Title: Semi Classical Quantization of Space Times with Event Horizons

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Abstract:

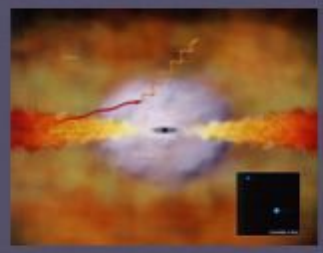
Quantum Mechanical Entropy

- Quantum mechanics, entropy arises from a loss of information about a part of the system
- Or the system is abinitio in a mixed thermodynamical ensemble, microcanonical, canonical or even grand canonical

Both the above interpretations point towards tracing over some degrees of freedom.

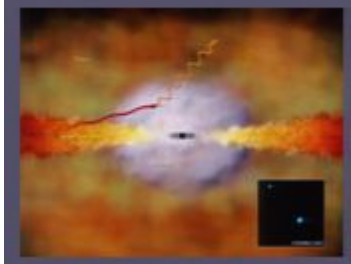
horizon entropy seems obviously associated with the causal structure, or the tipping of the light cone whereby the space-time beyond the horizon is lost to the outside observer.

Causal structures are classical



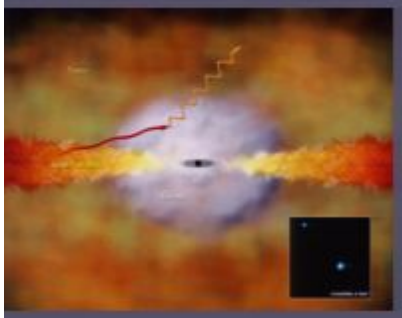
- The light cone is a classical concept.
- Quantum mechanically there is at most a fuzzy generalisation
- Why should the classical boundary conditions (be it apparent, isolated event horizon) be imposed on the quantum mechanical wavefunction?
- In particular Hawking radiation suggests a tunneling even at the semiclassical approximation,

Which defies the classical one way causal flow.



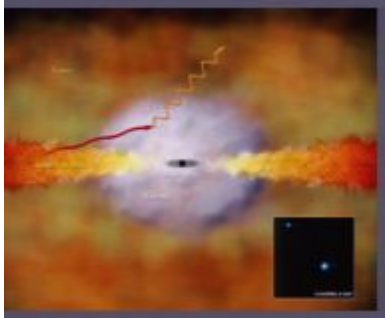
Classical horizons

- Entropy should be measured for classical horizons
- The loss of information is as observed by a classical observer, for whom the space-time beyond the horizon is totally disconnected.
- **The origin of entropy must be from semi-classical states: Counting the number of ways to build a classical horizon starting from a quantum mechanical set up**



Semiclassical States

- Coherent states, in which the expectation values of the operators are closest to their classical values.
- Coherent states in ***non-perturbative*** gravity framework, as horizons and causal structures emerge in that limit.



Coherent states

- In quantum mechanics coherent states are in the Hilbert space with minimum uncertainty
- When the Hilbert space is restricted to these states classical dynamics is recovered!
- A quantum mechanical action principle

$$I_{quantum} = \int [\ < \psi \mid i \frac{d}{dt} \mid \psi > - < \psi \mid H \mid \psi > \ ] dt$$

$$\frac{\delta I}{\delta \psi} = 0 \Rightarrow \frac{i \partial}{\partial t} |\psi\ \rangle = H\ |\psi\ \rangle$$

Schrodinger's Equation

Coherent states

$$|\psi\rangle = e^{-iqP} e^{ipQ} |0\rangle$$

$$I_{resquantum} = \int [p\dot{q} - < H >] dt$$

Classical Equation

Coherent states in gravity

- Defined for Canonical Sen-Ashtekar-Barbero-Immirzi variables using the Hall transform
- The classical limit of these variables are also

defined on graphs, with edges and dual 2 - dimensional surfaces.

$$h = P\exp(i\int A)$$

$$\mathrm{P}_{\mathrm{e}}^{\mathrm{I}} = -\frac{1}{a}\mathrm{Tr}\left[\mathrm{T}^{\mathrm{I}}\mathrm{h}_{\mathrm{e}}\int {}^{*}E\,\mathrm{h}_{\mathrm{e}}^{-1}\right]$$

$$\psi^{t}\left(gh^{-1}\right) = \rho^{t}(h)_{h\to g}$$

Analytically continued Heat Kernel

$$g_{e} = \exp\left(-iP^{I}T^{I}\right)\mathrm{h}_{\mathrm{e}}(A)$$

Complexified SL(2,C) label

The quantum gravity action principle

- The quantum gravity action principle does not exist.
- The Wheeler Dewitt equation, is derived from the classical action principle and as such does not qualify as a ab-initio Schrodinger equation.
- Further the coherent states derived for canonical gravity are kinematical.

