

Title: Cohomology groups in mirror symmetry

Date: Nov 21, 2004 11:30 AM

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Abstract:

Mirror Symmetry for CY 3-folds

①

$$X, Y \quad Y = X^* \Rightarrow$$

$$h^{1,1}(X) = h^{2,1}(Y)$$

$$h^{2,1}(X) = h^{1,1}(Y)$$

$H^1(X)$ contains an integral lattice
 $H^1(X, \mathbb{Z})/\text{torsion} \cong H^1(X, T_X)$
 has an integral structure
 Same for complex
 $H^1(Y, \mathbb{Z})/\text{torsion}$ and $H^1(Y, T_Y)$
 Another mirror grading
 $H^3(X, \mathbb{Z})/\text{torsion} \cong H^3(Y, \mathbb{Z})$ is
 not graded

Mirror Symmetry for CY 3-folds

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$$X, Y \quad Y = X^* \Rightarrow$$

$$h^{1,1}(X) = h^{2,1}(Y)$$

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$H^4(X)$ carries an integral lattice

$H^2(X, \mathbb{Z})/\text{torsion}$ but $H^4(X) \cong h^2(X, \mathbb{Z})$

has no integral structure

But we can compare

$$H^2(X, \mathbb{Z})/\text{torsion} \text{ and } H^2(Y, \mathbb{Z})/\text{torsion}$$

Another problem: grading

$$H^4(X, \mathbb{Z})/\text{torsion} = h^2(X, \mathbb{Z})$$

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$H^{2,1}(X)/\text{hor}$ but $H^{2,1}(X) \cong h^{1,1}(Y)$

has no integral structure

Better to compare

$H^{1,1}(X, \mathbb{Z})/\text{hor}$ and $H^{2,1}(Y, \mathbb{Z})/\text{hor}$

Another problem: grading

$$H^{1,1}(X, \mathbb{Z})/\text{hor} \cong H^3(Y, \mathbb{Z})/\text{hor}$$

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$$\bigvee H^2(X, \mathbb{Z}) / \text{tor}, \quad \text{but } H^{2,1}(X) \cong h^1(X, T_X)$$

has an integral structure

Settin to compare

$$H^2(X, \mathbb{Z}) / \text{tor} \quad \text{and} \quad H^1(X, \mathbb{Z}) / \text{tor}$$

Another problem: grading

$$H^{2,1}(X, \mathbb{Z}) / \text{tor} = H^3(X, \mathbb{Z}) / \text{tor}$$

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Better to compare

$$H^{\text{ev}}(X, \mathbb{Z}) / \text{tor} \quad \text{and} \quad H^{\text{odd}}(Y, \mathbb{Z}) / \text{tor}$$

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$$H^{\text{odd}}(Y, \mathbb{Z}) / \text{tor} = H^3(Y, \mathbb{Z}) \text{ is}$$

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not graded

$$H^{\text{even}}(X, \mathbb{Z}) \hookrightarrow K_0(X)$$

$$H^{\text{odd}}(X, \mathbb{Z}) \xrightarrow{\text{K-Theory}} K_1(X)$$

probably the best choice!

②

The case of Calabi-Yau varieties
 $Y = X^n$ of dimension $d > 3$
 is more complicated

$$H^*(X) = H^*(Y)$$

again, we have problems with integral
 structures.

$$K(X) \cong \bigoplus H^{2i}(X)$$

Limited category of coherent sheaves
 on X

$$\bigoplus H^{2i}(Y)$$

Finite category of vector
 bundles on Y

$$H^{\text{even}}(X, \mathbb{Z}) \xrightarrow{\quad} K_0(X)$$

$$H^{\text{odd}}(X, \mathbb{Z}) \xrightarrow{\quad} K_1(X)$$

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K-Theory

The case of Calabi-Yau varieties
 $X \subset \mathbb{P}^n$ of dimension $d > 3$
 is more complicated
 $H^*(X) = H^*(X)$

we have problems with integral
 structure

$$K_0(X) \cong H^{2d}(X)$$

derived category of coh sheaves
 on X

$$\bigoplus_{i=0}^d H^{2i}(X) \cong \text{Bridgeland category of complexes on } X$$

$$H^{\text{even}}(X, \mathbb{Z}) \hookrightarrow K_0(X)$$

$$H^{\text{odd}}(X, \mathbb{Z}) \longrightarrow K_1(X)$$

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K-Theory

$$H^{\text{even}}(X, \mathbb{Z}) \longmapsto K_0(X)$$

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$$H^{\text{odd}}(X, \mathbb{Z}) \longrightarrow K_1(X)$$

K-Theory

probably the best choice!

