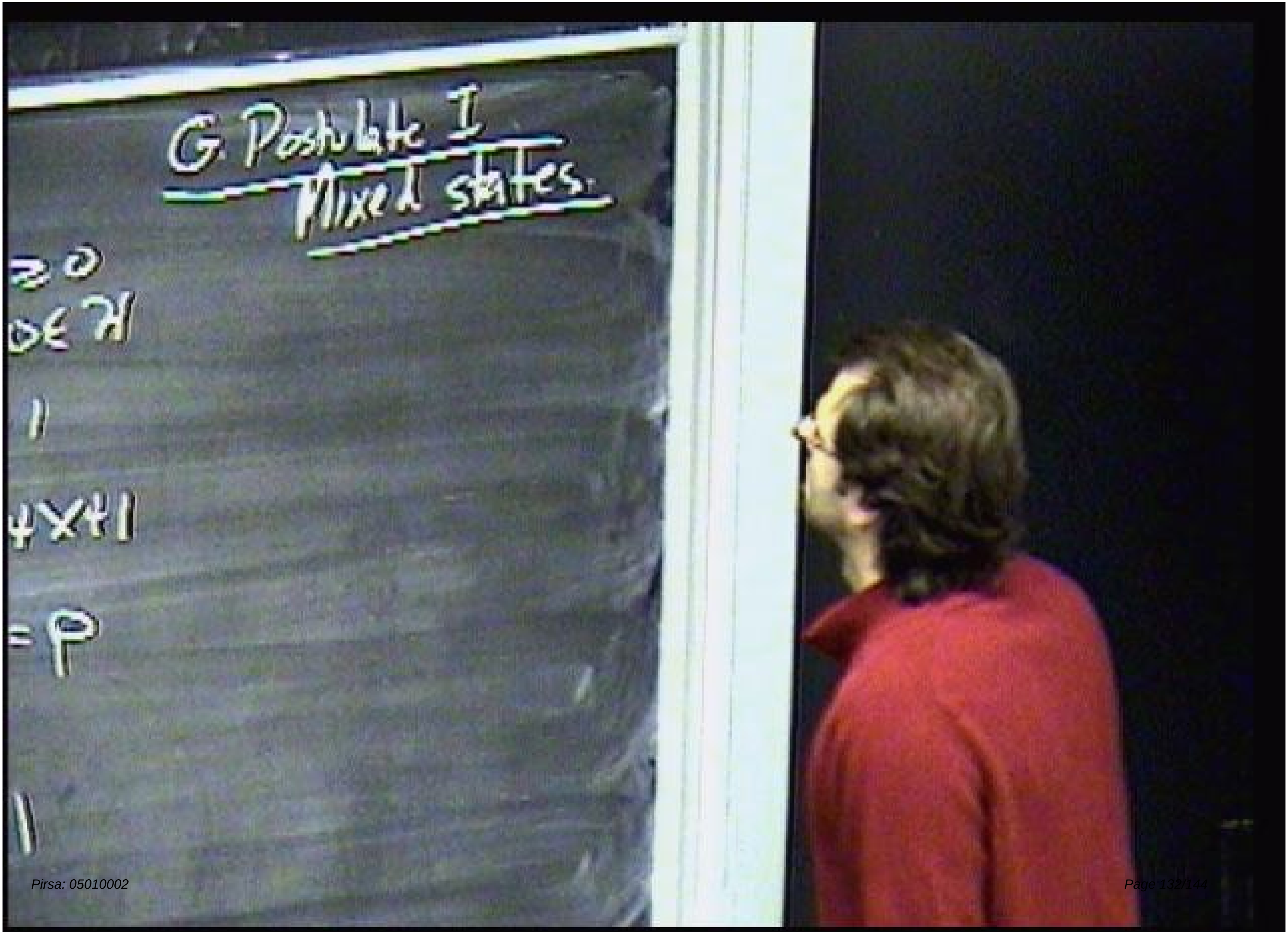

(3'')
$\rho^2 = \rho$
$tr(\rho^2) < 1$
$\rho^2 \neq \rho$

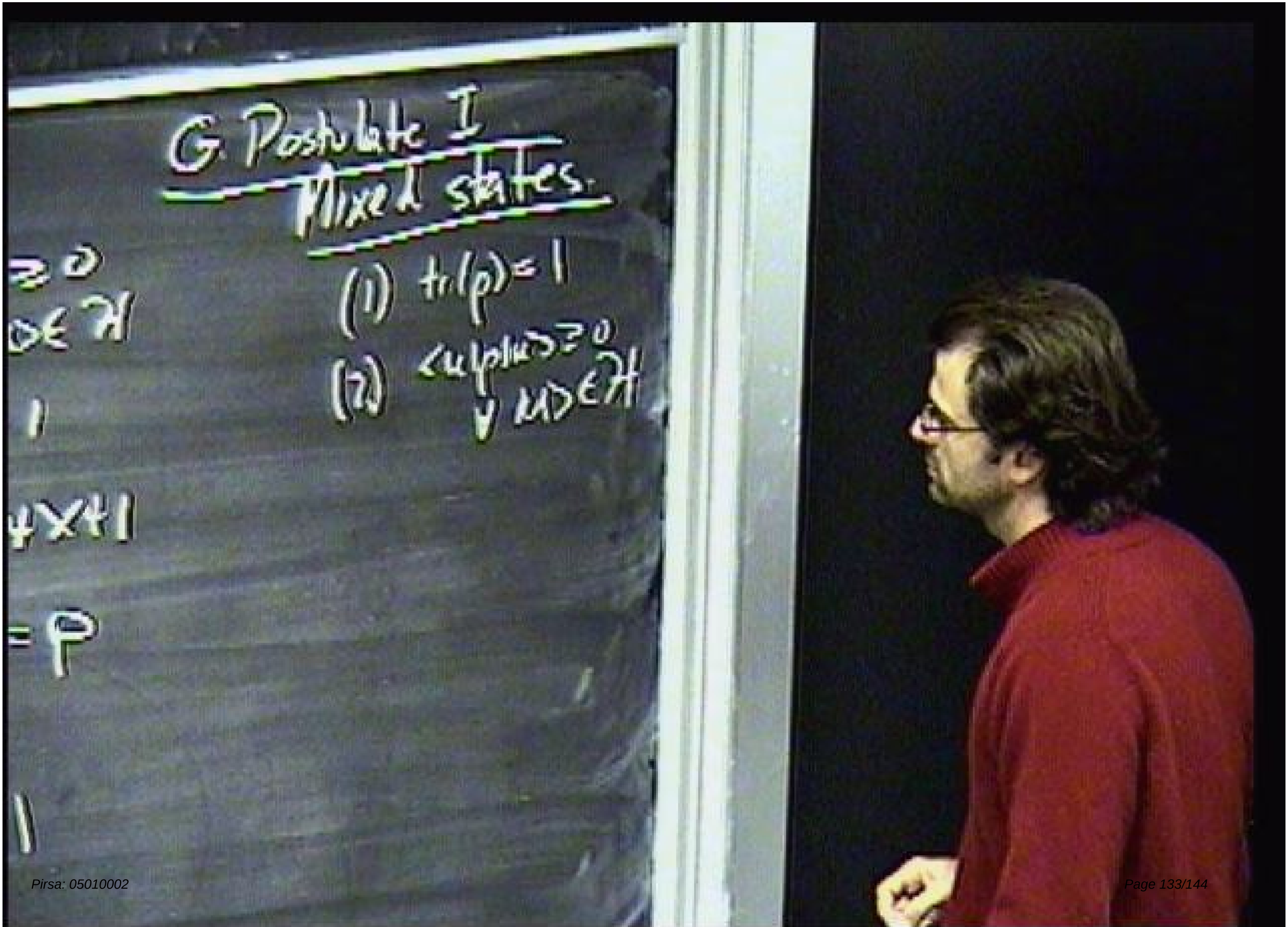
$\lfloor \cdot (3'')$ $\rho^2 = \rho$

$\mathrm{tr}(\rho^2) < 1$

$\rho^2 \neq \rho$


Mixed states.


G Postulate I
Mixed states.


