Title: The general boundry formulation of quantum mechanics: Motivations and elementary examples.

Date: Nov 02, 2004 02:00 PM

URL: http://pirsa.org/04110000

Abstract:

THE GENERAL BOUNDARY FORMULATION OF QUANTUM MECHANICS: MOTIVATIONS AND ELEMENTARY EXAMPLES

Robert Oeckl

Instituto de Matemáticas
UNAM, Morelia, Mexico

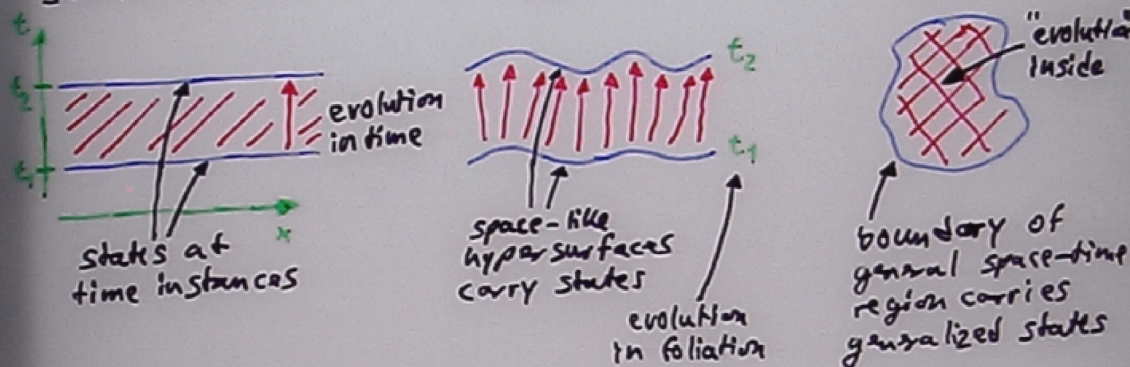
gr-qc/0306007
hep-th/0306025
gr-qc/0307118
gr-qc/0312081
+
work in progress

THE GENERAL BOUNDARY FORMULATION

on axiomatic level

$$\boxed{\text{QM} + \text{TQFT} = \text{general boundary QM}}$$

standard QM | curved space-time QM | general boundary QM



▷ associate generalized state spaces to boundaries of regions of space-time

▷ associate "transition" amplitudes to regions ~~them~~selves

features

▷ avoid interpretational problems of combining GR with standard QM (notably problem of time)

▷ preserve standard QM where applicable

▷ local description of measurement process

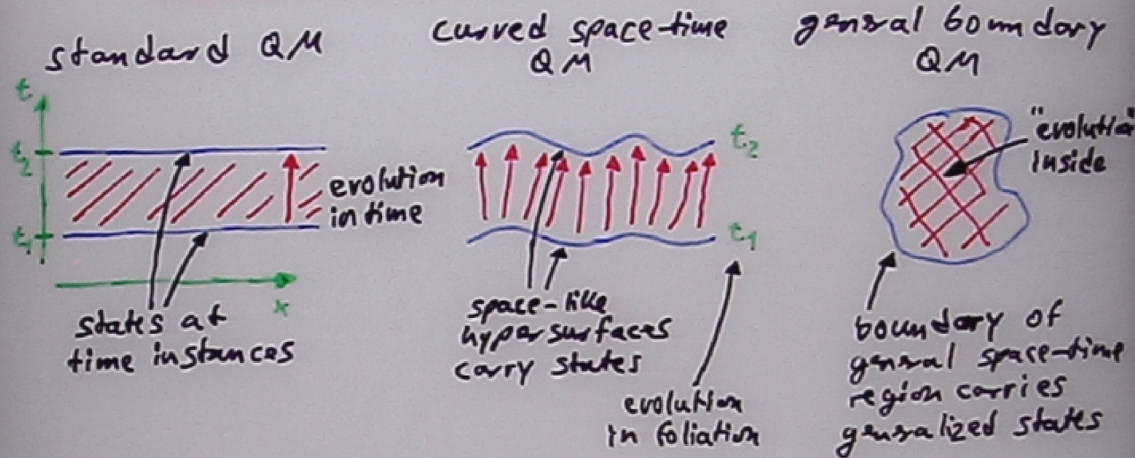
▷ distinction between "in" and "out" states and between "preparation" and "observation" disappears

▷ interpretation: "collapse of wavefunction" is delocalized in time

THE GENERAL BOUNDARY FORMULATION

on axiomatic level

$$\boxed{QM + TQFT = \text{general boundary } QM}$$



- ▷ associate generalized state spaces to boundaries of regions of space-time
- ▷ associate "transition" amplitudes to regions ~~them~~selves

features

- ▷ avoid interpretational problems of combining GR with standard QM (notably problem of time)
- ▷ preserve standard QM where applicable
- ▷ local description of measurement process
- ▷ distinction between "in" and "out" states and between "preparation" and "observation" disappears
- ▷ interpretation: "collapse of wavefunction" is delocalized in time

