Title: Interpretation of Quantum Theory: Lecture 1

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Abstract:

The Interpretation of Quantum Theory: Current Status and Future Directions

A Perimeter Institute lecture series and University of Waterloo special topics course.

Location: Perimeter Institute (room 405).
Organizers: Joseph Emerson and Ray Laflamme.
Lectures: Tuesdays and Thursdays from 2:15-3:45 from Jan. 4th to April 5th.

Course Outline and List of Lecturers:

<u>Lecture week 1 (Jan. 4, 6):</u> **The Structure of Quantum Theory.**
A. Postulates of Quantum Theory.
B. Operationalism and Generalized Axioms of the Quantum Theory.
<u>Lecturer:</u> J. Emerson

<u>Lecture weeks 2 and 3 (Jan. 11, 13, 18, 20):</u> **Basic Problems of Interpretation.**
A. Basics of Interpretation: Ontic vs Epistemic Classical Theories.
B. Orthodox and Copenhagen Interpretations (following von Neuman, Dirac, and Bohr).
C. The Measurement Problem (Schrodinger's cat) and the Projection Postulate (collapse of the wavefunction).
D. The EPR Paradox (state realism vs non-locality, the possibility of incompleteness).
E. Constraints on Hidden Variables: Bell's Theorem (non-locality) and the Kochen-Specker Theorem (contextuality).
<u>Lecturer:</u> J. Emerson

<u>Lecture week 4 (Jan. 25, 27):</u> **Many Worlds Interpretation.**
<u>Lecturer:</u> D. Wallace

<u>Lecture week 5 (Feb. 1, 3):</u> **The deBroglie-Bohm Interpretation.**
<u>Lecturer:</u> S. Goldstein

<u>Lecture week 6 (Feb. 8, 10):</u> **The Statistical Interpretation.**
<u>Lecturer:</u> L.E. Ballentine

<u>Lecture week 7 (Feb. 15, 17):</u> **Spontaneous Collapse Models.**
<u>Lecturer:</u> P. Pearle

University of Waterloo Reading Week (Feb 22, 24): No Lectures.

<u>Lecture week 8 (March 1, 3):</u> **Experimental Interlude I: Interference of Macro Molecules.**
<u>Lecturer:</u> A. Zeilinger

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Lecturer: R. Griffiths

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Lecture week 11 (March 22, 24): **Advanced Topics in deBroglie-Bohm Theory.**
Lecturer: A. Valentini

Lecture Week 12 (March 29, 31): **Epistemic Features of the Quantum State.**
A. Chaos and Quantum Classical Correspondence in the Macroscopic Limit.
Lecturer: J. Emerson
B. Quantum Properties from Constraints on Classical Knowledge: A Toy Theory.
Lecturer: R. Spekkens

Lecture Week 13 (April 5): **Physical Axioms for Quantum Theory.**
Lecturer: L. Hardy

Evaluation:
30% Class Participation
30% Assignments
40% Term Project

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U. Waterloo Course PHY490/PHY773, Winter 2005
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Lecture 1 Jan 4, 2005 **Lecturer:** Joseph Emerson

Standard Postulates of Quantum Mechanics

Postulate I. A physical preparation (or state) is described by an operator ρ that is non-negative (and Hermitian) with unit trace. Rank-one projectors, $\tilde{\rho} = |\psi\rangle\langle\psi|$, called pure states, correspond to states of maximal knowledge.

In many applications, it is adequate to specify the quantum state using only vectors $|\psi\rangle$, where these vectors are elements of a Hilbert space.

A Hilbert space $\mathcal{H}$ is a linear vector space with an inner product defined on it, $(\psi, \phi) \in \mathbb{C}$, or in Dirac notation, $\langle\psi|\phi\rangle \in \mathbb{C}$. (We will see later that for a vector space to qualify as an infinite dimensional Hilbert space we must specify a further condition.)

The dimension of $\mathcal{H}$ is the maximum number of linearly independent vectors.

A linearly independent set of vectors spanning $\mathcal{H}$ is called a basis.

Any vector can be expressed as a linear combination of basis vectors, e.g., let $\{\phi_j\}$ be a basis of $\mathcal{H}$, $|\psi\rangle \in \mathcal{H}$, then $|\psi\rangle = \sum_j c_j|\phi_j\rangle$.

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Example 1. *A linearly independent set of column vectors form a basis for a discrete Hilbert space.*

Example 2. *The space of differentiable functions can form a Hilbert space.*

An *inner product* is defined by the properties:

i) $(\psi, \phi) \in \mathbb{C}$

ii) $(\phi, \psi) = (\psi, \phi)^*$ ($*$ denotes complex conjugation)

iii) $(\phi, c_1\psi_1 + c_2\psi_2) = c_1(\phi, \psi_1) + c_2(\phi, \psi_2)$

iv) $||\psi||^2 = (\psi, \psi) >= 0$

In Dirac's notation (i) takes the form: $\langle\psi|\phi\rangle \in \mathbb{C}$

An orthonormal basis $\{\phi_j\}$ has

$$(\phi_j, \phi_i) = \langle\phi_j|\phi_i\rangle = \delta_{ij} \tag{1}$$

where δ_{ij} is the Kronecker delta-function.

