Title: On Potential Effects of Modified Dispersion Relations

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Abstract:

On Potential Effects of Modified Dispersion Relations

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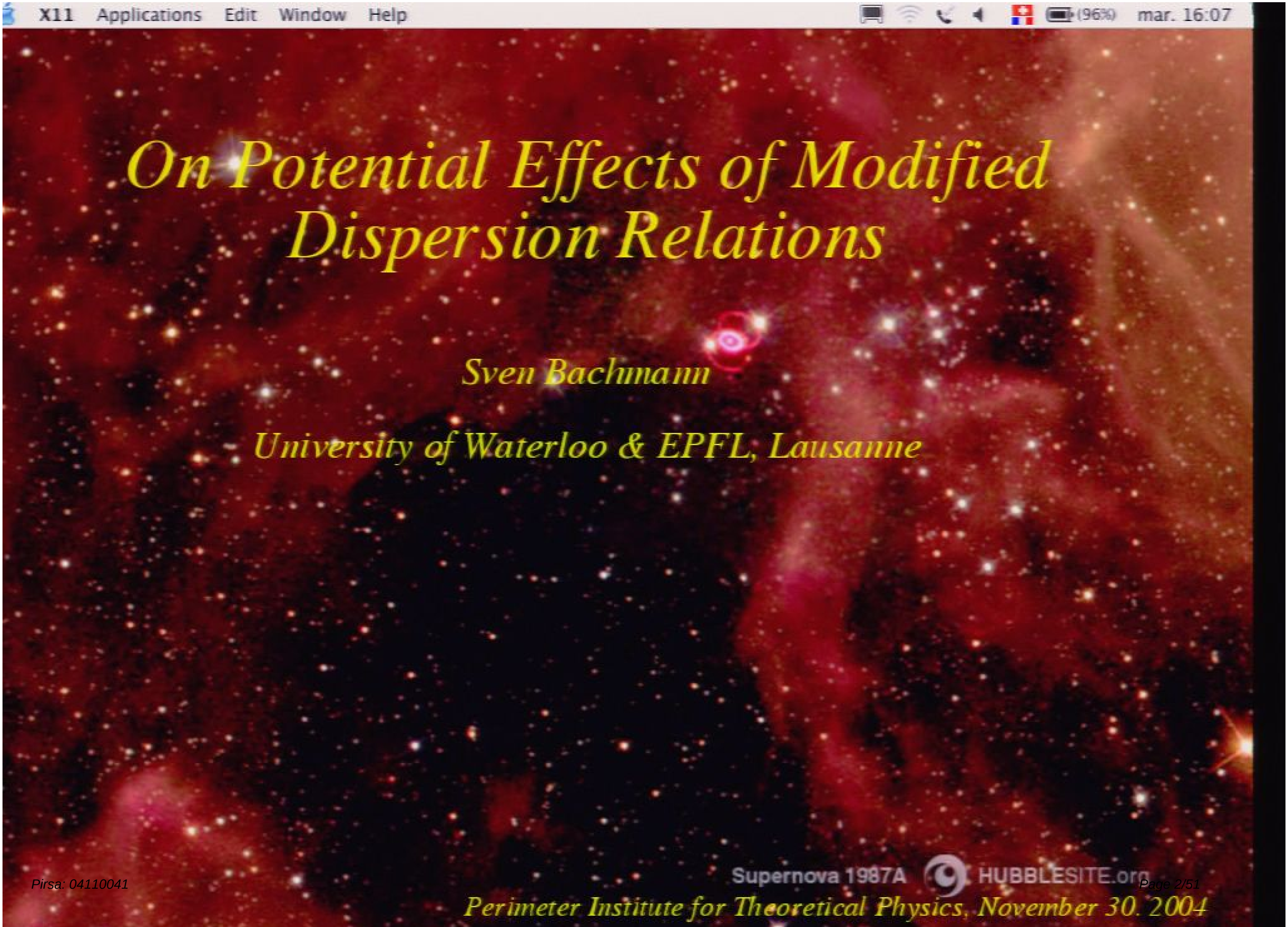

Perimeter Institute for Theoretical Physics, November 30. 2004

MODIFIED DISPERSION RELATION 1

Thought experiments & various models of quantum gravity

$\Rightarrow$ *Planck scale physics could manifest itself through <u>modified dispersion relation</u>*

Has been shown to be compatible with a generalized Principle of Relativity (e.g. Smolin)

Usual program:

- *Choose a class of modified dispersion relation*
- *Implement it in calculations involving UV physics*
- *Determine if and how the modification in UV physics affects observable predictions*

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Inflation (>200 papers)

- *Sensitive to Planck scale physics for some dispersion relation*
- *Level of sensitivity not clearly identified*
- *Backreaction problem*
- *Recently suggested: the backreaction contribution could be absorbed into a redefinition of inflaton potential (Brandenberger et al.)*

$\Rightarrow$ *Possible window to Planck scale physics!*

MODIFIED DISPERSION RELATION III

Other UV-sensitive problems:

- *Black hole radiation spectrum (Unruh, etc.)*
- *Gamma Ray Bursts (Amelino-Camelia, Sakharov & Ellis)*

BUT

- *Ad hoc choice of dispersion relation or low order corrections not general classes of dispersion relation*
- *Regularization / renomalization scheme inependence?*

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MODIFIED DISPERSION RELATION IV

$\Longrightarrow$ *New program:*

- *Choose structurally simplest phenomenon involving UV physics:*

  the Casimir effect
- *Use generic classes of dispersion relation*
- *Make sure the results are independent of the regularization scheme*
- *Draw most general conclusions about UV/IR coupling*

Most probably no measurable effects, but could act as a model:

- *Find a precise mapping: dispersion relation $\Longrightarrow$ Casimir force*
- *What mathematical methods succeed?*

Canonical Field Quantization:

- *Decompose the field into Fourier modes in a box*
- *Consider each mode as a quantum harmonic oscillator*
- *Impose canonical commutation relations*

$\Rightarrow$ *Each mode fluctuates even in ground state*

$\Rightarrow$ *Infinite vacuum energy:* $E_0 = \frac{1}{2} \sum_{n=0}^{\infty} \omega_n$

Key fact: Boundary conditions change

$\Rightarrow$ *set of allowed modes change* $\Rightarrow \Delta E_0$

Casimir force $F = -\frac{\partial}{\partial L} E_0$

Model (general)

And we shall let $M \to \infty$

With the unmodified spectrum between the plates $\{k_n\}_{n=1}^{\infty} = \left\{\frac{n\pi}{L}\right\}_{n=1}^{\infty}$

But: UV modified dispersion relation: $\omega(k) = k_{pl} f\left(\frac{k}{k_{pl}}\right)$

where k_{pl} *~ scale of new physics, Planck scale*

$f(x)$*: Some generic function, such that:*

- $f(x) \approx x, x \ll 1$
- $f(x) \geq 0$

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Energy density generally still infinite $\Longrightarrow$ *UV regulator (UV cutoff)*

For simplicity, use e.g. exponential damping function

with parameter α *(must be removed later)*

$$E_0^{\text{inside}} = \frac{1}{2}\sum_{n=1}^{\infty} k_{pl}\, f\left(\frac{n\pi}{k_{pl}L}\right) e^{-\alpha k_{pl} f\left(\frac{n\pi}{k_{pl}L}\right)}$$

The energy density on the other side of the plates is

$$D_0^{\text{outside}} = \frac{k_{pl}^2}{2\pi}\int_0^{\infty} dx\, f(x)\, e^{-\alpha k_{pl} f(x)}$$

$\Longrightarrow$ *The force is thus:* $F_\alpha = -\frac{\partial}{\partial L} E_0^{\text{inside}} + D_0^{\text{outside}}$

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Fortunately we have the Euler MacLaurin formula:

$$\sum_{n=a+1}^{b} u(n) = \int_a^b u(t)\,dt + \sum_{r=0}^{k} \frac{(-1)^{r+1} B_{r+1}}{(r+1)!}\left[u^{(r)}(b) - u^{(r)}(a)\right] + \cdots$$

Note:

- *There is an integral rest*
- *If* u *is infinitely differentiable, can take the limit* $k \to \infty$
- *Careful of convergence of the series / the integral rest*
- *Careful with limit* $b \to \infty$
- $B_{2r-1} = 0,\ r > 1,$ *only the odd derivatives appear*

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Apply E.-ML here:

$$F_\alpha = \frac{k_{pl}}{2}\left[\sum t_n f'(t_n) e^{(-\alpha k_{pl} f(t_n))}\left[1-\alpha k_{pl} f(t_n)\right] + \frac{k_{pl}}{\pi}\int_0^\infty f(x) e^{-\alpha k_{pl} f(x)} dx\right]$$

Integrate by parts $\Rightarrow \sim -\int_0^\infty f(x) e^{-\alpha k_{pl} f(x)} + \cancelto{0}{\text{boundary}}$

$\Rightarrow$ *The two integrals cancel out each other!*

Remarks:

- *Analytic dispersion relation: sum to infinity*
- *Arrive at single series with Bernoulli numbers*

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Finally:

- *Having introduced a damping function, could control infinities*
- *Now, remove cutoff by taking* $(\alpha \to 0)$
- *Hence, the Casimir force is:*

$$\boxed{F = \frac{k_{pl}}{2\pi L} \sum_{r=1}^{\infty} (-1)^r (2r-1) \left(\frac{1}{2 k_{pl} L} \right)^{2r-1} \zeta(2r) f^{(2r-1)}(0)}$$

Encodes all possible information!

