

Title: Generalized Kahler geometry and T-duality (no audio)

Date: Nov 19, 2004 02:00 PM

URL: <http://pirsa.org/04110016>

Abstract: Please Note: There is no audio for most presentations from November 18th due to technical issues. We appologize to presenters and interested parties for this inconvenience.

Geometry of $T \oplus T^*$

- Natural split-signature metric $\langle X + \xi, Y + \eta \rangle = \frac{1}{2}(\xi(Y) + \eta(X))$ determines $O(n, n)$ structure
- Sub-bundles T and T^* are maximal isotropic
- Spinors for $T \oplus T^*$ are given by $S = \wedge^\bullet T^*$
- Exterior derivative d generates the *Courant bracket*

$$[X + \xi, Y + \eta] = [X, Y] + L_X \eta - L_Y \xi - \frac{1}{2}d(\eta(X) - \xi(Y))$$

Generalized complex structures $O(2n, 2n) \rightarrow U(n, n)$

Definition: complex structure $\mathcal{J} \in O(T \oplus T^*)$ such that $[\mathcal{J}, \mathcal{J}] = 0$.

Examples: $\mathcal{J}_J = \begin{pmatrix} -J & \\ & J^* \end{pmatrix}$, $\mathcal{J}_\omega = \begin{pmatrix} & -\omega^{-1} \\ \omega & \end{pmatrix}$

Local invariant:

- $\mathcal{J} = \begin{pmatrix} A & \pi \\ \sigma & -A^\sharp \end{pmatrix}$, and π is a Poisson structure.

- $k := n - \frac{1}{2} \text{rk } \pi$, called **type** of GCS.

complex $\rightarrow n, n-1, \dots, 1, 0 \leftarrow$ **symplectic**

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Kähler geometry

Definition: triple (g, J, ω) such that

$$\begin{array}{ccc} T & \xrightarrow{g} & T^* \\ & \searrow J & \nearrow \omega \\ & T & \end{array}$$

Equivalently: pair (J, ω) such that $(-\omega J)^* = -\omega J$ and $-\omega J > 0$

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$$\begin{pmatrix} -J & \\ & J^* \end{pmatrix} \begin{pmatrix} & -\omega^{-1} \\ \omega & \end{pmatrix} = \begin{pmatrix} & -\omega^{-1} \\ \mathcal{J}_\omega & \end{pmatrix} \begin{pmatrix} -J & \\ & J^* \end{pmatrix} = \begin{pmatrix} & g^{-1} \\ g & \end{pmatrix} > 0$$

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Generalized Kähler geometry $U(n, n) \rightarrow U(n) \times U(n)$

Definition: pair $(\mathcal{J}_A, \mathcal{J}_B)$ of generalized complex structures such that

$$\mathcal{J}_A \mathcal{J}_B = \mathcal{J}_B \mathcal{J}_A = G > 0$$

Properties:

- $\text{type}(\mathcal{J}_A) + \text{type}(\mathcal{J}_B) \begin{cases} \leq n \\ \equiv n \pmod{2} \end{cases}$
- $C_{\pm} := \pm 1$ -eigenspaces of G . These are **complex** for $\mathcal{J}_A, \mathcal{J}_B$ and **\pm -definite**. By projection, this induces a Riemannian metric g , a 2-form b , and two Hermitian almost complex structures J_{\pm} on the tangent bundle T .

Theorem 1 [MG]: Algebraic equivalence $(\mathcal{J}_A, \mathcal{J}_B) \Leftrightarrow (J_+, J_-, b, g)$, given by

$$\mathcal{J}_{A/B} = \frac{1}{2} \begin{pmatrix} 1 & \\ b & 1 \end{pmatrix} \begin{pmatrix} J_+ \pm J_- & -(\omega_+^{-1} \mp \omega_-^{-1}) \\ \omega_+ \mp \omega_- & -(J_+^* \pm J_-^*) \end{pmatrix} \begin{pmatrix} 1 & \\ -b & 1 \end{pmatrix}$$

Theorem 2 [MG]: Generalized Kähler structure $(\mathcal{J}_A, \mathcal{J}_B)$ is integrable if and only if J_{\pm} are integrable and

$$d_-^c \omega_- = -d_+^c \omega_+ = db + H$$

Equivalently: $\nabla^{\pm} J_{\pm} = 0$, where $\nabla^{\pm} = \nabla \pm \frac{1}{2}g^{-1}(db + H)$, and $db + H$ is of type $(2, 1) + (1, 2)$.