G Postulate I
Mixed states.
(1) tr(ρ)=1
(2) ⟨u|ρ|u⟩≥0
∀ |u⟩∈ℋ

≥ 0

$\rangle \in \mathcal{H}$

1

$\psi\rangle\langle\psi|$

$= \rho$

1

G. Postulate I

Mixed states

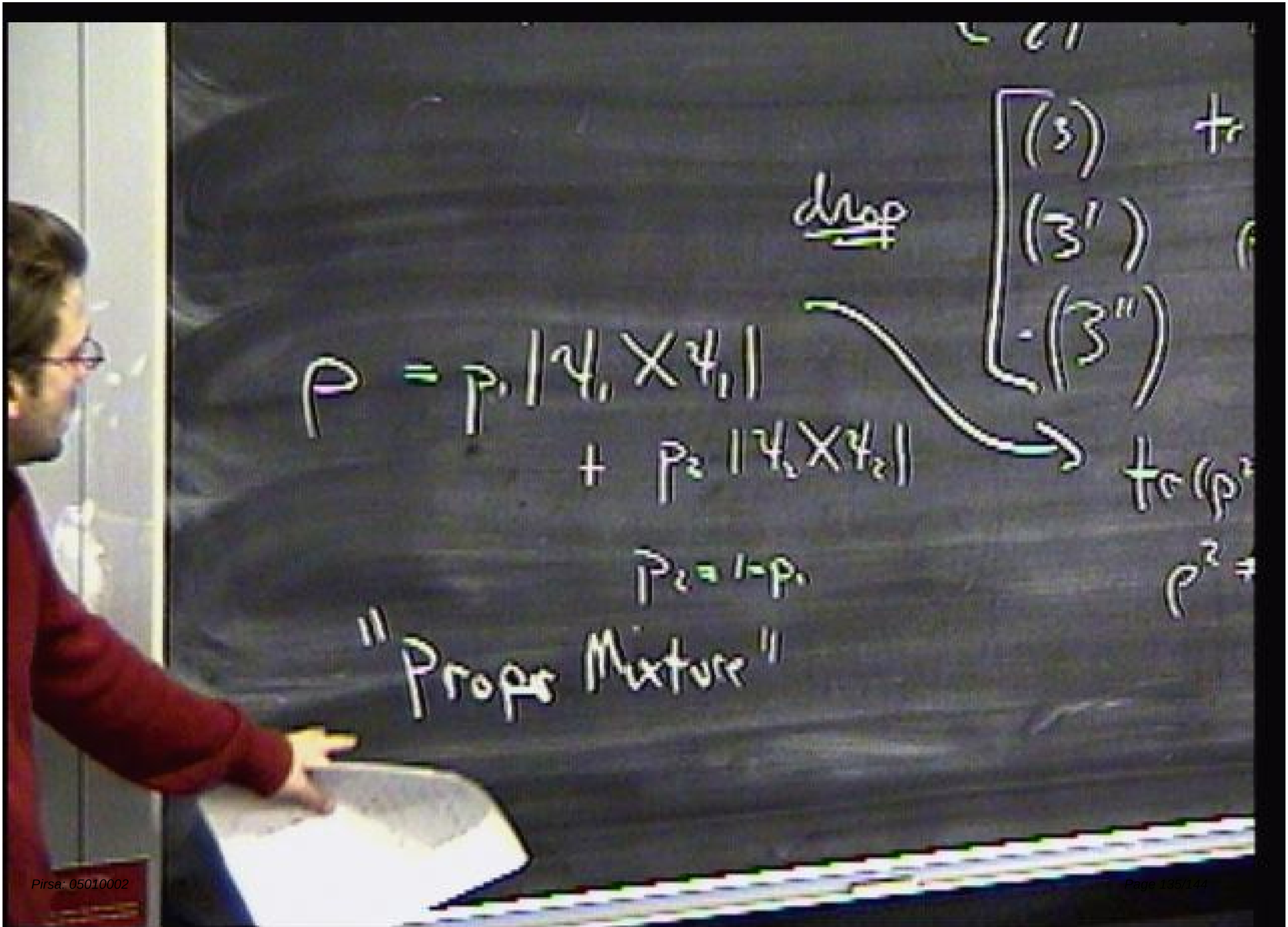
(1) $tr(\rho) = 1$

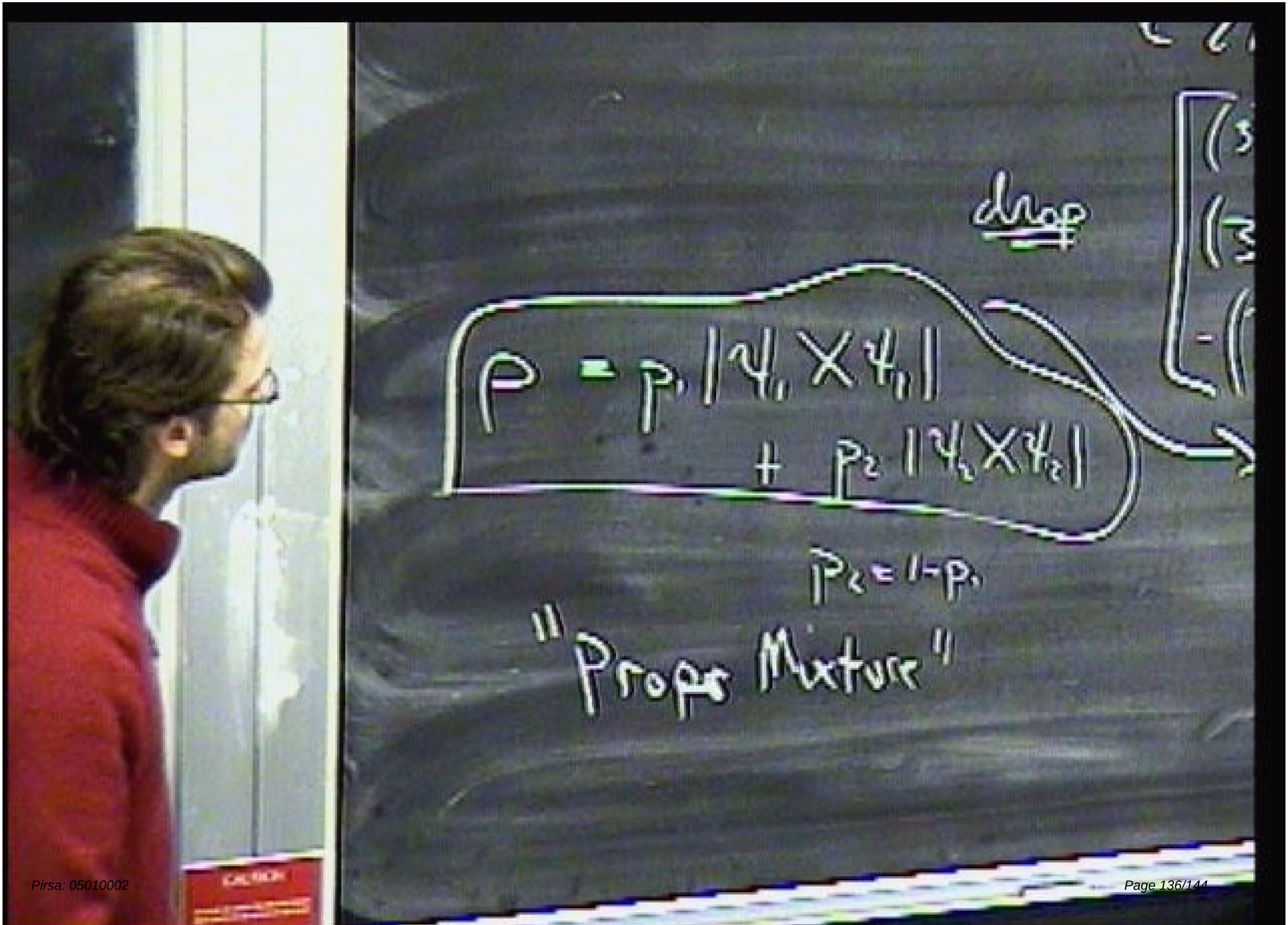
(2) $\langle u|\rho|u\rangle \geq 0$
$\forall\, |u\rangle \in \mathcal{H}$

