Title: A Semiclassicality Criterion for States of Quantum Geometry

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Abstract:

A semiclassicality criterion for states of quantum geometry

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PLAN

0. INTRODUCTION AND OVERVIEW
1. STATIC POTENTIAL FROM CONTINUUM QFT
2. A SEMICLASSICALITY CRITERION FOR KINEMATICAL STATES OF LQG
3. TEST: NEWTON POTENTIAL FROM POLYMER PICTURE OF LINEARIZED QUANTUM GRAVITY
4. SUMMARY

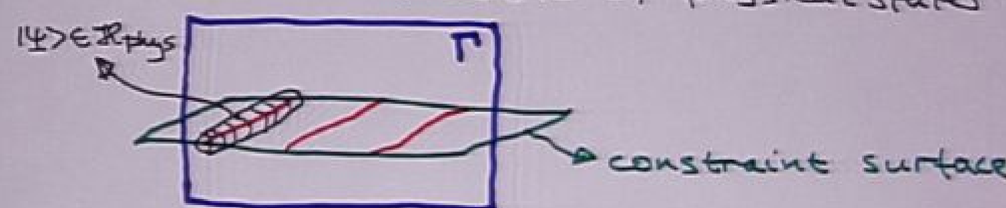
0. INTRODUCTION AND OVERVIEW

- Most candidate semiclassical states so far: in kinematical Hilbert space of LQG

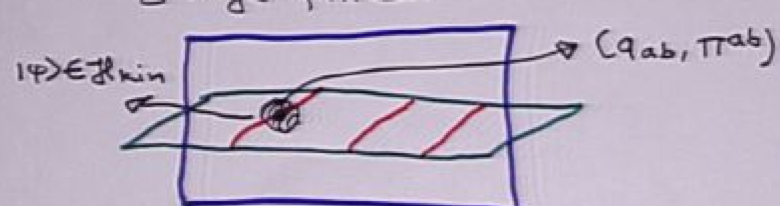
- What we would like:

  states that
  - * solve the constraints
  - * are peaked around diff-class of metrics

- Viewpoint: there is a sense in which kinematical semiclassical states are "gauge-fixed" versions of physical states



Gauge-fixed:



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- However, gauge-fixed states should still "know" about constraints

In particular: viable family of states in $\mathcal{H}_{kin}$ should "know" about the Newtonian potential

- In ordinary gauge theories: static potential occurs in "Coulomb phase": Wilson and 't Hooft loops should decay exponentially with perimeter

and

subleading term in exponent from Wilson loop should yield static potential

- Wilson loop order parameter can be reformulated in terms of overlap of appropriately chosen coherent states

→ semiclassicality criterion for states of quantum geometry (i.e. states in $\mathcal{H}_{kin}$ of LQG)

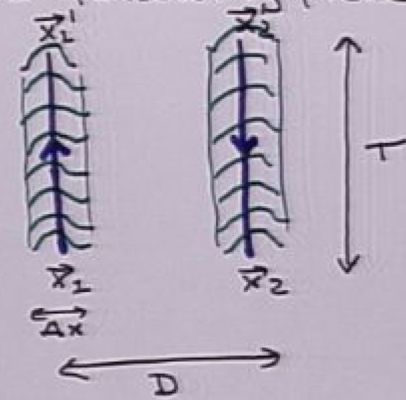
- "Test": polymer picture of linearized quantum gravity

As we shall see, can get the Newtonian potential there.

1. THE STATIC POTENTIAL FROM CONTINUUM QFT

- When do we expect "Coulombic" behavior?

