Title: Strominger-Yau-Zaslow revisited

Date: Nov 21, 2004 10:00 AM

URL: http://pirsa.org/04110024

Abstract:

①

Strominger-Yau-Zaslow (SYZ), 1996:

Let Y be a Calabi-Yau 3-fold.

with a mirror partner X. (both compact)

$$\{D0 \text{ on } Y\} \longleftrightarrow \{D3 \text{ on } X\}$$

0-brane charge $=1$

Some family of D3-branes

$$\mathcal{M}_{D0,\,\text{charge } 1} = Y \longleftrightarrow \mathcal{M}_{D3}$$

$\uparrow$ Some particular family

Semi-
Classical moduli space of Dp-branes: ②

- geometric moduli (as submanifolds)
- bundle moduli (gauge field)

$H^1(M,\mathbb{R})/H^1(M,\mathbb{Z})$

BPS D3-branes on Calabi-Yau 3-folds:

Special Lagrangian Cycles.

ω = Kähler form on X

Ω = holomorphic 3-form on X

$$\omega|_M = 0 \text{ and } \mathrm{Im}(\Omega)|_M = 0.$$

Geometric moduli (McLean):

has dimension $b_1(M)$, for M a manifold.

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has dimension $b_1(M)$, for M a manifold.

So for M a manifold,

$$\dim \mathcal{M}_M = 6$$

requires

$$b_1(M) = 3.$$

Y has an open set Y^0 mapping to a 3-manifold B^0 with fibers $\check{T}^3 = H^1(M,\mathbb{R}) / H^1(M,\mathbb{Z})$

$$\begin{tikzcd}
\check{T}^3 \arrow[r] & Y^0 \arrow[d] & \subseteq & Y \arrow[d] \\
 & B^0 & \subseteq & B
\end{tikzcd}$$

Supersymmetric T^3-fibration (singular fibers allowed).

Mirror symmetry is symmetric:

$$\begin{tikzcd} T^3 \arrow[r] & X^0 \arrow[d] & \subseteq & X \arrow[d] \\ & B^0 & \subseteq & B \end{tikzcd}$$

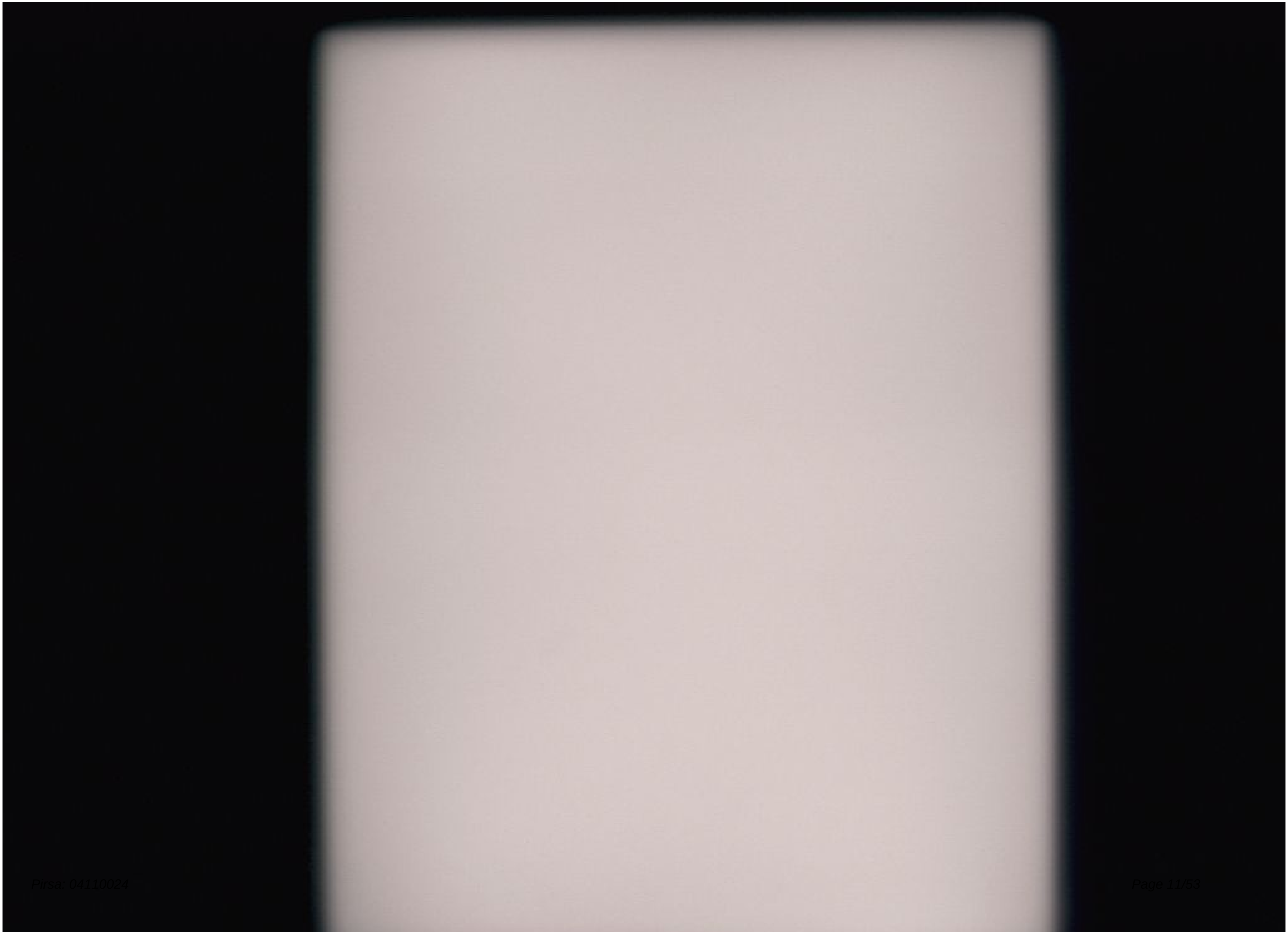
Questions

1) What are the singular fibers?

2) How does the mirror operation work?

3) What other data is needed to specify the fibration?

Generalization to Calabi-Yau n-folds, T^n fibrations



Example 1

$$X = T^2, \quad M = S^1.$$

$$\begin{array}{ccc} S^1 & \rightarrow & T^2 \\ & & \downarrow \\ & & S^1 \end{array}$$

No singular fibers.

Example 2

$$X = K3, \quad M = T^2.$$

$$\begin{array}{ccc} T^2 & \rightarrow & X \\ & & \downarrow \\ & & S^2 \end{array}$$

- holomorphic fibration w.r.t. same complex structure
- Kodaira singular fibers
- generic: 24 I_1 fibers.

