

Title: Quasi-normal modes for black-holes with generic singularities

Date: Dec 14, 2004 02:15 PM

URL: <http://pirsa.org/04120007>

Abstract:



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Perturbations of a Black Hole

$$\frac{d^2 R(r)}{dr^2} + [\omega^2 - V(r)] R(r) = 0$$

$$\omega = \Re(\omega) + i\Im(\omega)$$

$$\delta A = \delta(16\pi M^2) = 32\pi M \delta M \equiv 32\pi M \omega_{QNM}$$

$$\uparrow$$

$$[\delta A = 8\pi\gamma\sqrt{j_m(j_m+1)} \Rightarrow \gamma = \frac{4M\omega_{QNM}}{\sqrt{j_m(j_m+1)}} \leftarrow \text{Immirzi parameter}]$$

$$S = N \ln(2j_m + 1) = \left[\frac{A}{\delta A}\right] \ln(2j_m + 1) = \frac{A}{32\pi M \omega_{QNM}} \ln(2j_m + 1)$$

$$= \frac{A}{4} \frac{\ln(2j_m+1)}{8\pi M \omega_{QNM}} = S_{BH} \frac{\ln(2j_m+1)}{8\pi M \omega_{QNM}}$$

Question: $\ln(2j_m + 1) = 8\pi M \omega_{QNM}$?

Yes: $\omega_{QNM} = \frac{\ln(3)}{8\pi M} \Rightarrow j_m = 1$

O. Dreyer (gr-qc/0211076)

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QNM and Black Hole Area Spectrum

$$\delta A = 4 \ln(3) \rightarrow A = 4n \ln(3) \rightarrow \Omega_{BH} = \exp(A/4) = 3^n \text{ (integer)}$$

What About Other Black Holes?

First: $\ln(3) \rightarrow \ln(k)$

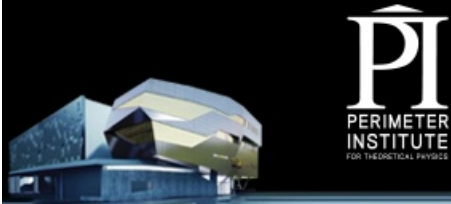
Second: $S_{BH} = \frac{A}{4} = [\ln(k)] \mathbf{n} \leftarrow \text{Adiabatic invt (Bekenstein)?}$

If so, then: Periodic system: $S_{BH} \sim \int \frac{dE}{\omega(E)}$

Third: $\omega_{QNM} = \frac{\ln(3)}{8\pi M} \rightarrow T_H \ln(k)$

$$\Rightarrow \text{Adia Invt.} = \frac{1}{\ln(k)} \int \frac{dE}{T_H} \propto S_{BH}$$

J. D. Bekenstein, Phys. Rev. **D7** (1973) 2333;
 S. Hod, gr-qc/9812002; G. Kunstatter, gr-qc/0212014.





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Horizon Area = Adiabatic Invariant?

Reduced action for a Sph Symm BH:

$$I_{Red} = \int dt \left(P_M \dot{M} - \frac{H(M)}{N \circ P_M} \right)$$

Can. Trans. $(M, P_M) \rightarrow (X, \Pi_X)$ to incorporate periodicity ($\sim 1/T_H$)

$$X = \sqrt{\frac{B(M)}{\pi}} \cos(2\pi P_M T_H(M))$$

$$\Pi_X = \sqrt{\frac{B(M)}{\pi}} \sin(2\pi P_M T_H(M))$$

$\{X, \Pi_X\} = T_H \frac{\partial B(M)}{\partial M} \Rightarrow B(M) \equiv S_{BH}(M)$ (From First Law)

Square & add: $\hat{S}_{BH} = \pi (\hat{X}^2 + \hat{\Pi}_X^2) \Leftarrow$ SHO w/ $m = 1/2, \omega = 2$

$$\Rightarrow S_{BH} = 2\pi \left(n + \frac{1}{2} \right)$$

Adiabatic invt: $= \int \frac{dM}{T_H} = S_{BH}$ (First law)
 $\propto n + \dots$ (Bohr-Sommerfeld-Wilson)

A. Bravinsky, S. Das, G. Kunstatter, hep-th/0209039;

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$$I_{Ret} = \int dt \left(P_M \dot{M} - \frac{H(M)}{N_0 P_M} \right)$$

Can. Trans. $(M, P_M) \rightarrow (X, \Pi_X)$ to incorporate periodicity ($\sim 1/T_H$)

$$X = \sqrt{\frac{B(M)}{\pi}} \cos(2\pi P_M T_H(M))$$

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A. Bravinsky, S. Das, G. Kunstatter, hep-th/0209039;
 G. Gour, A. J. M. Medved, gr-qc/0211089;
 S. Das, H. Mukhopadhyay, P. Ramadevi, hep-th/0407151 .