The kinematic action principle

Retaining only the Kinetic term

$$I_{quantum} = \int \langle \psi | \frac{id}{dt} | \psi \rangle$$

Restricting now to the Coherent state sector, does one obtain a classical action principle? This equation will be true for all SU(2) coherent states,

And for one copy of SU(2) for gravity, i.e. the coherent state for one edge.

$$|\psi\rangle \equiv \sum_j d_j e^{-\frac{tj(j+1)}{2}} \chi_j(gh^{-1})$$

$$I_{resquantum} = ?$$

Kinematic action principle..

- Normalised wavefunction

$$\langle\psi|\frac{d}{dt}|\psi\rangle=\int d\mu(h)\frac{\dot{\lambda}}{\lambda}\left[\left(\frac{\lambda+\lambda^{-1}}{\lambda-\lambda^{-1}}\right)\overline{\psi}\psi+\overline{\psi}\left(\sum_j(2j+1)^2e^{-tj(j+1)}\chi_j\right)\right]-\frac{1}{\|\psi\|}\frac{d}{dt}\|\psi\|$$

$$I_{resquantum}=\int dtTr\left[T^I\ h_e^{-1}\dot{h}_eP_e^I\ \right]\qquad\chi_j(gh^{-1})=\frac{\lambda^{2j+1}-\lambda^{-2j-1}}{\lambda-\lambda^{-1}}$$

Precisely the kinetic term for the classical action

Although not obvious, this confirms the fact that the coherent states contain informatiom about the classical symplectic structure, and the quantum action principle should be applicable to all kinematic states in LQG, and perhaps, there indeed exists a Schrodinger equation for the QG wavefunction

Coherent state structure….

- The state thus can describe any classical space-time, except that the SU(2) valued classical label will be different.
- Also, due to the structure of the wavefunction, it is unclear the classical labels g are continuous across the edges.
- Hence this classical continuity needs to be imposed as an extra condition.

The classical phase space of a spherically symmetric space-time

- A graph
- A dual graph
- Corresponding holonomy and momentum
- For spherically symmetric space-times, the time slices are chosen such that the intrinsic metric is flat. Thus, for any generic space-time, the graph has to be on a manifold with

$$ds^2 = dr^2 + r^2 d\Omega$$

Schwarzschild Black Hole

- The extrinsic curvature will isolate specific metrics. For the Schwarzschild black hole, the time slices are from the metric

$$ds^2 = -d\tau^2 + \frac{dR^2}{\left[\frac{3}{2r_g}(R-\tau)\right]^{2/3}} + \left[\frac{3}{2}(R-\tau)\right]^{4/3} r_g^{2/3}(d\theta^2 + \sin^2\theta d\phi^2)$$

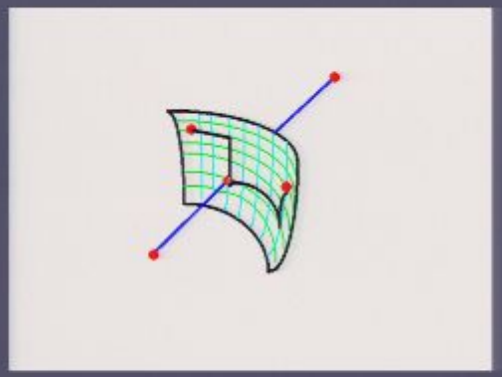
$$R - \tau = \frac{2}{3} r_g, \qquad R - \tau = 0$$

$$r_g = \frac{2GM}{c^2} \quad , c, \hbar = 1$$

$$\text{Extrinsic Curvature} : K = q^{ab} K_{ab} = -\frac{3}{2}\sqrt{\frac{r_g}{r^3}}$$

The momentum

$$P_e^I = -\frac{1}{a}\,\mathrm{Tr}\left[T^I h_e \int * E\, h_e^{-1}\right]$$



$$\alpha = \sqrt{\frac{r_g}{r}},\quad \gamma = \frac{\delta}{r_g}$$

$$P^1 = X_1 \quad P^2 = \frac{X_3}{\sqrt{\alpha^2+1}}\left[\alpha \sin(\gamma\alpha^3) + \cos(\gamma\alpha^3)\right]$$

$$P^3 = \frac{X_3}{\sqrt{\alpha^2+1}}\left[-\alpha\cos(\gamma\alpha^3) + \sin(\gamma\alpha^3)\right]$$

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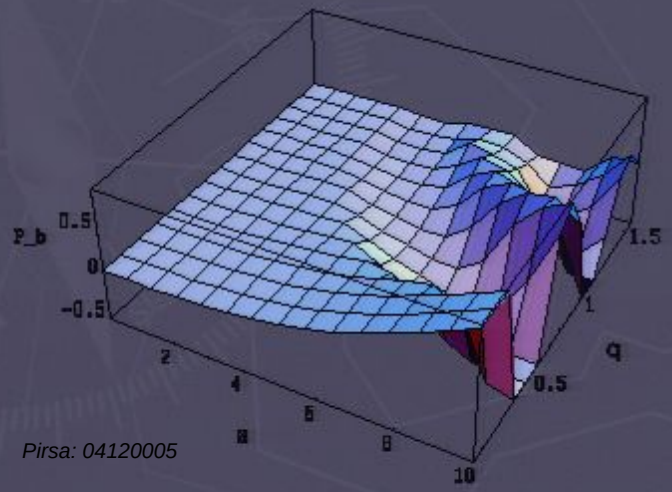

