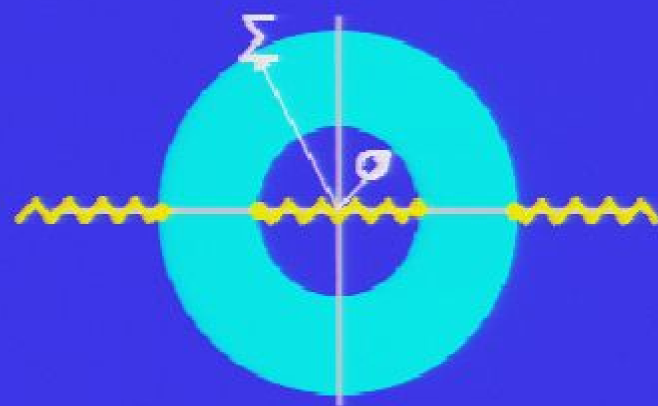
Title: Classical limit of quantum gravity in an accelerating universe.

Date: Nov 23, 2004 04:05 PM

URL: http://pirsa.org/04110037

Abstract:

Classical limit of quantum gravity in an accelerating universe



Frederic P. Schuller
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Schuller and Wohlfarth: [1] Nucl Phys B (2004) [2] gr-qc/0411076

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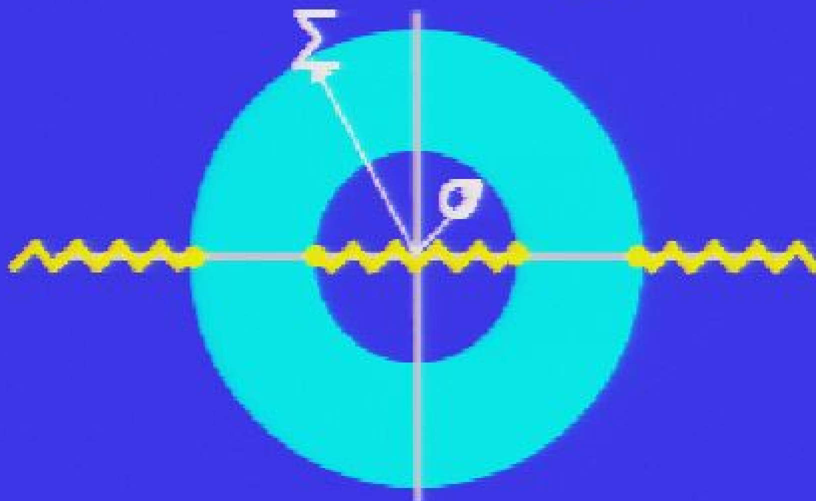

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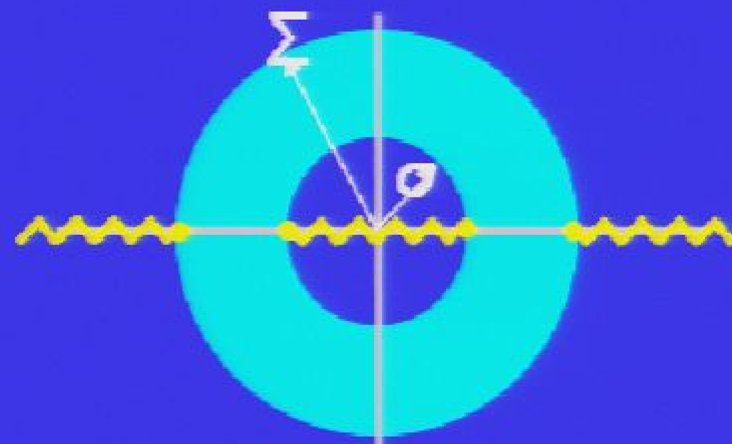

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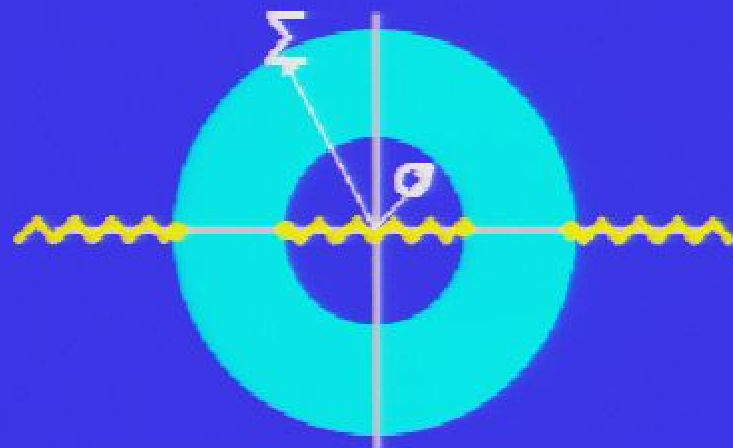

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COSMOLOGICAL PUZZLE

Einstein-Hilbert gravity: $R_{ab} - \frac{1}{2} R\, g_{ab} = T_{ab}$

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recent observations: universe expands at accelerating rate

Einstein-Hilbert gravity: $R_{ab} - \frac{1}{2} R\, g_{ab} = T_{ab} + \Lambda g_{ab}$ $\quad \Lambda > 0$

strong energy condition $\Rightarrow \Lambda = \text{const.}$

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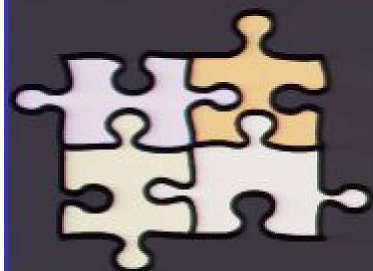

Cosmological Constant Puzzle:

What is the physical origin

of such a ubiquitous vacuum energy ?

e.g. quantum vacuum fluctuations of standard model fields: off by $> 10^{40}$

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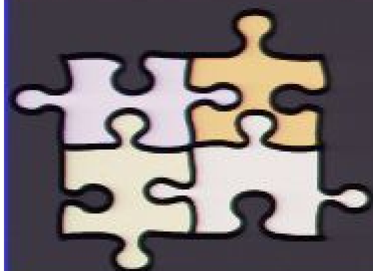

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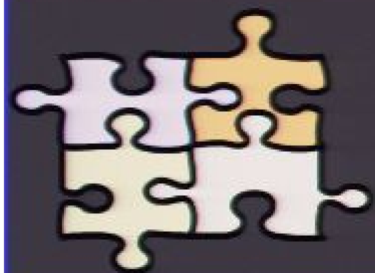

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unknown — unclear → — Λ classical concept

Quantum structure → Lorentzian manifold

← hints & hunches

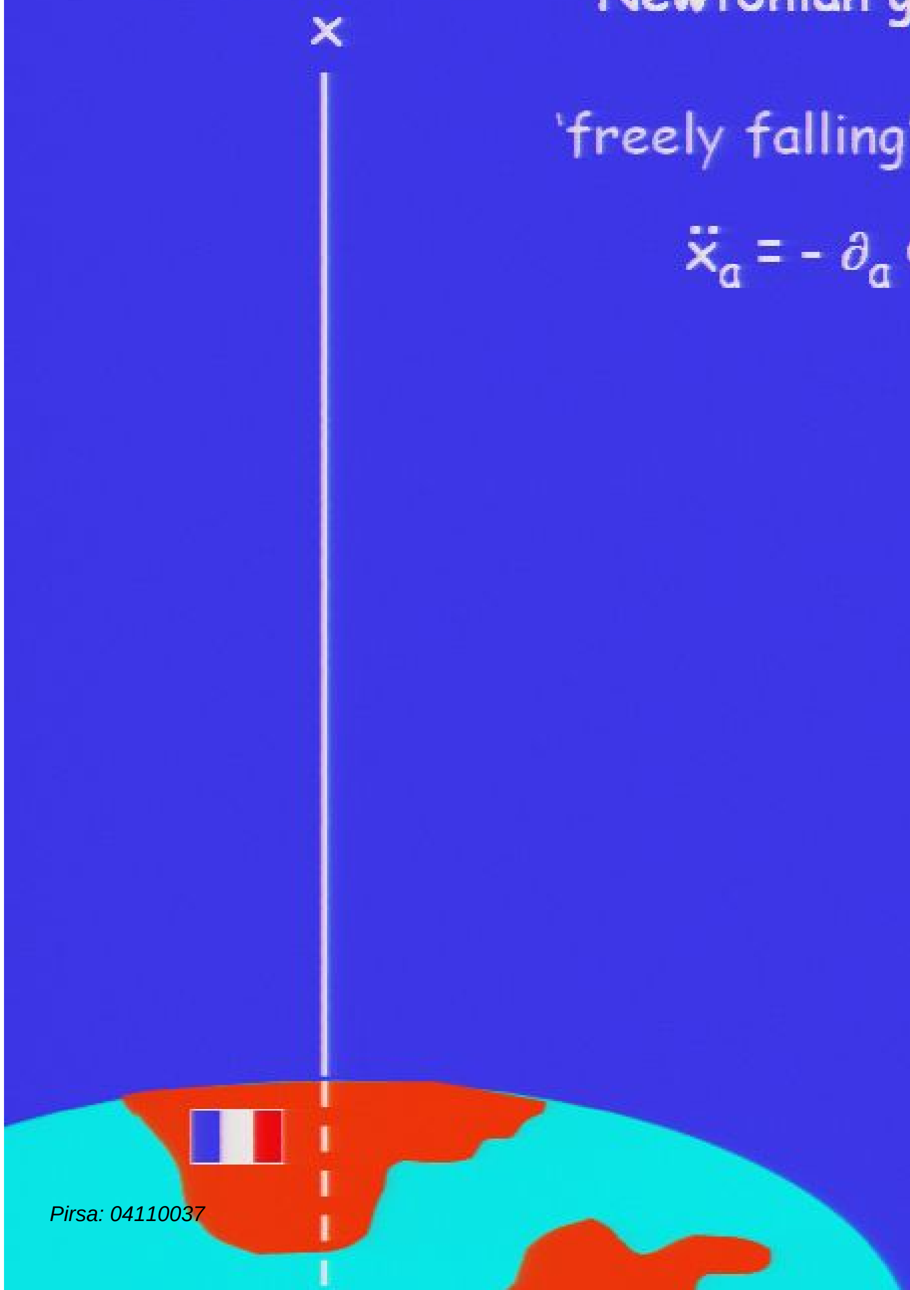
laim: possible classical signatures of quantum gravity surprisingly limited