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Spherically Symmetric Single Horizon Black Holes

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{g(r)} + \rho^2(r)d\Omega_D^2$$

$$= f(r)[-dt^2 + dx^2] + \rho^2(r)d\Omega_D^2$$

$$x = \int \frac{dr}{\sqrt{f(r)g(r)}} \text{ (Tortoise)}$$

Near horizon (r_h)

$$f(r) = f'(r_h)(r - r_h); g(r) = g'(r_h)(r - r_h)$$

$$T_H = \frac{1}{4\pi} \sqrt{g'(r_h) f'(r_h)}$$

$$x = c_0(r_h) \ln(r - r_h)$$

For example: $f(r) = g(r) = 1 - \left(\frac{r_h}{r}\right)^{D-1}$. $T_H = \frac{(D-1)}{4\pi r_h}$ (Schwarzschild)

$$f(r) = g(r) = \frac{(r^{D-1} - r_h^{D-1})^2}{r^{2(D-1)}} \text{ (Extremal RN)}$$

Near the Singularity ($y = \beta t$)

$$ds^2 \stackrel{r \rightarrow 0}{\simeq} \eta r^{2p/q} dy^2 - \frac{4\eta}{q^2} r^{2(p-q+2)/q} dr^2 + r^2 d\Omega_D^2$$

$$= \eta x^p (dy^2 - dx^2) + x^q d\Omega_D^2 \text{ [Generic Power Law Singularity]}$$

($\eta = 1, 0, -1$ Space/Null/Timelike)

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$$= f(r) \left[-dt^2 + dx^2 \right] + \rho^2(r) d\Omega_D^2$$

$$x = \int \frac{dr}{\sqrt{f(r)g(r)}} \quad (\text{Tortoise})$$

Near horizon (r_h)

$$f(r) = f'(r_h) (r - r_h); \quad g(r) = g'(r_h) (r - r_h)$$

$$T_H = \frac{1}{4\pi} \sqrt{g'(r_h) f'(r_h)}$$

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$$= \eta x^p (dy^2 - dx^2) + x^q d\Omega_D^2 \quad [\text{Generic Power Law Singularity}]$$

($\eta = 1, 0, -1$ Space/Null/Timelike)

$$R_{abcd} R^{abcd} = \frac{e x^{-2p+dx-2q+4+ex-p-q+2}}{x^4}$$

P. Szekeres, V. Iyer, Phys. Rev. **D47** (1993) 4362.

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$$x = \int \frac{dr}{\sqrt{f(r)g(r)}} \text{ (Tortoise)}$$

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P. Szekeres, V. Iyer, Phys. Rev. **D47** (1993) 4362.

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$$x = c_0(r/r_h) \ln(r/r_h)$$

For example: $f(r) = g(r) = 1 - \left(\frac{r_h}{r}\right)^{D-1}$. $T_H = \frac{(D-1)}{4\pi r_h}$ (Schwarzschild)

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$\Downarrow$ ($\eta = 1, 0, -1$ Space/Null/Timelike)

$$R_{abcd} R^{abcd} = \frac{c x^{-2p+dx-2q+4} + e x^{-p-q+2}}{x^4}$$

P. Szekeres, V. Iyer, Phys. Rev. **D47** (1993) 4362.

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For example:

$$p = \frac{1-D}{D} ; \quad q = \frac{2}{D} ; \quad x = \frac{r^D}{\beta} \quad (\text{Schwarzschild})$$

$$p = -2 \left(\frac{D-1}{2D-1} \right) ; \quad q = \frac{2}{2D-1} ; \quad x = \frac{r^{2D-1}}{\beta} \quad (\text{Non-Extr RN})$$

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For example:

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In general:

$$f(r) = -\eta \beta^2 r^{2p/q}; \quad \frac{1}{g(r)} = -\frac{4\eta}{q^2} r^{2(p-q+2)/q}; \quad \rho(r) = r; \quad x = \frac{r^{2/q}}{\beta}$$