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Example 2. *The space of differentiable functions can form a Hilbert space.*

An *inner product* is defined by the properties:

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Example 3. *For column vectors with elements,*

$$|\phi\rangle \rightarrow \begin{bmatrix} a_1 \\ a_2 \\ \vdots \end{bmatrix}, \quad |\psi\rangle \rightarrow \begin{bmatrix} b_1 \\ b_2 \\ \vdots \end{bmatrix}$$

the inner product is expressed as follows:

$$\langle \psi | \phi \rangle = \sum_j b_j^* a_j$$

Note that bra vectors (e.g. $\langle \psi |$) are elements of a dual space $\mathcal{H}^\dagger$, which consists of linear functionals mapping elements of the Hilbert space to complex scalars.

Example 4. *Let $\psi(x)$ and $\phi(x)$ be complex functions, then the inner product takes the form:* $\langle \psi | \phi \rangle = \int d\mu(x) \psi^*(x) \phi(x)$

An infinite dimensional $\mathcal{H}$ has to be complete in the norm – that is, all vectors obtained from limits of Cauchy sequences are contained in $\mathcal{H}$. Given a Cauchy sequence $\{\psi_m\}$, $||\psi_m\rangle - |\psi_n\rangle| \rightarrow 0$ as $m, n \rightarrow \infty$, $|\psi\rangle = lim_{m \rightarrow \infty} |\psi_m\rangle \in \mathcal{H}$, and $\|\psi\|^2 < \infty$.

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An important example of a continuous Hilbert space is $L^2(a,b)$, that is, the set of square integrable complex functions, or

$$\int_a^b dx|\psi(x)|^2 < \infty. \tag{3}$$

In practice it is convenient to make use of non-square integrable and generalized functions which do not fit in the Hilbert space framework, for example,

$$\psi(x) = \langle x|p\rangle = \frac{1}{\sqrt{2\pi\hbar}} \exp\left(-i\frac{px}{\hbar}\right).$$

and the Dirac 'delta-function':

$$\delta(x - x_o) = \langle x|x_o\rangle,$$

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To accommodate these elements we can use the *rigged Hilbert space* formalism and treat the following inner products as well-defined:

$$\langle x|x_0\rangle = \delta(x - x_0) \quad (4)$$

$$\langle p|p_0\rangle = \delta(p - p_0) \quad (5)$$

A state operator $\hat{\rho}$ must be non-negative. An operator is non-negative iff $\langle\mu|\hat{\rho}|\mu\rangle \geq 0$ for all $|\mu\rangle \in \mathcal{H}$.

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For a normalized state operator $\hat{\rho}$ ($\mathrm{tr}(\hat{\rho}) = 1$) there are three equivalent definition of purity (maximal knowledge):

1) $\hat{\rho}^2 = \hat{\rho}$, which means that ρ is projector.
2) $\mathrm{tr}(\hat{\rho}^2) = 1$.
3) $\hat{\rho} = |\psi\rangle\langle\psi|$, defining a projector onto a one-dimensional subspace of $\mathcal{H}$.

The *trace* of an operator $\hat{A}$ is defined by

$$\mathrm{tr}(\hat{A}) = \sum_j \langle\phi_j|\hat{A}|\phi_j\rangle$$

where $\{|\phi_j\rangle\}$ is any (convenient) normalized orthogonal basis.

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Postulate 2.a) is responsible for the novel structural aspects of quantum theory. Operators with discrete spectra are "quantized" (in the sense that they are discretized). Examples of this are the atomic energy levels, angular momentum, and electromagnetic radiation can only exchange discrete amounts of energy with some systems (i.e. "photons").

Postulate 2.b) provides the statistical/probabilistic/indeterministic character of quantum predictions. It is known as **the Born rule.**

The *trace* of an operator $\hat{A}$ is defined by

$$\mathrm{tr}(\hat{A}) = \sum_j \langle \phi_j | \hat{A} | \phi_j \rangle$$

where $\{|\phi_j\rangle\}$ is any (convenient) normalized orthogonal basis.

Postulate II. Each physical observable is represented by a Hermitian operator $\hat{O}$. Let $\hat{O}$ be a Hermitian operator with eigenvalues λ_i and eigenvectors $|\lambda_i\rangle$.

a) **The set of possible observable outcomes is determined from $\{\lambda_i\}$.**

b) **The probability of outcome λ_i, is given by $\Pr(\lambda_i) = \mathrm{tr}(|\lambda_i\rangle\langle\lambda_i|\hat{\rho})$ when the physical configuration is described by $\hat{\rho}$.**

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