Gates, Hull, Roček (1984): General target space geometry for $N = (2, 2)$ sigma model

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$$\begin{array}{c}
 \{\text{Generalized Kähler}\} \longleftarrow \mathcal{J}_A, \mathcal{J}_B \text{ commute in } T \oplus T^* \\
 \updownarrow \\
 \{\text{bihermitian (2,2)}\} \longleftarrow J_+, J_- \text{ need not commute in } T
 \end{array}$$

Remark: J_+, J_- commute if and only if $\text{type}(\mathcal{J}_A) + \text{type}(\mathcal{J}_B) = n$.

Examples of generalized Kähler

- any Kähler manifold ($J_{\pm} = J$, $b = 0$), **type = $(n, 0)$ or $(0, n)$**
- hyperKähler, where $(J_+, J_-, b, g) = (I, J, \omega_K, g)$, **type = $(0, 0)$**
- primary Hopf surfaces and finite covers thereof, **type = $(1, 1)$ or $(0, 0)$ with jumping to $(2, 0)$ along anticanonical divisor**
- even semi-simple Lie group $(J_+, J_-, b, g) = (J_R, J_L, 0, \text{Kill})$, $H = \text{Cartan}$, **type jumping**
- (N. Hitchin) moduli space of instantons on a generalized Kähler manifold is generalized Kähler

Hodge theory for generalized Kähler

- $(\mathcal{J}_A, \mathcal{J}_B)$ commuting endomorphisms act via Spin representation on $S = \wedge^\bullet T^*$.
- decomposition of forms into (ir, is) -eigenspaces for $(\mathcal{J}_A, \mathcal{J}_B)$:

$$\wedge^\bullet T^* \otimes \mathbb{C} = \bigoplus_{\substack{|r+s| \leq n \\ r+s \equiv n \pmod{2}}} U^{r,s}$$

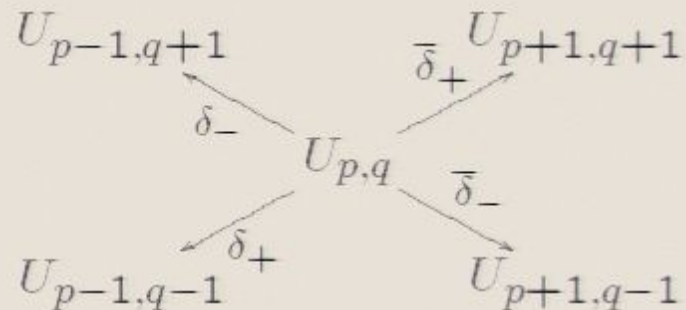
- decomposition is orthogonal in the **Born-Infeld metric** $(,)$ on forms depending on $g + b$.

$$\begin{array}{ccccccc}
 & & & & U_{0,n} & & \\
 & & & \dots & & \dots & \\
 & U_{-n,0} & U_{-n+1,1} & & & U_{n-1,1} & U_{n,0} \\
 & & \dots & & & & \\
 & U_{-n+1,-1} & & & & U_{n-1,-1} & \\
 & & \dots & & & \dots & \\
 & & & U_{0,-n} & & & \\
 & & & \text{☞} & & &
 \end{array}$$

- exterior derivative d (or $d_H = d + H$) decomposes into 4 differentials:

$$d = \delta_+ + \delta_- + \bar{\delta}_+ + \bar{\delta}_-,$$

where the differential operators act as follows:



Lemma (generalized Kähler identities):

$$\bar{\delta}_+^* = -\delta_+ \quad \text{and} \quad \bar{\delta}_-^* = \delta_-.$$