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$$H^{\text{odd}}(X, \mathbb{Z}) \xrightarrow{\text{K-Theory}} K_1(X)$$

probably the best choice!

$$H^{\text{even}}(X, \mathbb{Z}) \hookrightarrow K_0(X)$$

(2)

$$H^{\text{odd}}(X, \mathbb{Z}) \xrightarrow{\text{K-Theory}} K_1(X)$$

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The case of Calabi-Yau varieties
 $X, Y = X^*$ of dimension $d > 3$
 is more complicated

$$h^{p,q}(X) = h^{d-p,q}(Y)$$

again we have problems with integral
 structures

$$H^{\text{even}}(X, \mathbb{Z}) \longmapsto K_0(X)$$

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 structures

$$K_0(X), \text{ or } \bigoplus H^{p,p}(X)$$

derived category of coh. sheaves
 on X

$$\bigoplus_{p+q=d} H^{p,q}(X)$$

Fukaya category of some
 d -cycles on Y

$$H^{\text{even}}(X, \mathbb{Z}) \longmapsto K_0(X)$$

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$$H^{\text{odd}}(X, \mathbb{Z}) \xrightarrow{\text{K-Theory}} K_1(X)$$

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Aim: Mathematical experiments with
integral cohomology of Calabi-Yau
3-folds using toric construction

③

This is a joint work with
Max Kerner

(work in progress)

Toric terminology

$$M = T^n, \quad H^*(M, \mathbb{Z}) = \mathbb{Z}[x_1, \dots, x_n]$$

$$H^*(M, \mathbb{Q}) = \mathbb{Q}[x_1, \dots, x_n]$$

$$H^*(M, \mathbb{R}) = \mathbb{R}[x_1, \dots, x_n]$$

$$H^*(M, \mathbb{C}) = \mathbb{C}[x_1, \dots, x_n]$$

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Toric terminology

$$M \cong \mathbb{Z}^d \quad N = \text{Hom}(M, \mathbb{Z})$$

$$T^d := \text{Spec } \mathbb{C}[M]$$

$$M = \text{Hom}_{\text{alg}}(\mathbb{T}^d, \mathbb{G}_m) \quad \text{characters}$$

$$N = \text{Hom}_{\text{alg}}(\mathbb{G}_m, \mathbb{T}^d) \quad \begin{array}{l} \text{1-param.} \\ \text{subgroups} \end{array}$$

$$\langle *, * \rangle : M \times N \rightarrow \mathbb{Z}$$

pairing

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pairing

$\Delta \subseteq M_{\mathbb{R}} = M \otimes \mathbb{R}$ lattice

(4)

polytope of $\dim = d$

$\mathbb{P}_{\Delta}$ toric compactification of Δ
(singular)

Δ is called reflexive if

(1) $\text{Int } \Delta \cap M = \{0\}$

(2) $\Delta^* = \{y \in N_{\mathbb{R}} : \langle x, y \rangle \geq -1 \ \forall x \in \Delta\}$

is again a (reflexive) lattice
polytope

$(\Delta^*)^* = \Delta$ polar duality

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Δ is called reflexive if
 $\Delta^{\circ} = \Delta^{\vee} = \Delta^*$

$\Delta^{\circ} = \{ \gamma \in M \mid \langle \gamma, \alpha \rangle \geq 1 \text{ for all } \alpha \in \Delta \}$
 $\Delta^{\vee} = \{ \gamma \in M \mid \langle \gamma, \alpha \rangle \geq 0 \text{ for all } \alpha \in \Delta \}$
 $\Delta^* = \{ \gamma \in M \mid \langle \gamma, \alpha \rangle \in \mathbb{Z} \text{ for all } \alpha \in \Delta \}$