TIDAL ACCELERATIONS

Newtonian gravity

'freely falling' particle

$$\ddot{x}_a = -\partial_a \Phi(x)$$



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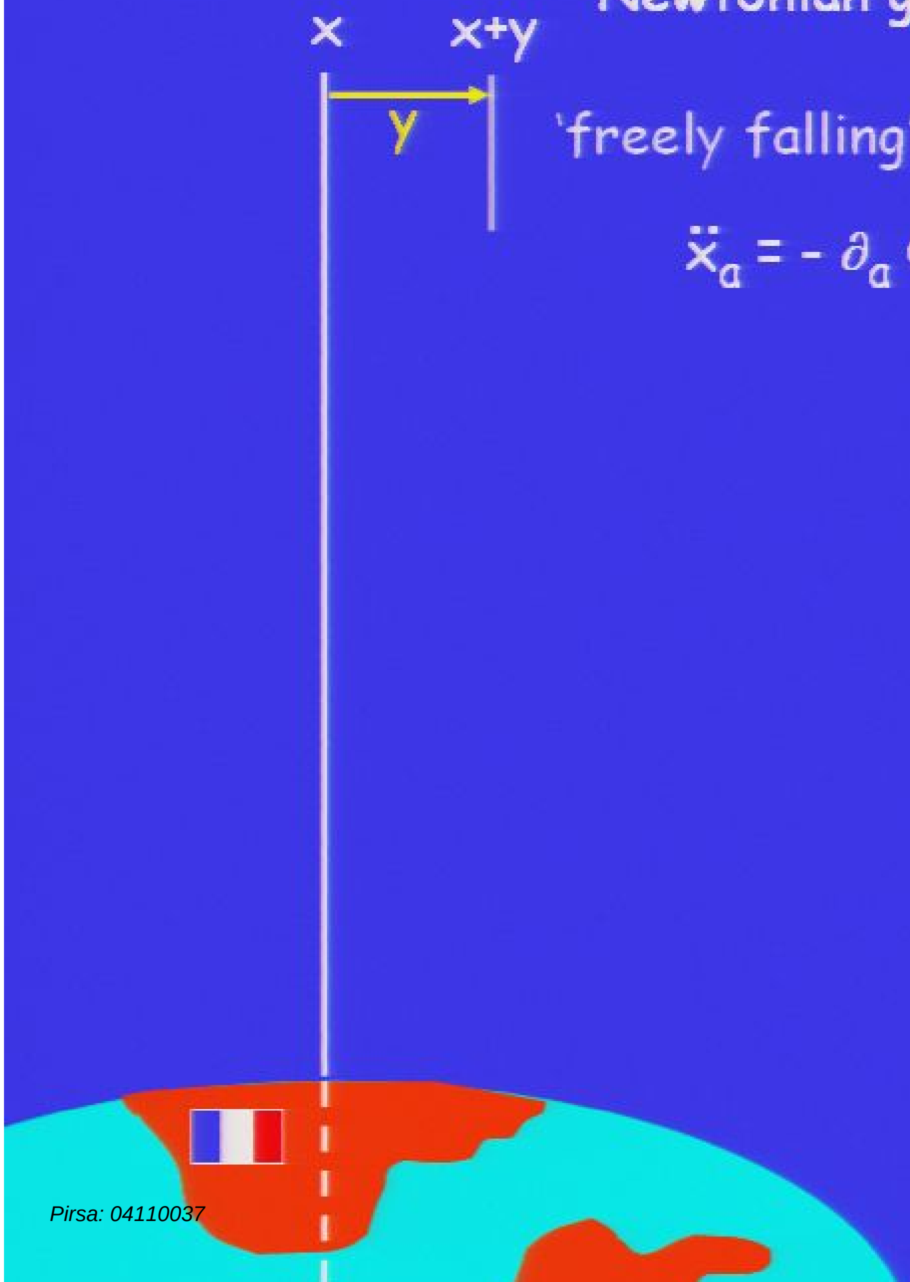

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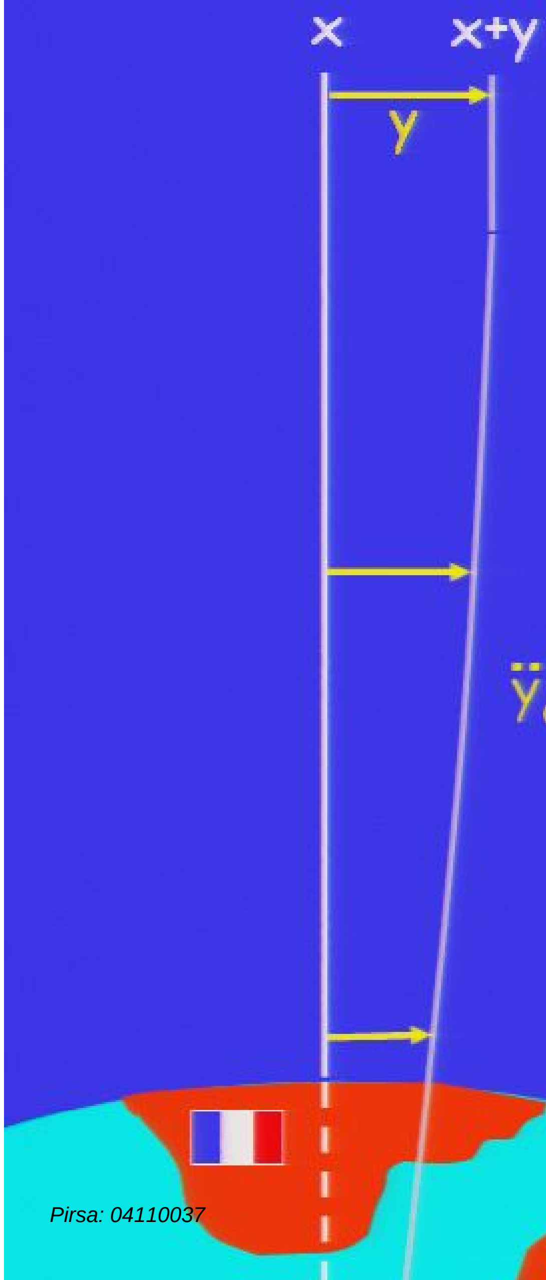

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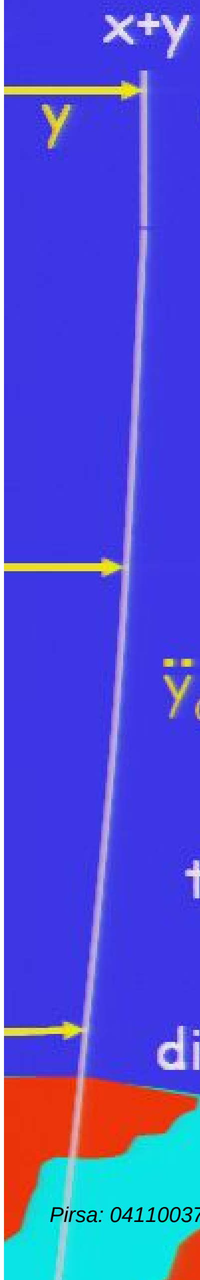

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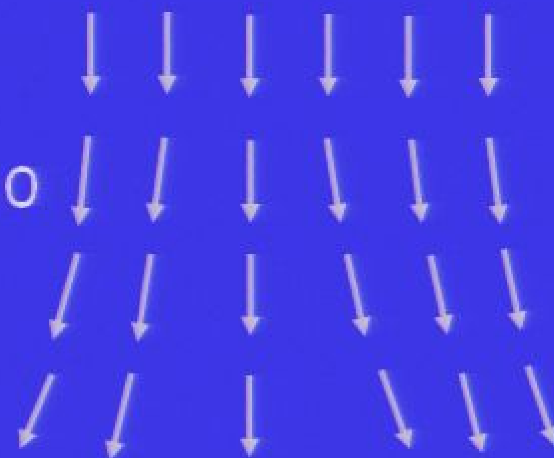

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x+y

Y

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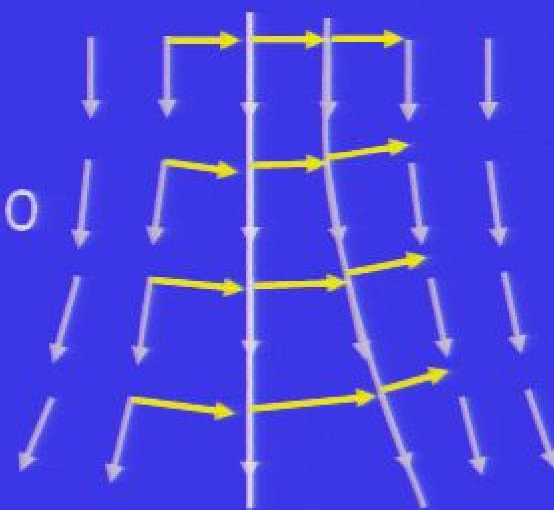

$[Y, X] = 0$

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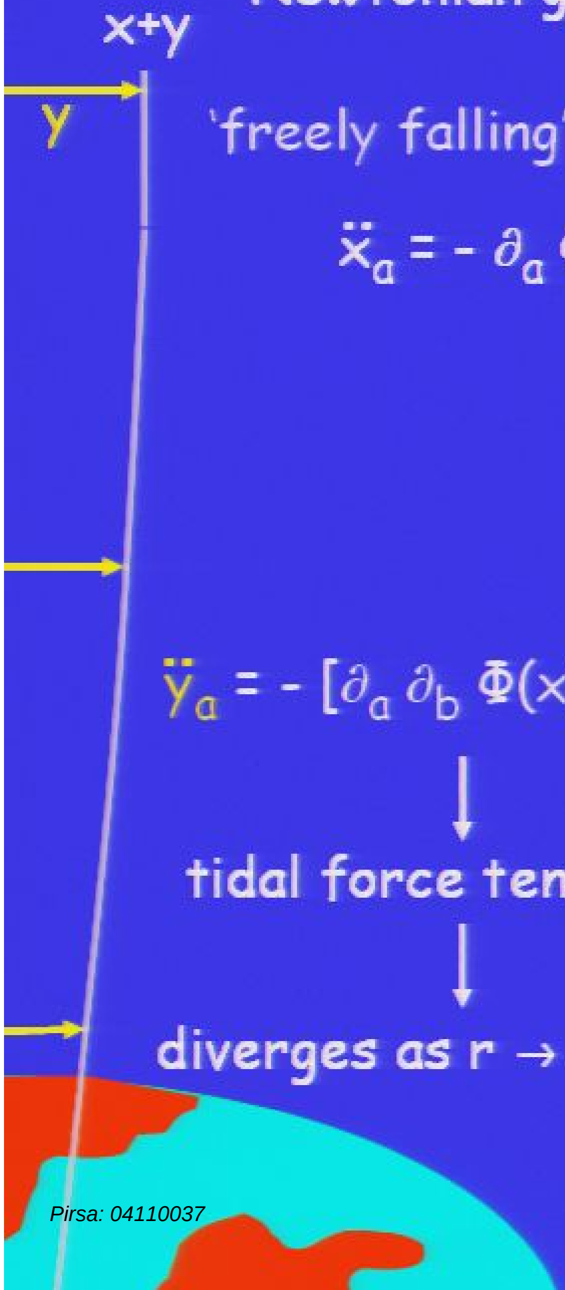