Quasi Normal Modes

$$\square \Phi \equiv \frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu \Phi) = 0,$$

$$\frac{d^2 R(r)}{dx^2} + [\omega^2 - V(r)] R(r) = 0$$

$$V(r) = \frac{l(l+D-1)}{r^2} f(r) + \left(\frac{D}{2} \right) \rho(r)^{-\frac{D}{2}} \sqrt{f(r) g(r)} \times \frac{d}{dr} \left(\rho(r)^{\frac{D-2}{2}} \frac{d\rho(r)}{dr} \sqrt{f(r) g(r)} \right) \text{ [Regge-Wheeler potential]}$$

$$V(r) \underset{r \rightarrow 0}{\underset{r \rightarrow r_h}{\simeq}} \left[\frac{l(l+D-1)}{\rho^2(r_h)} f'(r_h) + \frac{D}{2} f'(r_h) g'(r_h) \frac{\rho'(r_h)}{\rho(r_h)} \right] (r - r_h) + \mathcal{O}[(r - r_h)^2]$$

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$$V(r) \underset{r \rightarrow r_h}{\simeq} \left[\frac{l(l+D-1)}{\rho^2(r_h)} f'(r_h) + \frac{D}{2} f'(r_h) g'(r_h) \frac{\rho'(r_h)}{\rho(r_h)} \right] (r - r_h) + \mathcal{O}[(r - r_h)^2]$$

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$$J(r) = -\eta(r) \frac{1}{g(r)} \frac{d}{dr} \left(\frac{1}{q^2} \right), \quad \rho(r) = r, \quad x = \frac{r}{\beta}$$

Quasi Normal Modes

$$\square \Phi \equiv \frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu \Phi) = 0,$$

$$\frac{d^2 R(r)}{dx^2} + [\omega^2 - V(r)] R(r) = 0$$

$$V(r) = \frac{l(l+D-1)}{r^2} f(r) + \left(\frac{D}{2}\right) \rho(r)^{-\frac{D}{2}} \sqrt{f(r)g(r)} \times \frac{d}{dr} \left(\rho(r)^{\frac{D-2}{2}} \frac{d\rho(r)}{dr} \sqrt{f(r)g(r)} \right) \text{ [Regge-Wheeler potential]}$$

$$V(r) \underset{r \rightarrow r_h}{\simeq} \left[\frac{l(l+D-1)}{\rho^2(r_h)} f'(r_h) + \frac{D}{2} f'(r_h) g'(r_h) \frac{\rho'(r_h)}{\rho(r_h)} \right] (r - r_h) + \mathcal{O}[(r - r_h)^2]$$

$$\underset{r \rightarrow 0}{\simeq} \underbrace{\left(\frac{q\beta}{2} \right)^2 \frac{D}{2} \left(\frac{D}{2} - \frac{2}{q} \right) r^{-4/q}}_{\text{Dominates } (q > 0, p - q + 2 > 0)} - \eta\beta^2 l(l+D-1) r^{2(p-q)/q}$$

7

Solution (with $R(x) \sim e^{\pm i\omega x}$ as $x \rightarrow \mp\infty$)

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$$\square \Psi = \frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu \Psi) = 0,$$

$$\frac{d^2 R(r)}{dx^2} + [\omega^2 - V(r)] R(r) = 0$$

$$V(r) = \frac{l(l+D-1)}{r^2} f(r) + \left(\frac{D}{2}\right) \rho(r)^{-\frac{D}{2}} \sqrt{f(r) g(r)} \times \frac{d}{dr} \left(\rho(r)^{\frac{D-2}{2}} \frac{d\rho(r)}{dr} \sqrt{f(r) g(r)} \right) \text{ [Regge-Wheeler potential]}$$

$$V(r) \stackrel{r \rightarrow r_h}{\simeq} \left[\frac{l(l+D-1)}{\rho^2(r_h)} f'(r_h) + \frac{D}{2} f'(r_h) g'(r_h) \frac{\rho'(r_h)}{\rho(r_h)} \right] (r - r_h) + \mathcal{O}[(r - r_h)^2]$$