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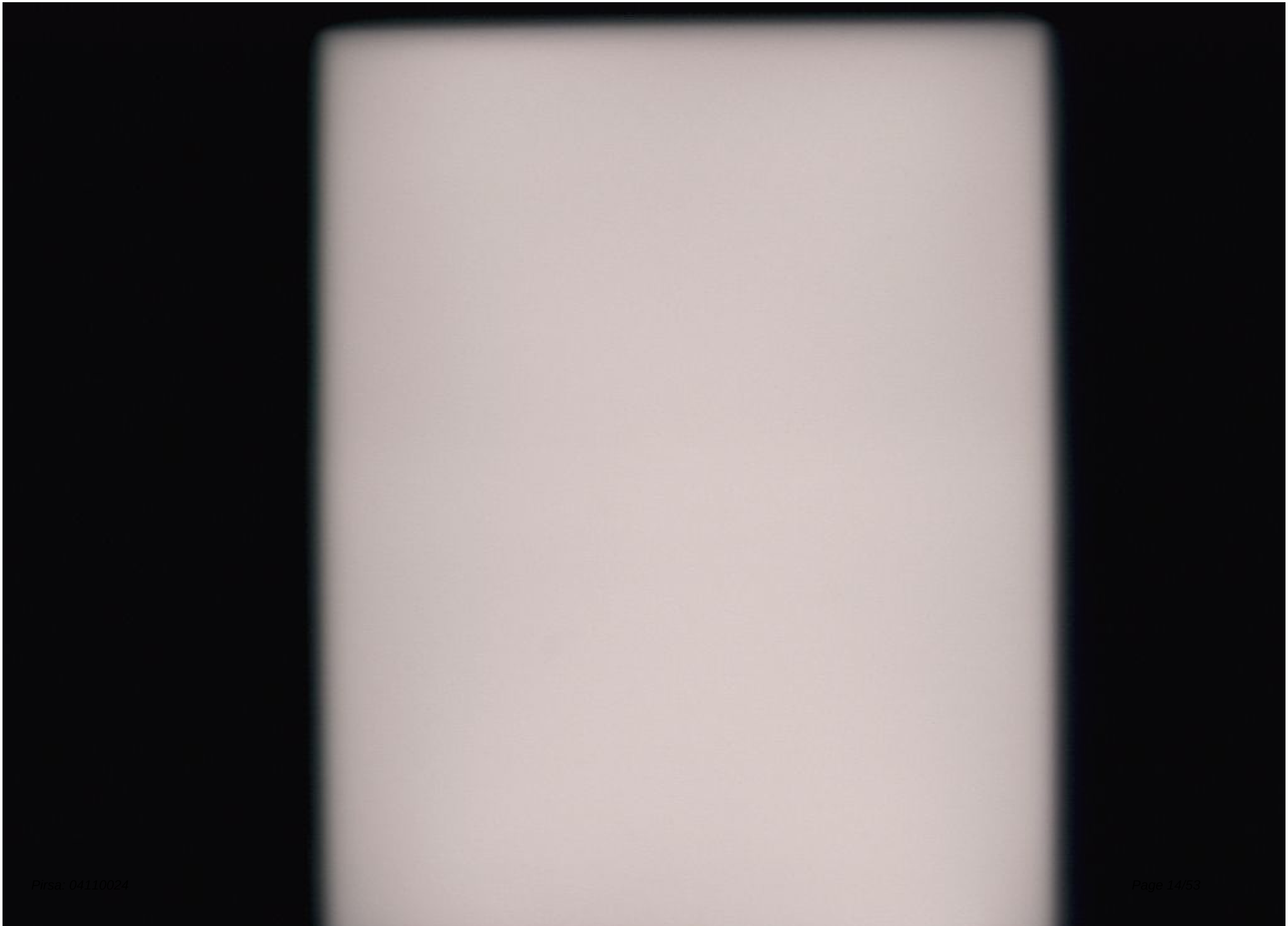

