Title: StringsTBA

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Abstract:

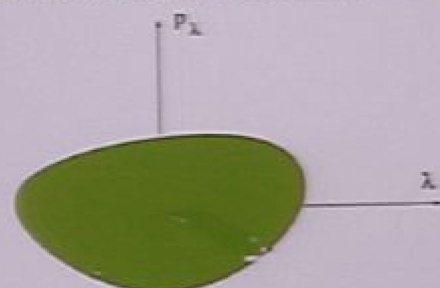
Outline

- 1/2 BPS states in field theory & Fermi liquid.
- Technique for constructing gravity solutions.
- Solutions of IIB SUGRA and Laplace equation.
- Solutions of 11D SUGRA and Toda equation.
- Summary.

Half–BPS states in $\mathcal{N}=4$ SYM

- $\mathcal{N}=4$ SYM on $S^3 \times R$:
  - chiral primaries:
    $$\mathrm{Tr}(Z^{n_1})\ldots\mathrm{Tr}(Z^{n_k}). \qquad Z=\phi_1+i\phi_2$$
  - symmetry: $S^3 \times SO(4)$
- Action in 1/2 BPS sector:
  $$S=\int dt\,\mathrm{Tr}\left[\frac{1}{2}|D_tZ|^2-\frac{1}{2R^2}|Z|^2\right]$$
- Matrix model description: harmonic oscillator
  - set of harmonic oscillators: $\alpha_n^\dagger=\mathrm{Tr}[(a^\dagger)^n]$
  - eigenvalue basis & Fermi liquid

Berenstein '04



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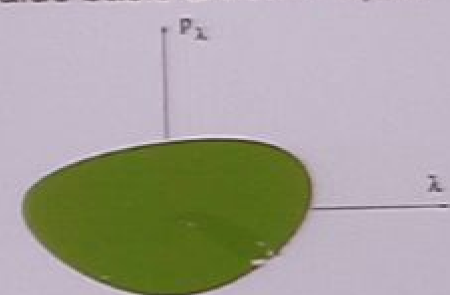

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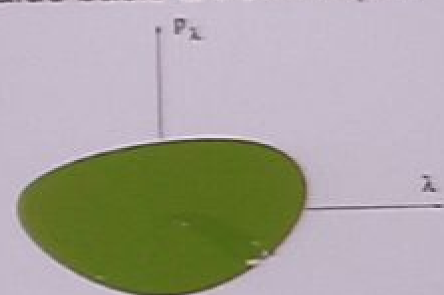

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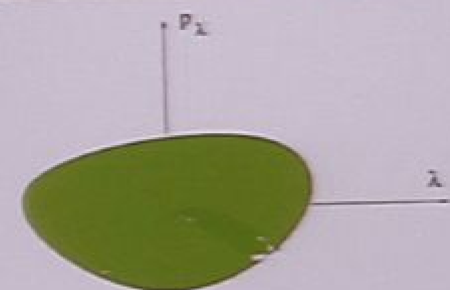

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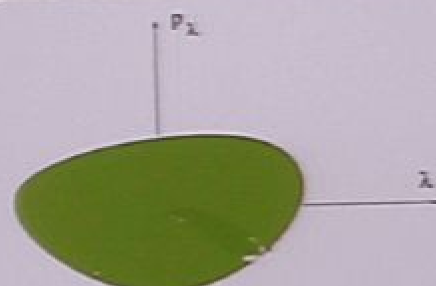

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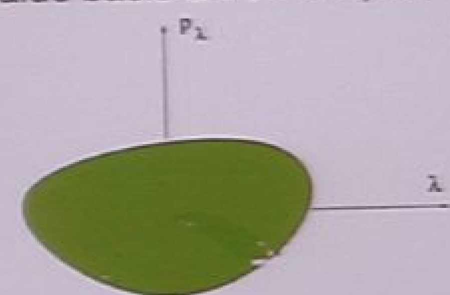

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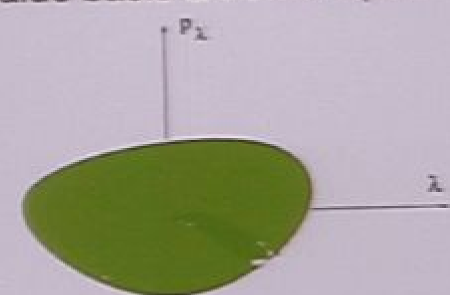

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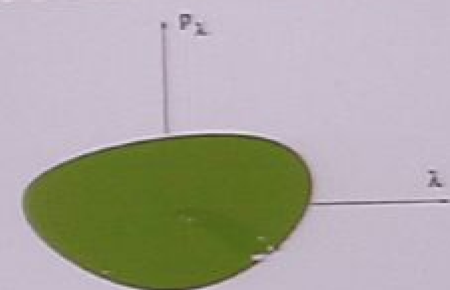

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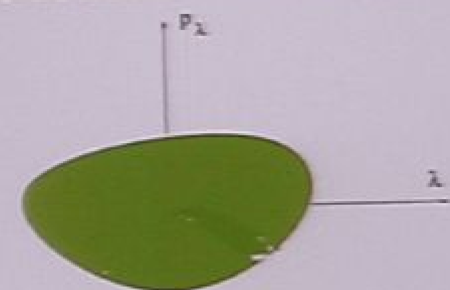

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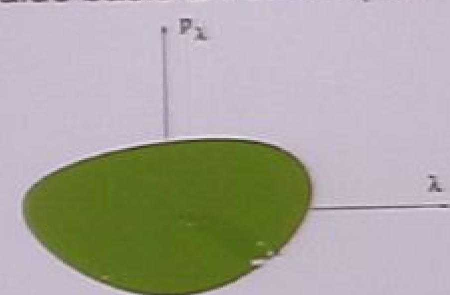

Branes and states in field theory

- States with small $\Delta = J$:
  - field theory: $\mathrm{Tr}(Z^{n_1}) \ldots \mathrm{Tr}(Z^{n_k})$
  - small ripples on the Fermi sea
  - perturbative states in string theory
- Supersymmetric branes
  - giant gravitons expanding on S^3 or $\tilde{S}^3$.