Consider following process in QED:



Ingoing state : $|\underbrace{\Psi_2}_{\text{2 fermions}}, \underbrace{\phi}_{\text{state for EM field}}\rangle$

$$\frac{\hbar}{M} \ll \Delta x \ll D \ll T \ll T_{coll}$$

$$\hookrightarrow \sim \frac{\sqrt{M}\, D^{3/2}}{e}$$

If mass sufficiently large,
all inequalities can be satisfied
→ physics dominated by static potential

[Gravity: not so obvious
increase mass → increase force betw. particles

Indeed, $T_{coll} \sim \frac{D^{3/2}}{\sqrt{GM}}$

To still have $T \ll T_{coll}$, need
$GM \ll D$]

With these assumptions, in QED

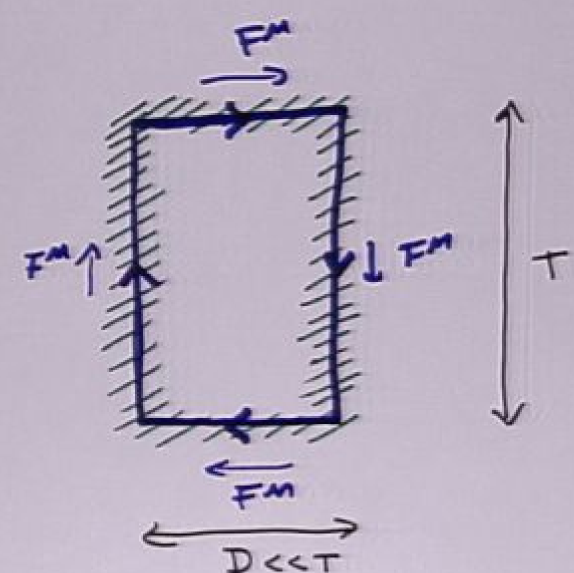
$$\text{Amplitude} \sim \mathcal{W}[\hat{A}]\, G_F(\vec{x}_1', \vec{x}_2'; \vec{x}_1, \vec{x}_2)$$

where $\mathcal{W}[\hat{A}]$ is Wilson loop:

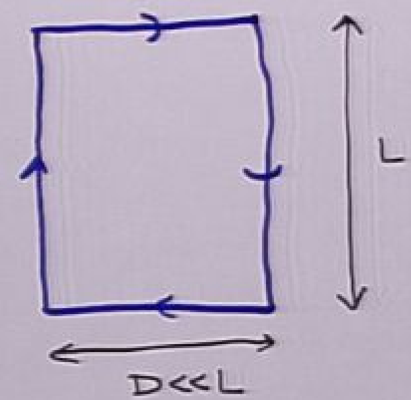
$$\mathcal{W}[\hat{A}] = \langle 0 | \exp\left[\frac{ie}{\hbar} \int d^3x\, \hat{A}_\mu(x) F^\mu(x) \right] | 0 \rangle$$

Fock vacuum for photons

Form factor $F^\mu(x)$ 3D Gaussian smeared around a rectangular loop; smearing width Δx



For our purposes: better to consider spatial loop

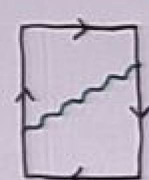


$$W_E[\hat{A}] = \langle 0 | \exp\left[\frac{ie}{\hbar}\int d^3x\, \hat{A}_a(\vec{x})\, F^a(\vec{x})\right] | 0 \rangle$$