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(4)

$\mathbb{P}_{\Delta}$ toric compactification of $\mathbb{A}^d$
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⑤

Let $\Delta \subset M_{\mathbb{R}}$ be reflexive
polytope

$$f(x) = \sum_{m \in \Delta \cap M} a_m x^m \quad \text{Laurent polynomial}$$

$$Z_f = \{f=0\} \subset \mathbb{T}^d \quad \text{hypersurface}$$

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$$\bar{Z}_f \subset \mathbb{P}_{\Delta} \quad \text{closure of } Z_f \text{ in } \mathbb{P}_{\Delta}$$

5

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$$\bar{Z}_f \quad \text{normal projective with trivial} \\ \text{canonical sheaf } \omega_{\bar{Z}_f} = \mathcal{O}_{\bar{Z}_f}$$

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$Z_f = \{f=0\} \subset \mathbb{T}^d$ hypersurface

$\bar{Z}_f \subset \mathbb{P}_{\Delta}$ closure of Z_f in $\mathbb{P}_{\Delta}$

$\bar{Z}_f$ normal projective with trivial
canonical sheaf $\omega_{\bar{Z}_f} = \mathcal{O}_{\bar{Z}_f}$

$\exists$ desingularization $\hat{Z}_f$ of $\bar{Z}_f$
which still has
trivial canonical class ($d \leq 4$)

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⑥

Hodge numbers of $\mathbb{Z}_1^1$

$\ell(P) := \#$ of lattice points in P

$\ell^*(P) := \#$ of lattice points in $\text{Int } P$.

⑥

Hodge numbers of $\hat{Z}_f$

$\ell(P) := \#$ of lattice points in P

$\ell^*(P) := \#$ of lattice points in $\text{Int } P$.

$d=4$

$$h^{2,1}(\hat{Z}_f) = \ell(\Delta) - 5 - \sum_{\substack{\dim \theta = 3 \\ \theta \subset \Delta}} \ell^*(\theta) + \sum_{\substack{\dim \theta = 2 \\ \theta \subset \Delta}} \ell^*(\theta) \cdot \ell^*(\theta^*)$$

⑥

Hodge numbers of $\hat{Z}_f$

$\ell(P) := \#$ of lattice points in P

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$d=4$

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Hodge numbers of $\hat{Z}_f^1$

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$d=4$

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⑥

Hodge numbers of $\hat{Z}_f$

$l(P) := \#$ of lattice points in P

$l^*(P) := \#$ of lattice points in $\text{Int } P$.

$d=4$

$$h^{2,1}(\hat{Z}_f) = l(\Delta) - 5 - \sum_{\substack{\dim \theta = 3 \\ \theta \subset \Delta}} l^*(\theta) + \sum_{\substack{\dim \theta = 2 \\ \theta \subset \Delta}} l^*(\theta) \cdot l(\theta^*)$$

$$h^{1,1}(\hat{Z}_f) = l(\Delta^*) - 5 - \sum_{\substack{\dim \theta^* = 3 \\ \theta^* \subset \Delta^*}} l^*(\theta^*) + \sum_{\substack{\dim \theta^* = 2 \\ \theta^* \subset \Delta^*}} l^*(\theta^*) \cdot l(\theta)$$

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Hodge numbers of $\hat{Z}_f$

$l(P) := \#$ of lattice points in P

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$$h^{2,1}(\hat{Z}_f) = l(\Delta) - 5 - \sum_{\substack{\dim \theta = 3 \\ \theta \subset \Delta}} l^*(\theta) + \sum_{\substack{\dim \theta = 2 \\ \theta \subset \Delta}} l^*(\theta) \cdot l(\theta^*)$$

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$$l(P) = (\dim P) \cdot d(P) \in \mathbb{Z}$$

⑥

Hodge numbers of $\hat{Z}_f$

$l(P) := \#$ of lattice points in P

$l^*(P) := \#$ of lattice points in $\text{Int } P$.