    McGreevy, Susskind, Toumbas '00
  - field theory: excitation of a single eigenvalue:

$$Z \sim e^{i\omega t} \mathrm{diag}\left(\eta, -\frac{\eta}{N-1}, \ldots -\frac{\eta}{N-1}\right)$$

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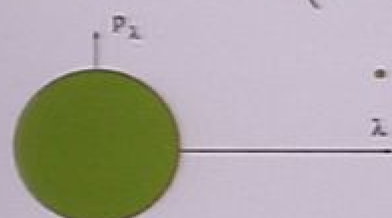

- Our goal: exact solutions of SUGRA
- $AdS_7 \times S^4$: giant gravitons with $S^5 \times S^2$.

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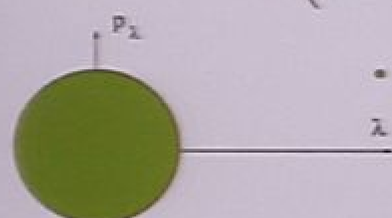

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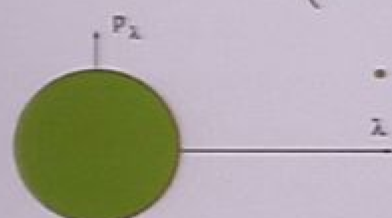

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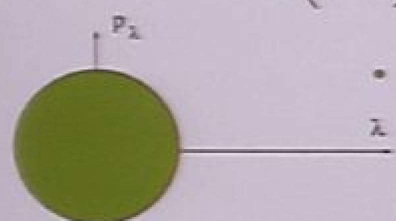

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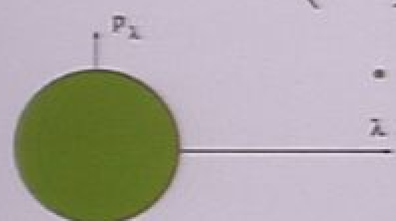

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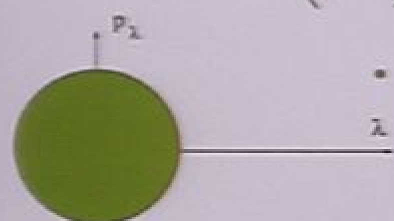

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Technique for constructing gravity solutions

- Assumptions
  - bosonic symmetries: $SO(4) \times SO(4)$
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- Reduction on $S^3 \times S^3$: spinor in 4D interacting with gauge field and 2 scalars
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Gauntlett, Gutowski, Martelli, Pakis, Reall, Sparks, Waldrum '02–'04

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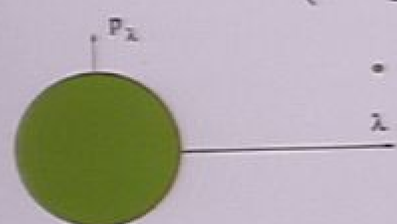

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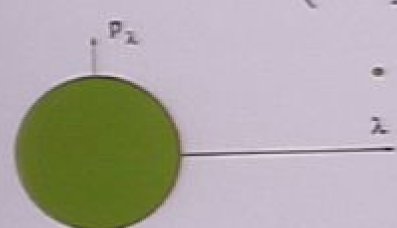

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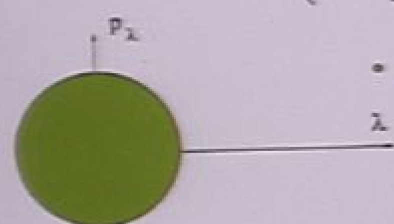

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- Assumptions
  - bosonic symmetries: $SO(4) \times SO(4)$
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1/2 BPS geometries in Type IIB SUGRA

- Explicit geometry and Laplace equation

$$ds^2 = -h^{-2}(dt + V_i dx^i) + h^2(dy^2 + dx^i dx^i)$$
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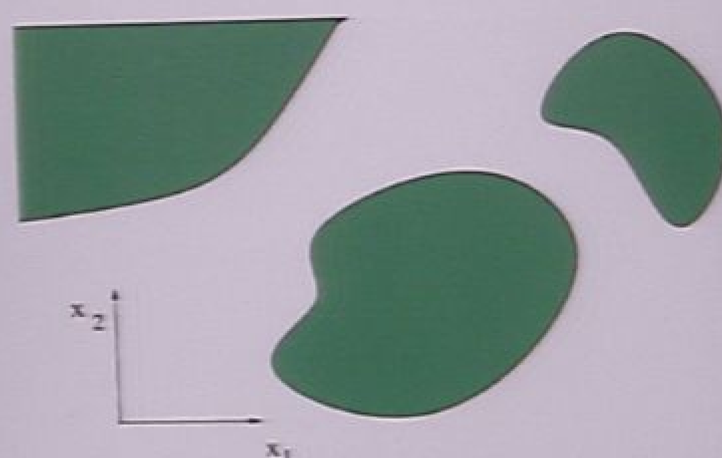
Regular solutions: general description

- Laplace equation and boundary conditions
  - 6D Laplace equation for $\Phi = \frac{z}{y^2}$
  - regularity at $y = 0$: $z = \pm\frac{1}{2}$

$$h^2 dy^2 + y e^{-G} d\bar{\Omega}_3^2 \sim \frac{1}{c(x)}(dy^2 + y^2 d\bar{\Omega}_3^2)$$