In the linear case: $F_{Cas} = -\frac{\pi}{24 L^2}$ *ok!*

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Example: analytic dispersion relation $f(x)=\sum \nu_s x^s$

We get $F \sim \frac{1}{L}\sum_r (-1)^r \left(\frac{L_{pl}}{L}\right)^{(2r-1)} (2r-1)!\,\nu_{2r-1}$

Important features:

- *Asymptotically goes to zero as* $L \to \infty$
- *This has alternating signs*
- *The coefficients grow factorially faster*
- $L_{pl}/L \sim 10^{-27}$ *If L is a measurement scale and* L_{pl} *at Planck scale*

so this ratio decreases rapidly as r *grows*

$\Rightarrow$ *In order to have non zero radius of convergence, we need to have* $\nu_{2r-1} \sim [(2r-1)!]^{-1}$ *when* $r \to \infty$ *(at least)*

$\Rightarrow$ *Competing effects in r: polynomial decrease vs factorial blowup*

Example: polynomial dispersion relation:

- *The sum is truncated: No convergence issues!*
  The force is defined for short L
  BUT might show new behaviour close to k_{pl}

- *Two different cases:*

i. The ν_s *are of order* $[(2r-1)!]^{-1} \Longrightarrow$ *polynomial decrease of* $F(L) \Longrightarrow$ *No measureable effects*

ii. There is at least one ν_{2r-1} *which is of order 1 and comes with a high power so that the factorial dominates* $\Longrightarrow$ *measureable effects!*
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In any case, the result is independent of the choice of cutoff function

Choose any γ_α *such that*

i. Regularizes the sum: $E_0 \sim \sum f(\xi_n)\gamma_\alpha[f(\xi_n)] < \infty$

ii. It is a cutoff function:

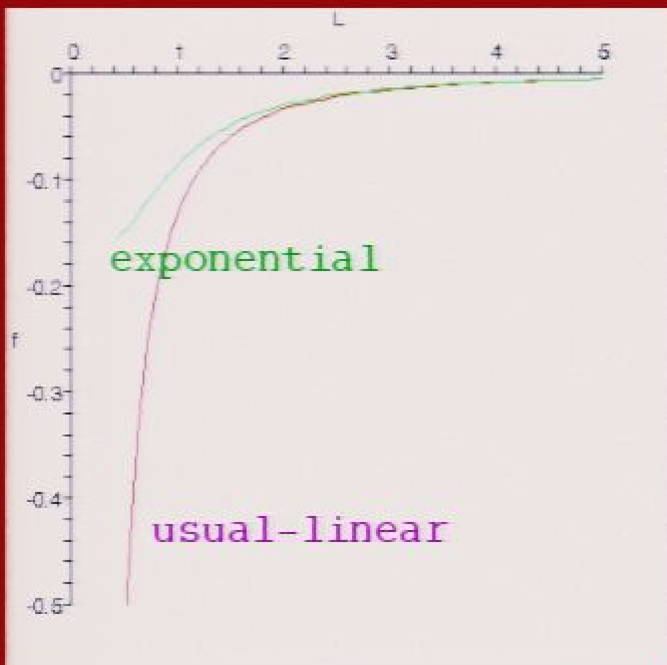
- $\gamma_\alpha \approx 1$ *when* $x \ll 1$
- $\gamma_\alpha \approx 0$ *when* $x \gg 1$
- $\gamma_\alpha \in C^\infty$ $\lim_{\alpha \to 0} \gamma_\alpha \equiv 1$

$\longrightarrow$ *Euler-MacLaurin can be used*

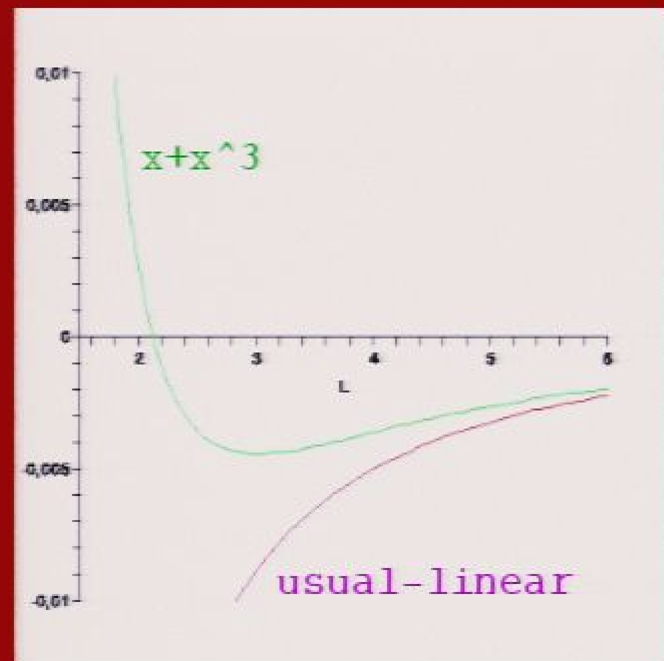
& full divergent series recovered as $(\alpha \to 0)$

Concrete examples of dispersion relations

i. Exponential



ii. Polynomial



- *Very fast convergence above the Planck length*
- *Finite radius of convergence*

$\Longrightarrow$ *suggests minimal length scale*

- *Fast convergence above* L_{pl}
- *Well defined at all distances, but new behaviour at cutoff scale*