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7

Solution (with $R(x) \sim e^{\pm i\omega x}$ as $x \rightarrow \mp\infty$)

$$R(z) = A_+ c_+ \sqrt{\omega z} J_{\nu}(\omega z) + A_- c_- \sqrt{\omega z} J_{-\nu}(\omega z) \quad (\nu = (Dq - 2)/4)$$

$$c_{\pm} \sqrt{\omega z} J_{\pm\nu}(\omega z) \sim 2 \cos(\omega z - \alpha_{\pm}) \quad \text{as } \omega z \rightarrow \infty; \quad (\alpha_{\pm} \equiv \frac{\pi}{4} [1 \pm 2\nu])$$

$$A_+ e^{-i\alpha_+} + A_- e^{-i\alpha_-} = 0$$

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$$J(r) = -\eta \rho(r), \quad g(r) = \frac{1}{q^2}, \quad \rho(r) = r, \quad x = \frac{r}{\beta}$$

Quasi Normal Modes

$$\square \Phi \equiv \frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu \Phi) = 0,$$

$$\frac{d^2 R(r)}{dx^2} + [\omega^2 - V(r)] R(r) = 0$$

$$V(r) = \frac{l(l+D-1)}{r^2} f(r) + \left(\frac{D}{2}\right) \rho(r)^{-\frac{D}{2}} \sqrt{f(r)g(r)} \\ \times \frac{d}{dr} \left(\rho(r)^{\frac{D-2}{2}} \frac{d\rho(r)}{dr} \sqrt{f(r)g(r)} \right) \quad [\text{Regge-Wheeler potential}]$$

$$V(r) \stackrel{r \rightarrow r_h}{\simeq} \left[\frac{l(l+D-1)}{\rho^2(r_h)} f'(r_h) + \frac{D}{2} f'(r_h) g'(r_h) \frac{\rho'(r_h)}{\rho(r_h)} \right] (r - r_h) + \mathcal{O}[(r - r_h)^2]$$

$$\stackrel{r \rightarrow 0}{\simeq} \underbrace{\left(\frac{q\beta}{2} \right)^2 \frac{D}{2} \left(\frac{D-2}{2} - \frac{2}{q} \right) r^{-4/q}}_{\text{Dominates } (q > 0, p - q + 2 > 0)} - \eta \beta^2 l(l+D-1) r^{2(p-q)/q}$$

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Solution (with $R(x) \sim e^{\pm i\omega x}$ as $x \rightarrow \mp\infty$)

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Solution (with $R(x) \sim e^{\pm i\omega x}$ as $x \rightarrow \mp\infty$)

$$R(z) = A_+ c_+ \sqrt{\omega z} J_{+\nu}(\omega z) + A_- c_- \sqrt{\omega z} J_{-\nu}(\omega z) \quad (\nu = (Dq - 2)/4)$$

$$c_{\pm} \sqrt{\omega z} J_{\pm\nu}(\omega z) \sim 2 \cos(\omega z - \alpha_{\pm}) \quad \text{as } \omega z \rightarrow \infty; \quad (\alpha_{\pm} \equiv \frac{\pi}{4} [1 \pm 2\nu])$$

$$A_+ e^{-i\alpha_+} + A_- e^{-i\alpha_-} = 0$$

$$R(z) \sim [A_+ e^{+i\alpha_+} + A_- e^{+i\alpha_-}] e^{-i\omega z} \quad \text{as } \omega z \rightarrow \infty$$

Rotate: $\Im(\omega z) \rightarrow \infty \rightarrow \Im(\omega z) \rightarrow -\infty$
 $r \rightarrow r \exp(i3\pi q/2) \Rightarrow z \rightarrow \exp(i3\pi q/2 \times 2/q) = \exp(3i\pi)z$
 $J_{\nu}(ze^{im\pi}) = e^{im\nu\pi} J_{\nu}(z), \text{ (as } \omega z \rightarrow -\infty)$

$$R(z) \sim (A_+ e^{5i\alpha_+} + A_- e^{5i\alpha_-}) e^{-i\omega z} + (A_+ e^{7i\alpha_+} + A_- e^{7i\alpha_-}) e^{+i\omega z}$$

$$\text{Monodromy} = \frac{A_+ e^{5i\alpha_+} + A_- e^{5i\alpha_-}}{A_+ e^{i\alpha_+} + A_- e^{i\alpha_-}}$$

$$= \frac{e^{6i\alpha_+} - e^{6i\alpha_-}}{e^{2i\alpha_+} - e^{2i\alpha_-}} = \frac{\sin(3\pi\nu)}{\sin(\pi\nu)} = 1 + 2 \cos\left(\frac{\pi}{2}[Dq - 2]\right)$$