- Boundary condition for a generic state



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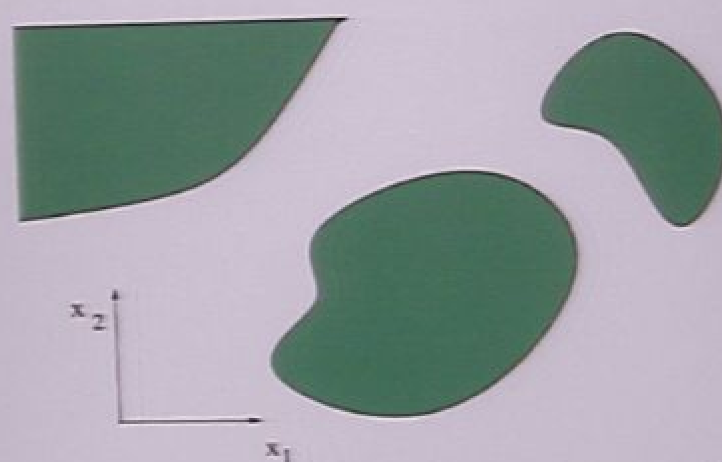

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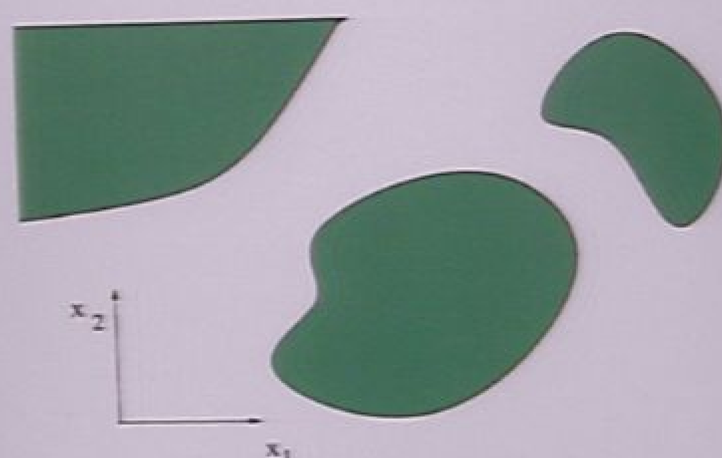

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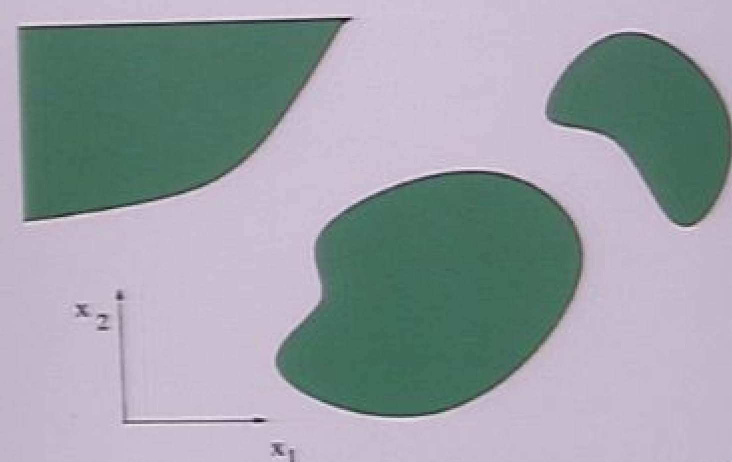

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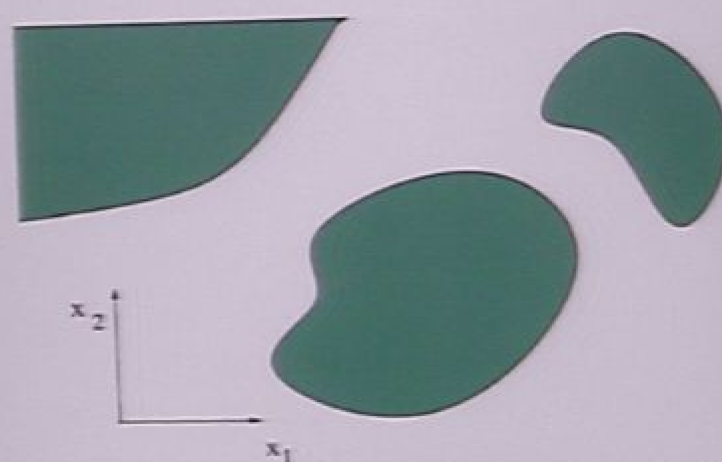

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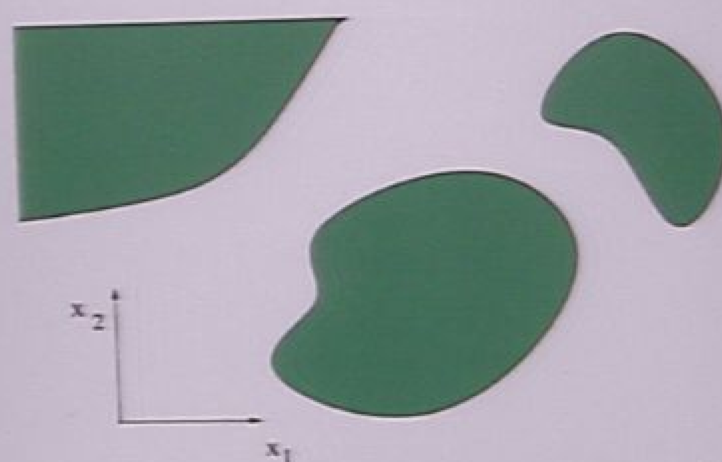

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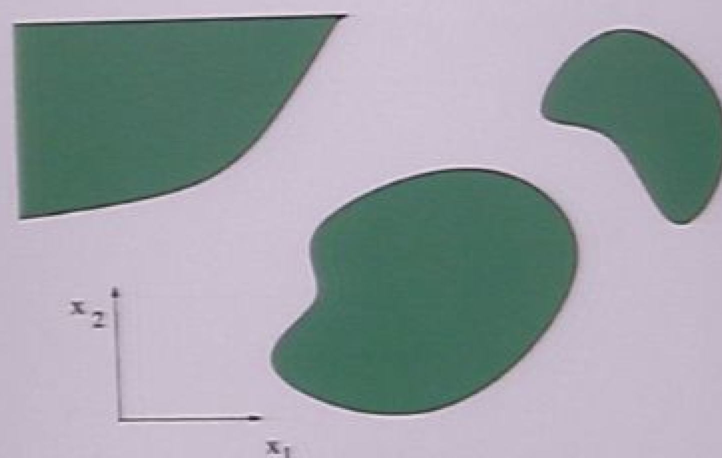