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From a functional analysis point of view:

$$K:\{\text{dispersion relations}\}\rightarrow\{\text{Casimir forces}\}$$

$$f(k)\rightarrow F(L)$$

$$K=\frac{k_{pl}}{2\pi L}\sum(-1)^r(2r-1)\left(\frac{1}{2k_{pl}L}\right)^{2r-1}\zeta(2r)\frac{d^{(2r-1)}}{dk^{(2r-1)}}(k=0)$$

We have K ***explicitely!***

$\Longrightarrow$ *Encodes all information in a straightforward way!*

We find an ***integral kernel*** *representation:*

$$\boxed{K[f](L)=\frac{k_{pl}^2}{\pi}\Re\int_0^\infty i^{(-1)}f(i\kappa)(1-\kappa\Lambda)e^{(-\kappa\Lambda)}d\kappa}$$

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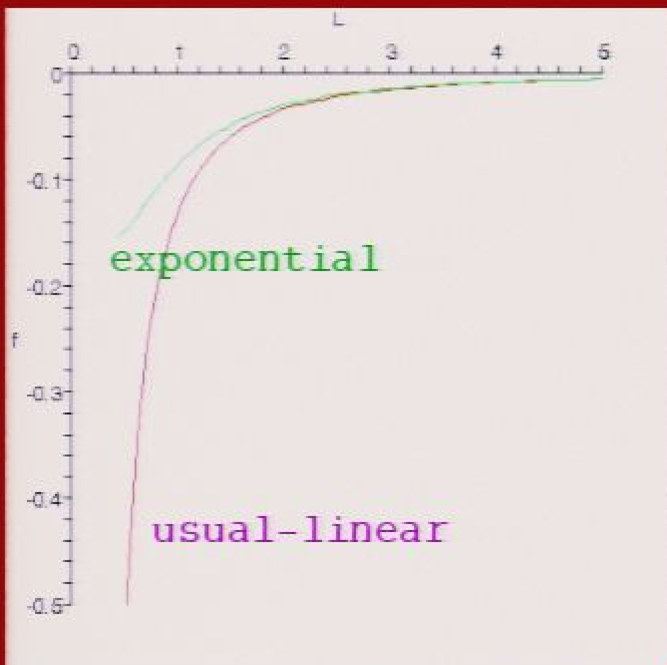

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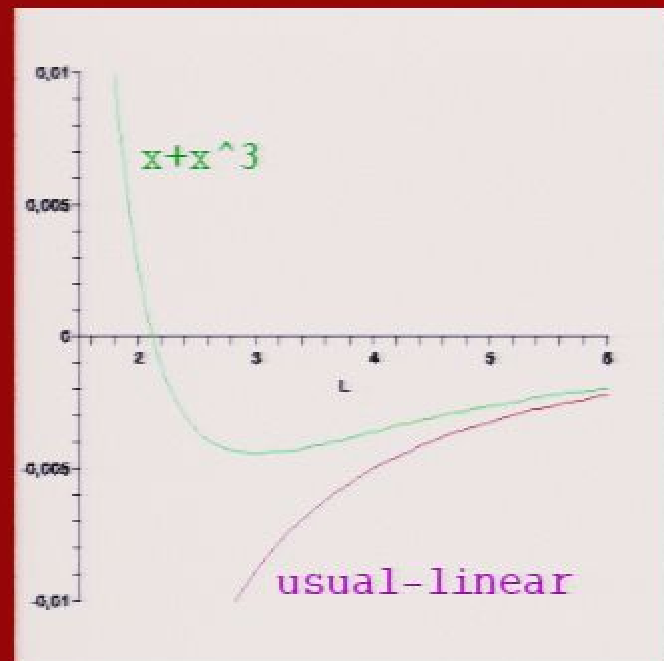

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INTEGRAL KERNEL II

- *Neglecting $\zeta(2r)$, this is exact as soon as the integral converges*
- *Essentially the Laplace transform of the Wick rotated disp. relation*
- *Downsides:*

  i. Involves analytic continuation into the complex plane

  ii. Not very intuitive

Solution: It suffices to write:

$$f(x)=xg(x^2)$$

Then:

$$K[f](L)=\frac{k_{pl}}{\pi}\int_0^\infty \kappa\, g(-\kappa^2)(1-\kappa\Lambda)e^{(-\kappa\Lambda)}d\kappa$$

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INTEGRAL KERNEL III

This expression is:

- *Much more convenient: analytic continuation in the negative reals only*
- *This can be written as an exact Laplace transform:*

$$\boxed{K[f](L) = \frac{k_{pl}}{\pi}\left[1 + \Lambda \frac{d}{d\Lambda}\right] \int_0^\infty \kappa\, g(-\kappa^2)\, e^{(-\kappa \Lambda)}\, d\kappa}$$

(Note: Analytic contintution is unique: $g(x^2)$ *determines* $g(-x^2)$

All information about the force already encoded in the physical part of the dispersion relation)