Near horizon ($z \rightarrow -\infty$):

$$R(z) \sim e^{+i\omega z} \sim \exp[i\alpha_+ \ln(r - r_+)] \quad \text{Monodromy} = \exp(4\pi i \nu)$$

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Solution (with $R(x) \sim e^{\pm i\omega x}$ as $x \rightarrow \mp\infty$)

$$R(z) = A_+ c_+ \sqrt{\omega z} J_{+\nu}(\omega z) + A_- c_- \sqrt{\omega z} J_{-\nu}(\omega z) \quad (\nu = (Dq - 2)/4)$$

$$c_{\pm} \sqrt{\omega z} J_{\pm\nu}(\omega z) \sim 2 \cos(\omega z - \alpha_{\pm}) \quad \text{as } \omega z \rightarrow \infty; \quad (\alpha_{\pm} \equiv \frac{\pi}{4} [1 \pm 2\nu])$$

$$A_+ e^{-i\alpha_+} + A_- e^{-i\alpha_-} = 0$$

$$R(z) \sim [A_+ e^{+i\alpha_+} + A_- e^{+i\alpha_-}] e^{-i\omega z} \quad \text{as } \omega z \rightarrow \infty$$

Rotate: $\Im(\omega z) \rightarrow \infty \rightarrow \Im(\omega z) \rightarrow -\infty$
 $r \rightarrow r \exp(i3\pi q/2) \Rightarrow z \rightarrow \exp(i3\pi q/2 \times 2/q) = \exp(3i\pi)z$
 $J_{\nu}(ze^{im\pi}) = e^{im\nu\pi} J_{\nu}(z), \text{ (as } \omega z \rightarrow -\infty)$

$$R(z) \sim (A_+ e^{5i\alpha_+} + A_- e^{5i\alpha_-}) e^{-i\omega z} + (A_+ e^{7i\alpha_+} + A_- e^{7i\alpha_-}) e^{+i\omega z}$$

$$\text{Monodromy} = \frac{A_+ e^{5i\alpha_+} + A_- e^{5i\alpha_-}}{A_+ e^{i\alpha_+} + A_- e^{i\alpha_-}}$$

$$= \frac{e^{6i\alpha_+} - e^{6i\alpha_-}}{e^{2i\alpha_+} - e^{2i\alpha_-}} = \frac{\sin(3\pi\nu)}{\sin(\pi\nu)} = 1 + 2 \cos\left(\frac{\pi}{2}[Dq - 2]\right)$$

Near horizon ($z \rightarrow -\infty$):

$$R(z) \sim e^{+i\omega z} \sim \exp[i c_0 \omega \ln(x - x_h)], \quad \text{Monodromy} = \exp(4\pi\omega c_0)$$

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$$c_{\pm} \sqrt{\omega z} J_{\pm \nu}(\omega z) \sim 2 \cos(\omega z - \alpha_{\pm}) \quad \text{as } \omega z \rightarrow \infty; \quad (\alpha_{\pm} \equiv \frac{\pi}{4} [1 \pm 2\nu])$$

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Equating:

$$\omega_{QNM} = \frac{i}{2\alpha_0} \left[n + \frac{1}{2} \right] \pm \frac{1}{4\pi\alpha_0} \log \left[1 + 2 \cos\left(\frac{\pi}{2} [Dq - 2]\right) \right]$$

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$$\Re(\omega_{QNM}) = \pm \frac{1}{4\pi c_0} \log \left[1 + 2 \cos \left(\frac{\pi}{2} [Dq - 2] \right) \right]$$

$\uparrow$ $\uparrow$
 Horizon Singularity (no p)

Note:

$\Re(\omega_{QNM}) = \text{Integer, iff}$

(a) $\left(\frac{Dq - 2}{2} \right) \pi = 2j\pi$ where $j \in \mathcal{Z}$

(b) $\left(\frac{Dq - 2}{2} \right) \pi = \frac{m\pi}{3}$ where $m \in \mathcal{Z}$