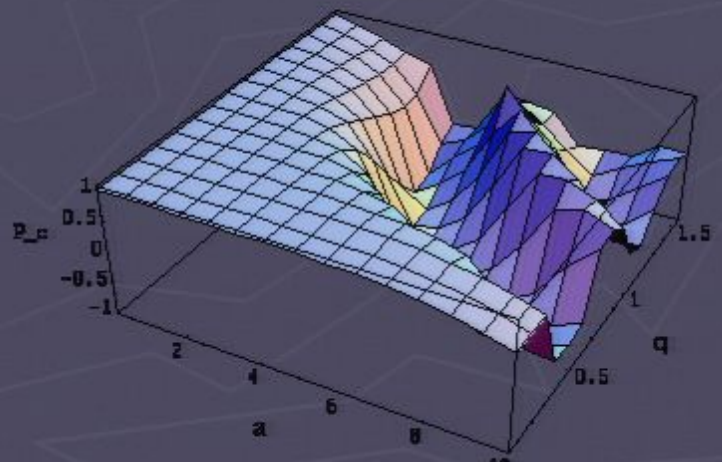

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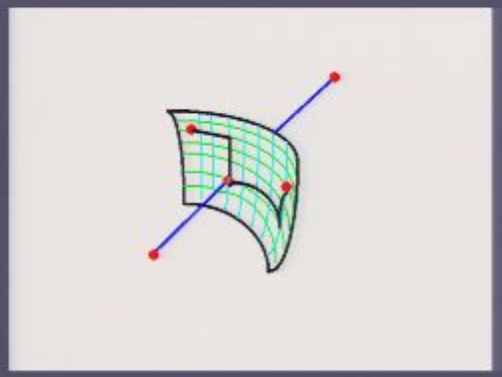

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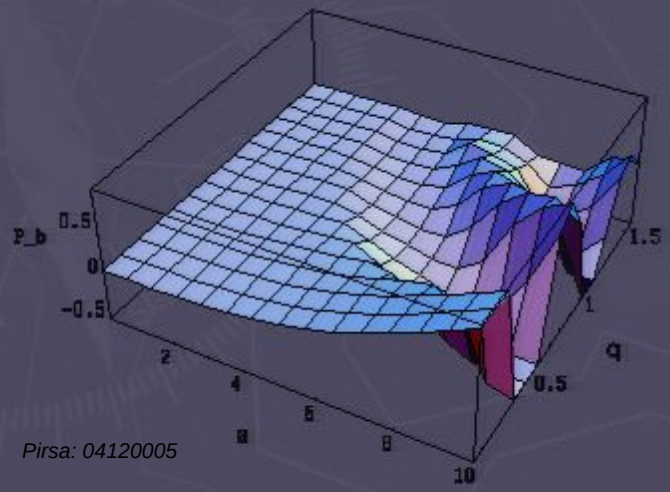
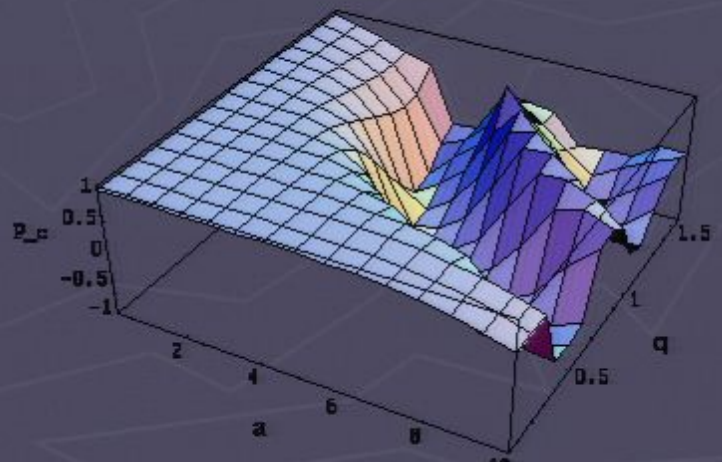
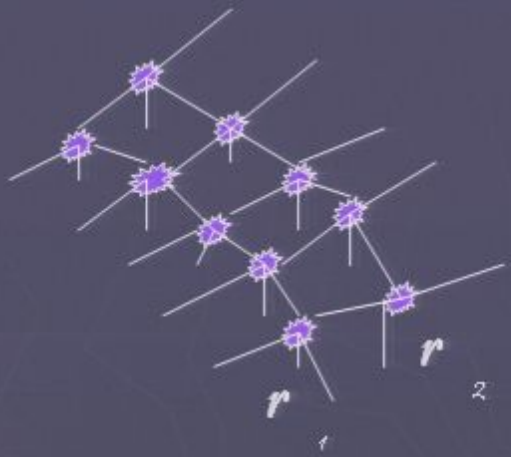