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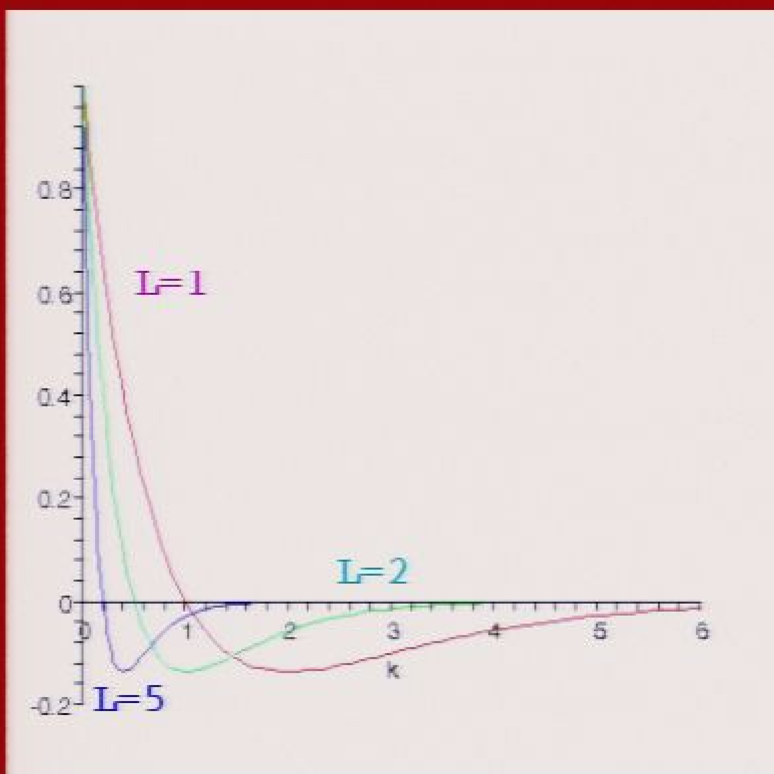
Properties of the transformation K :

- *As a Laplace transform: supresses heavily the large* k.
- *The kernel as a function of* k :



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F(L) *depends only on the energy* *difference*

ii. Changes sign at $k = \dfrac{1}{2L}$

$\Longrightarrow$ *short wavelengths tend to cancel out long ones.*

iii. As L *grows, we probe the* *long wavelengths*

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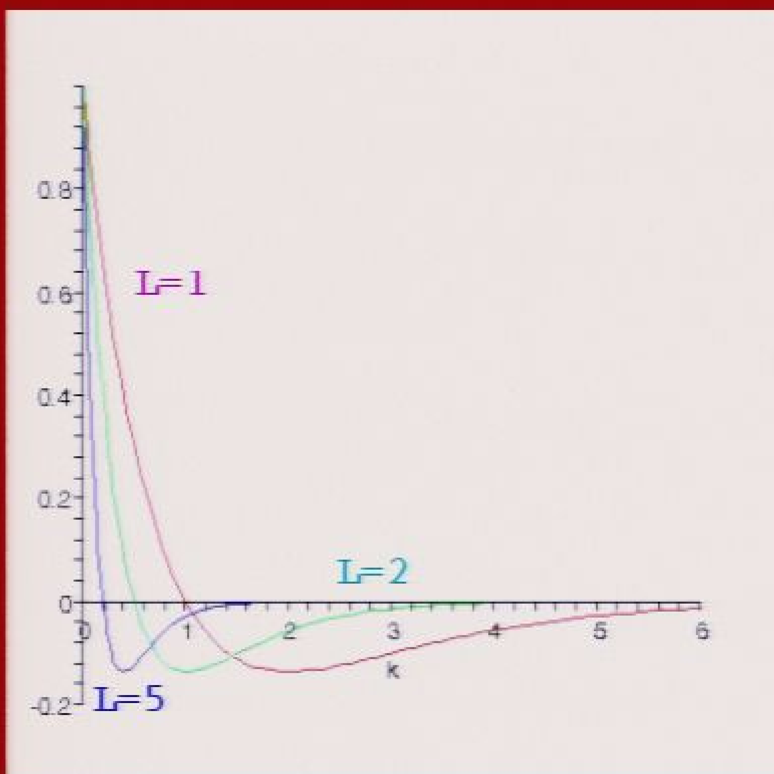