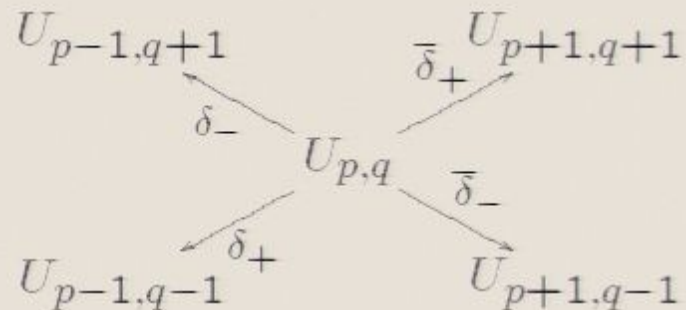
\Rightarrow classical $N = (2, 2)$ SUSY algebra representation on $\Omega^\bullet(M, \mathbb{C})$

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Theorem [MG]: The H -twisted cohomology of a compact generalized Kähler manifold carries a Hodge decomposition:

$$H_H^\bullet(M, \mathbb{C}) = \bigoplus_{\substack{|p+q| \leq n \\ p+q \equiv n \pmod{2}}} \mathcal{H}^{p,q},$$

Corollary: A 4-dimensional generalized Kähler manifold with $H = 0$ must have b_1 even and b_2^+ odd, and to admit type $= (1, 1)$ it must also have b_2 even.

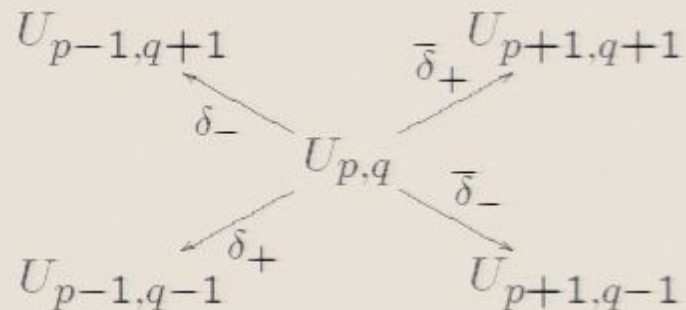
Note: In the usual Kähler case, this decomposition is *not* the usual (p, q) -decomposition. It was first discovered by Michelsohn and is called the Clifford decomposition. if $(\mathcal{J}_A, \mathcal{J}_B) = (\mathcal{J}_J, \mathcal{J}_\omega)$, then

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Geometric T-duality (joint with G. Cavalcanti)

- our approach: T-duality of any generalized geometry, assuming *invariance*, but allowing flux. Based on the topological approach of Bouwknecht, Evslin, Mathai.
- early ideas of Minasian on the use of $Spin(n, n)$ “bispinors” to mirror symmetry.
- recent work of Ben-Bassat on T-duality of generalized complex structures where invariance is relaxed to **semi-flatness**.

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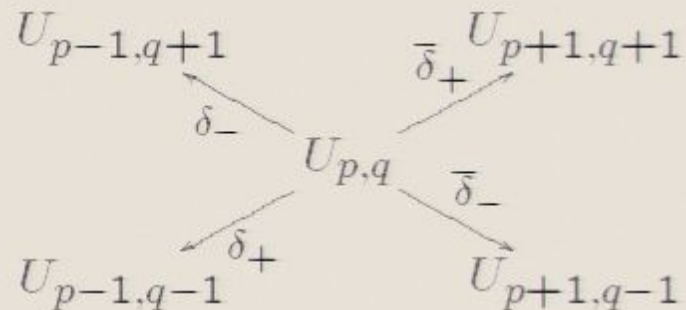
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First step: T-duality between principal S^1 -bundles

Data:

$$\begin{cases} \pi : P \longrightarrow B \text{ a principal } S^1\text{-bundle} \\ u \in \Omega^1(P) \text{ a connection, with curvature } du = F \\ H \in \Omega_{cl}^3(P)^{S^1} \text{ a NS 3-flux, } [H] \in H^3(M, \mathbb{Z}). \end{cases}$$

write $H = u \wedge \tilde{F} + h$, $h \in \Omega^3(B)$.