3D spatial Gaussian smearing factor; width ε

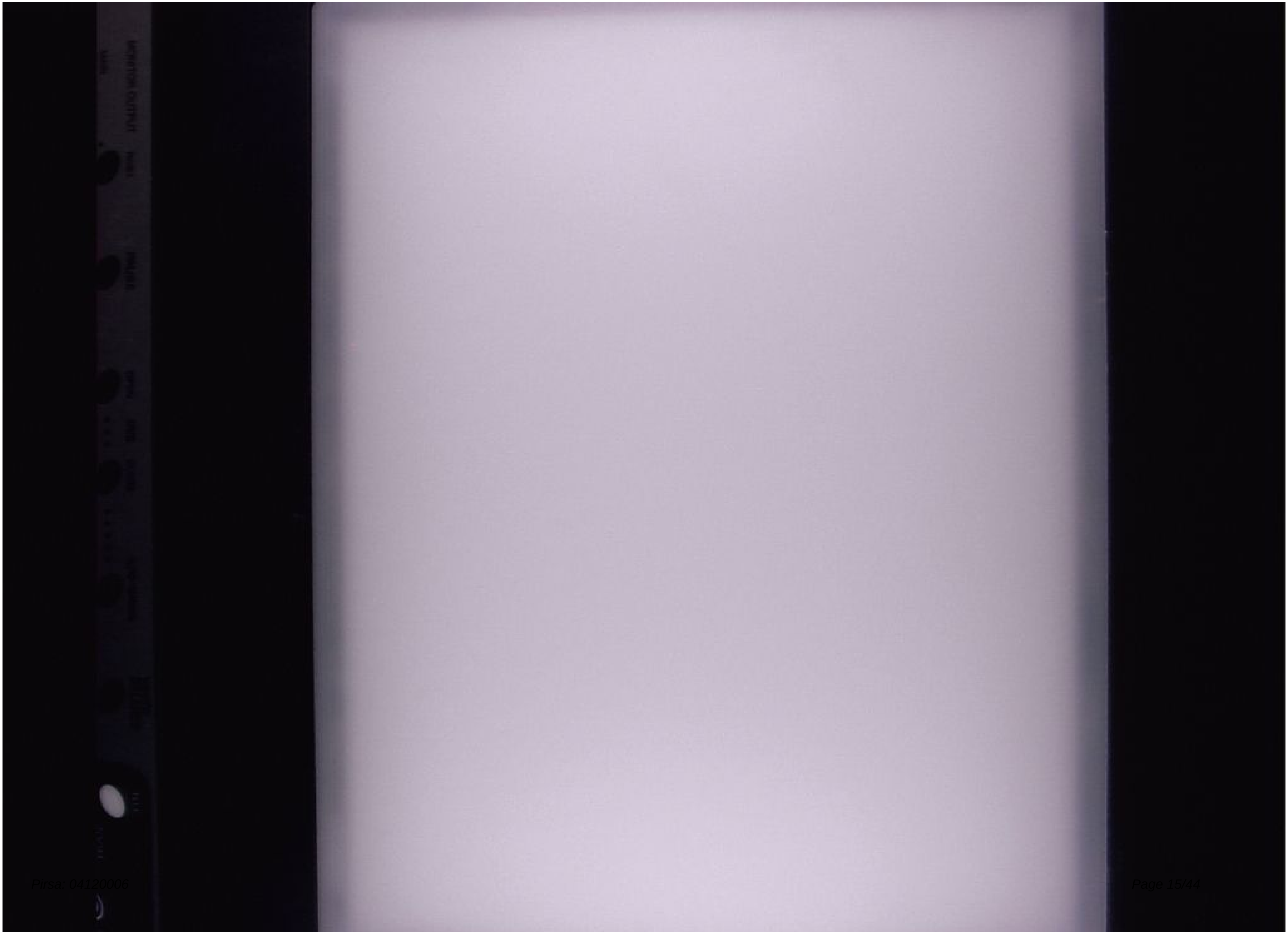
$$\simeq \exp\left[-\frac{(L+D)}{\hbar}\left(\underbrace{E_{self}}_{\mathcal{O}\left(\frac{e^2}{\varepsilon}\right)} + \underbrace{V_C(D)}_{= -\frac{e^2}{4\pi D}} + \underbrace{\cdots}_{\mathcal{O}\left(\frac{e^2\varepsilon}{D^2}\right)}\right)\right]$$

Why does Wilson loop "know" about static potential even though no fermions?