The holonomy



The radial holonomy

$$h_{er} = \cos\left(\frac{\sqrt{r_g}}{2\sqrt{r_1}} - \frac{\sqrt{r_g}}{2\sqrt{r_2}}\right) - i\sigma^1 \sin\left(\frac{\sqrt{r_g}}{2\sqrt{r_1}} - \frac{\sqrt{r_g}}{2\sqrt{r_2}}\right)$$

A regularisation of the singularity

$$\cos\left(\frac{\sqrt{r_g}}{2\sqrt{r_1}}\right)\cos\left(\frac{\sqrt{r_g}}{2\sqrt{r_2}}\right) + \sin\left(\frac{\sqrt{r_g}}{2\sqrt{r_1}}\right)\sin\left(\frac{\sqrt{r_g}}{\sqrt{r_2}}\right)$$

Take values from -1...1 as $r_{1(2)} \to 0$

The angular holonomy

$$h_{e_\theta} = \left[\frac{\sqrt{\alpha^2+1}}{2\sqrt{2}}(\theta_0 - \theta)\right] - \frac{i}{\sqrt{\alpha^2+1}}(\alpha\sigma^2 + \sigma^3)\sin\left[\frac{\sqrt{\alpha^2+1}}{2\sqrt{2}}(\theta_0 - \theta)\right] \qquad \alpha = \sqrt{\frac{r_g}{r}}$$

The momentum

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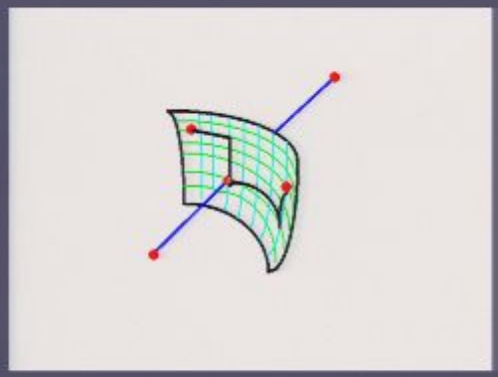

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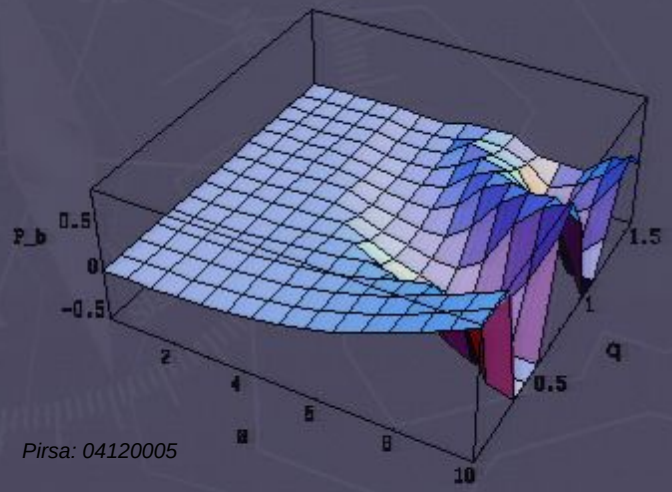
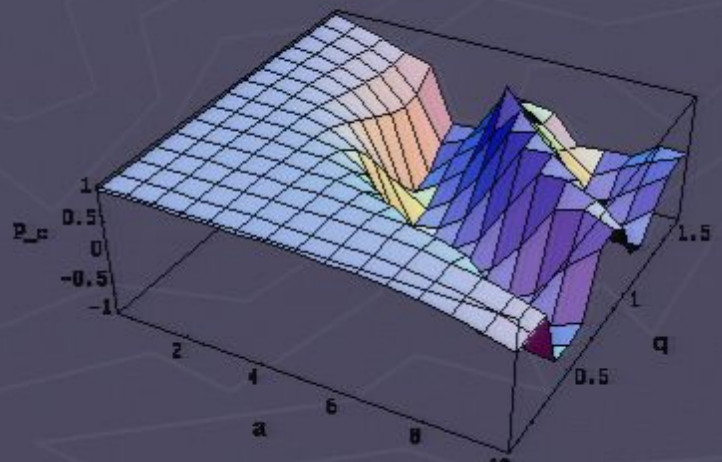