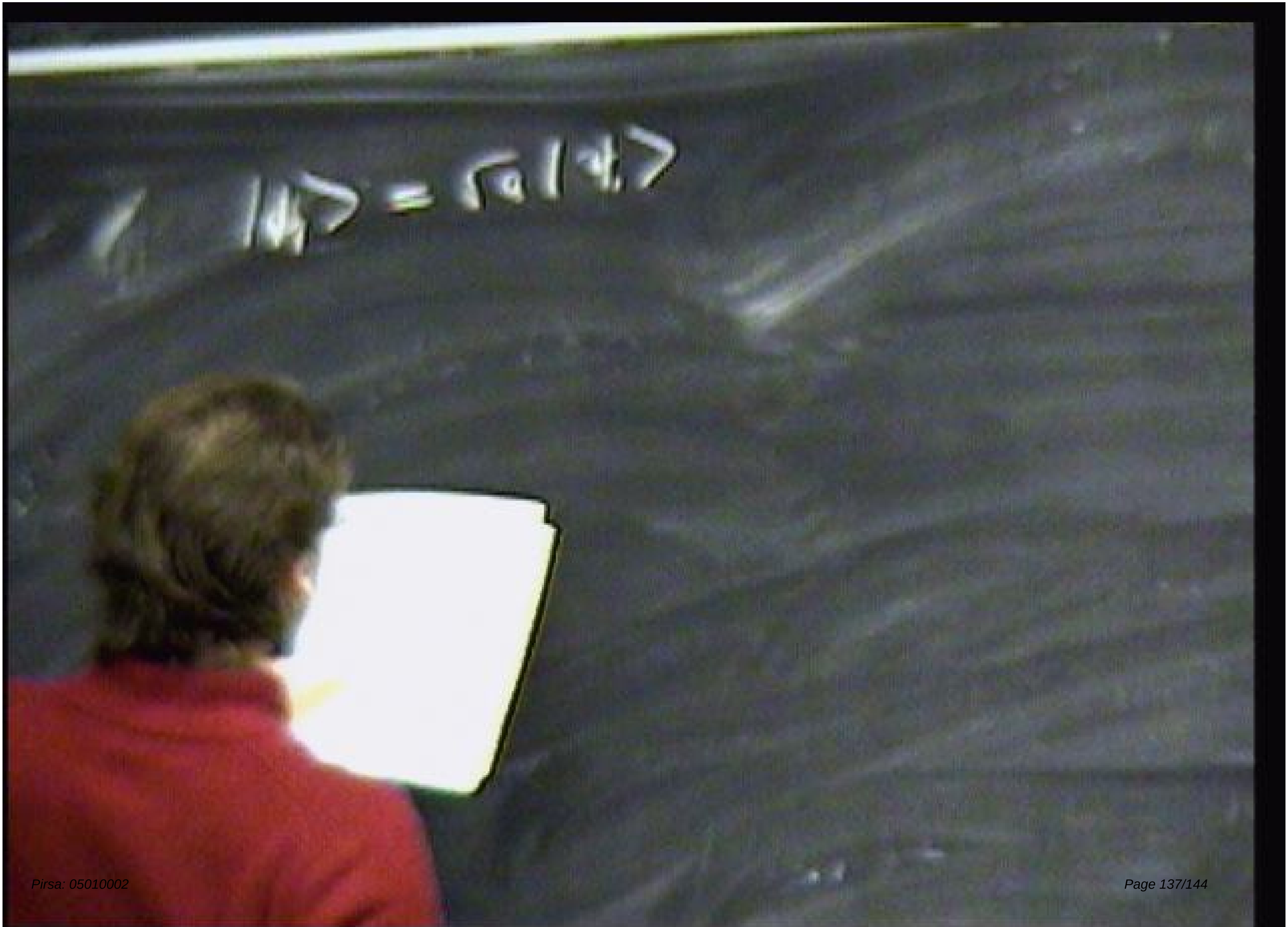
$\Rightarrow tr(\rho^2) < 1$

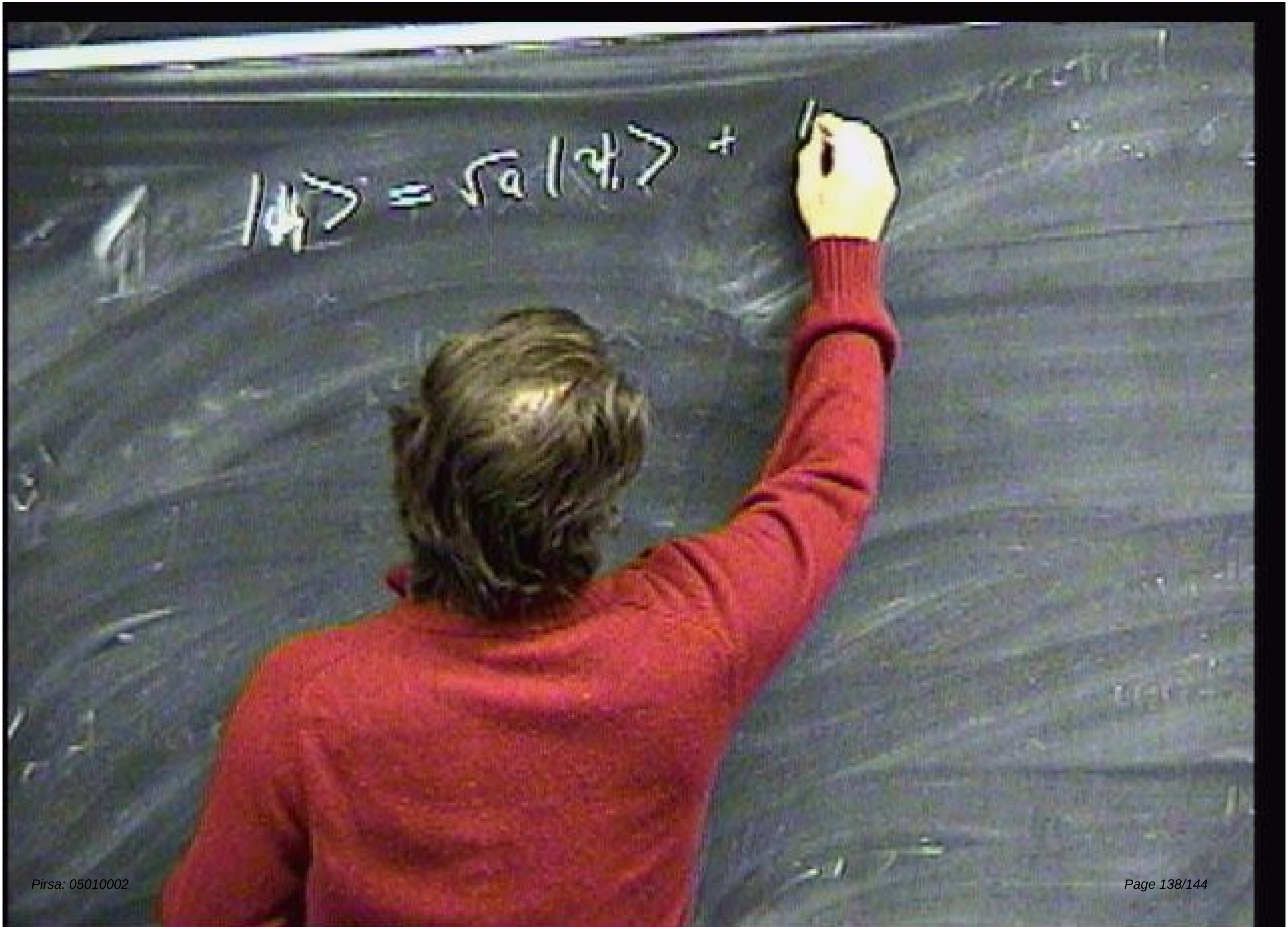
In finite $\dim(\mathcal{H}) = D$

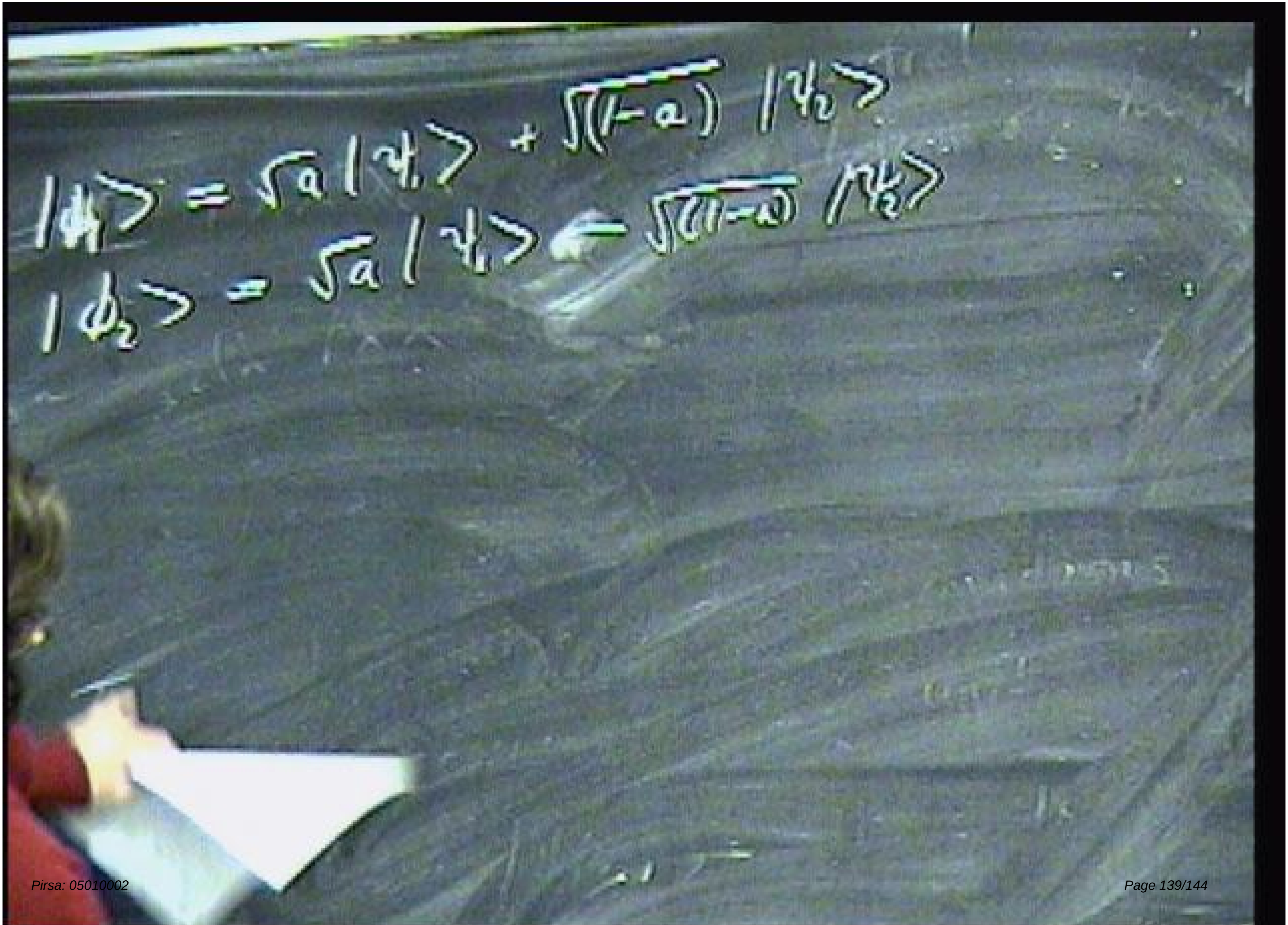
$\frac{1}{D} < tr(\rho^2) < 1$


drop
(3)
(3')
(3'')
$\rho = p_1 |\psi_1\rangle\langle\psi_1| + p_2 |\psi_2\rangle\langle\psi_2|$
$p_2 = 1 - p_1$
"Proper Mixture"


drop