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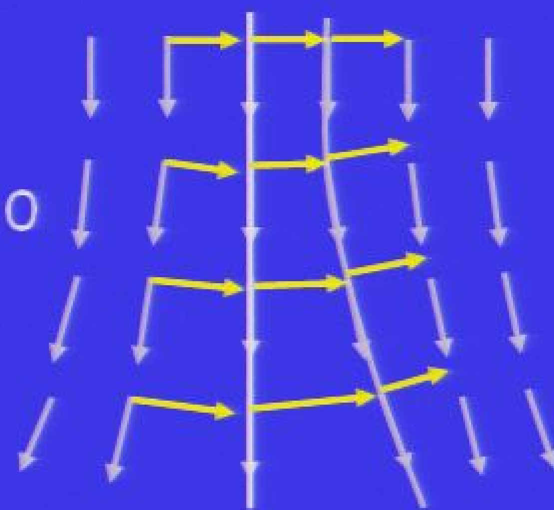

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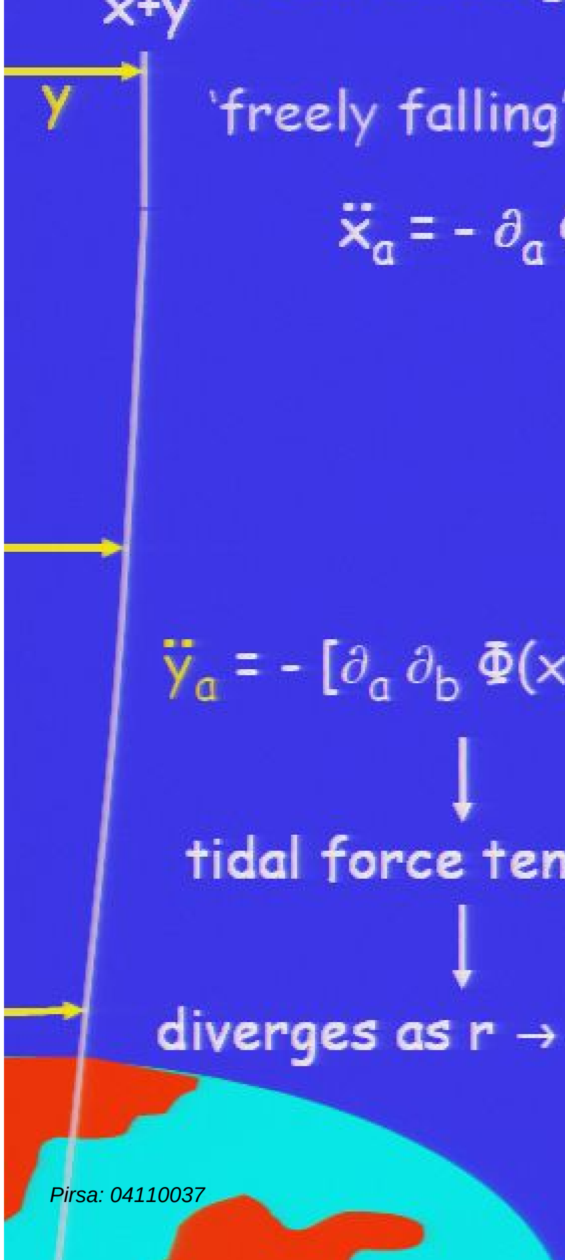

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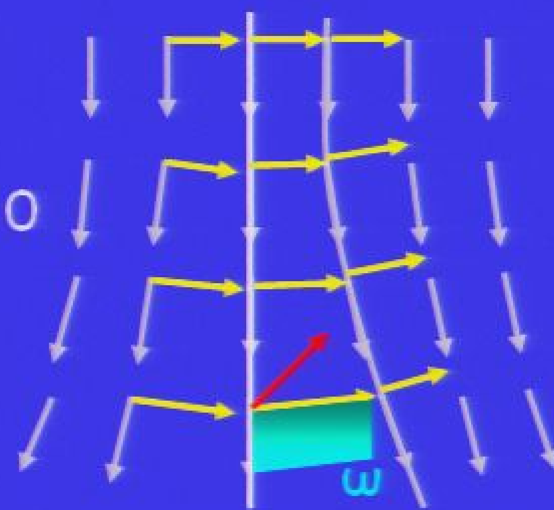

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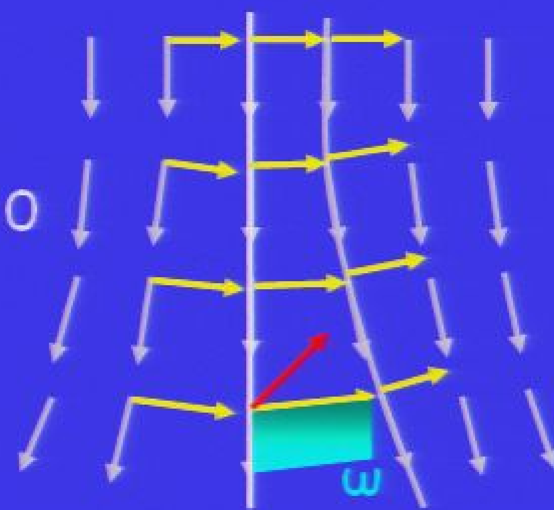

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frequency GL(2,R) invariant

(only depends on 2-plane ω)

⇒ physically meaningful

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THOUGHT EXPERIMENT

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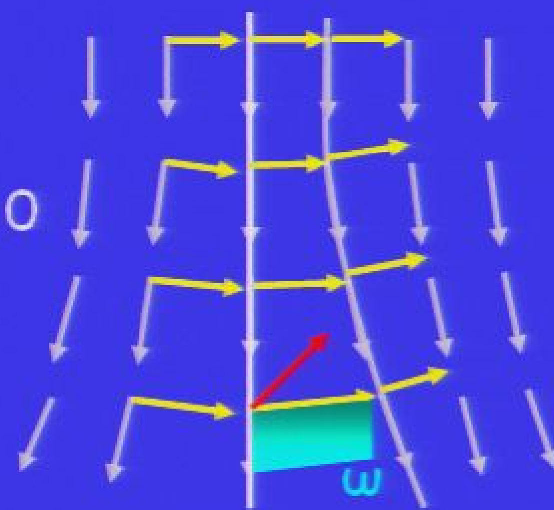
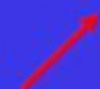
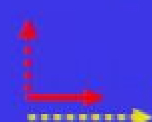


 $= \nabla_X \nabla_X Y = R(X,Y)X$

 $= [\nabla_X \nabla_X Y]_Y = \dfrac{R(X,Y,X,Y)}{G(X,Y,X,Y)} Y$

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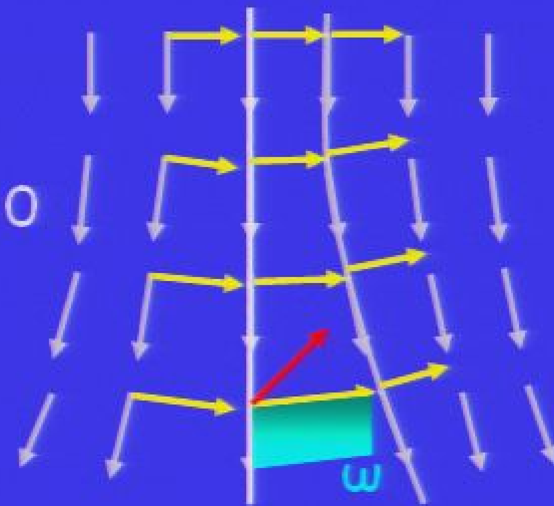

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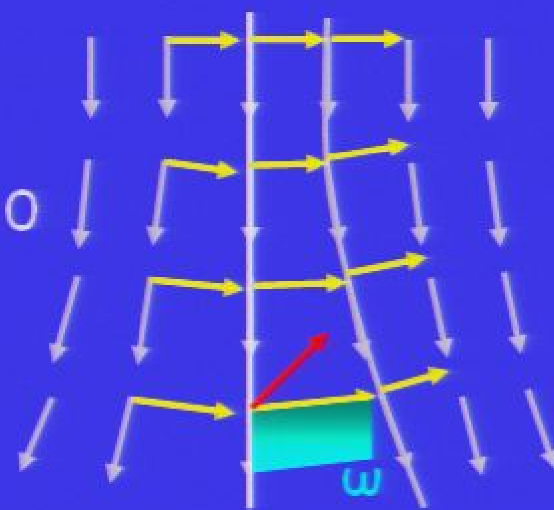

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causal sets — breakdown smooth→quantum
small length scale ↔ large length scale

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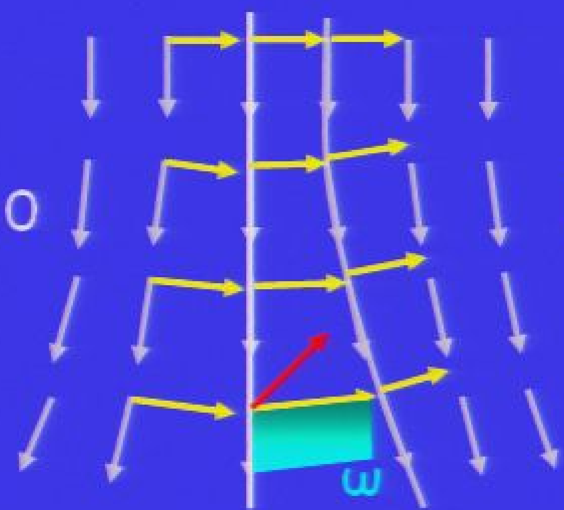