L. Motl, A. Neitzke, hep-th/0301173;
 SD, S. Shankaranarayanan, hep-th/0410209.

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$$c_{\pm} \sqrt{\omega z} J_{\pm\nu}(\omega z) \sim 2 \cos(\omega z - \alpha_{\pm}) \quad \text{as } \omega z \rightarrow \infty; \quad (\alpha_{\pm} \equiv \frac{\pi}{4} [1 \pm 2\nu])$$

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Rotate: $\Im(\omega z) \rightarrow \infty \rightarrow \Im(\omega z) \rightarrow -\infty$
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$$\Re(\omega_{QNM}) = \pm \frac{1}{4\pi c_0} \log \left[1 + 2 \cos \left(\frac{\pi}{2} [Dq - 2] \right) \right]$$

$\uparrow$ $\uparrow$
 Horizon Singularity (no p)

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L. Motl, A. Neitzke, hep-th/0301173;
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Applications

$(D + 2)$ -dimensional Schwarzschild

$$f(r) = g(r) = 1 - \left(\frac{r_h}{r}\right)^{D-1} ; T_H = \frac{(D-1)}{4\pi r_h}$$

$(D + 2)$ -dimensional Extremal RN

$$f(r) = g(r) = \frac{(r^{D-1} - r_h^{D-1})^2}{r^{2(D-1)}} ; T_H = 0$$

2-Dim Stringy BH

$$f(r) = g(r) = 1 - \frac{M}{\lambda} e^{2\lambda r}$$

$$r_H = \frac{1}{2\lambda} \ln\left(\frac{\lambda}{M}\right) ; T_H = \frac{\lambda}{2\pi}$$

E. Witten, Phys. Rev. **D44** (1991) 314;
 G. Mandal et al, Mod. Phys. Lett. **A6** (1991) 1685;
 CGHS hep-th/9111056

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(D + 2)-dimensional Extremal RN

$$f(r) = g(r) = \frac{(r^{D-1} - r_h^{D-1})^2}{r^{2(D-1)}} ; T_H = 0$$

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$$f(r) = g(r) = 1 - \frac{M}{\lambda} e^{2\lambda r}$$

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E. Witten, Phys. Rev. **D44** (1991) 314;
 G. Mandal et al, Mod. Phys. Lett. **A6** (1991) 1685;
 CGHS hep-th/9111056

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4-Dim Stringy BH

$$f(r) = g(r) = \left(1 - \frac{r_h}{r}\right) \left(1 - \frac{r_0}{r}\right)^{(1-\alpha^2)/(1+\alpha^2)};$$

$$\rho(r) = r \left(1 - \frac{r_0}{r}\right)^{\alpha^2/(1+\alpha^2)} \quad [\alpha = 0 \rightarrow \text{RN}]$$

$$M = \frac{r_h}{2} + \left[\frac{1-\alpha^2}{1+\alpha^2}\right] \frac{r_0}{2}; \quad Q = \left[\frac{r_h r_0}{1+\alpha^2}\right]^{1/2}$$

$$T_H = \frac{1}{4\pi r_h} \left(1 - \frac{r_0}{r_h}\right)^{(1-\alpha^2)/(1+\alpha^2)}$$

Garfinkle, Horowitz, Strominger, Phys. Rev. **D43** (1991) 3140 etc

5-Dim Stringy BH

$$f(r) = F^{-2/3} \left(1 - \frac{r_h^2}{r^2}\right); \quad \rho(r) = F^{1/6} r;$$

$$g(r) = F^{-1/3} \left(1 - \frac{r_h^2}{r^2}\right),$$

and

$$F = \left[1 + \frac{r_h^2 \sinh^2 \alpha}{r^2}\right] \left[1 + \frac{r_h^2 \sinh^2 \gamma}{r^2}\right] \left[1 + \frac{r_h^2 \sinh^2 \sigma}{r^2}\right].$$

$$Q_1 = \frac{V r_h^2}{2g} \sinh 2\alpha; \quad Q_5 = \frac{r_h^2}{2g} \sinh 2\gamma \quad (\# \text{ of 1\&5-branes})$$

$$n = \frac{R^2 V r_h^2}{2g^2} \sinh 2\sigma \quad (\text{Momentum on 1-brane})$$

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$$T_H = \frac{1}{4\pi r_h} \left(1 - \frac{r_0}{r_h} \right)^{(1-\alpha^2)/(1+\alpha^2)}$$