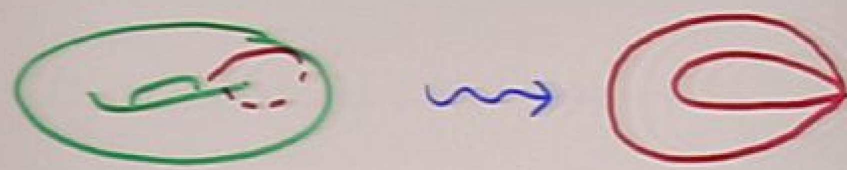

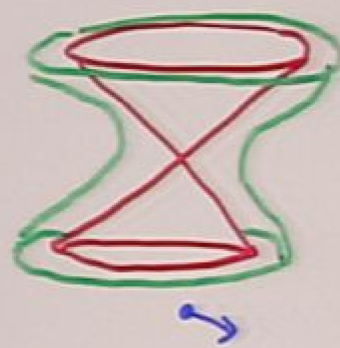

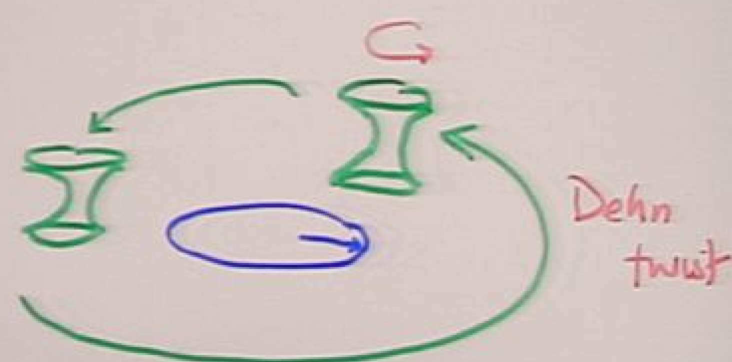
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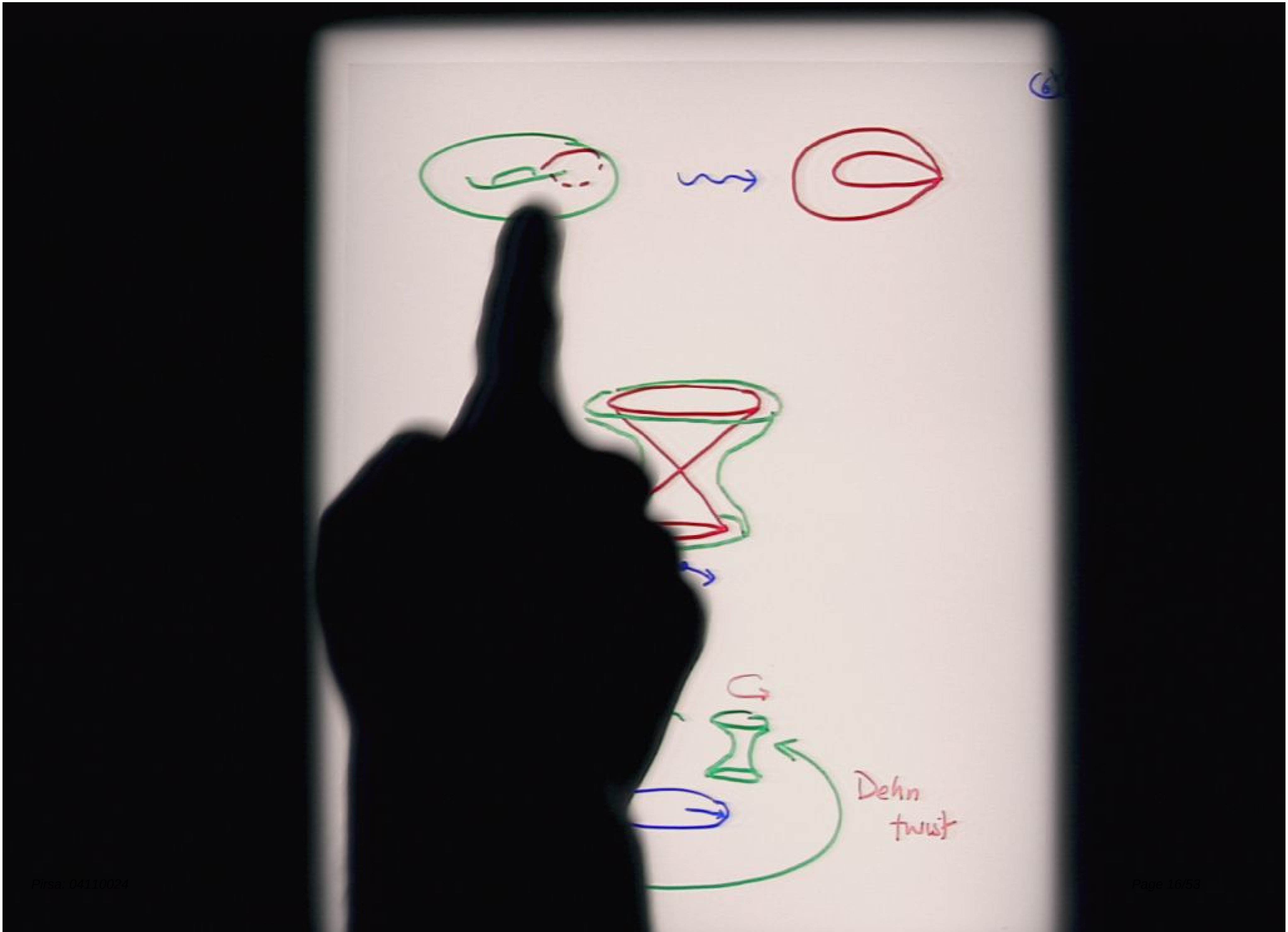
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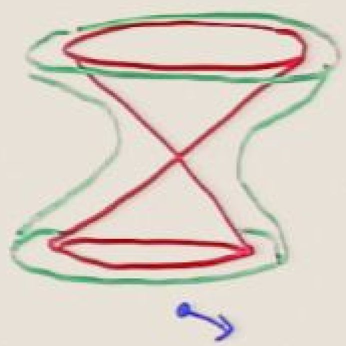
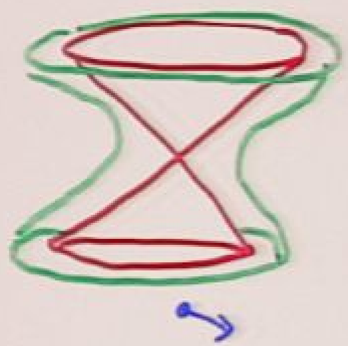
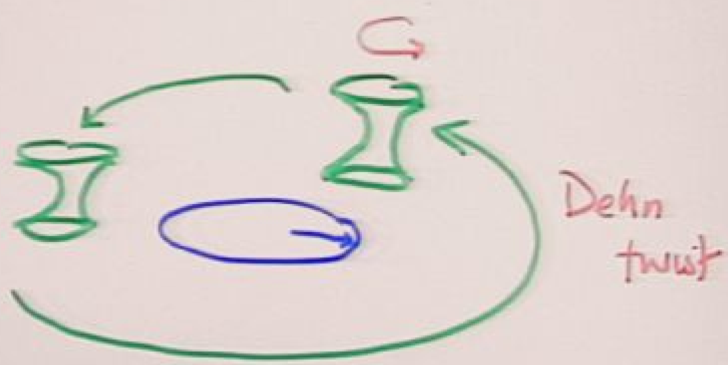


Dehn twist

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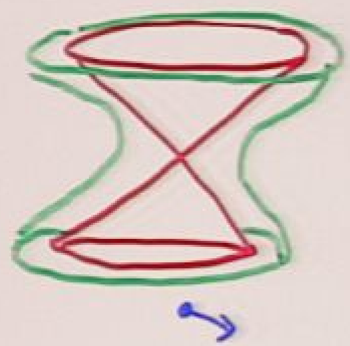
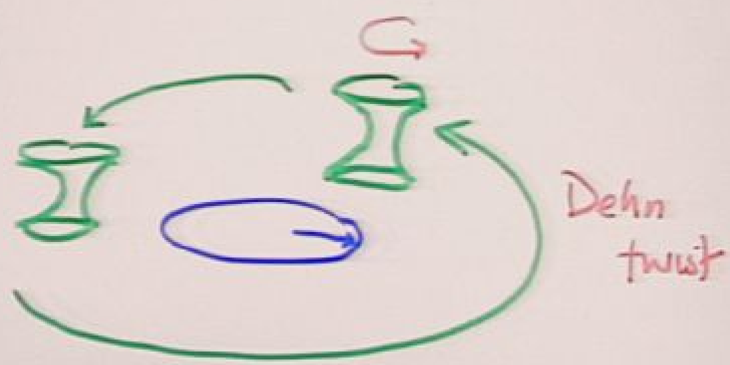




Monodromy: $\sim \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

- Singular point of fiber has codim 4

Taub-NUT

- Local U(1) action:

M theory on X (locally)

= IIA on $X/U(1)$

with D6-brane

at singular point

$$\begin{array}{ccc} X & \supseteq & S \\ \downarrow & & \downarrow \\ B & \supseteq & D \end{array}$$

- S has real codim 4

  speculation: <u>complex subvariety</u>

- Codim 6 singularities are transverse triple points

$\mathrm{Sing}(z_1 z_2 z_3 = 0)$.

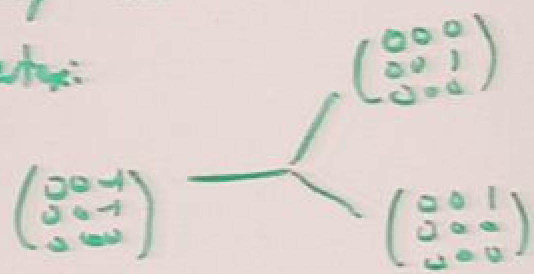
- Image of S in B retracts to a trivalent graph Γ

- Images of triple points are <u>positive</u> vertices

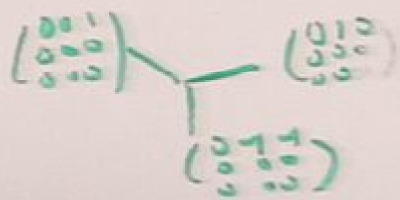
<u>Negative</u> vertices are retractions of pairs of points