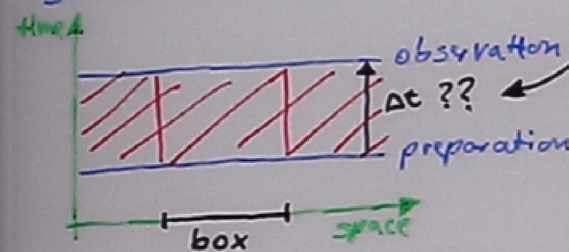
AVOIDING THE PROBLEM OF TIME

what is the analogue of time evolution in a quantum theory of gravity ?

▷ consider a typical quantum mechanical experiment in a box:

1. prepare initial state
2. wait for time Δt
3. observe final state

standard QM

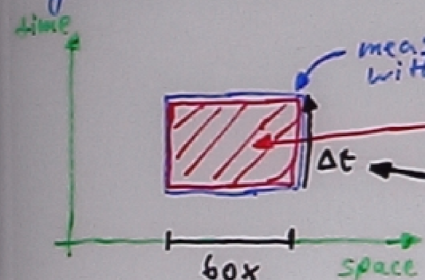


space-time here is supposed to be quantum mechanical

no classical Δt !

?

general boundary QM



measurement associated with closed boundary

space-time quantum mechanical

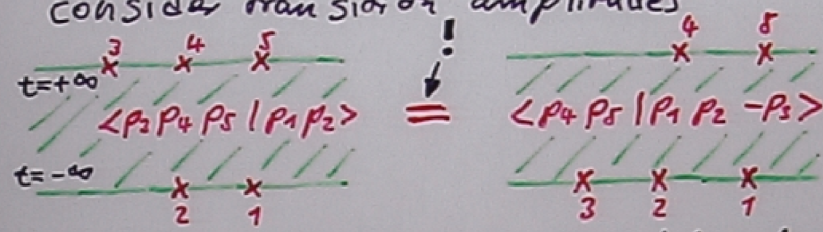
almost classical nature of space-time encoded in state including time duration Δt !

If gravitational component in boundary state is (almost) classical it encodes classical geometry of the boundary (and thus Δt)

MOTIVATION FROM QFT

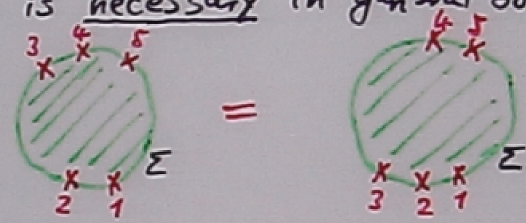
(a) compatibility from LSZ reduction

consider transition amplitudes

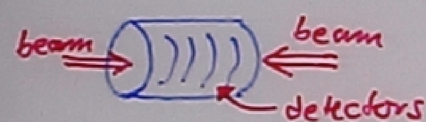


equality is <u>necessary</u> in general boundary formulation

since

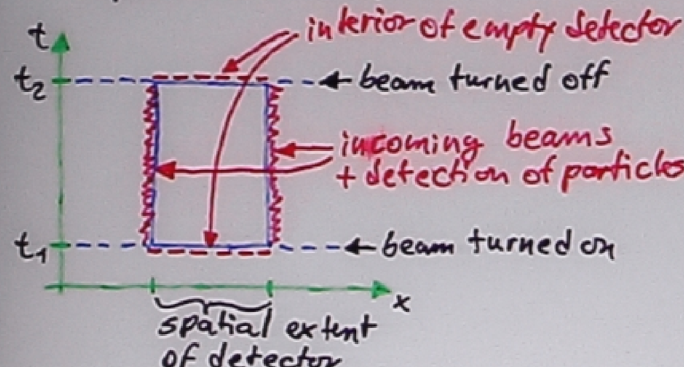


(b) typical measurement setup



collision experiment at accelerator

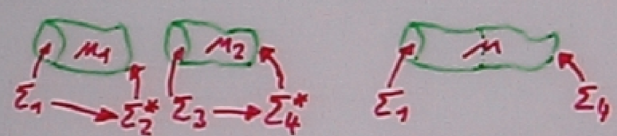
space-time diagram



interesting stuff happens at <u>time-like</u> boundaries

FORMALIZATION

- state spaces $\mathcal{H}_\Sigma$ associated to boundaries Σ of space-time regions M
- if $\Sigma = \Sigma_1 \dot\cup \Sigma_2$ with Σ_1, Σ_2 disconnected then $\mathcal{H}_\Sigma = \mathcal{H}_{\Sigma_1} \otimes \mathcal{H}_{\Sigma_2}$
- Σ^* the same hypersurface as Σ but with opposite orientation $\Rightarrow$ $\mathcal{H}_{\Sigma^*} = \mathcal{H}^*_\Sigma$
- associated with M is the amplitude function $\rho_M : \mathcal{H}_\Sigma \to \mathbb{C}$ where $\Sigma = \partial M$
- composition rule : given $M = M_1 \cup M_2$