$\rho = p_1 |\psi_1\rangle\langle\psi_1| + p_2 |\psi_2\rangle\langle\psi_2|$
$p_2 = 1 - p_1$
"Proper Mixture"




$|\psi\rangle = \sqrt{a}\,|\psi_1\rangle +$


$|\phi_1\rangle = \sqrt{a}\,|\psi_1\rangle + \sqrt{(1-a)}\,|\psi_2\rangle$
$|\phi_2\rangle = \sqrt{a}\,|\psi_1\rangle - \sqrt{(1-a)}\,|\psi_2\rangle$

$$|\phi_1\rangle = \sqrt{a}\,|\psi_1\rangle + \sqrt{(1-a)}\,|\psi_2\rangle$$

$$|\phi_2\rangle = \sqrt{a}\,|\psi_1\rangle - \sqrt{(1-a)}\,|\psi_2\rangle$$

$$\rho = \frac{1}{2}\,|\phi_1\rangle\langle\phi_1| + \frac{1}{2}\,|\phi_2\rangle\langle\phi_2|$$

$$|\phi_1\rangle = \sqrt{a}\,|\psi_1\rangle + \sqrt{(1-a)}\,|\psi_2\rangle$$

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$$\rho = \tfrac{1}{2}\,|\phi_1\rangle\langle\phi_1| + \tfrac{1}{2}\,|\phi_2\rangle\langle\phi_2|$$