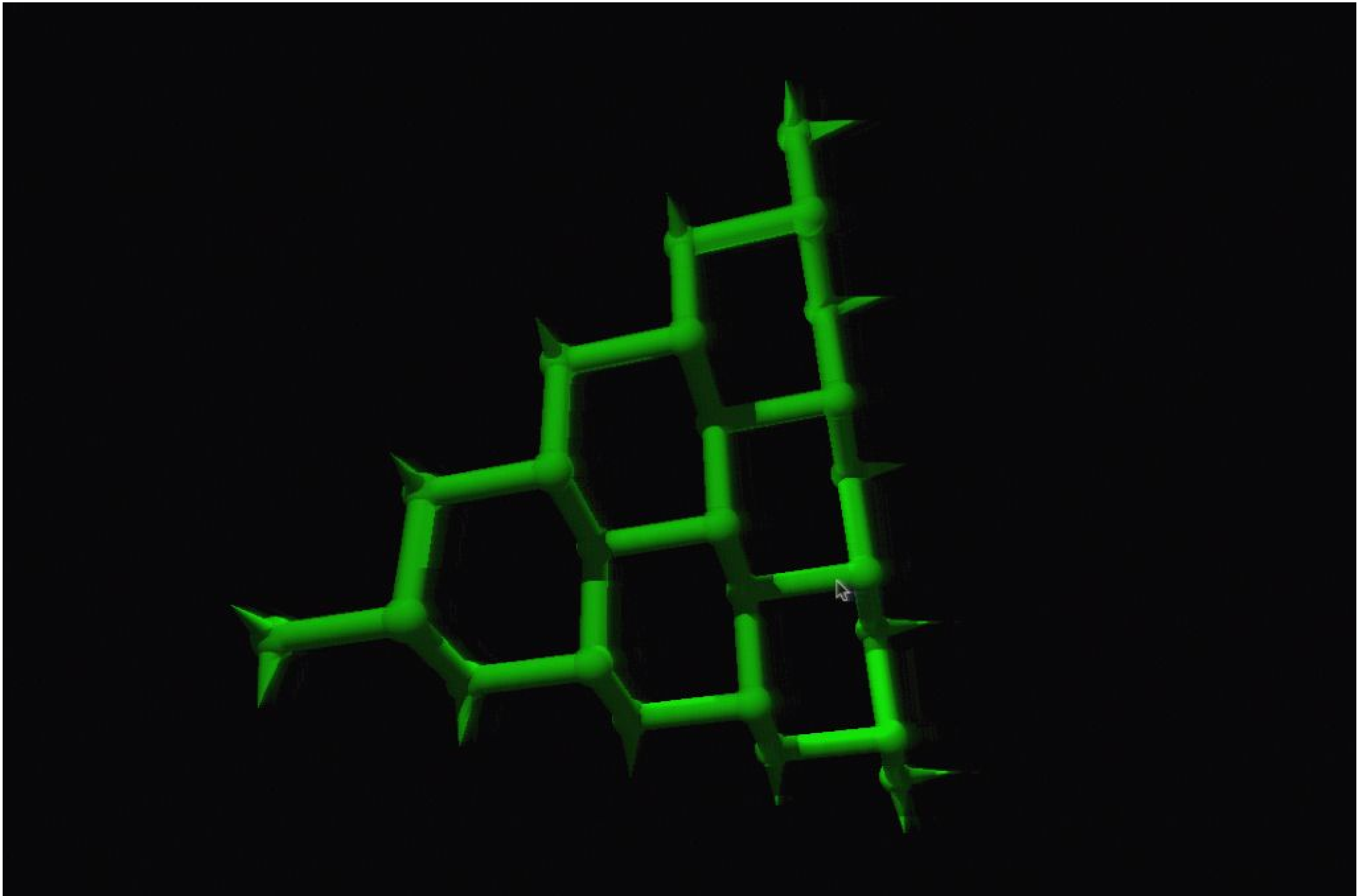
- Monodromy near vertices:

  + vertex:



  − vertex





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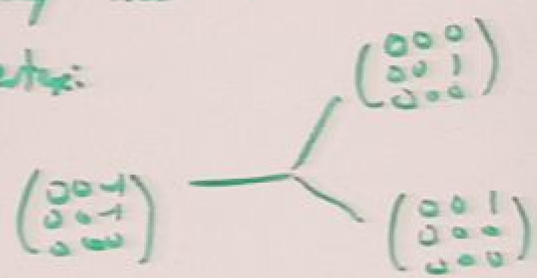

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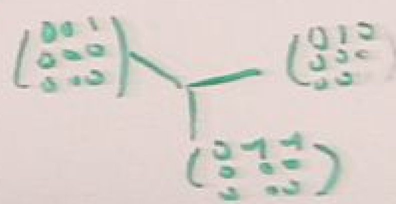

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Old speculation:

- Discriminant locus in B has codimension 2

Joyce (2000):

- Generic special Lagrangian fibration has discriminant locus of codimension 1

Joyce (2002):

- Explicit description of conjectured behavior for discriminant locus.

Codimension 1 discriminant locus had also been observed by Ruan.

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Special Lagrangian fibrations of $\mathbb{C}^3$

Harvey-Lawson: $(r,s,t)\in\mathbb{R}^3$

$$M_{r,s,t} = \{ |z_1|^2-|z_3|^2 = r,\ |z_2|^2-|z_3|^2 = s,\ \mathrm{Im}(z_1 z_2 z_3) = t \}$$

$T^2\times\mathbb{R}$, except along coordinate axes in $\mathbb{C}^3$
(mapping to: $r=t=0$, $s=t=0$, $r-s=t=0$)

Disk instantons when $t=0$:
fix θ

$$D_\theta = \{ |z_1|^2 \le r,\ z_2 = s e^{i\theta},\ z_3 = 0 \}$$

$\partial D_\theta \subseteq M_{r,s,t}$, area πr

An S^1 of disk instantons near the
$r=t=0$ curve

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An S^1 of disk instantons near πa

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Joyce $(a, c) \in \mathbb{R} \times \mathbb{C}$

$$N^+_{a,c} = \{ |z_1|^2 - a = |z_2|^2 + a = |z_3 - c|^2 + |a|,$$
$$\mathrm{Im}\,(z_1 z_2 (z_3 - c)) = 0$$
$$\mathrm{Re}\,(z_1 z_2 (z_3 - c)) \geq 0 \}$$

$$N^-_{a,c} = \{ |z_1|^2 - a = |z_2|^2 + a = |z_3 - c|^2 + |a|,$$
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each is $S^1 \times \mathbb{R}^2$ unless $a = 0$.
(singular in codim 1)

Disk instantons:

$$D = \{ |z_1|^2 \leq 2a,\ z_2 = 0,\ z_3 = c \}$$

$\partial D \subseteq N^+_{a,c}$ and $\partial D \subseteq N^-_{a,c}$ area $2\pi a$

Single disk instanton near $a = 0$.

(14)

Joyce

Abstract family, parametrised by (a,c).