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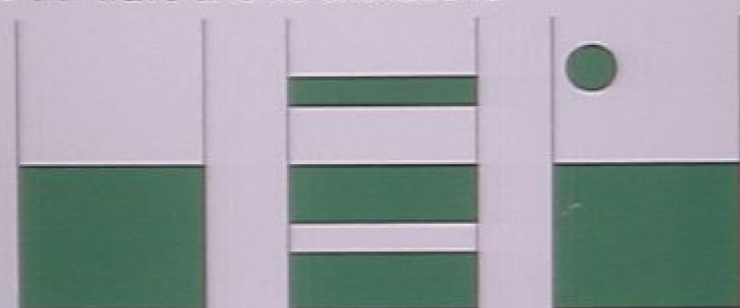
Examples

- $AdS_5 \times S^5$

$$z = \frac{r^2 - r_0^2 + y^2}{2\sqrt{(r^2 + r_0^2 + y^2)^2 - 4r^2 r_0^2}}$$

- (Giant) gravitons

- PP wave and its excitations



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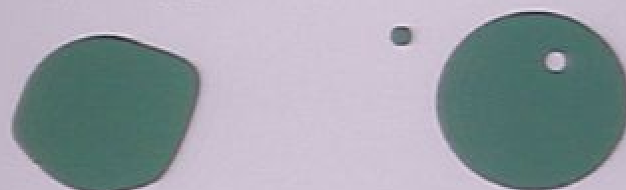

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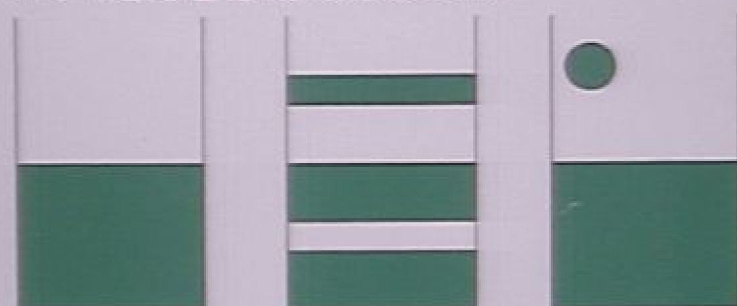

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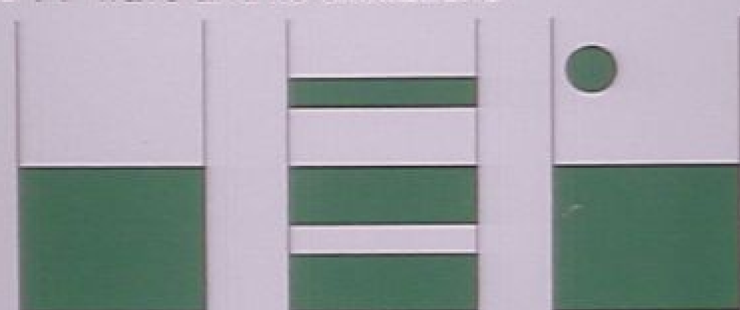

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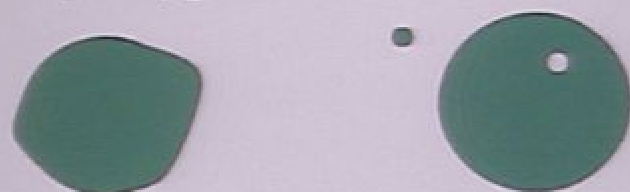

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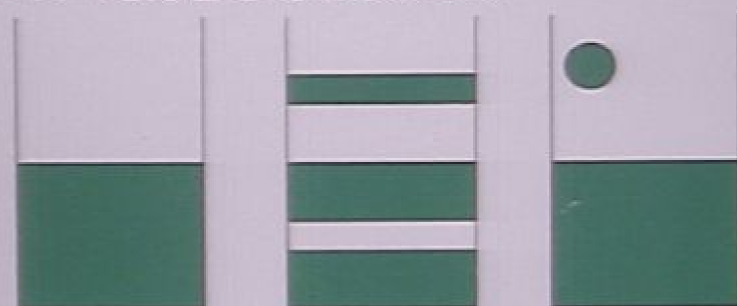

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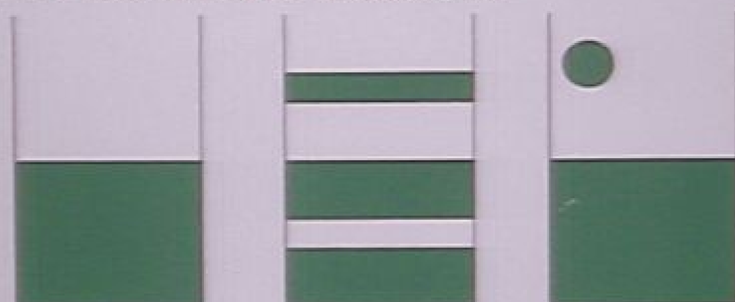

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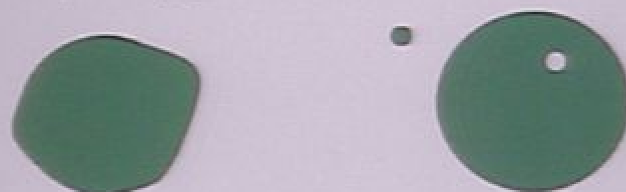