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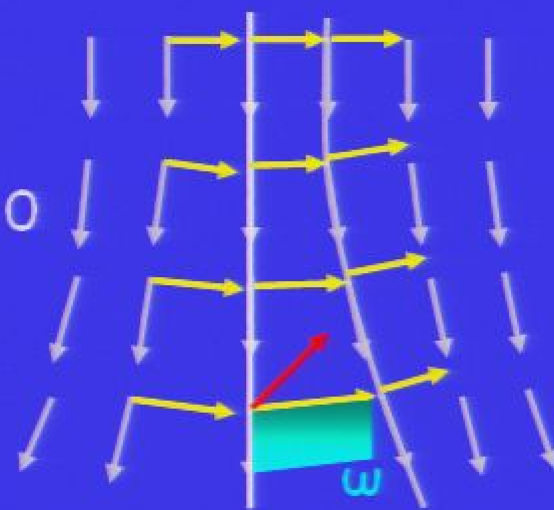

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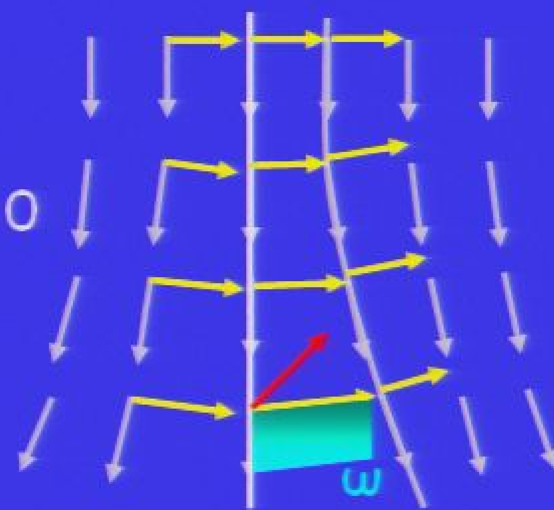

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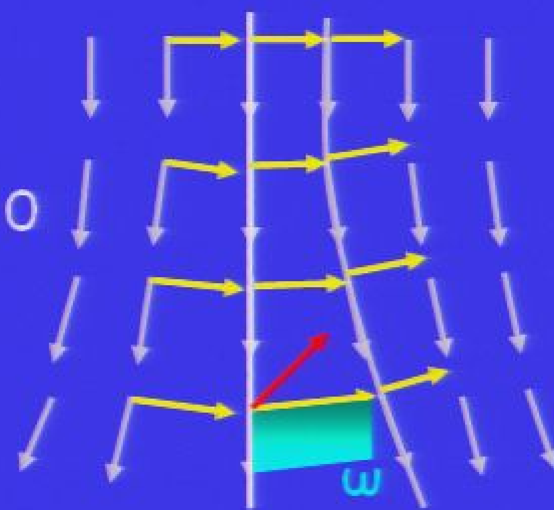

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embedding by family of polynomials

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varieties = closed sets $\mathcal{A}$ of Zariski topology on V

(i) $\emptyset, V \in \mathcal{A}$

(ii) $A_i \in \mathcal{A} \Rightarrow \cap_i A_i \in \mathcal{A}$

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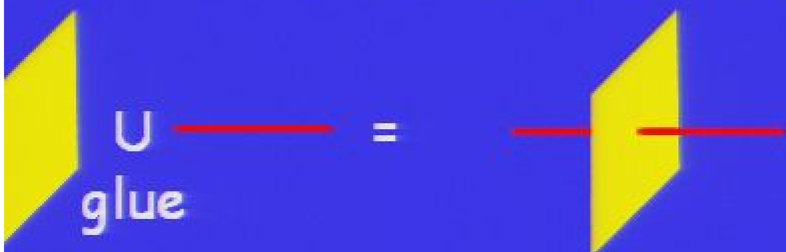

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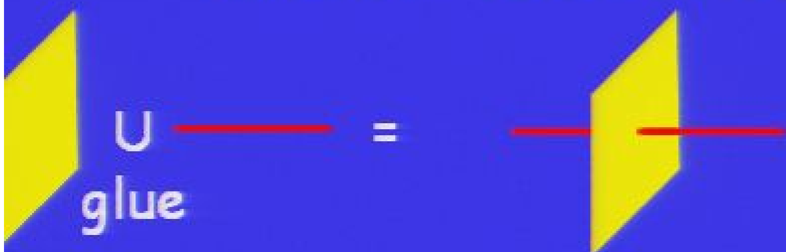

pace of oriented areas is a variety $\Omega = X \wedge Y$ for some vectors X, Y

$$\Updownarrow$$

$$(TM \oplus TM)/SL(2,R) = \{\Omega \in \Lambda^2(TM) \mid \Omega \wedge \Omega = 0\}$$

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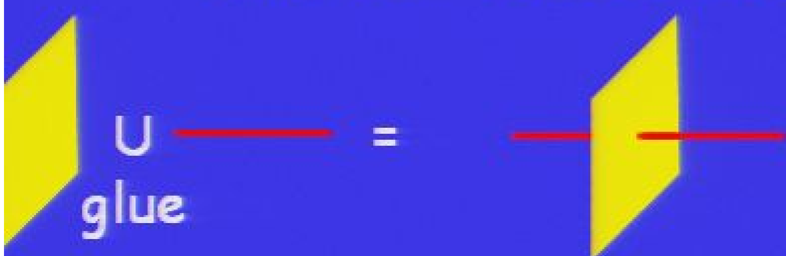

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$\mathbb{P}V = V/_\sim$ where $v \sim w \Leftrightarrow \exists \lambda \neq 0: v = \lambda w$

(PROJECTIVE) VARIETIES

ariety $\mathbb{V} \subseteq V$ V vector space

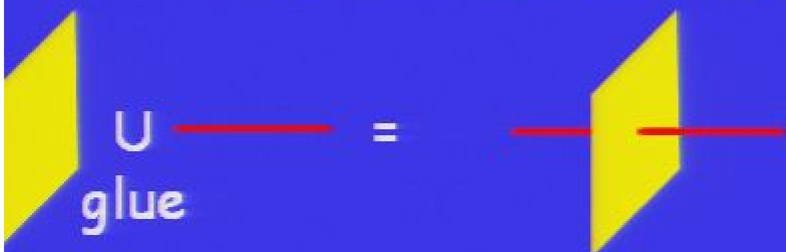
embedding by family of polynomials

$$\mathbb{V} \xrightarrow[P_\alpha]{} V$$

$V = \mathbb{R}^3$; $P(v) = \eta(v,v)$

$\mathbb{V}$ =

varieties = closed sets $\mathcal{A}$ of Zariski topology on V



(i) $\emptyset, V \in \mathcal{A}$

(ii) $A_i \in \mathcal{A} \Rightarrow \cap_i A_i \in \mathcal{A}$

(iii) $A_1,\ldots,A_n \in \mathcal{A} \Rightarrow \cup_i A_i \in \mathcal{A}$

pace of oriented areas is a variety $\Omega = X \wedge Y$ for some vectors X, Y

$$\Updownarrow$$

$$(TM \oplus TM)/SL(2,R) = \{\Omega \in \Lambda^2(TM) \mid \Omega \wedge \Omega = 0\}$$

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rassmannian (space of planes) is projective variety:

$$(TM \oplus TM)/GL(2,R) = \{\Omega \in \mathbb{P}\Lambda^2(TM) \mid \Omega \wedge \Omega = 0\} = Gr2(TM)$$

SECTIONAL CURVATURE

<u>ectional curvature</u> $S: Gr_2 \longrightarrow R$

$$S(\Omega) = \frac{R(X,Y,X,Y)}{G(X,Y,X,Y)} \qquad \Omega = (X \oplus Y)/GL(2,R)$$

G = induced metric on $\Lambda^2 TM$ (anti-symmetric 2-tensors)

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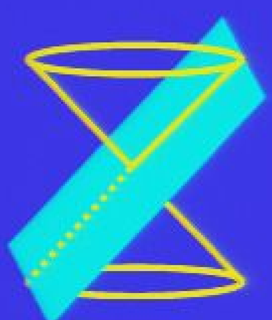

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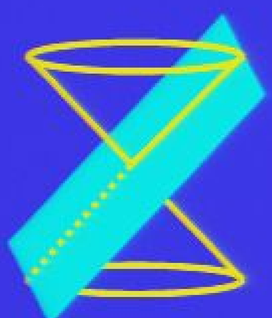

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Nomizu-Harris rigidity roots in this unnatural restriction

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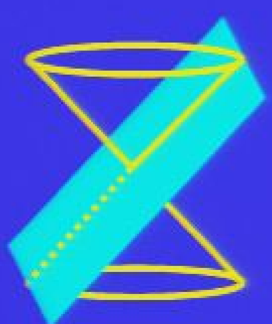

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circumvent rigidity theorems by restriction to

maximal subvariety $\mathcal{V} \subseteq Gr_2 \cap \{\Omega \mid G(\Omega,\Omega) \neq 0\}$

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CONSTRUCT maximal subvariety $\mathcal{V}$ of non-null planes

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lectromagnetic analogy: invariants: $F_{ab}F^{ab} = E^2 - B^2$, $F_{ab}{*F}^{ab} = 2\, E.B$

cannot bound both E and B due to Lorentz signature

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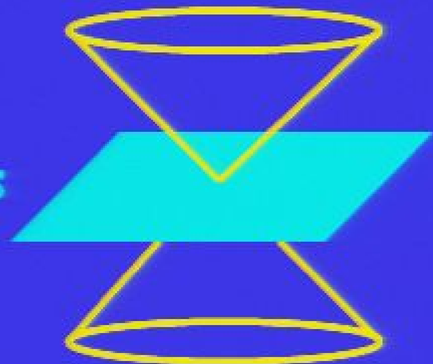
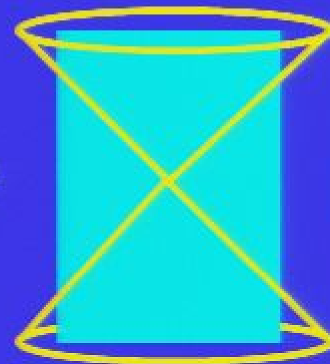


$\in \mathcal{V} \ni$



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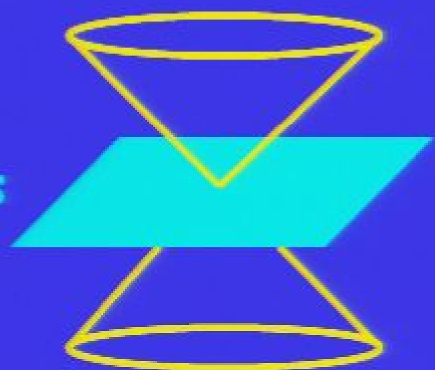