Singular when $a=0$ and $|\mathrm{Im}\,c|\leq 1$

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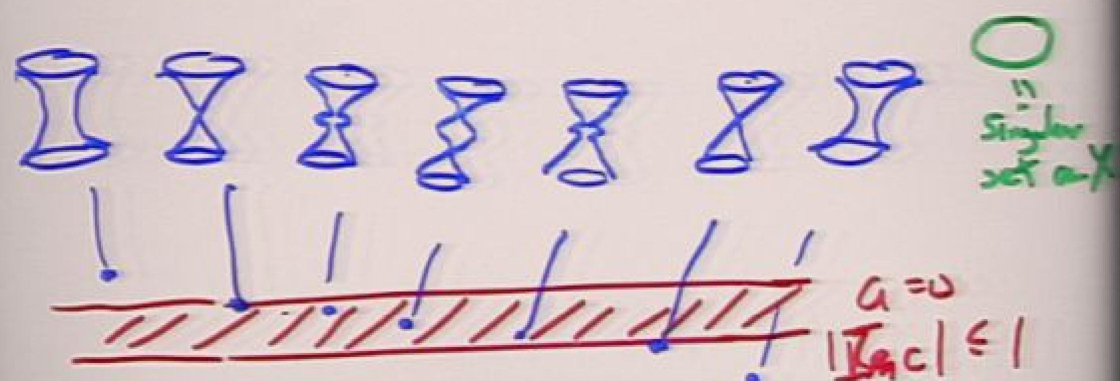

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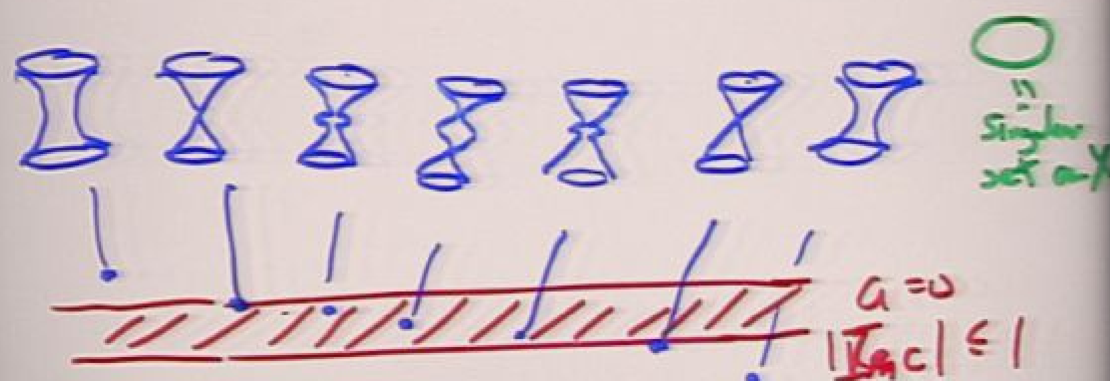

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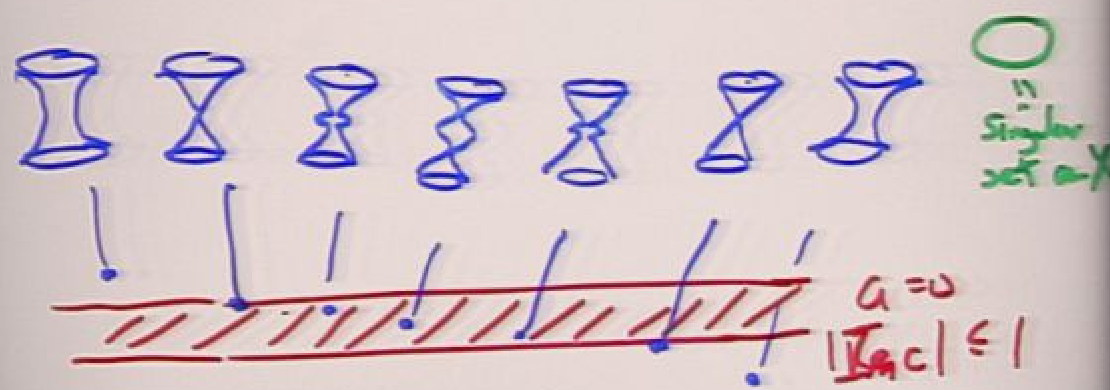

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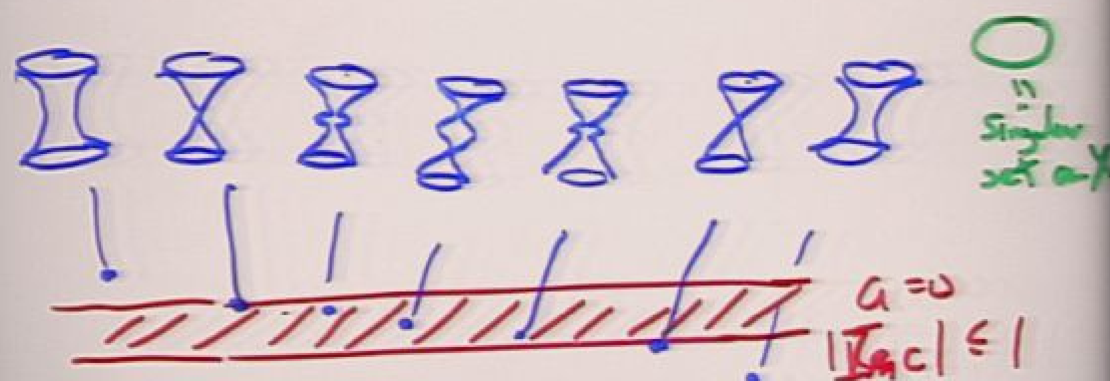

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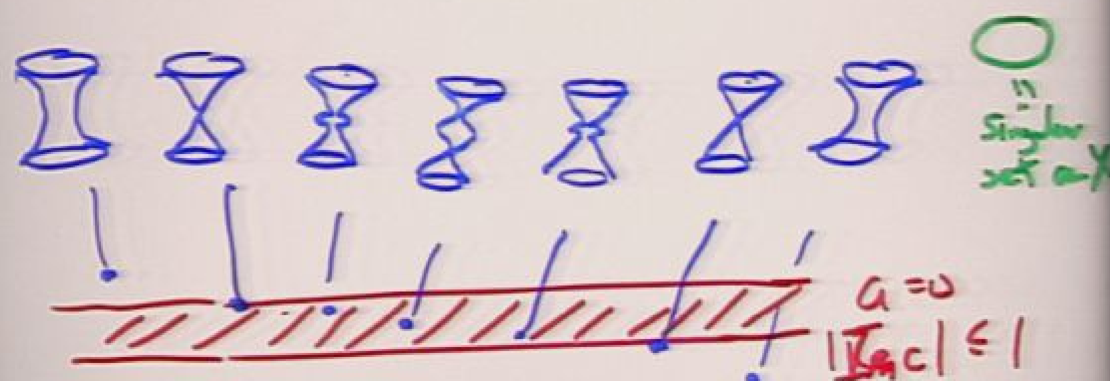