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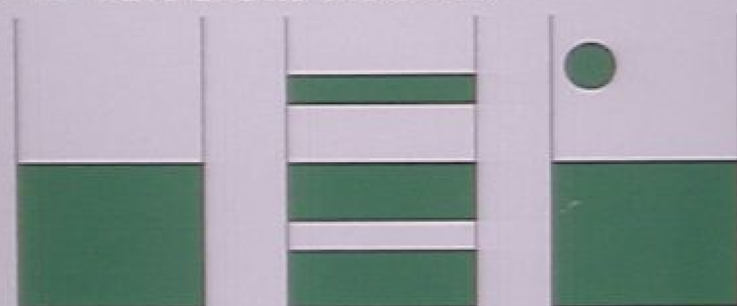

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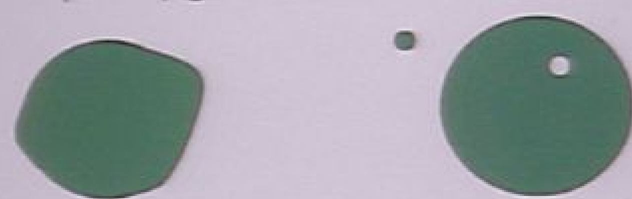

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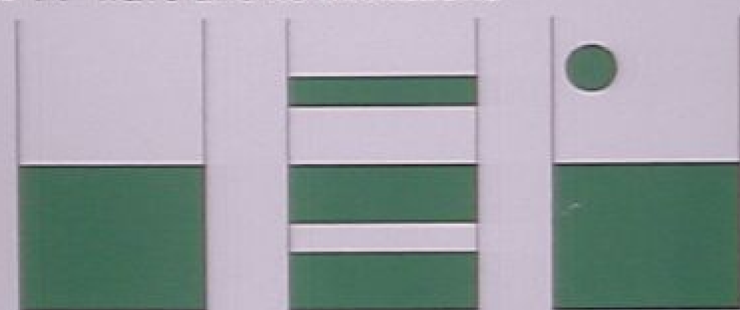

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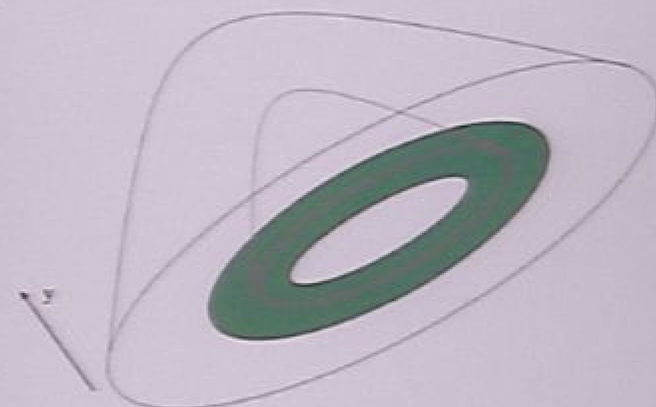
8

Topology and fluxes

- Two types of closed five–manifolds



- Different topologies: non–contractible spheres
- Quantization of fluxes:

$$\tilde{N} = -\frac{1}{2\pi^2 l_p^4}\int d\hat{\tilde{B}} = \frac{(\mathrm{Area})_{z=-\frac{1}{2}}}{4\pi^2 l_p^4}$$

- Energy and higher moments

$$\Delta = J = \int \frac{d^2x}{2\pi\hbar}\frac{\frac{1}{2}(x_1^2+x_2^2)}{\hbar} - \frac{1}{2}\left(\int_D \frac{d^2x}{2\pi\hbar}\right)^2$$

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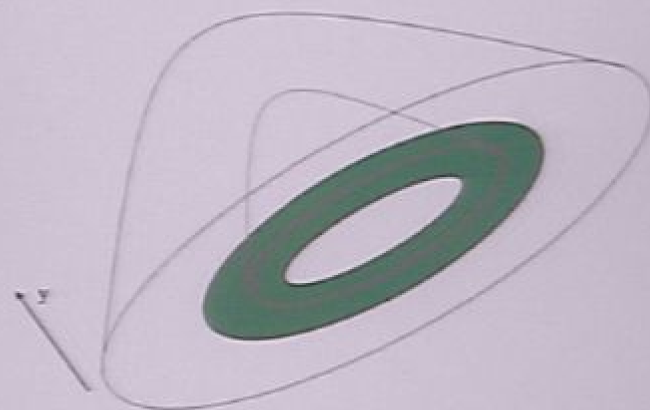

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- Two types of closed five–manifolds

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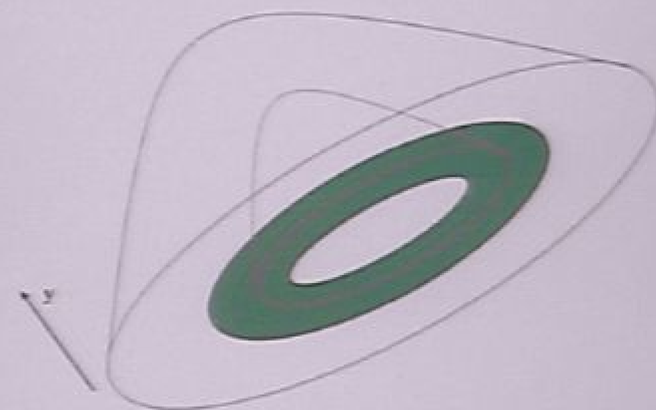

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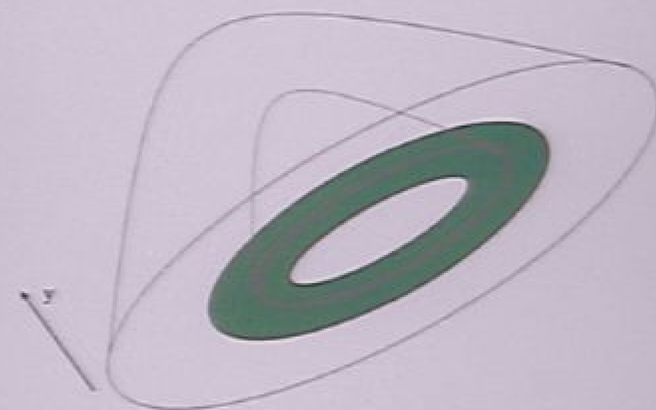