Garfinkle, Horowitz, Strominger, Phys. Rev. **D43** (1991) 3140 etc

5-Dim Stringy BH

$$f(r) = F^{-2/3} \left(1 - \frac{r_h^2}{r^2} \right); \quad \rho(r) = F^{1/6} r;$$

$$g(r) = F^{-1/3} \left(1 - \frac{r_h^2}{r^2} \right),$$

and

$$F = \left[1 + \frac{r_h^2 \sinh^2 \alpha}{r^2} \right] \left[1 + \frac{r_h^2 \sinh^2 \gamma}{r^2} \right] \left[1 + \frac{r_h^2 \sinh^2 \sigma}{r^2} \right].$$

$$Q_1 = \frac{V r_h^2}{2g} \sinh 2\alpha; \quad Q_5 = \frac{r_h^2}{2g} \sinh 2\gamma \quad (\# \text{ of } 1\&5 - \text{branes})$$

$$n = \frac{R^2 V r_h^2}{2g^2} \sinh 2\sigma \quad (\text{Momentum on } 1\text{-brane})$$

$$\sigma = 0 \Rightarrow r = 0 \text{ singular} \rightarrow T_H = (2\pi r_h \cosh \alpha \cosh \gamma)^{-1}$$

Horowitz, Maldacena, Strominger, hep-th/9603109

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Space-Time	Near horizon properties		Near singularity properties			$\Re[\omega_{\text{QNM}}]$
	κ	c_0	p	q	$(Dq-2)/2$	
Non-degenerate Horizons	$\frac{\sqrt{f'(r_h)g'(r_h)}}{2}$	$\frac{1}{2\kappa} = \frac{1}{4\pi T_H}$	$p > q - 2$	$q > 0$	Any real value	$\frac{1}{4\pi c_0} \log \left[1 + 2 \cos \left(\frac{\pi(Dq-2)}{2} \right) \right]$
$(D+2)$ -dimens. Schwarzschild	$\frac{(D-1)}{2r_h}$	$\frac{r_h}{D-1}$	$\frac{1-D}{D}$	$\frac{2}{D}$	0	$T_H \log 3$
4-dimensional Stringy BH	$\frac{1}{2r_h} \left[1 - \frac{r_0}{r_h} \right]^{\frac{1-\alpha^2}{1+\alpha^2}}$	$\frac{1}{2\kappa}$	$\frac{1-\alpha^2}{2\alpha^2}$	1	0	$T_H \log 3$
5-dimensional Stringy BH	$\frac{r_h^{-1}}{\cosh \alpha \cosh \gamma}$	$\frac{1}{2\kappa}$	$-\frac{2}{3}$	$\frac{2}{3}$	0	$T_H \log 3$
2-dimensional Stringy BH	1	$\frac{1}{2}$	1	0	-1	0
$(D+2)$ -dimens. Extremal RN	0	$\frac{D}{(D-1)^2} r_h$	$-2 \left(\frac{D-1}{2D-1} \right)$	$\frac{2}{2D-1}$	$\left(\frac{1-D}{2D-1} \right)$	$\frac{1}{8\pi r_h} \log 2$

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$$\Re(\omega_{QNM}) = \pm \frac{1}{4\pi c_0} \log \left[1 + 2 \cos \left(\frac{\pi}{2} [Dq - 2] \right) \right]$$

$\uparrow$ $\uparrow$
 Horizon Singularity (no p)

Note:

$\Re(\omega_{QNM}) = \text{Integer, iff}$

(a) $\left(\frac{Dq - 2}{2} \right) \pi = 2j\pi$ where $j \in \mathbb{Z}$

(b) $\left(\frac{Dq - 2}{2} \right) \pi = \frac{m\pi}{3}$ where $m \in \mathbb{Z}$

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Solution (with $R(x) \sim e^{\pm i\omega x}$ as $x \rightarrow \mp\infty$)

$$R(z) = A_+ c_+ \sqrt{\omega z} J_{+\nu}(\omega z) + A_- c_- \sqrt{\omega z} J_{-\nu}(\omega z) \quad (\nu = (Dq - 2)/4)$$

$$c_{\pm} \sqrt{\omega z} J_{\pm\nu}(\omega z) \sim 2 \cos(\omega z - \alpha_{\pm}) \quad \text{as } \omega z \rightarrow \infty; \quad (\alpha_{\pm} \equiv \frac{\pi}{4} [1 \pm 2\nu])$$

$$A_+ e^{-i\alpha_+} + A_- e^{-i\alpha_-} = 0$$

$$R(z) \sim [A_+ e^{+i\alpha_+} + A_- e^{+i\alpha_-}] e^{-i\omega z} \quad \text{as } \omega z \rightarrow \infty$$