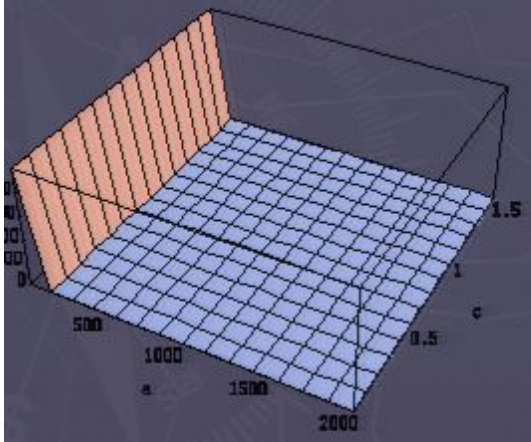


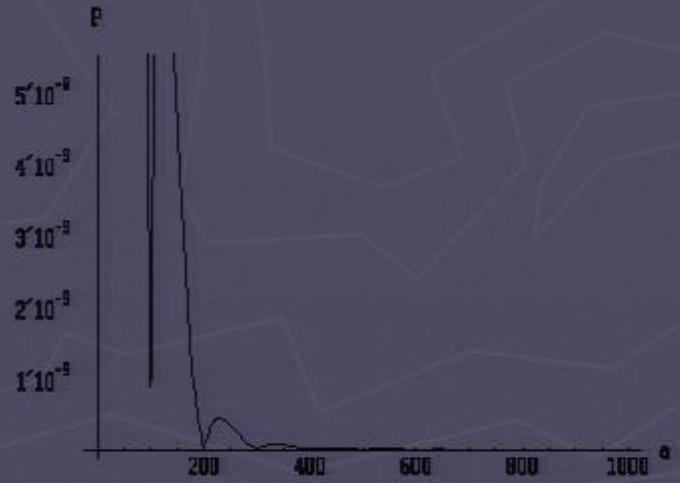
The gauge invariant momentum and area

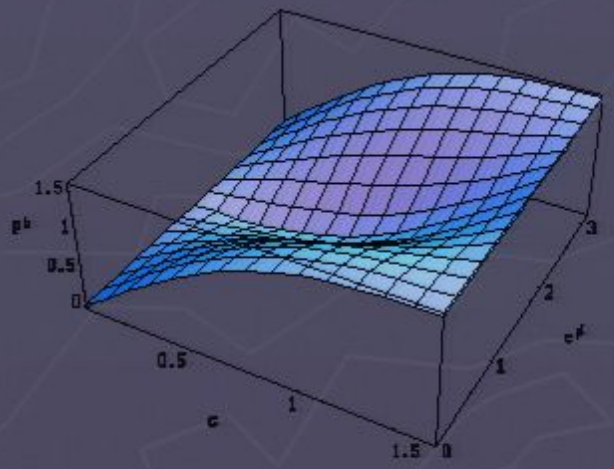
$$P=\sqrt{P^{I}P^{I}}$$

$$=\sqrt{X_1^{\,2}+X_2^{\,2}}$$

$$=\frac{r_g^{\,2}}{a\alpha^4}\left[\left(\frac{\sin(1-\alpha')\theta}{(1-\alpha')}\right)^2+\left(\frac{\sin(1+\alpha')\theta}{(1+\alpha')}\right)^2-2\cos(2\theta_0)\frac{\sin(1-\alpha')\theta}{(1-\alpha')}\frac{\sin(1+\alpha')\theta}{(1+\alpha')}\right]^{1/2}$$







The Coherent state and Area operator

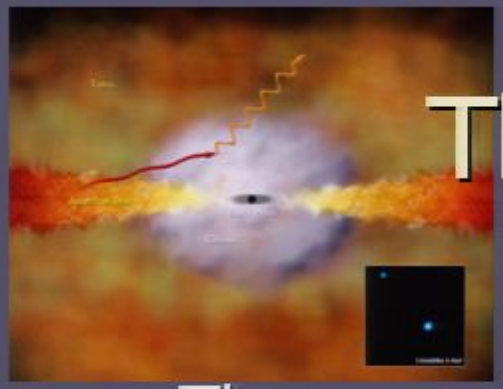
$$\hat{P}\left|jmn\right\rangle = \sqrt{j(j+1)t}\left|jmn\right\rangle$$

$$\left\langle jmn\left|\psi\right.\right\rangle = \exp\left(-tj(j+1)/2\right)\pi_j(g)_{mn}$$

$$\left\langle\psi\left|\hat{P}\right|\psi\right\rangle = \sum_j \sqrt{j(j+1)t}\,\overline{\pi}_j(g)_{mn} e^{-tj(j+1)}\pi_j(g)_{mn}$$

$$= \sum_j [(j+\frac{1}{2}) - \frac{1}{8(j+\frac{1}{2})^2} + \ldots.] \exp(-\frac{1}{t}((j+1/2)t-P)^2)$$