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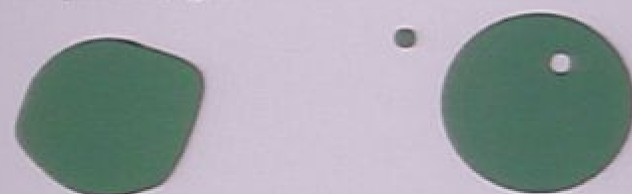

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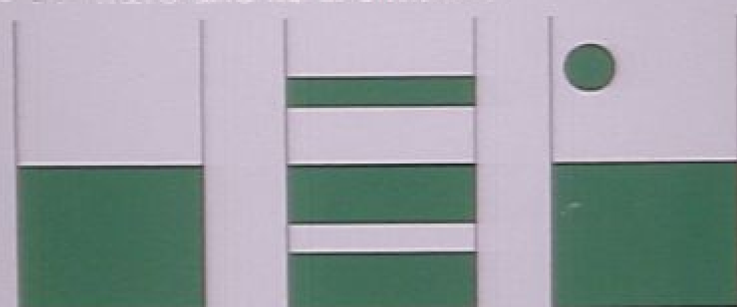

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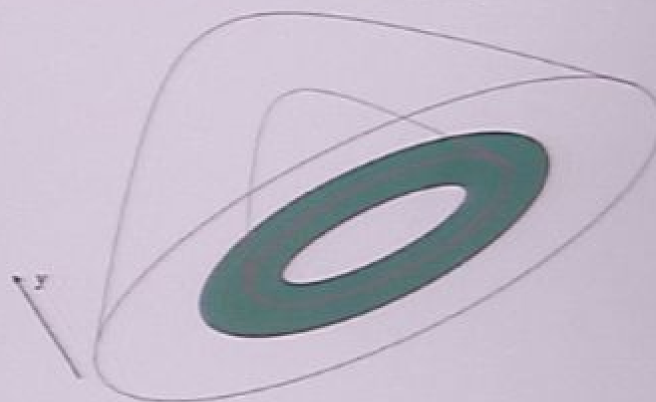

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$1/2$ BPS geometries in M theory

- Bosonic symmetries: $SO(6)\times SO(3)$

$$ds_{11}^2 = \frac{e^{2\lambda}}{m^2}d\Omega_5^2 + \frac{y^2e^{-4\lambda}}{4m^2}d\tilde{\Omega}_2^2 - \frac{e^{2\lambda}h^2}{m^2}(dt+V_i dx^i)^2 + \frac{e^{-4\lambda}}{4m^2h^2}\left(dy^2+e^D dx^2\right)$$

$$h = 1 + y^2 e^{-6\lambda}, \qquad e^{-6\lambda} = \frac{\partial_y D}{y(1-y\partial_y D)}$$

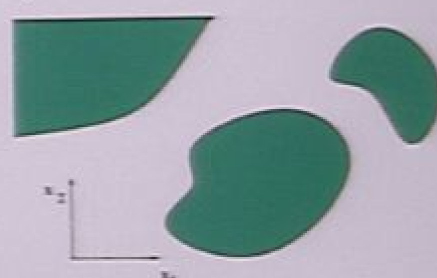
- Solution is parameterized by one function D which satisfies 3D Toda equation:

$$\Delta D + \partial_y^2 e^D = 0$$

- Boundary conditions at $y=0$

$\partial_y D = 0, \quad R_2 \to 0$

$D \sim \log y, \quad R_5 \to 0$



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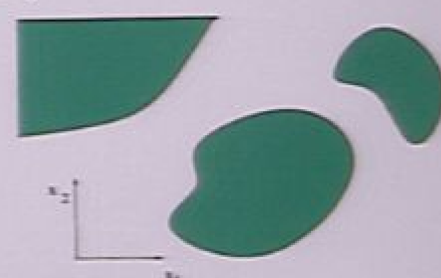

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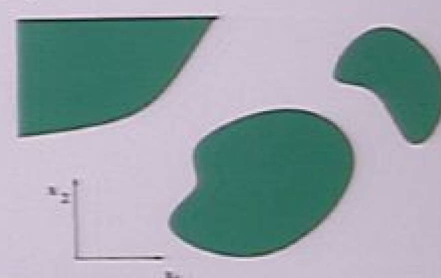

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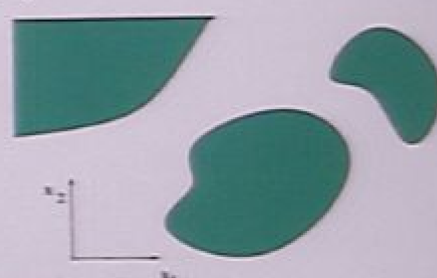

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Solutions of Toda equation

- $AdS_7 \times S^4$

$$e^D = \frac{r^2 L^{-6}}{4+r^2}, \quad x = (1+\frac{r^2}{4})\cos\theta, \quad 4y = L^{-3} r^2 \sin\theta$$

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Ward '90

- Compactification of x_1: type IIA gravity duals of the BMN matrix model

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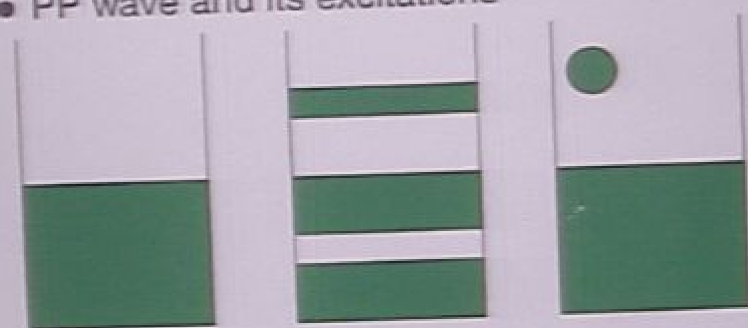