Dual data:

$$\begin{cases} \pi_*[H] \in H^2(B, \mathbb{Z}) \text{ determines a dual principal } S^1\text{-bundle } \tilde{P} \\ \text{choose a connection } \tilde{u} \text{ on } \tilde{P} \\ \tilde{H} := \tilde{u} \wedge F + h \end{cases}$$



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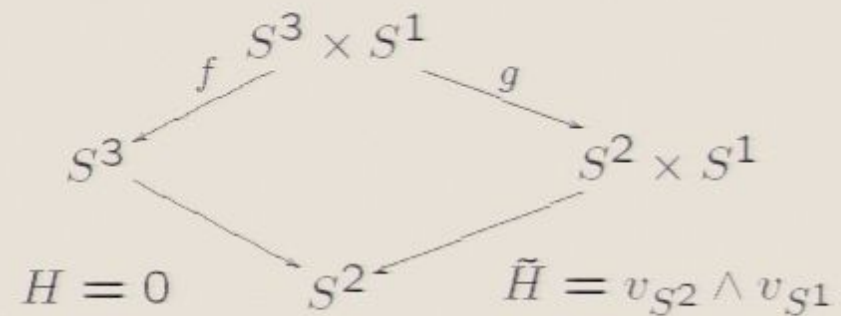
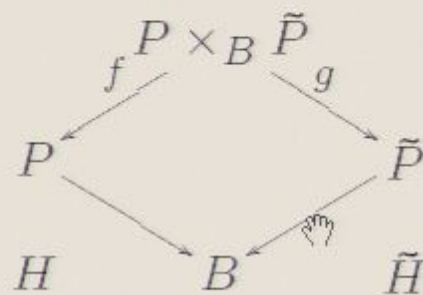
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Exchanging flux for topology



Theorem [BEM]: The H -twisted K-theory of P is isomorphic to the \tilde{H} -twisted K-theory of \tilde{P} .

Theorem [Cavalcanti, MG]: Let $F = u \wedge \tilde{u}$. The map

$$\mathcal{F} = g_* \circ e^F \circ f^*$$

induces an isomorphism of Courant algebroids

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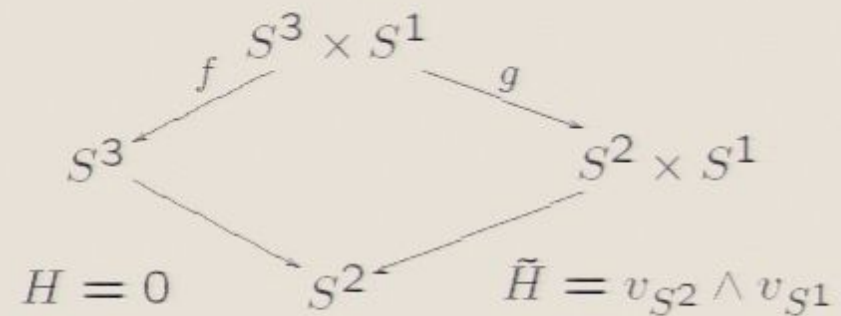
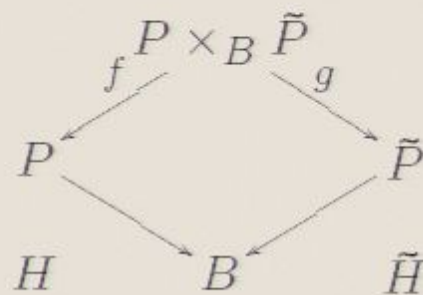
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Transport of geometric structure

The theorem can be used to transport generalized geometries from one manifold to the T-dual side.