$\Sigma_2^* = \Sigma_3$

ρ_{M_1} $\quad$ ρ_{M_2} $\quad$ $\boxed{\rho_M = \rho_{M_2} \circ \rho_{M_1}}$

TQFT

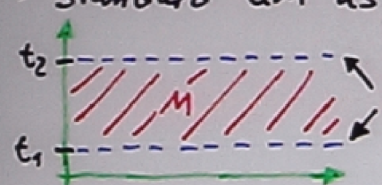
- probability rule : $\rho_M : \mathcal{H}_\Sigma \to \mathbb{C}$

$P = |\rho_M(\eta)|^2$

probability (density) $\quad$ (generalized) state $\eta \in \mathcal{H}_\Sigma$

▶ standard QM as special case

$M = \mathbb{R}^3 \times [t_1, t_2]$



Σ is the union of two components $\Rightarrow \mathcal{H}_\Sigma = H_{t_1} \otimes H^*_{t_2}$

$|\psi\rangle \in H_{t_1}$, $\quad \langle\varphi| \in H^*_{t_2}$

$$\rho_M(|\psi\rangle \otimes \langle\varphi|) = \langle\varphi| U(t_1, t_2) |\psi\rangle$$

final state $\quad$ time evolution operator $\quad$ initial state

WHAT ABOUT UNITARITY ?

assume we want to introduce inner products on state spaces, i.e. identify

$$*: \mathcal{H}_\Sigma \to \mathcal{H}_\Sigma^* = \mathcal{H}_{\Sigma^*}$$ antilinearly

motivated e.g. by boundary reversing symmetries like CPT in QFT

- recall amplitude interpreted in two ways

$$\rho : \mathcal{H}_\Sigma \to \mathbb{C} \quad \text{or} \quad \tilde{\rho} : \mathbb{C} \to \mathcal{H}_\Sigma^*$$

compatibility of inner product with amplitude is unitarity:

$$\begin{tikzcd} \mathcal{H}_\Sigma \arrow[r, "\bar{\rho}"] \arrow[dr, leftrightarrow, "*"'] & \mathbb{C} \arrow[d, "\tilde{\rho}"] \\ & \mathcal{H}_\Sigma^* \end{tikzcd}$$

diagram needs to commute

---

in standard QM: $\mathcal{H}_\Sigma = H_1 \otimes H_2^*$

initial final

amplitude becomes

$$U : H_1 \to H_2$$

$$\tilde{U} : H_2^* \to H_1^*$$

diagram

$$\begin{tikzcd} H_1 \arrow[r, "U"] \arrow[d, leftrightarrow, "*"'] & H_2 \arrow[d, leftrightarrow, "*"] \\ H_1^* & H_2^* \arrow[l, "\tilde{U}"] \end{tikzcd}$$

$$\tilde{U}^* = U^{-1} \quad \Leftarrow$$

standard unitarity

WHAT ABOUT UNITARITY ?

assume we want to introduce inner products on state spaces, i.e. identify

$$*: \mathcal{H}_\Sigma \to \mathcal{H}_\Sigma^* = \mathcal{H}_{\Sigma^*} \quad \text{antilinearly}$$

motivated e.g. by boundary reversing symmetries like CPT in QFT

- recall amplitude interpreted in two ways

$$\rho: \mathcal{H}_\Sigma \to \mathbb{C} \quad \text{or} \quad \tilde{\rho}: \mathbb{C} \to \mathcal{H}_\Sigma^*$$

compatibility of inner product with amplitude is <u>unitarity</u>:

$$\begin{tikzcd}
\mathcal{H}_\Sigma \arrow[r, "\bar{\rho}"] \arrow[dr, leftrightarrow, "*"'] & \mathbb{C} \arrow[d, "\tilde{\rho}"] \\
 & \mathcal{H}_\Sigma^*
\end{tikzcd}$$

diagram needs to commute

---

in standard QM: amplitude becomes

$$\mathcal{H}_\Sigma = H_1 \otimes H_2^*$$

initial, final

$$U: H_1 \to H_2$$
$$\tilde{U}: H_2^* \to H_1^*$$

diagram

$$\begin{tikzcd}
H_1 \arrow[r, "U"] \arrow[d, leftrightarrow, "*"'] & H_2 \arrow[d, leftrightarrow, "*"] \\
H_1^* & H_2^* \arrow[l, "\tilde{U}"]
\end{tikzcd}$$