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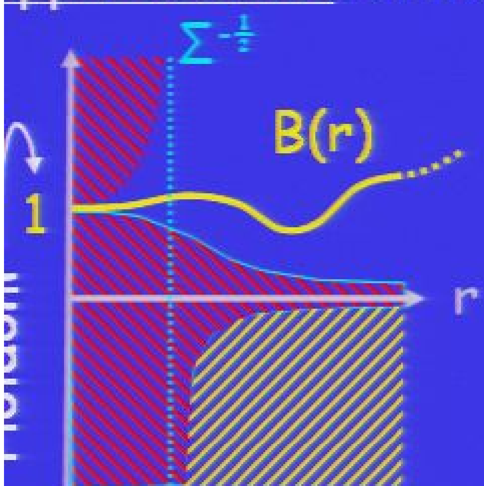
planes $\supset$ time axis

upper bounds on sectional curvature:



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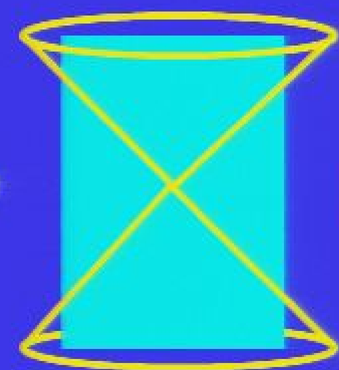

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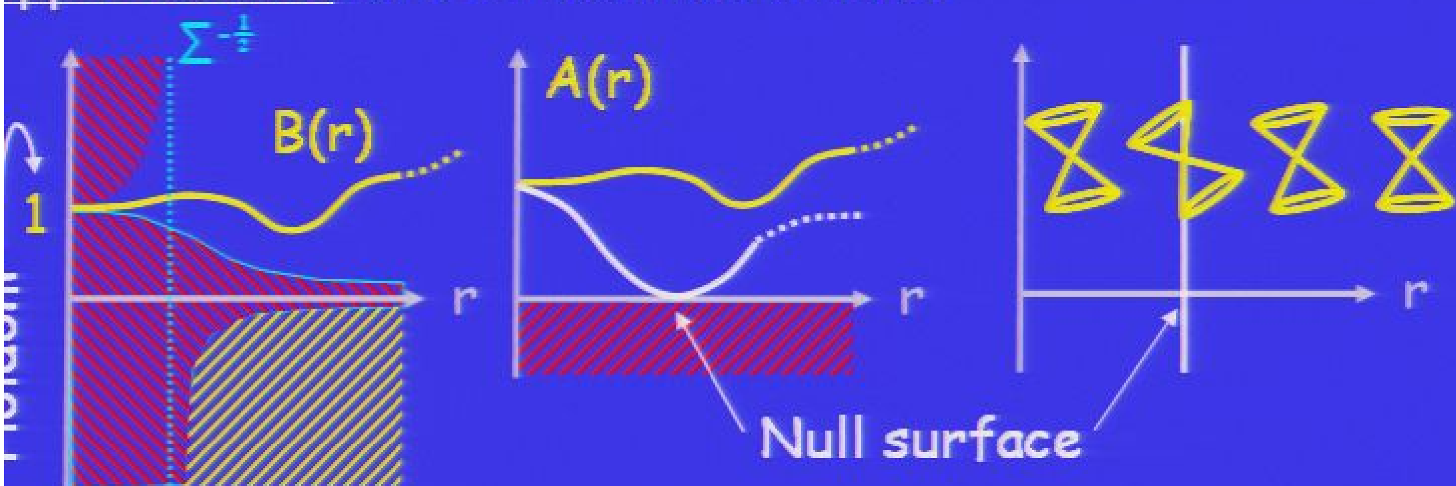

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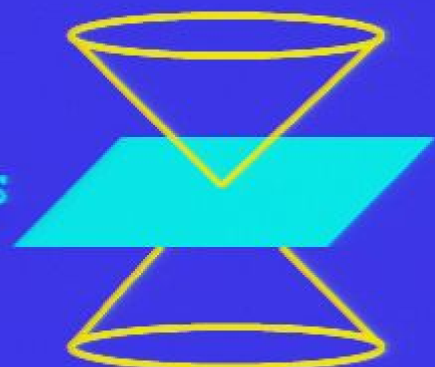
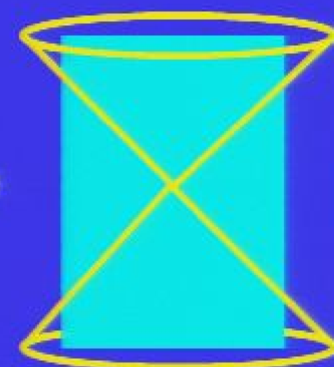
lanes ⊥ time axis  ∈ $\mathcal{V}$ ∋  planes ⊃ time axis

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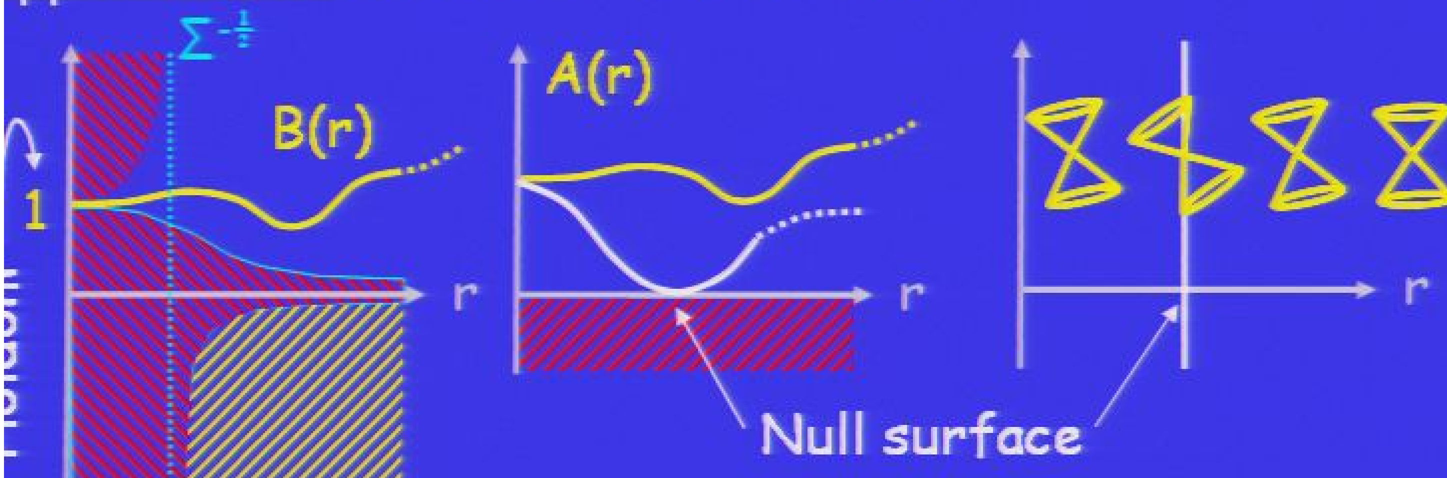

hm. ∄ space-like singularities in static spherically symmetric spacetimes

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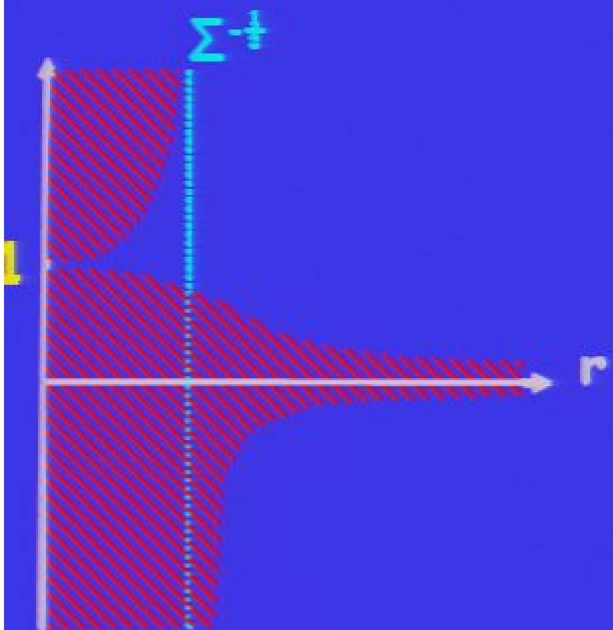

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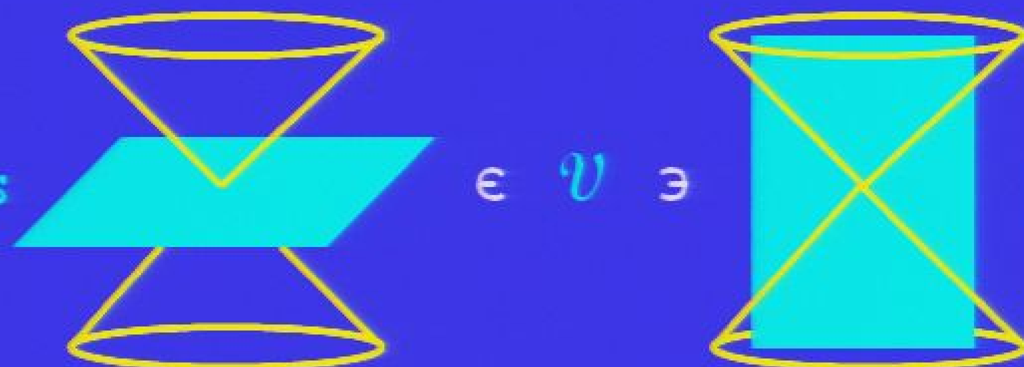

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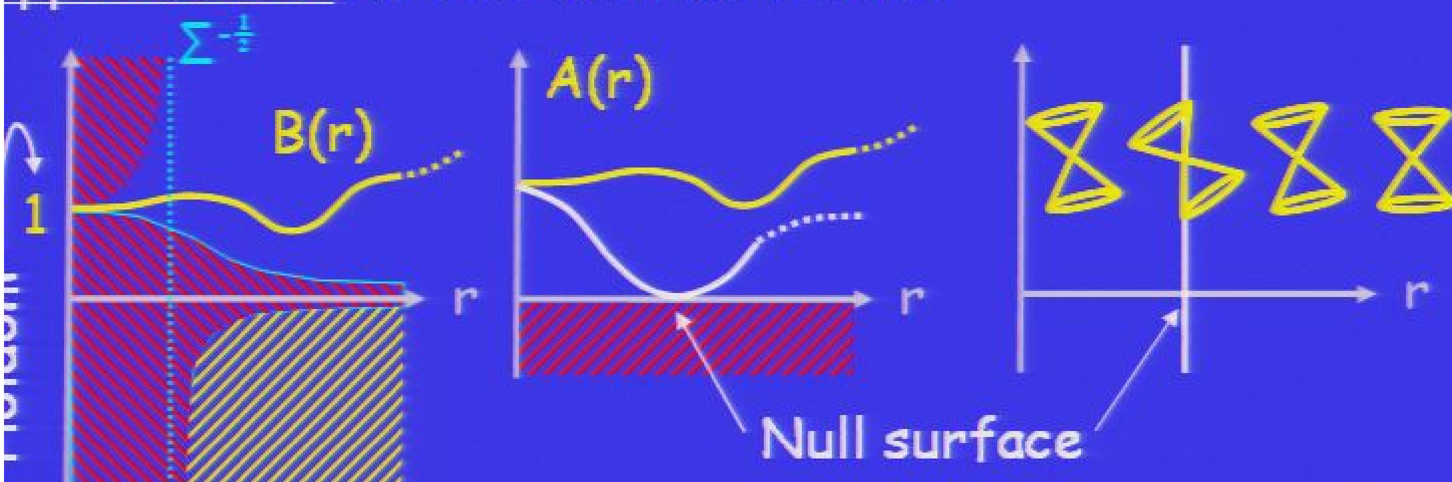