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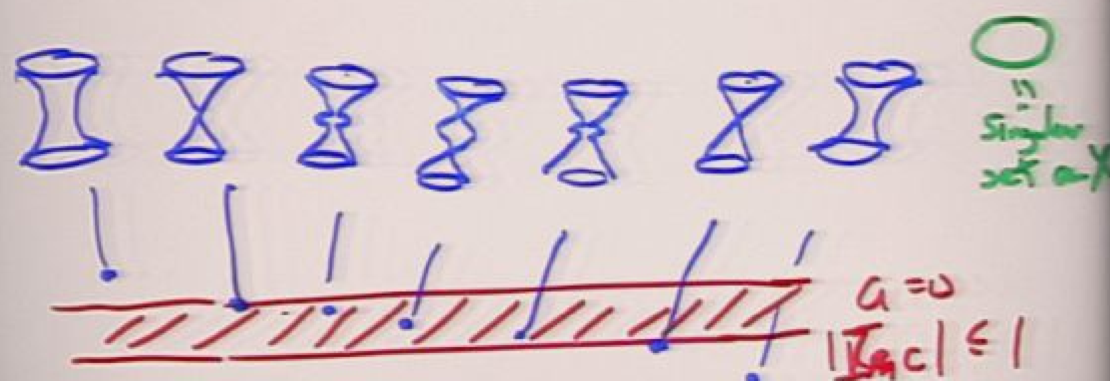

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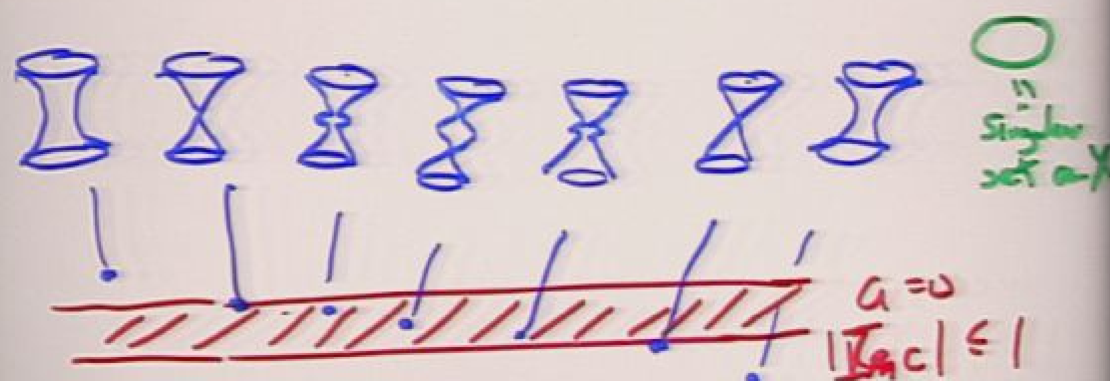