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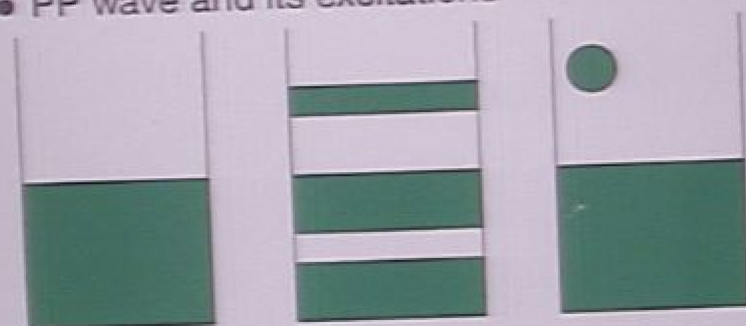

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- M theory on S^4: gauged SUGRA in 7D
  - field content: $SL(5,R)/SO(5)$ coset,
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    Perini, Pilch, van Nieuwenhuizen '84
  - 1/2 BPS black hole: symmetry group
    $SO(6) \times SO(3) \times SO(2) \times U(1)$ Liu, Minasian '99
- Our goal: regular supersymmetric solution
  - symmetry $SO(6) \times SO(3) \times U(1)$
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Relation to known solutions

- $\mathcal{N}=2$ superconformal field theories
  - 16 supercharges, $SO(4,2)\times SU(2)\times U(1)$
  - double analytic continuation of 11D solutions:

$$d\Omega_5^2 \to -ds^2_{AdS_5}, \qquad t \to \psi$$

  - different boundary conditions
  - example of a solution

$$e^D = \frac{1}{x_2^2}\left(\frac{1}{4} - y^2\right)$$

Maldacena, Nunez '00

  - new feature: space ends at $y = 1/2$:

$$\psi \sim \psi + 2\pi$$

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  - IIB with x_1 isometry $\longrightarrow$ IIA $\longrightarrow$ M theory
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Solution of gauged SUGRA

- M theory on S^4: gauged SUGRA in 7D
  - field content: $SL(5,R)/SO(5)$ coset, $SO(5)$ gauge field, five 3–forms

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- Our goal: regular supersymmetric solution
  - symmetry $SO(6) \times SO(3) \times U(1)$
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$$g = \exp(i\theta\sigma_2)\exp(-\rho\sigma_3) \in SL(2,R)/U(1)$$

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- $\mathcal{N} = 2$ superconformal field theories
  - 16 supercharges, $SO(4,2) \times SU(2) \times U(1)$
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$$d\Omega_5^2 \to -ds^2_{AdS_5}, \qquad t \to \psi$$

  - different boundary conditions
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$$e^D = \frac{1}{x_2^2}(\frac{1}{4} - y^2)$$

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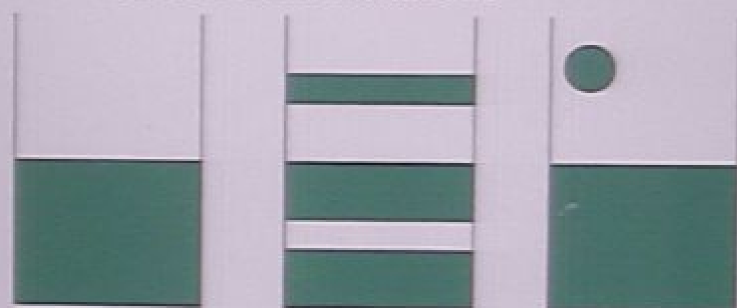
- (Giant) gravitons

- PP wave and its excitations



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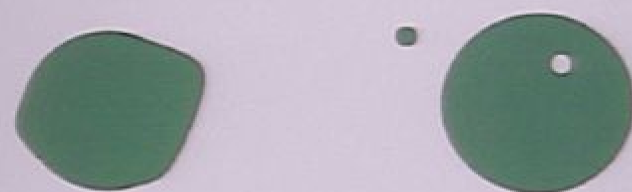

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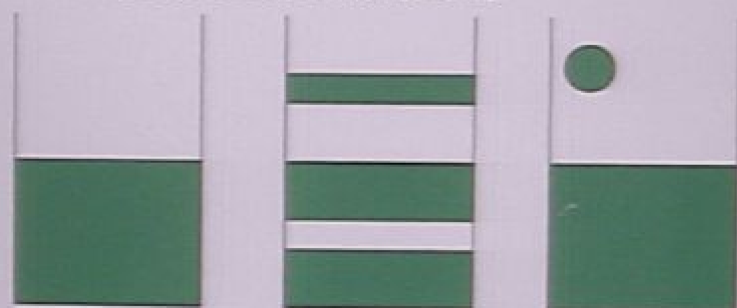

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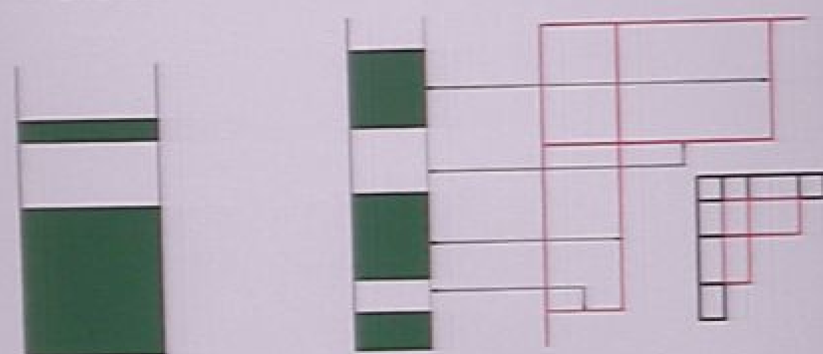