Examples:

- Let G be a generalized metric on P . Then $g_* \circ e^F \circ f^*(G)$ is a generalized metric $(\tilde{g} + \tilde{b})$ on \tilde{P} . These are known as the **Buscher rules**
- Let D be a Dirac structure on P . Then $g_* \circ e^F \circ f^*(D)$ is a Dirac structure on \tilde{P} . Operations f^* , f_* defined by **Weinstein**.
- Let \mathcal{J} be a generalized complex structure on P . Then $g_* \circ e^F \circ f^*(\mathcal{J})$ is a generalized complex structure $\tilde{\mathcal{J}}$ on \tilde{P} , **with** $\text{type}(\tilde{\mathcal{J}}) = \text{type} \mathcal{J} \pm 1$. E.g. in 6 dimensions, 3 T-dualizations are required to go from $\text{type} = 3$ to $\text{type} = 0$.
- Compare with gerbe T-duality of **Donagi and Pantev**.

Transport of generalized Kähler structure

$$(\mathcal{J}_A, \mathcal{J}_B) \xLeftrightarrow{T} (\tilde{\mathcal{J}}_A, \tilde{\mathcal{J}}_B)$$

$$H_c^{r,s}(M) \xLeftrightarrow{\quad} \tilde{H}_c^{r,s}(M)$$

If $(\mathcal{J}_A, \mathcal{J}_B)$ were a Kähler $2n$ -manifold and we could perform n T-dualities,

$$(\mathcal{J}_J, \mathcal{J}_\omega) \xLeftrightarrow{\quad} (\mathcal{J}_{\tilde{\omega}}, \mathcal{J}_{\tilde{J}})$$

$$H^{p,q} \xLeftrightarrow{\quad} H^{n-p,q}$$

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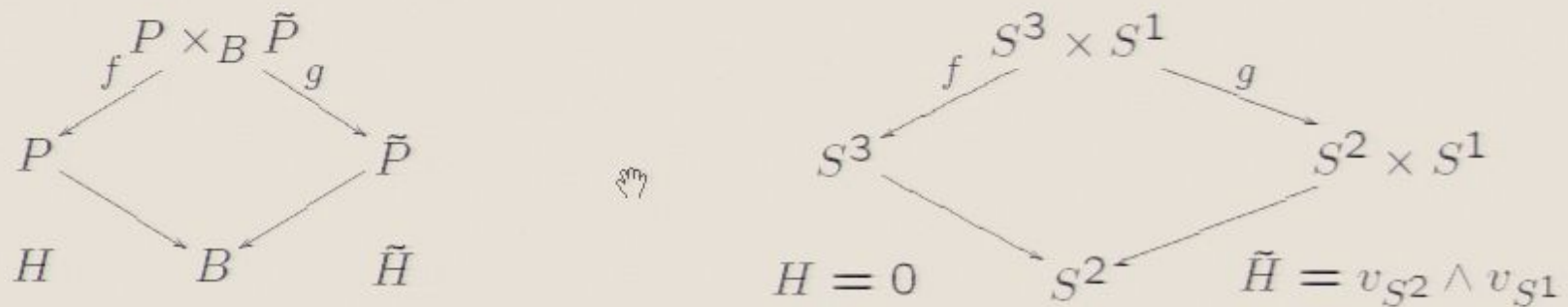
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write $H = u \wedge \tilde{F} + h$, $h \in \Omega^3(B)$.

Dual data:

$$\begin{cases} \pi_*[H] \in H^2(B, \mathbb{Z}) \text{ determines a dual principal } S^1\text{-bundle } \tilde{P} \\ \text{choose a connection } \tilde{u} \text{ on } \tilde{P} \\ \tilde{H} := \tilde{u} \wedge F + h \end{cases}$$

Exchanging flux for topology



Theorem [BEM]: The H -twisted K-theory of P is isomorphic to the \tilde{H} -twisted K-theory of \tilde{P} .

Theorem [Cavalcanti, MG]: Let $F = u \wedge \tilde{u}$. The map

$$\mathcal{F} = g_* \circ e^F \circ f^*$$

induces an isomorphism of Courant algebroids

$$\left(\frac{T_P \oplus T_P^*}{S^1}, H \right) \longrightarrow \left(\frac{T_{\tilde{P}} \oplus T_{\tilde{P}}^*}{S^1}, \tilde{H} \right)$$

Any questions?