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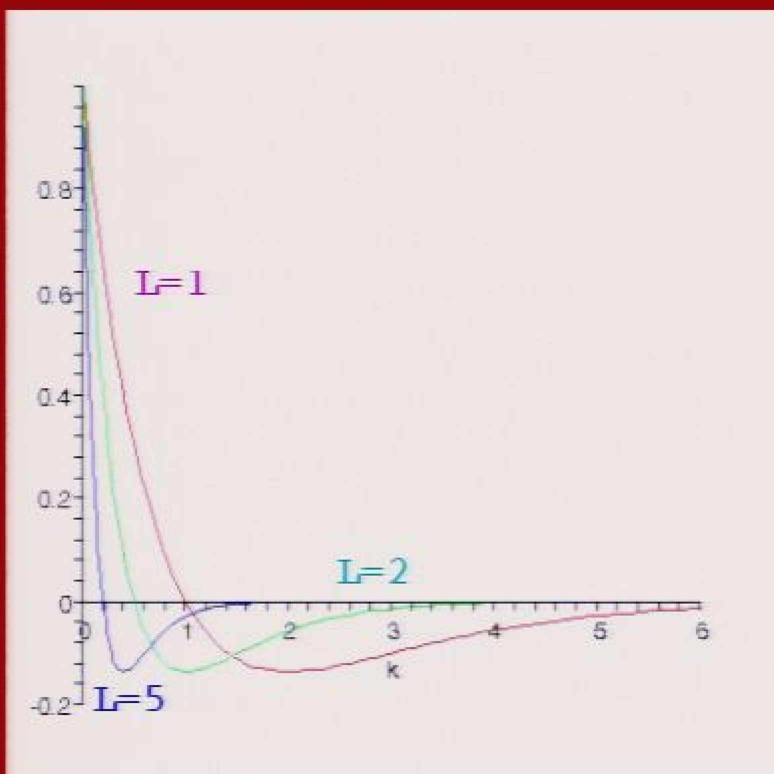

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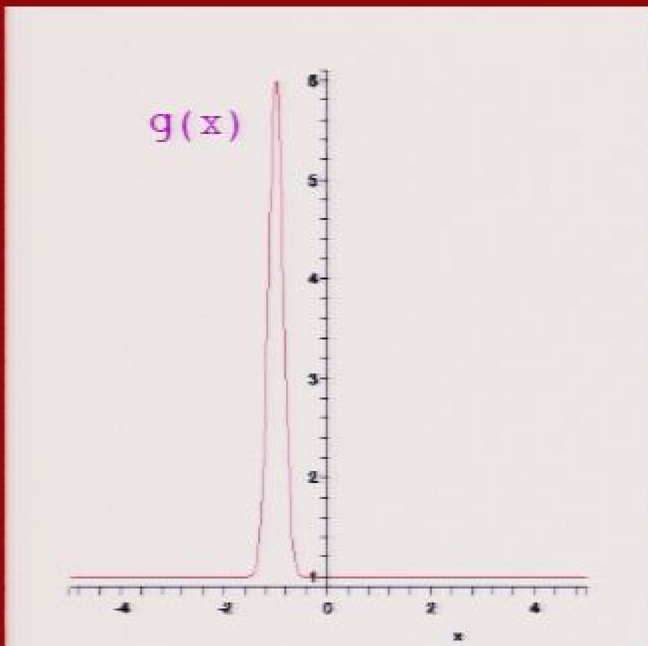
The analytic continuation is unique... But:



Recall: $f(x)=xg(x^2)$

Which translates into:

- *A dispersion relation essentially indistinguishable from the linear one*
- *A Casimir force that stays* *<u>orders of magnitude away</u>* *from the* L^{-2} *at any chosen scale*

$\Longrightarrow$ *Extreme case: violation of decoupling!*

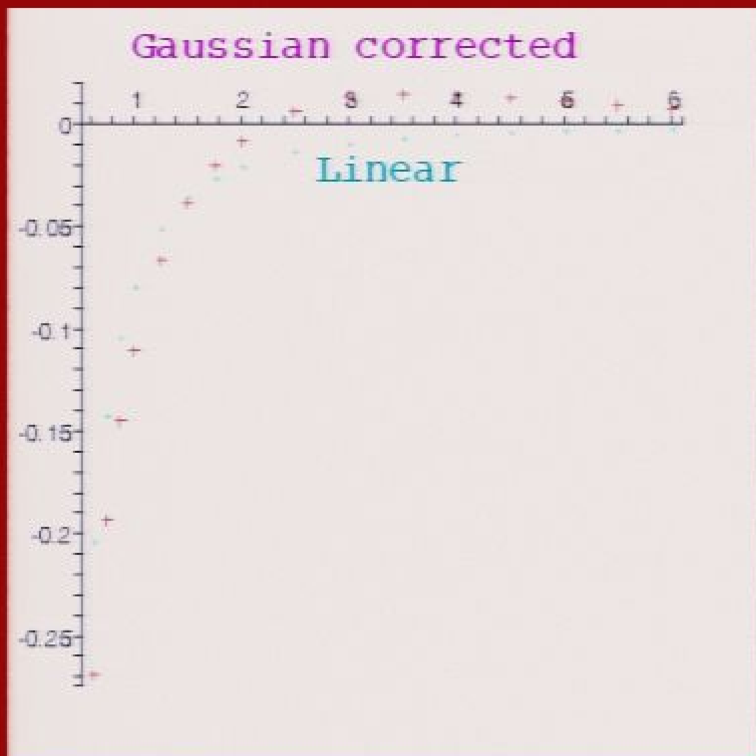
Example:

Let us choose g *to be a sharply peaked gaussian in the small negative* k

- *The dispersion relation is essentially the linear one*
- *The force function shows*

  i. a slightly different behaviour at short k

  ii. a very strong effect on the long distance: orders of magnitude!