$d=4$

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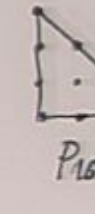
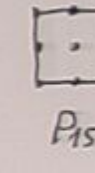
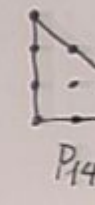
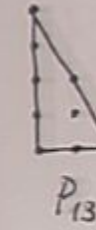
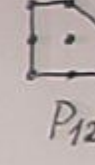
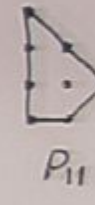
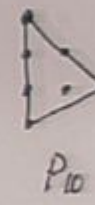
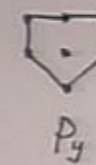
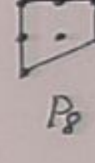
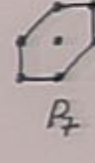
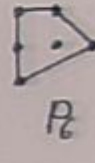
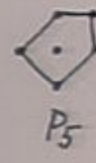
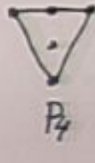
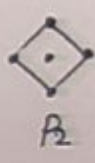
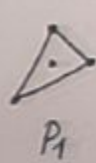
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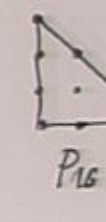
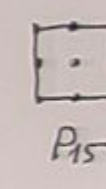
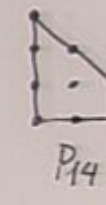
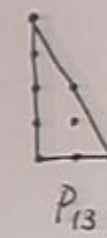
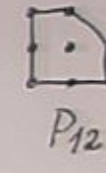
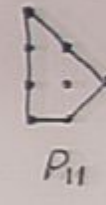
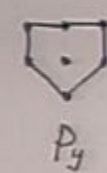
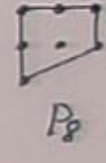
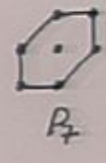
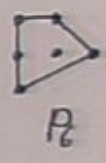
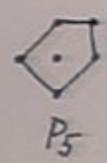
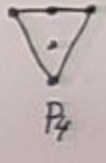
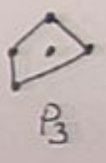
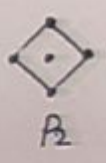
integral "volume".

8



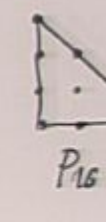
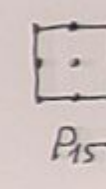
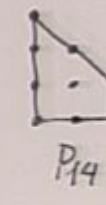
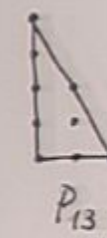
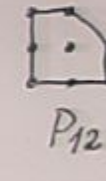
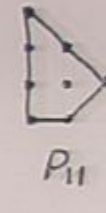
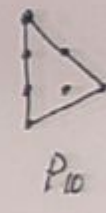
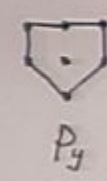
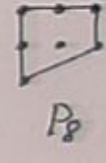
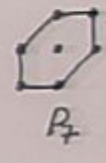
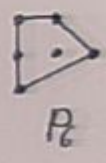
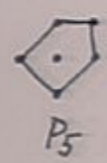
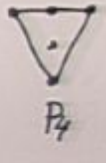
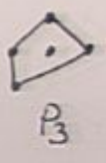
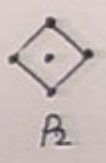
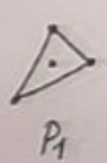
$$P_i \cong P_{16-i}^*$$

8



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Kreuzer and Skarke:

(7)

$\exists$ 473,800,776 reflexive
polytopes of $\dim = 4$

30,108 distinct pairs
of Hodge numbers
($h^{11}, h^{2,1}$)

$\exists$ 4319 reflexive polytopes
of $\dim = 3$

Computer data

(see homepage of Max Kreuzer)

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Integral cohomology
of a 3-fold $X/\mathbb{C}$