Ambiguity of mixtures

$\sqrt{a}\,|\psi_1\rangle + \sqrt{(1-a)}\,|\psi_2\rangle$

$\sqrt{a}\,|\psi_1\rangle - \sqrt{(1-a)}\,|\psi_2\rangle$

$|\phi_1\rangle\langle\phi_1| + \frac{1}{2}|\phi_2\rangle\langle\phi_2|$

$\rightarrow$ infinite set of different decompositions

ity of mixtures

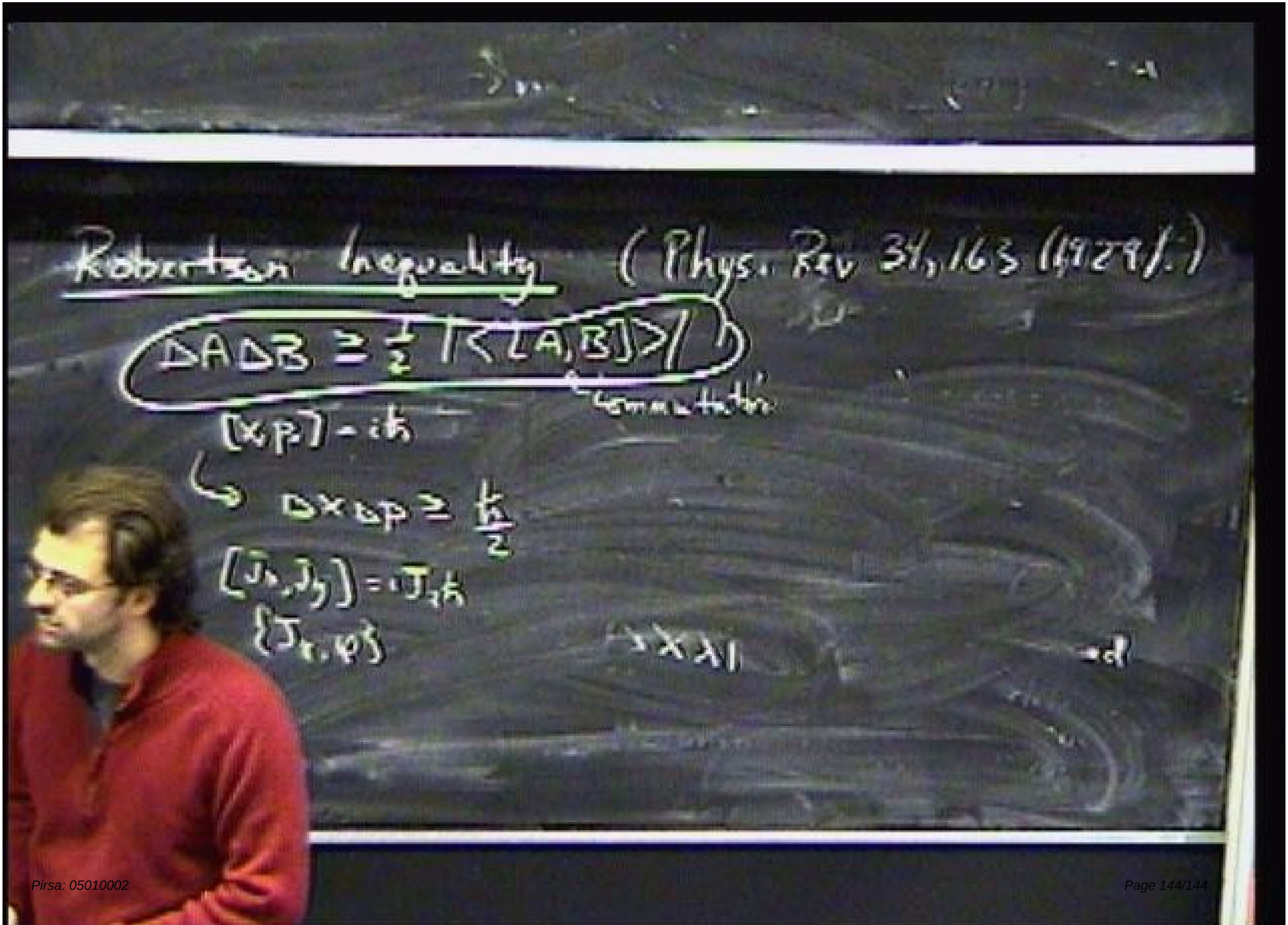
$|\psi_2\rangle$ } $\langle\phi_1|\phi_2\rangle=0$

$\langle\psi_1|\psi_2\rangle=0$

$\overline{-\omega}\ |\psi_2\rangle$ $\langle\phi_i|\psi_i\rangle\neq 0$

$\phi_2\rangle\langle\phi_2|$

t of different decompositions


Robertson Inequality (Phys. Rev 34, 163 (1929).)
ΔAΔB ≥ ½ |⟨[A,B]⟩|
commutator
[x,p] = iℏ
ΔxΔp ≥ ℏ/2
[Jx, Jy] = iJzℏ
{Jz, φ}