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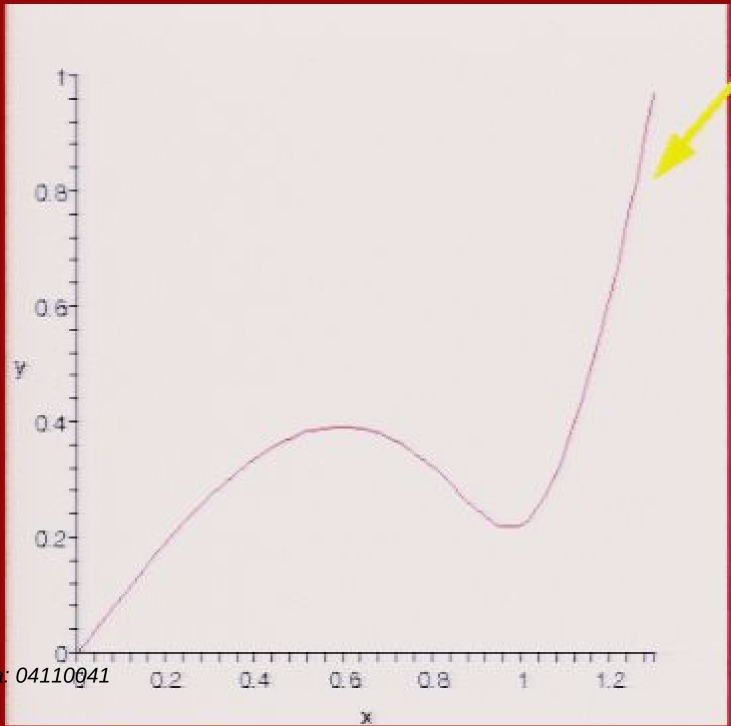
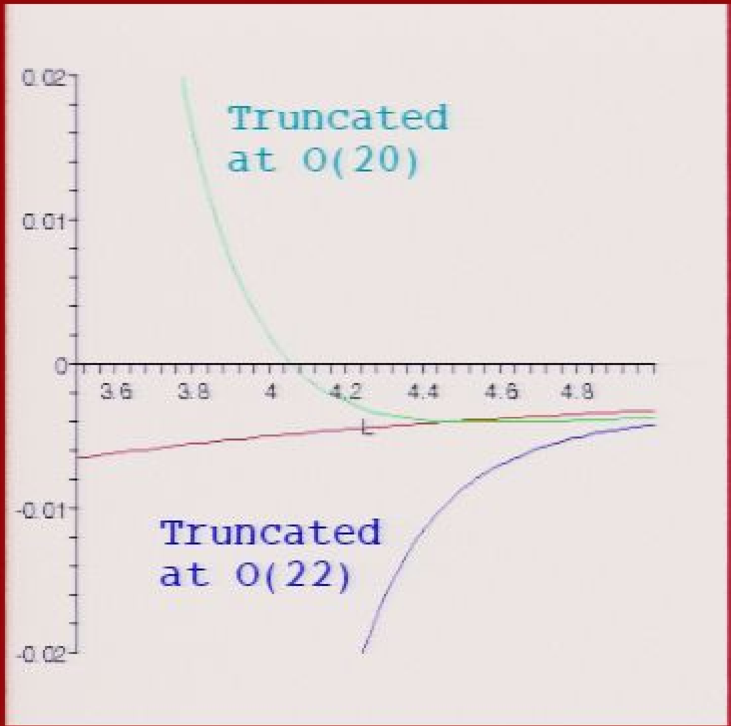
Final point, truncation effects:

- The force might depend heavily on the power at which one decides to cut a series $\Longrightarrow$ both UV and IR modifications!
- e.g. Brandenberger & Martin:

$$f(x)=\sqrt{x^2-\alpha x^4+\beta x^6}$$



$\Rightarrow$



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$\Longrightarrow$ *Classification of the possible effects w.r.t. UV/IR coupling*

- *Mathematically: dispersion relation determines Casimir force through*

  i. Analytic continuation

  ii. Operator K

- *However: can build functions with arbitrary UV/IR coupling!*

What does that mean?

Using suitable g, could fit ANY experimental data to arbitrary (but not complete) precision.

Theory has to come first!