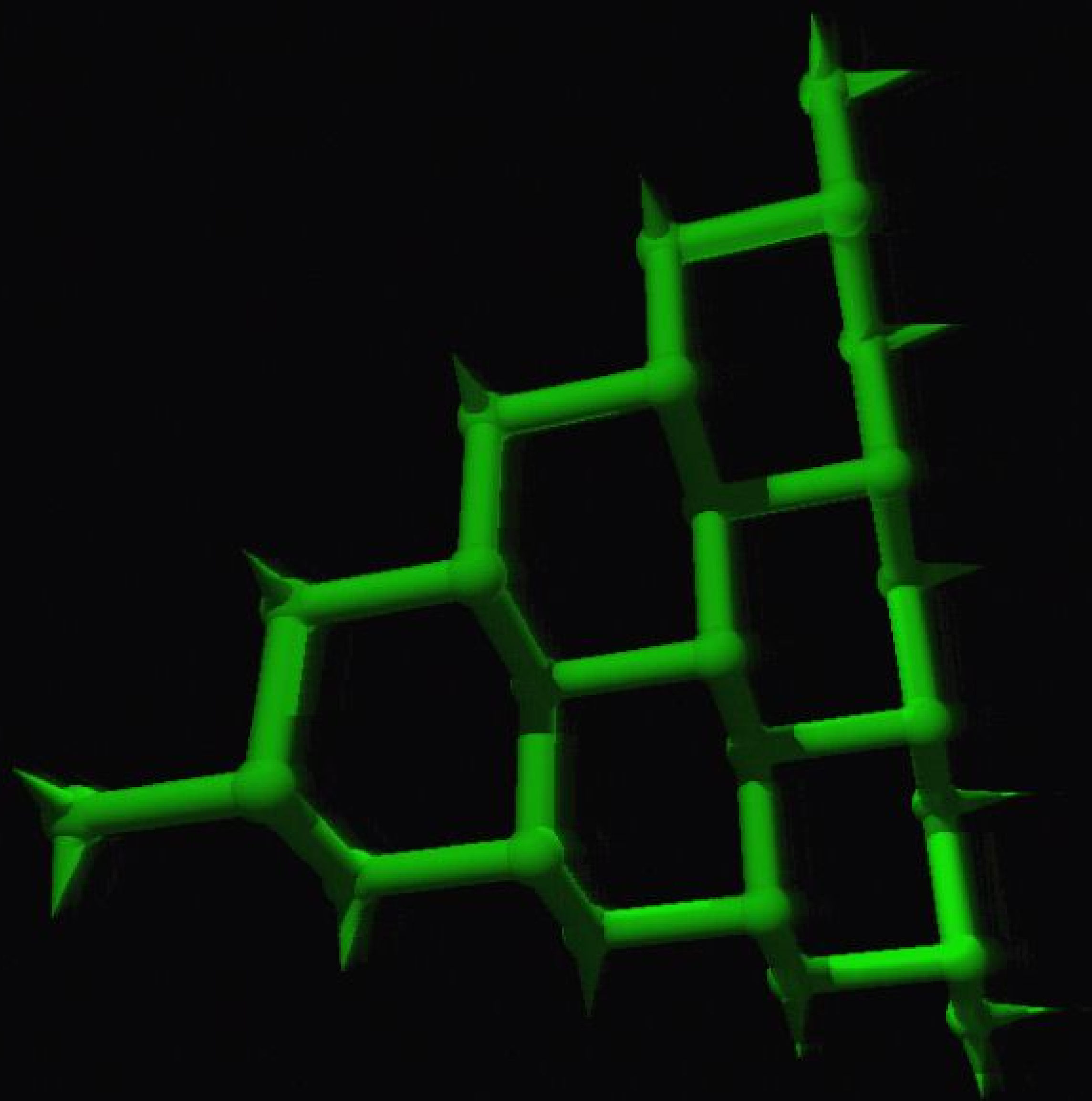

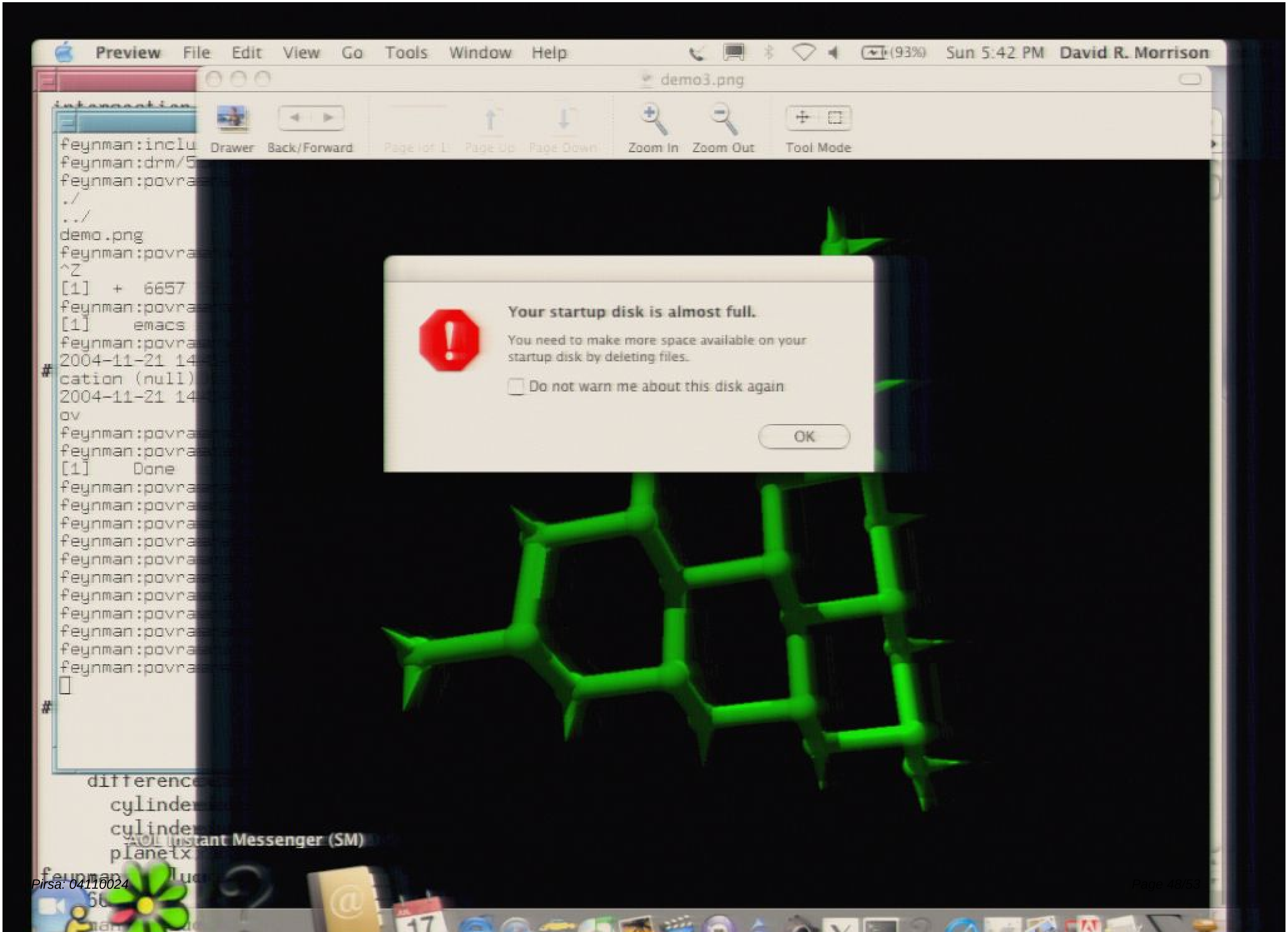
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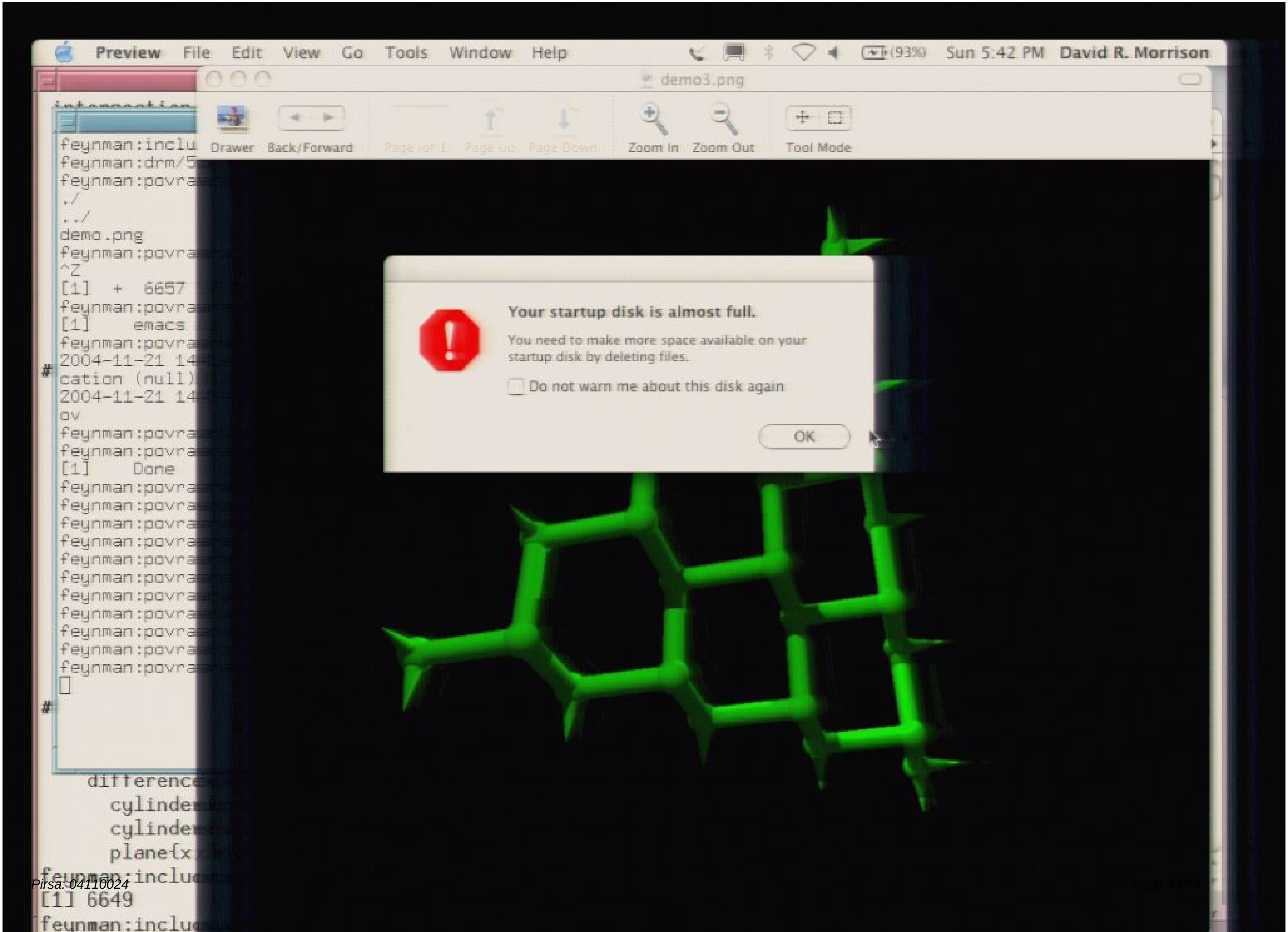
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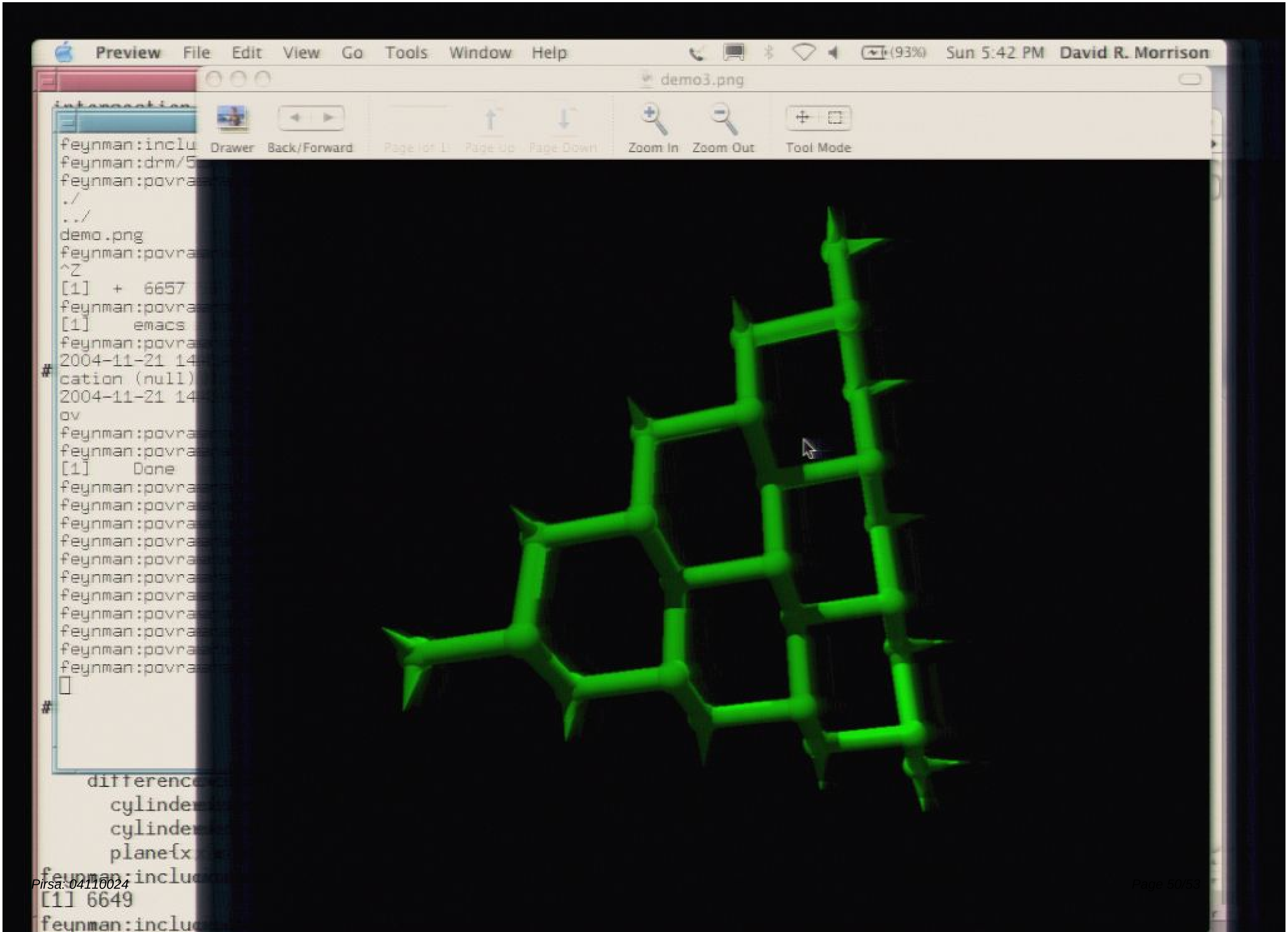