$$= \left(j_{cl}+\frac{1}{2}\right)t \qquad t\to 0$$



The Area Spectrum

- The area spectrum for j>> 1 equals =(j+1/2)t
- *Of course, in this limit, the area spectrum is almost continuous, and equals j t.*
- However, irrespective of the magnitude of P, the coherent state is peaked at P=(j+1/2)t, and hence the *classical area* seems to be the expectation value of a corrected self adjoint operator

$$\hat{A} = \sqrt{\hat{P}^2 + \frac{1}{2}}$$

$$\langle \psi | \hat{A} | \psi \rangle = P_{classical}$$

In the limit $t \to 0$

The curvature operator

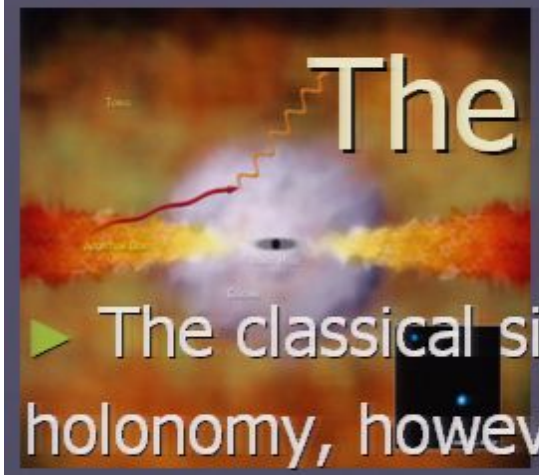
- The Riemann curvature tensor square

iverges at the center of the black hole, for flat spatial slicing
is is given by

$$R_{\mu\nu\lambda\sigma}R^{\mu\nu\lambda\sigma}$$

$$2\left[K^4 - K_{bc}K^{ac}K^d_a K^b_d\right]$$

ow, the extrinsic curvature is a function of the gauge
onnection, and the inverse triads. One option would be to
easure the expectation value of the holonomy, and then
valuate the classical curvature, using the regularised
ariables.



The classical singularity

- The classical singularity is regularised in the holonomy, however, taking the edge length to 0, it reappears.

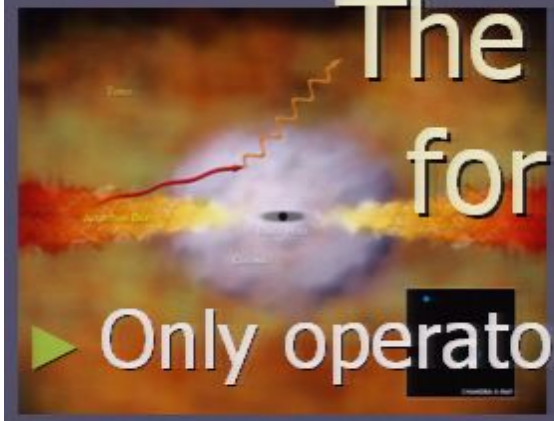
$$K_{rr} = A_r^I e_r^I$$

Obviously, if one measures holonomy and writes the Gauge connection as derived from there, and similarly for the inverse triad, one obtains:

$$A_r^I = -\frac{i}{r_1 - r_2}\mathrm{Tr}\left[\mathrm{T}^{\mathrm{I}}(h_{e_r} - 1)\right]$$

$$K_{rr} = \frac{\sqrt{r_g}}{\sqrt{r_1 r_2}\left(\sqrt{r_1} + \sqrt{r_2}\right)}$$

Diverges as $r \to 0$



The regularised operator for extrinsic curvature

- Only operators measured are h, P.

A certain regularisation of the extrinsic curvature operator gives the extrinsic curvature, with some extra terms which→0 at the singularity is the following

$$K_{ab} = \frac{1}{e_a(0) - e_a(1)} \mathrm{Tr}\left[h^{-1}_{e_b} \{ h_{e_b}, V \} h_{e_a} \right]$$

In terms of this, the curvature operator can be explicitly written also using a point Splitting method due to Thiemann as

$$\sqrt{g} R^2 = \left[\frac{1}{3} \varepsilon^{IJK} \varepsilon^{abc} \mathrm{Tr}\left[h^{-1}_{e_a} \{ h_{e_a}, V^{1/10} \} \right] \mathrm{Tr}\left[h^{-1}_{e_b} \{ h_{e_b}, V^{1/10} \} \right] \mathrm{Tr}\left[h^{-1}_{e_c} \{ h_{e_c}, V^{1/10} \} \right] \right.$$
$$\left. \times \mathrm{Tr}\left[h^{-1}_{e_d} \{ h_{e_d}, V^{1/10} \} h_{e_e} \right] \mathrm{Tr}\left[h^{-1}_{e_f} \{ h_{e_f}, V^{1/10} \} h_{e_g} \right] P^d_M P^e_M P^f_N P^g_N \right]^2$$