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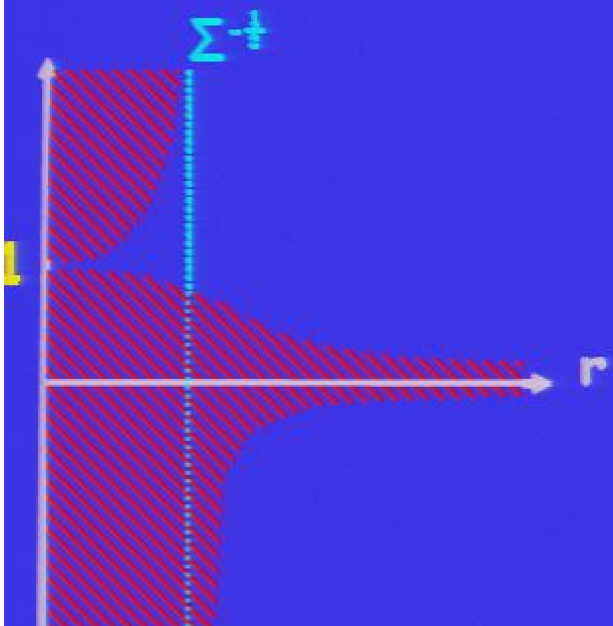
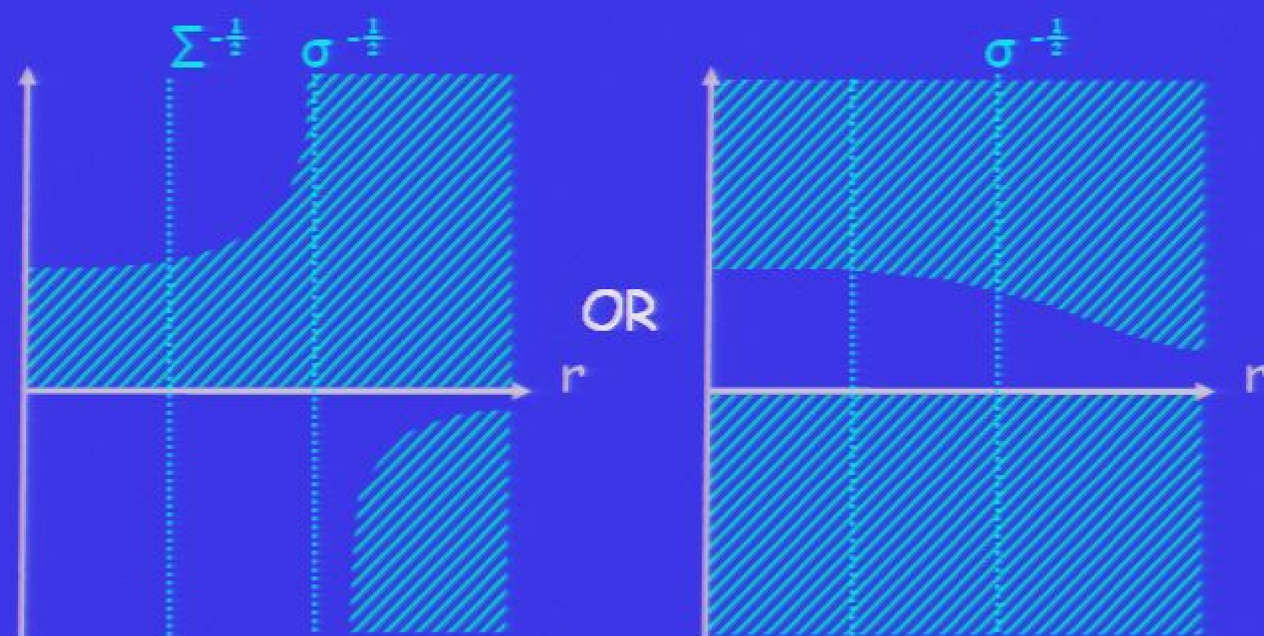


lower bounds:



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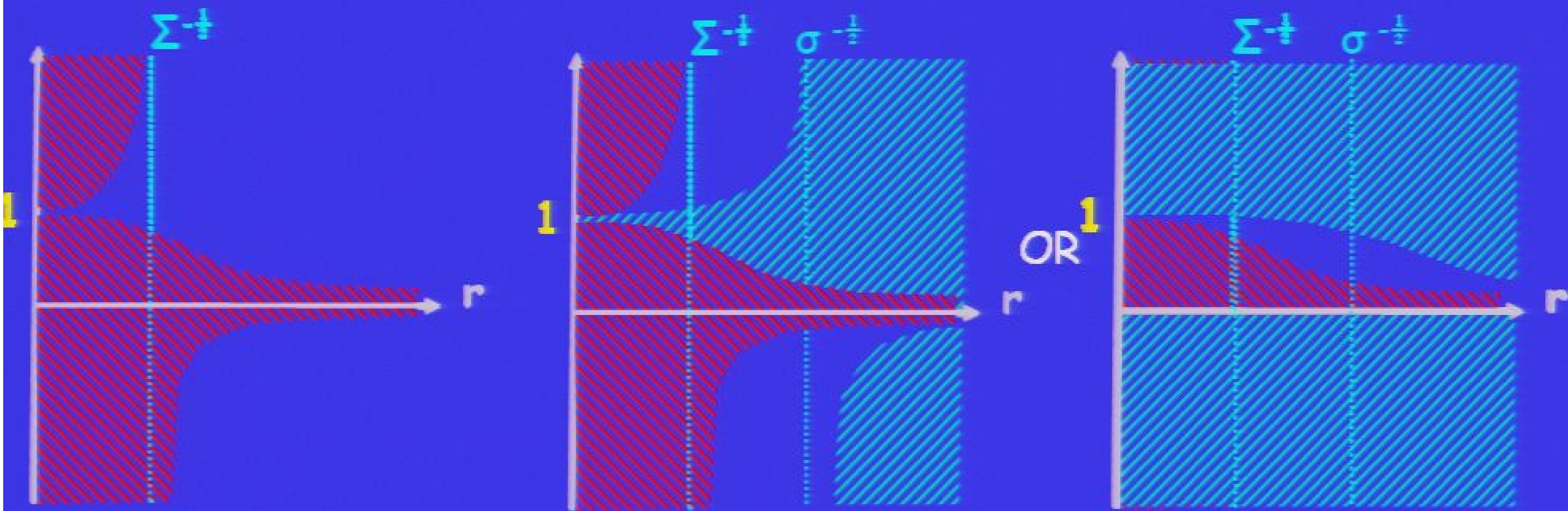

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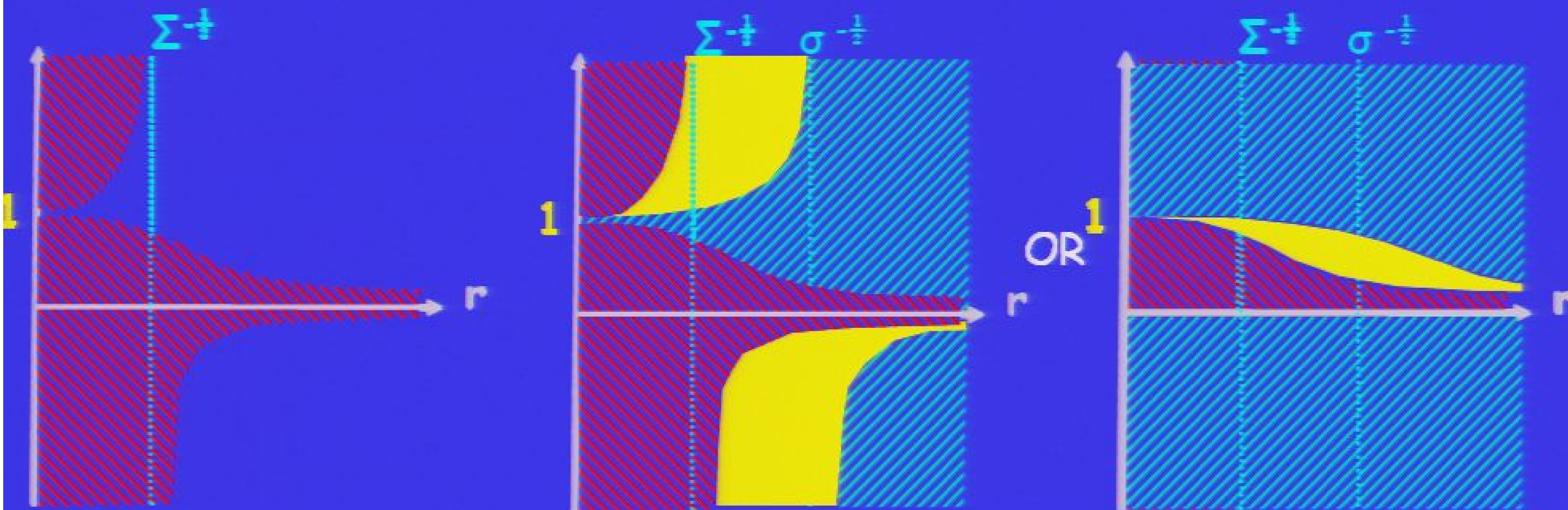


no Kepler region

Kepler region possible

no horizon-shielded singularities

lower and upper bounds $\Longrightarrow$ no static black holes if otherwise acceptable