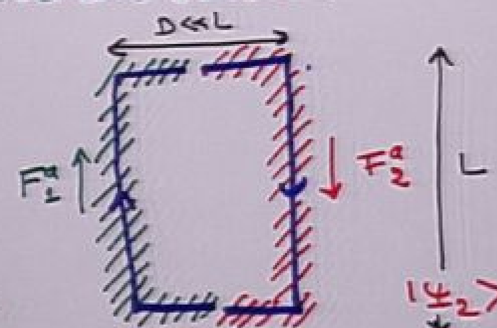


virtual photon exchange

$$V_C(D) \sim \int_{-L}^{L} d\tau\, \Delta_E^{00}(0,D,\tau,0)$$



Wilson loop reformulated in terms of coherent states:



$|\Psi_1\rangle$ peaked at $(A_a = 0,\ E^a \sim F_1^a)$

$|\Psi_2\rangle$ peaked at $(A_a = 0,\ E^a \sim F_2^a)$

Then $\langle \Psi_1 | \Psi_2 \rangle = W[\hat{A}]$

$$= \exp\left[-\frac{L}{\hbar} \left(E_{self} + V_{\bullet}(D) + \cdots \right) \right]$$

Expression that will be useful:

$$\langle \Psi_1 | \Psi_2 \rangle = \exp\left[-\frac{e^2}{\hbar} \int \frac{d^3k}{k} F^a(\vec{k}) F_a(\vec{k}) \right]$$

where $F^a(\vec{k}) = F_1^a(\vec{k}) + F_2^a(\vec{k})$

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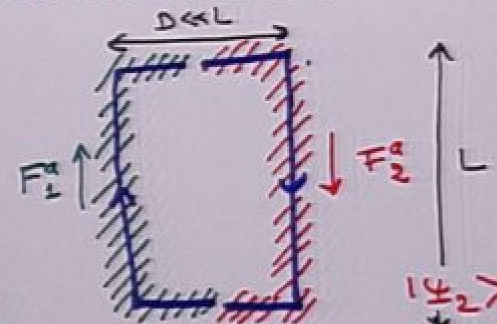

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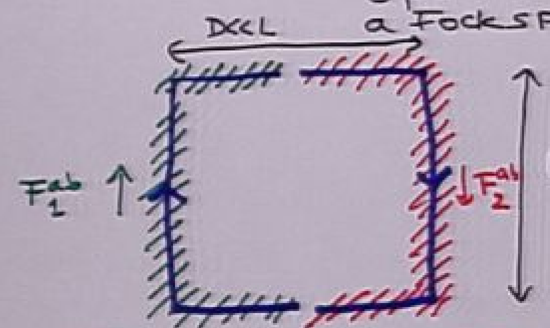
Same game in Linearized quantum gravity

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

Operator on a Fock space



$|\Psi_1\rangle$ peaked at $(h_{ab}=0, \pi_{ab} \sim F^1_{ab})$

$|\Psi_2\rangle$ peaked at $(h_{ab}=0, \pi_{ab} \sim F^2_{ab})$

Then

$$\langle \Psi_1 | \Psi_2 \rangle = \exp\left[-\frac{4\pi G M^2}{\hbar} \int \frac{d^3k}{k} \bar{F}^{ab}(\vec{k}) F_{ab}(\vec{k}) \right]$$

$$= \exp\left[-\frac{L}{\hbar} \left(\underbrace{E_{self}}_{\sim\, \mathcal{O}\left(\frac{GM^2}{\varepsilon}\right)} + \underbrace{V_N(D)}_{=\, -\frac{GM^2}{D}} + \underbrace{\cdots}_{\sim\, \mathcal{O}\left(\frac{GM^2 \varepsilon}{D^2}\right)} \right) \right]$$

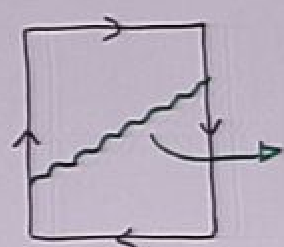
Important difference with quantum Maxwell theory:

- F^a automatically transverse
  $\longrightarrow$ forces transversality on $\hat{A}_a$

- F^{ab} not transverse-traceless
  $\longrightarrow$ will make a difference whether or not constraints imposed on $\hat{h}_{ab}$

To get $V_N(\Delta)$ in exponential: should not impose constraints

Consistent with the way Wilson loop emerges from QED calculation:



This feature will lead us to a criterion on states in kinematical state space of LQG