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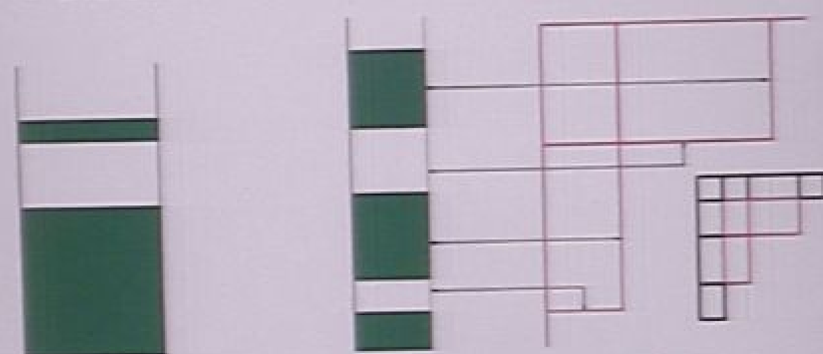

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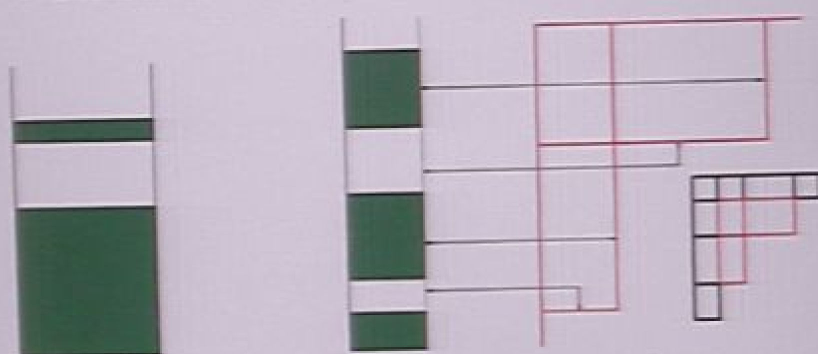

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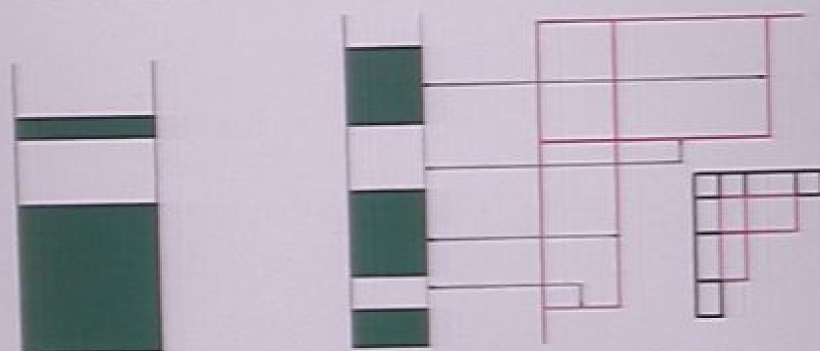

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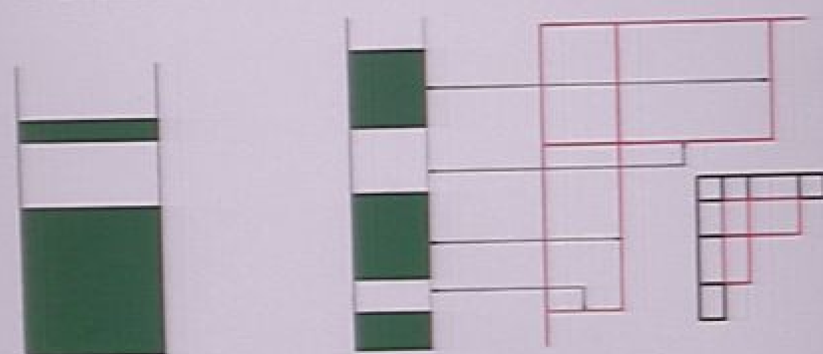

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  - boundary conditions and free fermions
  - explicit solutions in terms of integrals
  - fluxes and topologies
- All $1/2$ BPS solutions of 11D SUGRA
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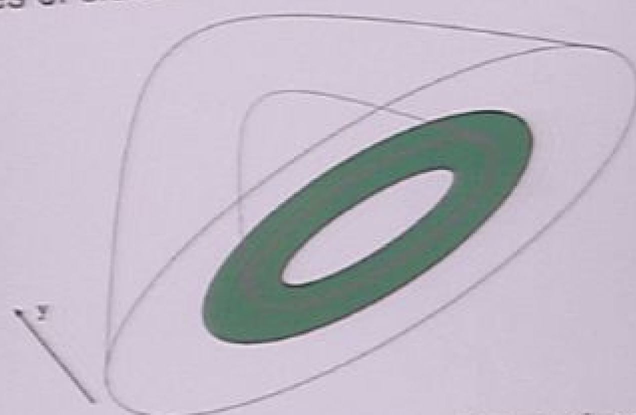
Topology and fluxes

- Two types of closed five–manifolds



- Different topologies: non–contractible spheres
- Quantization of fluxes:

$$\tilde{N} = -\frac{1}{2\pi^2 l_p^4} \int d\tilde{\tilde{B}} = \frac{(\text{Area})_{z=-\frac{1}{2}}}{4\pi^2 l_p^4}$$

- Energy and higher moments

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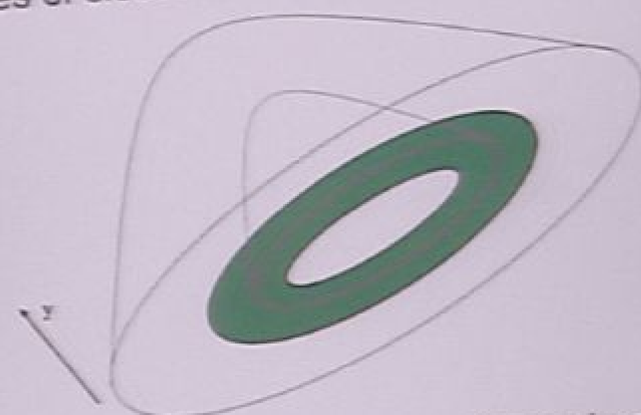

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