EXAMPLE 3

pp-wave spacetime

$$ds^2 = -H(u,x^i)\,du^2 - 2\,du\,dv + dx\,dx$$

$\partial/\partial v$ covariantly constant null Killing vector

well-known: all curvature invariants vanish

$$\mathrm{Tr}\,[R^{n\,A}{}_B] = 0 \quad \text{all } n \geq 1$$

$\downarrow$

$R^A{}_B$-eigenvalue equation $\quad 0 = \det\,(R^A{}_B - r\,\delta^A{}_B) = r^{\,d(d-1)/2}$

bounds $\sigma \leq |r| \leq \Sigma$ satisfied for any plane wave

null-singularities

unavoidable in gravity on Lorentzian spacetime

[Horowitz + Myers 1995]

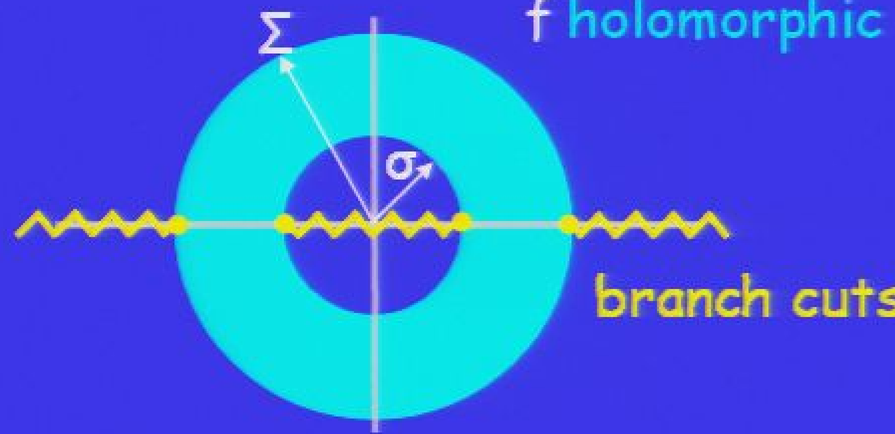
we could not bound all sectional curvatures,

hence we did not remove all singularities (!)

Thm. Sectional curvature bounds do not remove all (if any) null-singularities

gravitational action

$$S = \int \sqrt{-g}\ \mathrm{Tr}\ f(R^A{}_B)$$



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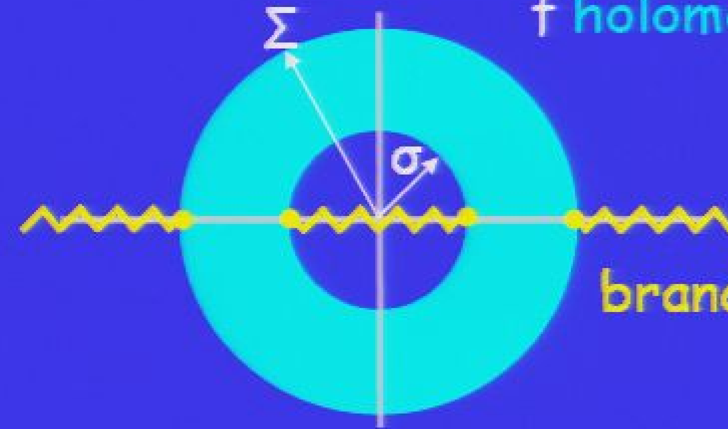
f-ambiguity reflects ignorance about exact quantum spacetime structure

f holomorphic



branch cuts

GRAVITATIONAL DYNAMICS

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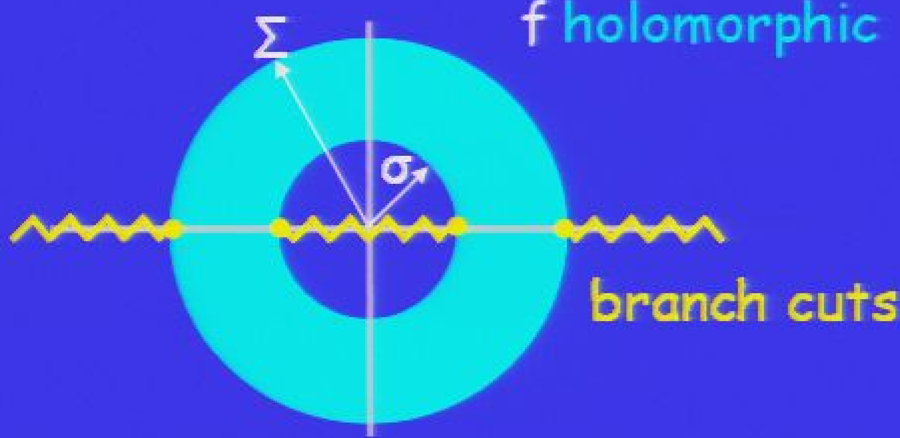

$$f(z) = \sum_{n\geq 0} \Sigma^{1-n}\, a_n\, z^n + \sigma^{1+n}\, a_{-n}\, z^{-n}$$

absolutely convergent !

(allows arbitrary re-ordering)

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$$\nabla_i \frac{\delta S}{\delta g_{ij}} = 0 \quad \text{(Noether constraint)}$$

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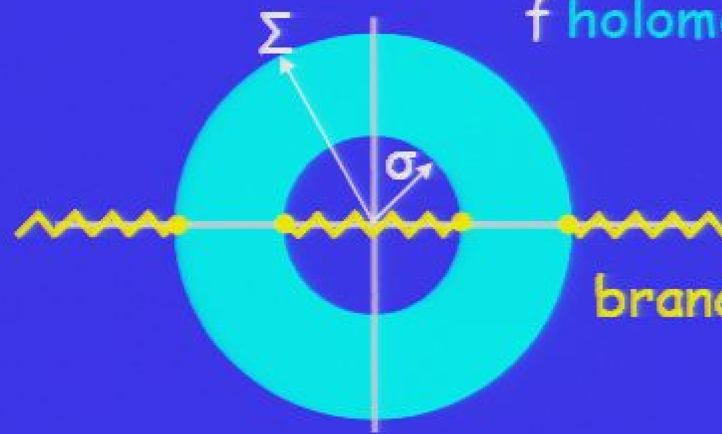

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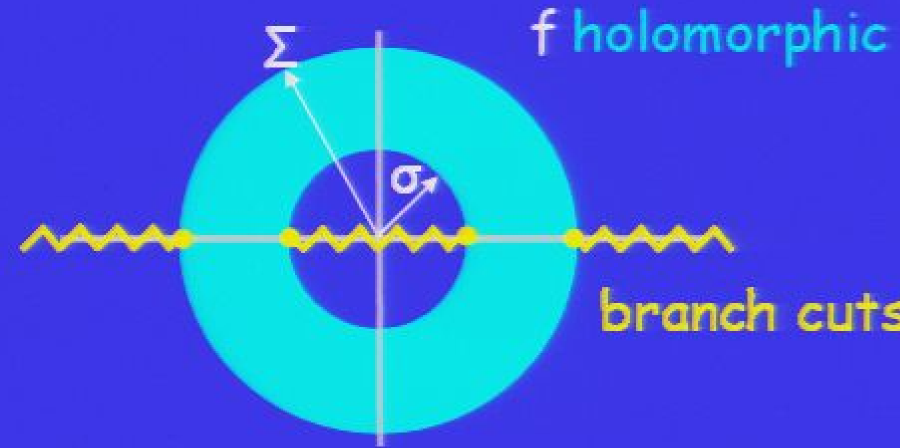

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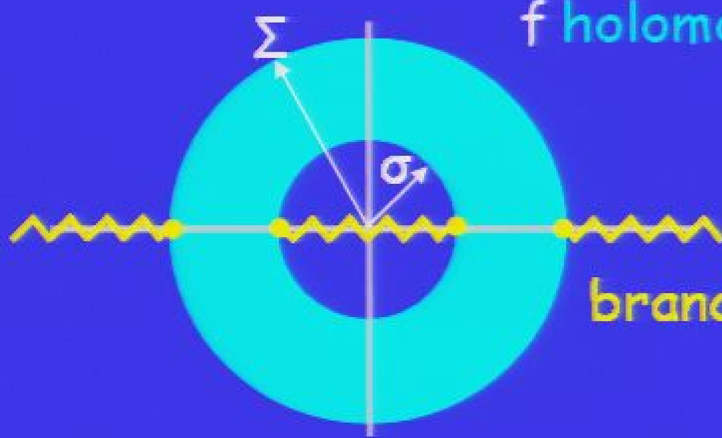

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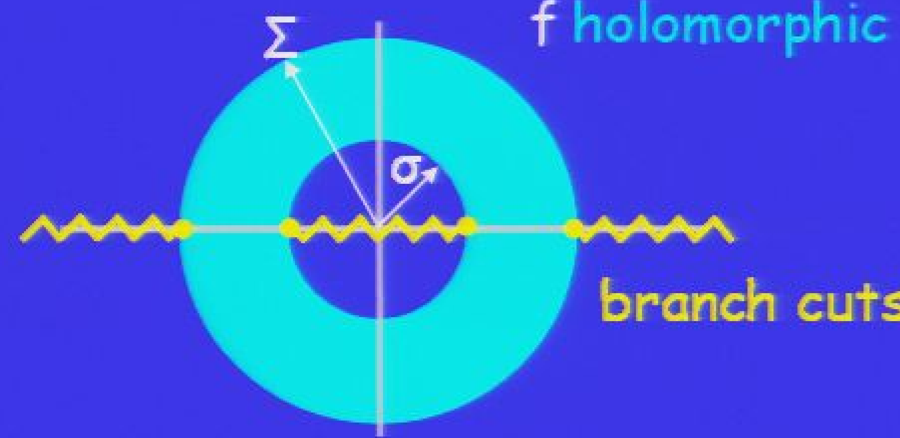

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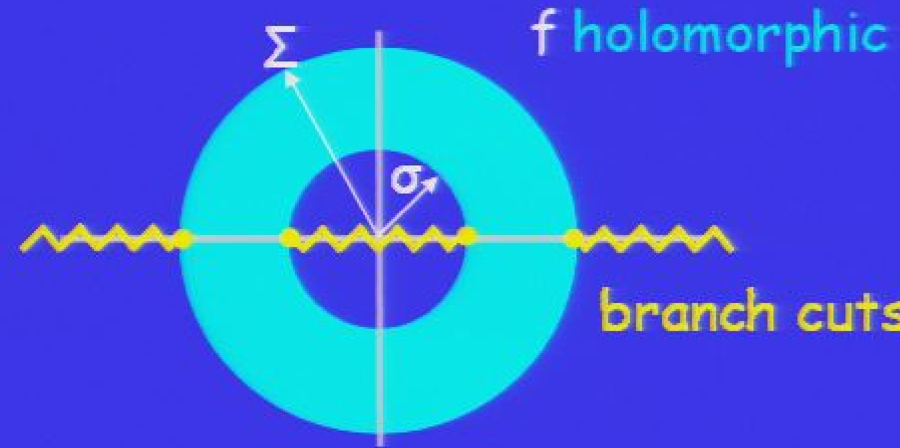