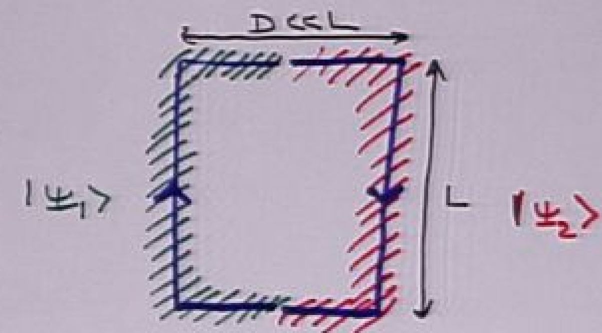
2. A SEMICLASSICALITY CRITERION FOR STATES OF QUANTUM GEOMETRY

Suppose $\mathcal{F} \subset \mathcal{H}_{kin}$ family of candidate semiclassical states in kinematical state space of LQG.

Then we should expect:

$\mathcal{F}$ contains states $|\Psi_1\rangle$, $|\Psi_2\rangle$

peaked at $(A^i_a \sim {}^2F^i_a, E^a_i \text{ flat})$; $(A^i_a \sim {}^2F^i_a, E^a_i \text{ flat})$



$$\langle \Psi_1 | \Psi_2 \rangle \simeq \exp\left[-\frac{L}{\hbar} \left(E_{self} + V_N(D) + \cdots \right) \right]$$

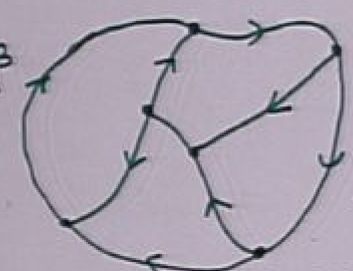
3. TEST: POLYMER PICTURE OF LINEARIZED QUANTUM GRAVITY

- Quantum configuration space: generalized $U(1)^3$ connections $\bar{A} \in \bar{\mathcal{A}}$

  e piecewise analytic edge

  $$\longmapsto \bar{A}(e) \in U(1)^3$$

- Cyl: Cylindrical functions on $\bar{\mathcal{A}}$ spanned by "flux network states"



$$\mathcal{N}_{\beta,\underline{n}}[\bar{A}] = \prod_{I \in E_\beta} [\bar{A}^1(e_I)]^{n_I^1} [\bar{A}^2(e_I)]^{n_I^2} [\bar{A}^3(e_I)]^{n_I^3}$$

where

$$\bar{A} = (\bar{A}^1, \bar{A}^2, \bar{A}^3)$$

$$\bar{A}^i \in U(1)\,; \quad i = 1,2,3$$

Ket notation: $|\beta, \underline{n}\rangle$

Inner product defined by Haar measure on $U(1)^3$.

$$Cyl \longrightarrow \mathcal{H}_{kin}$$

As in continuum theory: no constraints need to be imposed.

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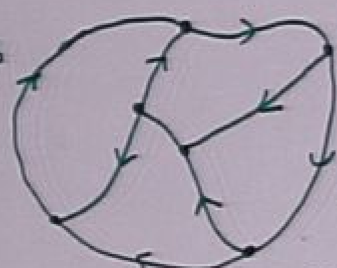

$I \quad (n_I^1, n_I^2, n_I^3)$

$$\mathcal{N}_{\beta,\underline{n}}[\bar{A}] = \prod_{I \in E_\beta} [\bar{A}^1(e_I)]^{n_I^1} [\bar{A}^2(e_I)]^{n_I^2} [\bar{A}^3(e_I)]^{n_I^3}$$