Quantum gravity theory ***predicts*** *dispersion relation* $\Rightarrow$ *(operator K) uniquely predicts Cas. force* $\Rightarrow$ *compare to experiment*

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UV/IR COUPLING IV

Final point, truncation effects:

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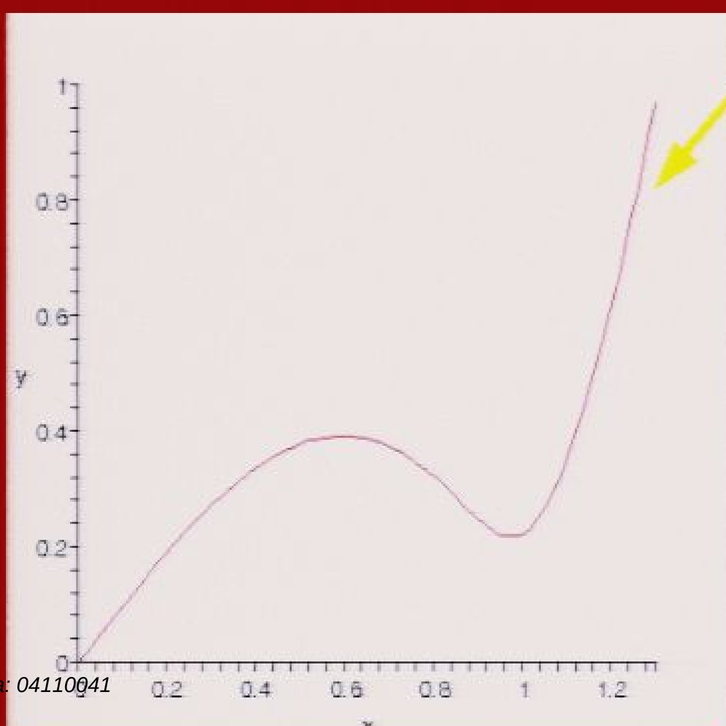
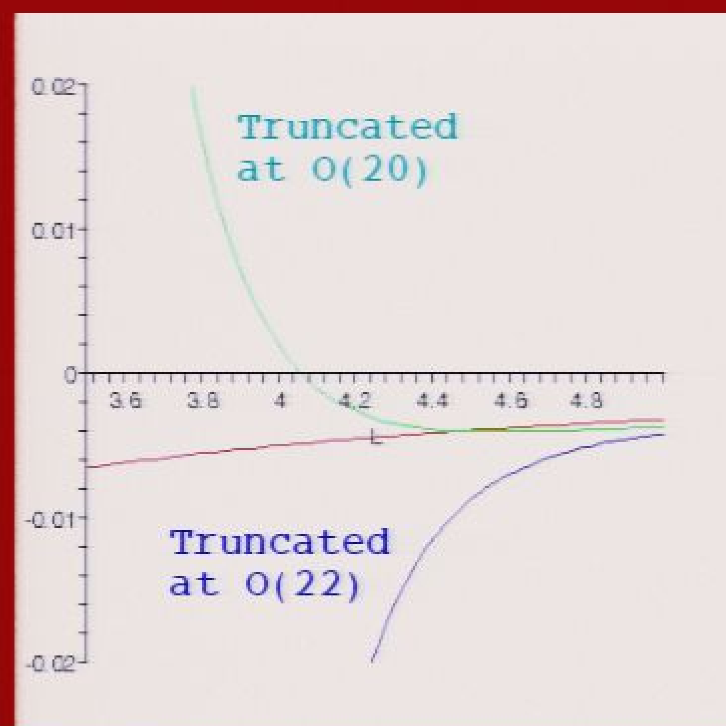
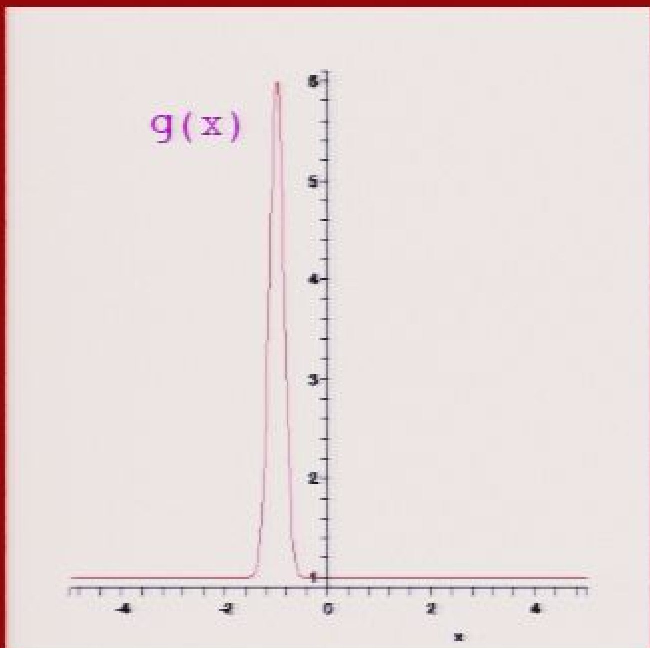


$\Longrightarrow$



UV/IR COUPLING II

The analytic continuation is unique... But:



Recall: $f(x)=xg(x^2)$

Which translates into:

- *A dispersion relation essentially indistinguishable from the linear one*
- *A Casimir force that stays orders of magnitude away from the* L^{-2} *at any chosen scale*

$\Longrightarrow$ *Extreme case: violation of decoupling!*