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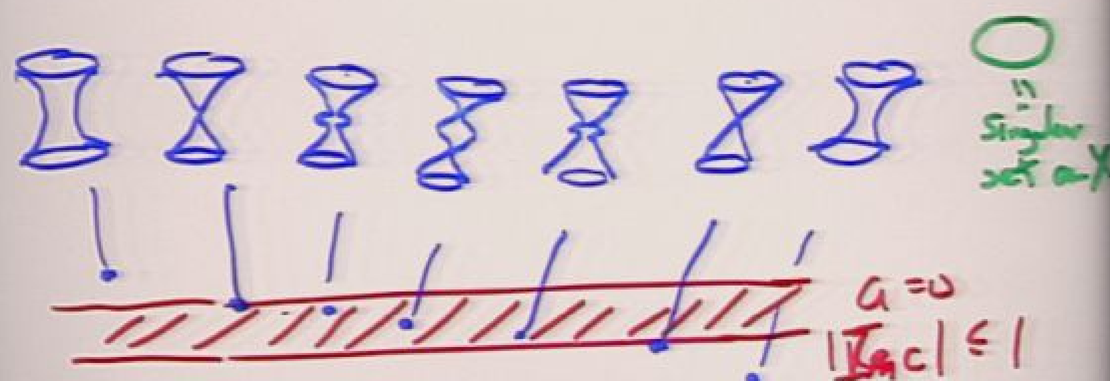

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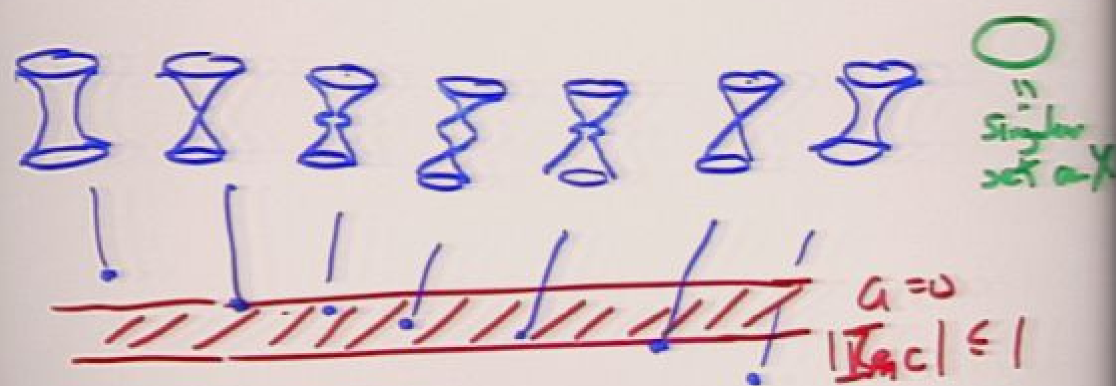

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