Towards Resolution…

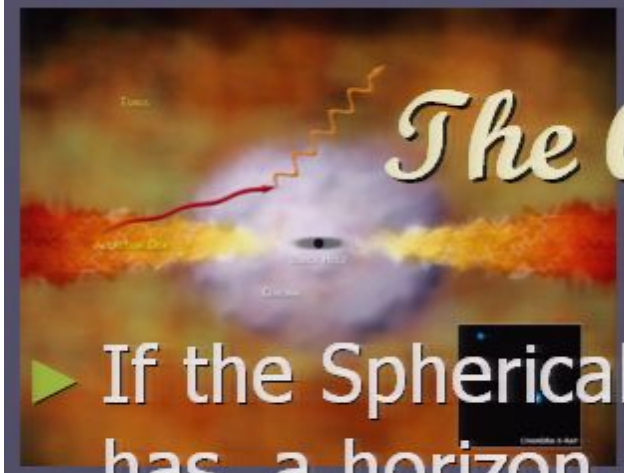
- Observation, classically space is spherically Symmetric, and hence:

$$V = P_{e_r}\left(e_r(1) - e_r(0)\right) \Rightarrow \langle\psi|\hat{V}|\psi\rangle \propto \langle\psi|\hat{P}|\psi\rangle$$

$$\langle\psi|\hat{R}|\psi\rangle \propto f(h^{cl}{}_{e_a}, \langle\psi|\hat{P}|\psi\rangle)$$

Bounded operator as $\langle\hat{P}\rangle \geq \frac{1}{2}t$

A manifestation of the minimum uncertainty principle



The Apparent Horizon

- If the Spherically symmetric spatial slice has a horizon, then the metric variables are restricted by the apparent horizon equation there (*no global information to detect the event horizon or isolated horizons*)

$$\nabla_a S^a + K_{ab} S^a S^b - K^2 = 0$$

In the spherically symmetric metric, the horizon S^2 will have a spatial normal (1,0,0)

$$-\Gamma^{\theta}_{\theta r} - \Gamma^{\phi}_{\phi r} + K_{rr} - K = 0$$

$$K = q^{ab} K_{ab} = q^{rr} K_{rr} + q^{\theta\theta} K_{\theta\theta} + q^{\phi\phi} K_{\phi\phi}$$

Since the intrinsic metric is diagonal,

$$K_{rr}\left(1 - q^{rr}\right) = \Gamma^{\theta}_{\theta r} + \Gamma^{\phi}_{\phi r} + q^{\theta\theta} K_{\theta\theta} + q^{\phi\phi} K_{\phi\phi}$$

The LHS is trivially 0 at every point in spatial slice

Horizon graph

- The apparent horizon equation imposes no extra condition on the radial edges crossing the horizon.

The graph at the horizon comprises solely of edges e_H peaked at radial edges crossing the classical horizon surfaces, which also induce it with area.

$$\sum_{e_H}(j_H+\frac{1}{2})\frac{l_p^2}{a}=\frac{A_H}{a}$$

In the final constraint, the undetermined constant a cancels !

The Schwarzschild observer

- If an observer static at infinity decides to slice the space-time, using the same flat slices, then there are no changes in the expressions for the holonomy or the momentum, except that the observer's time is infinite at the apparent horizon.
- This implies that the coherent state after the apparent horizon ceases to exist.

The Correlations

- The apparent horizon equation gives a difference equation for the angular edges

at the vertices immediately outside the horizon and those immediately inside.

These are *classical correlations*, and need to be added to the coherent state structure.

The gauge invariant coherent states will have additional Clebsch-Gordon coefficients.

Density Matrix

- Since, there are no correlations in the radial directions, the density matrix reduces to:

$$\rho = |\psi\rangle\langle\psi| = \prod_0 |\psi_0\rangle\langle\psi_0| \prod_I |\psi_I\rangle\langle\psi_I| \prod_{\nu\nu'} C_{\nu\nu'} |\psi_\nu\rangle |\psi_{\nu'}\rangle\langle\psi_{\nu'}| \langle\psi_\nu| C^*_{\nu\nu'}$$

$$\rho = \prod_{\nu\nu'} \rho^{\nu\nu'} = \prod_{\nu\nu'} C_{\nu\nu'} |\psi^o_\nu\rangle |\psi_{\nu'}{}^H\rangle |\psi^i_{\nu'}\rangle\langle\psi^i_{\nu'}| \langle\psi^H_{\nu'}| \langle\psi^o_\nu| C_{\nu\nu'}{}^*$$

The matrix elements of the density matrix are evaluated in a orthonormal basis

$$\prod_e \left| j_e \; m_e \; n_e \right\rangle$$

Since,

$$\left\langle jmn \middle| \psi_e \right\rangle = e^{-tj(j+1)/2} \pi_j (g_e)_{mn}$$