⑨

$$A := \text{tor}(H_1(X, \mathbb{Z}))$$

$$A^* = \text{Hom}(A, \mathbb{Q}/\mathbb{Z})$$

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$$H^2(X, \mathbb{Z}) = A^* \oplus \mathbb{Z}^a$$

$$H^3(X, \mathbb{Z}) = B^* \oplus \mathbb{Z}^{2b+2}$$

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$$H_{\text{tor}}^{\text{ev}}(X, \mathbb{Z}) \cong \left(H_{\text{tor}}^{\text{od}}(X, \mathbb{Z}) \right)^*$$

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Poincaré duality +

$$\text{tor}(H_i(X, \mathbb{Z})) = \left[\text{tor}(H^{i+1}(X, \mathbb{Z})) \right]^*$$

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$$\pi_1(\hat{Z}_f) \cong H_1(\hat{Z}_f, \mathbb{Z}) = N/N'$$

$$\sum_{i=1}^n \mathbb{Z} \alpha_i \subset N$$

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$$\pi_1(\hat{Z}_f) \cong H_1(\hat{Z}_f, \mathbb{Z}) = N/N!$$

where
 $\sum_{i=1}^n Z_i \in N$
and $N/N!$
is the quotient group
of N by the subgroup
generated by $N!$.
In this case, $N/N!$
is isomorphic to $\mathbb{Z}^3$.
The fundamental group
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$a = 3$ 2 examples

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$$a=5 \quad \bar{Z}_f \subset \mathbb{P}_\Delta = \mathbb{P}^4/\mu_5$$

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$\mu_5 = \langle \zeta \rangle$ acts by $(1, \zeta, \zeta^2, \zeta^3, \zeta^4)$
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(using SYZ-construction for mirrors)

$$H_{\text{tor}}^2(X, \mathbb{Z}) \cong H_{\text{tor}}^3(X^*, \mathbb{Z})$$

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relation between the fundamental
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Elements in $B(\mathbb{T}^d)$

(15)

$$\mathbb{T}^d = \text{Spec } \mathbb{C}[M]$$

Take $v \in \mathbb{N}$ and $\alpha, \beta \in M$

Symbol algebra $(\alpha, \beta)_v$ is generated by u, v with relations

$$u^v = \alpha, v^v = \beta \quad uv = \zeta vu$$

ζ is a primitive v -th root of unity

Ex: $\langle \alpha, \beta \rangle_2$ is a quaternionic algebra

$${}_v B(\mathbb{T}^d) = \{ v\text{-torsion elements in } B(\mathbb{T}^d) \}$$

${}_v B(\mathbb{T}^d)$ is a free $\mathbb{Z}/v\mathbb{Z}$ -module

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$m_1, m_2, \dots, m_d$ Basis of M

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Elements in $B(T^d)$

(15)

$$T^d = \text{Spec } \mathbb{C}[M]$$

Take $v \in \mathbb{N}$ and $\alpha, \beta \in M$

Symbol algebra $(\alpha, \beta)_v$ is generated by u, v with relations

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Brauer group of
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$$B(\hat{Z}_f) = B'(\hat{Z}_f) = \text{Hom}(N/N^4, \mathbb{Q}/\mathbb{Z})$$

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Computer program

17

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4-dimensional polytopes with $q_3 > 1$

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These 16 CY 3-folds
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of CY-3fold with nontrivial
cyclic fundamental group
 $\pi_1 = \mathbb{Z}/a\mathbb{Z} \quad a = 2, 3, 5$

In particular

Gross conjecture is true!

Example of CY 3-fold
with $B(X) \cong \mathbb{Z}/5\mathbb{Z}$

related to Aspinwall(-Morrison
example and a counterexample
to global Torelli theorem
(Baldus Szendroi)

$H := \mu_5 \times \mu_5$ action on $\mathbb{P}^4$
 $\begin{matrix} \downarrow & \downarrow \\ \xi_1 & \xi_2 \end{matrix}$

$$\xi_1 \rightarrow (1, \xi_1, \xi_1^2, \xi_1^3, \xi_1^4)$$

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$\mathbb{P}_\Delta = \mathbb{P}^4/H$ has singularities along 10
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$$\pi_1(\hat{\Sigma}_f) = 0, \text{ but}$$

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