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2. as many theories as Laurent series on annulus

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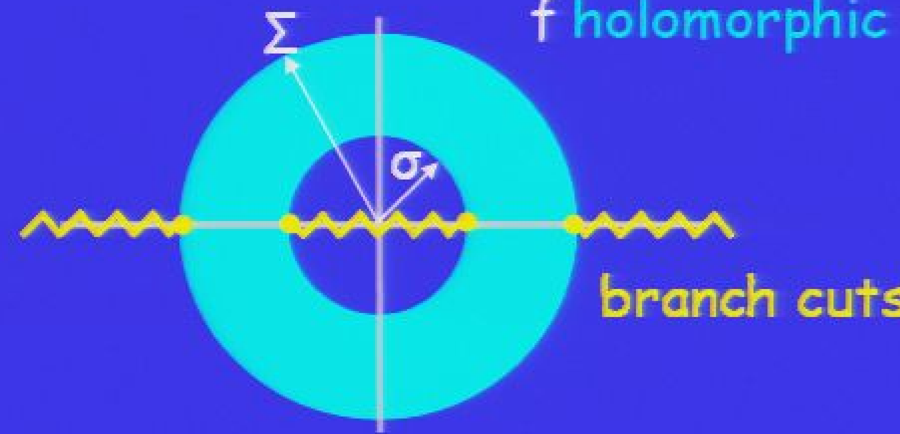

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absolutely convergent !

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GRAVITATIONAL DYNAMICS

gravitational action

$$S = \int \sqrt{-g}\ \mathrm{Tr}\ f(R^A{}_B)$$

f-ambiguity reflects ignorance about exact quantum spacetime structure

matter coupling as usual:

$$\nabla_i \frac{\delta S}{\delta g_{ij}} = 0 \quad \text{(Noether constraint)}$$

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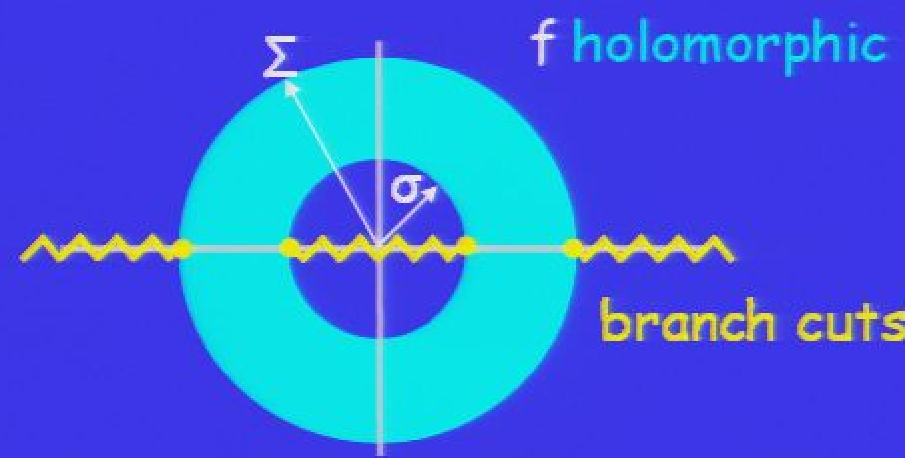

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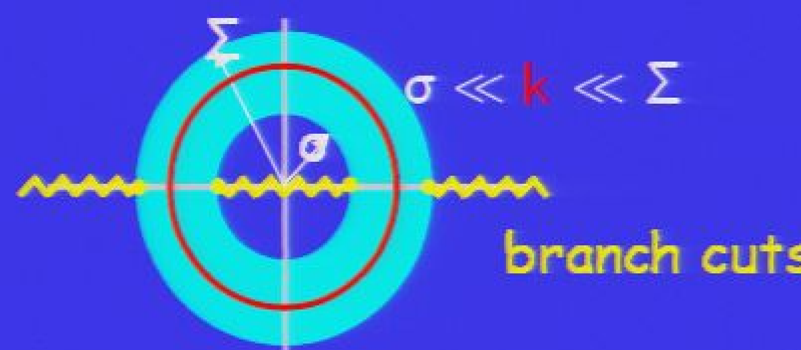
determines f_{-n} in terms of f_n

correct convergence $\Leftrightarrow \gamma = -\tfrac{1}{2}$



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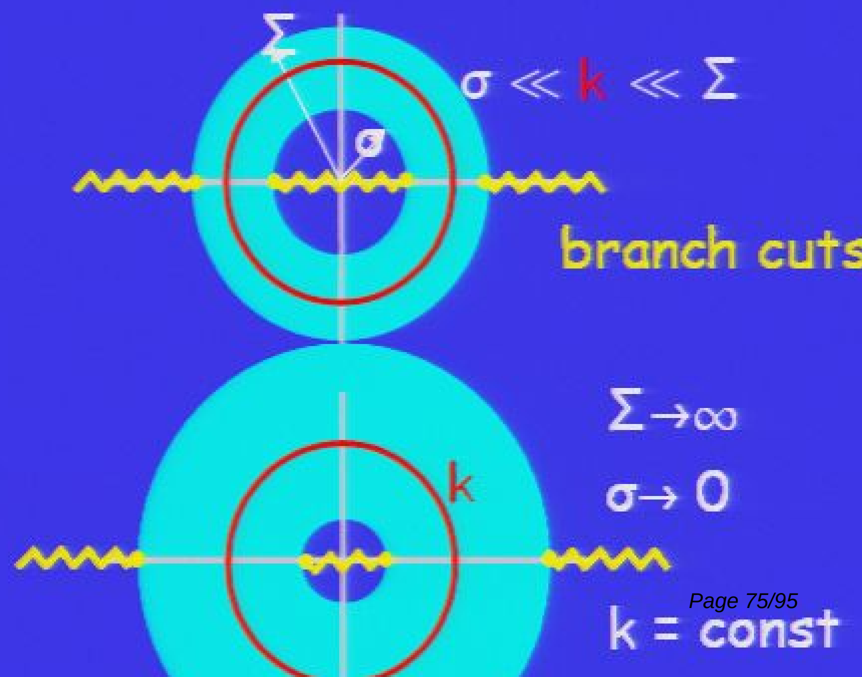

EINSTEIN'S SIBLINGS

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work in progress:

study of cosmological and spherically symmetric solutions

perturbation theory around dS   Galaxy rotation curves

Initial Value Formulation

Einstein-Hilbert:

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10 equations for 10 functions g_{ab}

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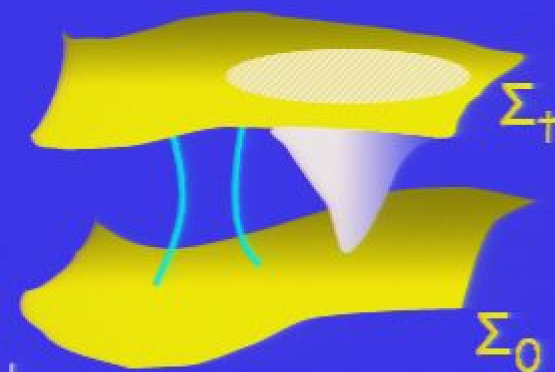

$G_{0b} = 0$ four constraints on Cauchy data

$G_{\alpha\beta} = 0$ six evolution equations

well-known: (i) only 6 degrees of freedom in g_{ab}

(ii) $G_{\alpha\beta} = 0$ partially linear hyperbolic

$\Longrightarrow$ locally unique causal solution

Einstein's siblings: $\int \sqrt{-g}\ \mathrm{Tr}\left[R^A{}_B + \delta \frac{k^2}{R^A{}_B}\right]$ initial value problem well-posed?

Bounded curvature theories: $\int \sqrt{-g}\ \mathrm{Tr}\ f(R^A{}_B)$ well-posed initial value problem further constrains function f

th order equations not prohibitive: $R + R^2$ well-posed IVF [Noakes 1982]

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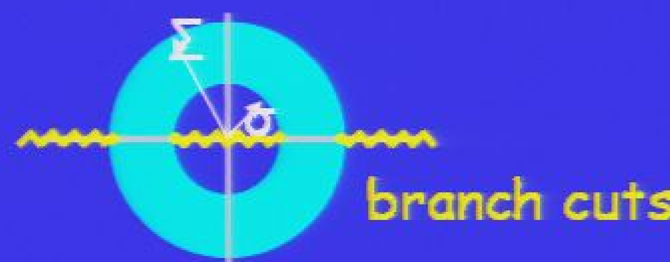

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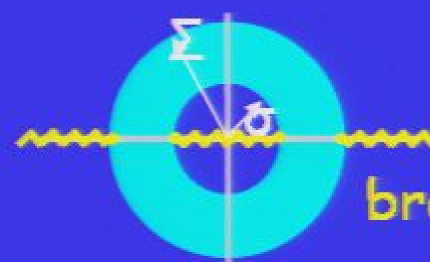

branch cuts

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fixes z^{-n} coefficients in terms of z^n coefficients

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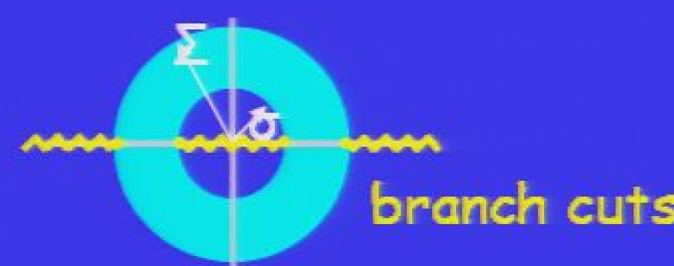

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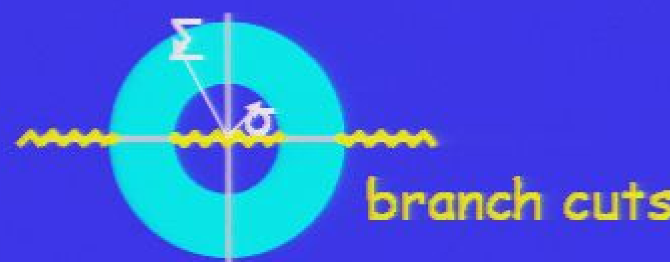

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open questions: Cauchy problem, stability, exact solutions: f(R) and R+1/R

To have and loose

is better than never having had

anonymous space-time manifold