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⑫

As shown by Varadarajan:
Fock states can be located in Cyl^*

- Rewrite

$$\hat{a}_{ab}(\vec{k})\,|0\rangle = 0$$

in terms of $\begin{cases}\text{triad perturbations}\\ \text{smeared holonomies}\end{cases}$

$$\exp\left[-\frac{1}{2}\frac{G\hbar}{16\pi}\gamma^2\int d^3k\; k\,|G^a_{(r)i}[\beta,\underline{n}](\vec{k})|^2\right]$$
$$\times \exp\left[-\frac{\gamma}{8\pi}\int d^3k\; k\,\bar{G}^i_{(r)a}[\beta,\underline{n}](\vec{k})\,\hat{e}^a_i(\vec{k})\right]|0\rangle$$
$$= \hat{A}^{+}_{(r)}[\beta,\underline{n}]\,|0\rangle$$

where $\hat{A}_{(r)}[\beta,\underline{n}] = \exp\left[i\int d^3k\,\bar{G}^a_{(r)i}[\beta,\underline{n}]\,\hat{A}^i_a(\vec{k})\right]$.

and $G^a_{(r)i}[\beta,\underline{n}](\vec{x}) = \sum_{I\in E_\beta} n^i_I \int_{e_I} dt\; f_r(\vec{x},\vec{e}_I(t))\,\dot{e}^a_I(t)$

Gaussian; width r

- Replace $\begin{cases}\text{triad pert.} \longrightarrow \text{smeared triad pert.}\\ \text{smeared hol.} \longrightarrow \text{unsmeared hol.}\end{cases}$

$$(\Psi^0|\,\exp\left[-\frac{1}{2}\frac{G\hbar}{16\pi}\gamma^2\int d^3k\; k\,|G^a_{(r)i}[\beta,\underline{n}](\vec{k})|^2\right]$$
$$\times\exp\left[-\frac{\gamma}{8\pi}\int d^3k\; k\,\bar{G}^i_{(r)a}[\beta,\underline{n}](\vec{k})\,\hat{e}^a_{(r)i}(\vec{k})\right]$$
$$= (\Psi^0|\,\hat{A}^{+}[\beta,\underline{n}]$$

- Solution:

$$(\Psi^0| = \sum_{\gamma, \underline{n}} c^0_{\gamma, \underline{n}} \langle \gamma, \underline{n}|$$

where

$$c^0_{\gamma, \underline{n}} = \exp\Big[-\tfrac{1}{2} \sum_{I,J \in E_\gamma} G_{IJ}\, n^i_I\, n^i_J\Big]$$

with

$$G_{IJ} = \frac{G\hbar}{k\pi}\, \gamma^2 \int d^3k\, k\, \bar{\tilde{F}}_I(\vec{k}) \cdot \tilde{F}_J(\vec{k})$$

$\tilde{F}^a_I$ smearing for edge I

- Similarly: locate any coherent state in Cyl^*

$$(\Psi_{(A \sim F, e=0)}|$$

$$= \sum_{\gamma, \underline{n}} \exp\Big[-\tfrac{1}{2} \sum_{I,J \in E_\gamma} G_{IJ}\, n^i_I\, n^i_J + \tfrac{1}{2}\, i G \gamma M \sum_{I \in E_\gamma} \int d^3x\, F_{(r)ab}\, \tilde{F}^a_I\, \delta^b_i\, n^i_I\Big] \langle \gamma, \underline{n}|$$

Shadow states

- $\langle\Psi| = \sum_{\gamma} \sum_{\underline{n}} c^{o}_{\gamma,\underline{n}} \langle\gamma,\underline{n}| \in Cyl^*$

pick particular graph β.

Shadow of $\langle\Psi|$ on β:

$$|\Psi\rangle = \sum_{\underline{n}} c^{o}_{\beta,\underline{n}} |\beta,\underline{n}\rangle$$

- Don't get cylindrical function by varying graph, but all shadow states together capture full information in $\langle\Psi|$.