With appropriate normalisations

Reduced Density Matrix

$$\rho = \sum_{ijkl} d_{ij} d^{*}{}_{kl} |i\rangle |j\rangle\langle k|\langle l|$$

$$Tr_{kl}\rho = \sum_{ijl} d_{ij} d_{jl}{}^{*} |i\rangle\langle l|$$

$$Tr(\rho_{red}) = 1$$

The internal direction trace here naturally gives the delta function peak, and The sum collapses to the jel

$$Tr_{I}\rho \propto \exp(-((j+1/2)t - P_{e_I})^2 / t)$$

The Matrix elements

Typical matrix elements are of the form

$$\psi_{(j)(\tilde{j})} = \frac{1}{\|\psi\|}\pi_{(j)}(g)e^{-tj(j+1)/2}\pi_{(\tilde{j})}(\bar{g})e^{-\tilde{t}\tilde{j}(\tilde{j}+1)/2}\, C_{(j)(j_a)}C^{*}_{(j_a)(\tilde{j})}$$

For large value of p, which is a semiclassical limit for the horizon, the off diagonal terms Go to zero due to a factor of $e^{-\frac{p^2}{t}}$ from the normalisation

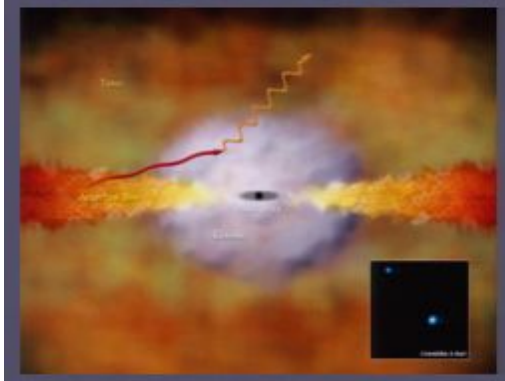
The diagonal elements in the limit t -->0 are delta function peaked at particular j(cl), and hence, one obtains:

$$\rho_{(j_{cl})(j_{cl}} = \left|c_{(j_{cl})(j_{cl})}\right|^2$$

Only at the horizon, the degeneracy in m and n contribute, as the states are free, And only observable is the induced area at the horizon, thus

$$Tr(\rho)=1 \Rightarrow \left|c_{(j_a)(j_a)'}\right|^2 = \frac{1}{2j_{cl}+1}$$

The Entropy



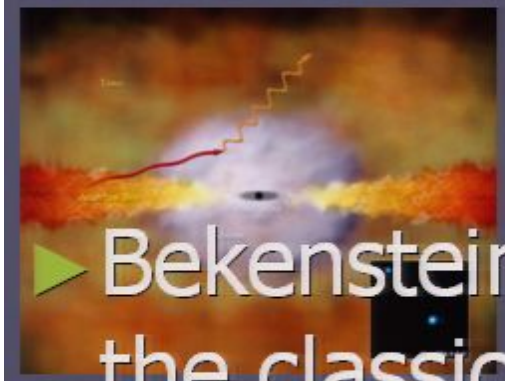
$$S = -\mathrm{Tr}\left[\rho \ln \rho\right]$$

$$= \prod \log(2j_{cl}+1) \qquad \sum (j_{cl}+\frac{1}{2}) l_p^2 = A_H$$

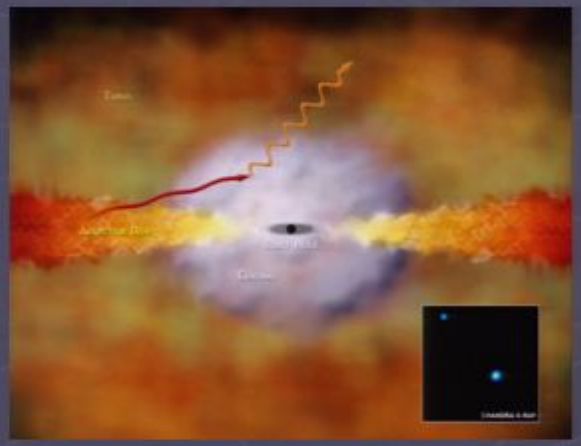
$$= \frac{A_H}{4l_p^2}\gamma$$

Bekenstein Hawking Entropy, with Immirzi parameter ambiguity

Conclusions



- Bekenstein-Hawking entropy obtained for the classical Horizon using semiclassical states
- Singularity appears to be excluded
- Hawking radiation?
- Unitary Evolution?



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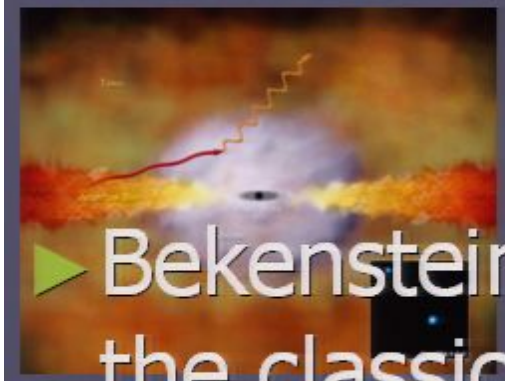
Conclusions



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