Rotate: $\Im(\omega z) \rightarrow \infty \rightarrow \Im(\omega z) \rightarrow -\infty$
 $r \rightarrow r \exp(i3\pi q/2) \Rightarrow z \rightarrow \exp(i3\pi q/2 \times 2/q) = \exp(3i\pi)z.$
 $J_{\nu}(ze^{im\pi}) = e^{im\nu\pi} J_{\nu}(z), \quad (\text{as } \omega z \rightarrow -\infty)$

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Non-degenerate Horizons	$\frac{\sqrt{f'(r_h)g'(r_h)}}{2}$	$\frac{1}{2\kappa} = \frac{1}{4\pi T_H}$	$p > q - 2$	$q > 0$	Any real value	$\frac{1}{4\pi\alpha_0} \log \left[1 + 2 \cos \left(\frac{\pi(Dq-2)}{2} \right) \right]$
D + 2)-dimens. Schwarzschild	$\frac{(D-1)}{2r_h}$	$\frac{r_h}{D-1}$	$\frac{1-D}{D}$	$\frac{2}{D}$	0	$T_H \log 3$
4-dimensional stringy BH	$\frac{1}{2r_h} \left[1 - \frac{r_0}{r_h} \right]^{\frac{1-\alpha^2}{1+\alpha^2}}$	$\frac{1}{2\kappa}$	$\frac{1-\alpha^2}{2\alpha^2}$	1	0	$T_H \log 3$
5-dimensional stringy BH	$\frac{r_h^{-1}}{\cosh \alpha \cosh \gamma}$	$\frac{1}{2\kappa}$	$-\frac{2}{3}$	$\frac{2}{3}$	0	$T_H \log 3$
2-dimensional stringy BH	1	$\frac{1}{2}$	1	0	-1	0
D + 2)-dimens. Extremal RN	0	$\frac{D}{(D-1)^2 r_h}$	$-2 \left(\frac{D-1}{2D-1} \right)$	$\frac{2}{2D-1}$	$\left(\frac{1-D}{2D-1} \right)$	$\frac{1}{8\pi r_h} \log 2$

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Implications for LQG

$$\Delta A = 4 \underbrace{\frac{\Re(\omega_{QNM})}{T_H}}_{\text{First Law}} = 4 \log \left[1 + 2 \cos \left(\frac{\pi(Dq - 2)}{2} \right) \right]$$

$$\Delta A = 8\pi\gamma\sqrt{j_m(j_m + 1)} \quad (\text{LQG})$$

$$\gamma = \frac{\log [1 + 2 \cos (\pi(q - 1))]}{2\pi\sqrt{j_m(j_m + 1)}} \quad (\text{Immirzi})$$

$$N = A/\Delta A$$

$$S = N \log(1 + 2j_m)$$

$$= \frac{A \ln(1 + 2j_m)}{4 \log [1 + 2 \cos (\pi(q - 1))]}$$

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$$N = A/\Delta A$$

$$S = \frac{N \log(1 + 2j_m)}{A \ln(1 + 2j_m)} = \frac{A \ln(1 + 2j_m)}{4 \log [1 + 2 \cos (\pi(q - 1))]}$$

$$j_m = \cos (\pi(q - 1))$$

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$$\Delta A = 8\pi\gamma\sqrt{j_m(j_m + 1)} \quad (\text{LQG})$$

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$$S = N \log(1 + 2j_m)$$

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$$j_m = \cos(\pi(q - 1))$$

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$$N = A/\Delta A$$

$$S = N \log(1 + 2j_m)$$

$$= \frac{A}{4} \frac{\ln(1 + 2j_m)}{\log [1 + 2 \cos (\pi(q - 1))]}$$

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Summary and Discussions

- QNM for generic singularity BHs
- $\log(\text{integer})$ for all examples. Why?
- Crucial: $\Re(\omega_{QNM}) \propto T_H$. In general true?
- Generalisation to **Multiple Horizons, Rotating BHs, Non AF**

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