- Can do this for any Fock state

- Here: specialize to shadow states on regular cubic lattices

$$\begin{cases} \text{lattice size } N \\ \text{edge length } \ell \end{cases}$$

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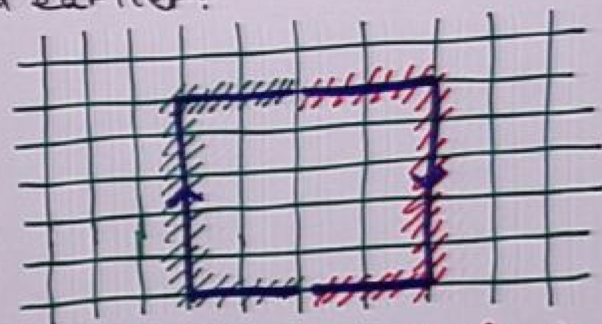
Shadows of particular coherent states we had earlier:



$|\Psi_1^{\beta_N, \ell}\rangle$ peaked at $(A_{ab} \sim F^1_{ab}, e_{ab} = 0)$

$|\Psi_2^{\beta_N, \ell}\rangle$ peaked at $(A_{ab} \sim F^2_{ab}, e_{ab} = 0)$

Then

$$\lim_{N \to \infty} \langle \Psi_1^{\beta_N, \ell} | \Psi_2^{\beta_N, \ell} \rangle$$

$$= \exp\left[- \frac{4\pi G M^2}{\hbar} \int_{C_\ell} \frac{d^3k}{k} \bar{F}^{ab}_{(r)}(\vec{k}) F_{(r)ab}(\vec{k}) \right]$$

where $F^{ab}_{(r)} = F^{ab}_{1(r)} + F^{ab}_{2(r)}$

and C_ℓ cube in momentum space with linear size $\frac{2\pi}{\ell}$

Next, $\ell \to 0$

$$\lim_{\substack{N \to \infty \\ \ell \to 0}} \langle \Psi_1^{\beta_N, \ell} | \Psi_2^{\beta_N, \ell} \rangle = \exp\left[- \frac{4\pi G M^2}{\hbar} \int \frac{d^3k}{k} \bar{F}^{ab}_{(r)}(\vec{k}) F_{(r)ab}(\vec{k}) \right]$$

→ Fock space result!

So,

$$\lim_{\substack{N\to\infty \\ \ell\to 0}} \langle \Psi_1^{\beta_{N,\ell}} | \Psi_2^{\beta_{N,\ell}} \rangle$$

$$= \exp\left[-\frac{4\pi G M^2}{\hbar} \int \frac{d^3k}{k} F_{(r)}^{ab}(\vec{k}) F_{(r)ab}(\vec{k}) \right]$$

$$= \exp\left[-\frac{L}{\hbar} \left(E_{self} + V_N(D) + \cdots \right) \right]$$

Hence, we retrieve Newton potential from linearized QG in polymer picture.

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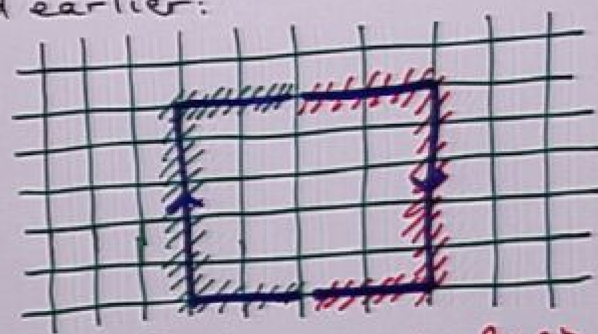

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→ **Fock space result!**

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$$= \exp\left[-\frac{4\pi G M^2}{\hbar} \int \frac{d^3k}{k} \bar{F}^{ab}_{(r)}(\vec{k}) F_{(r)ab}(\vec{k}) \right]$$

$$= \exp\left[-\frac{L}{\hbar} \left(E_{self} + V_N(D) + \cdots \right) \right]$$

Hence, we retrieve Newton potential from linearized GR in polymer picture.

4. SUMMARY

- Reformulated Wilson loop order parameter from continuum QFT to obtain semiclass. criterion for states of quantum geometry
- Have good control over polymer picture of Linearized QG ;
  criterion works very well there
  → Newton potential from polymer lin. grav.

NEXT:

- Do any of the states already proposed satisfy the criterion in Elton of LQG?
- Is there fuller sense in which semiclass. sector of LQG is like Coulomb phase?