$\tilde{U}^* = U^{-1} \Leftarrow$

standard unitarity

$*: \mathcal{H}_\Sigma \to \mathcal{H}^*_\Sigma = \mathcal{H}_{\Sigma^*}$ antilinearly

motivated e.g. by boundary reversing symmetries like CPT in QFT

- recall amplitude interpreted in two ways

$\rho: \mathcal{H}_\Sigma \to \mathbb{C}$ or $\tilde{\rho}: \mathbb{C} \to \mathcal{H}^*_\Sigma$

compatibility of inner product with amplitude is unitarity:

$$\begin{tikzcd} \mathcal{H}_\Sigma \arrow[r, "\bar{\rho}"] \arrow[dr, leftrightarrow, "*"'] & \mathbb{C} \arrow[d, "\tilde{\rho}"] \\ & \mathcal{H}^*_\Sigma \end{tikzcd}$$

diagram needs to commute

---

in standard QM: $\mathcal{H}_\Sigma = H_1 \otimes H_2^*$

initial final

amplitude becomes

$U: H_1 \to H_2$

$\tilde{U}: H_2^* \to H_1^*$

diagram

$$\begin{tikzcd} H_1 \arrow[r, "U"] \arrow[d, leftrightarrow, "*"'] & H_2 \arrow[d, leftrightarrow, "*"] \\ H_1^* & H_2^* \arrow[l, "\tilde{U}"] \end{tikzcd}$$

$\tilde{U}^* = U^{-1}$ $\Leftarrow$

standard unitarity

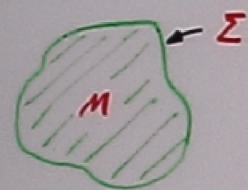
PATH INTEGRAL QUANTIZATION

How to produce quantum theories of the general boundary type ?

start with classical mechanical system / field theory :

- variables $\phi_1, \dots, \phi_n$ or fields $\phi(x)$
- action $S[\phi]$ (leading to equations of motion by extremization)

consider : space-time region M with boundary Σ



define :

K_Σ : space of (extended) configurations on Σ

e.g. – particle positions (or events) on Σ

– value of field on Σ

- state space $\mathcal{H}_\Sigma := C(K_\Sigma)$

space of complex functions on K_Σ

- amplitude function

$$\rho_M(\psi) := \int_{K_\Sigma} \mathcal{D}\tilde{\phi}\ \psi(\tilde{\phi}) \int_{\phi|_\Sigma = \tilde{\phi}} \mathcal{D}\phi\ e^{i/\hbar\, S[\phi]}$$

integral over configurations on the boundary

integral over configurations in M that match $\tilde{\phi}$ on the boundary

Remark :

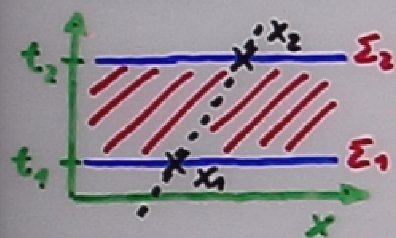
if $\Sigma = \Sigma_1 \dot{\cup} \Sigma_2$ (disjoint union)

then $K_\Sigma = K_{\Sigma_1} \times K_{\Sigma_2}$ and $\mathcal{H}_\Sigma = \mathcal{H}_{\Sigma_1} \otimes \mathcal{H}_{\Sigma_2}$

NON-RELATIVISTIC QM I

▸ consider free non-relativistic particle trajectory $x(t)$, action $S = -\frac{1}{2}\int dt\, m\, \dot{x}^2(t)$
solutions of e.o.m. $\ddot{x}(t) = 0$ are straight lines $x(t) = x_0 + vt$

▸ standard setup: choose t_1, t_2



$M = [t_1, t_2] \times \mathbb{R}^3$

boundary $\Sigma = \Sigma_1 \cup \Sigma_2$

▸ configuration space: pairs x_1, x_2 position at t_1, t_2

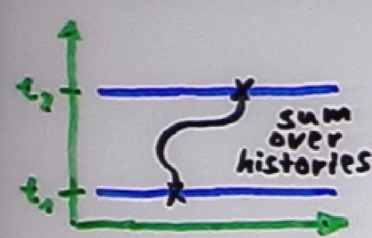
$K_\Sigma = K_1 \times K_2 \simeq \mathbb{R}^3 \times \mathbb{R}^3$

state space: functions on K_Σ

$\mathcal{H}_\Sigma = C(K_\Sigma) = C(K_1) \otimes C(K_2)$

space of wavefunctions at t_1 — at t_2

▸ amplitude for $\Psi(x_1, x_2) = \psi(x_1)\overline{\psi'(x_2)}$



$$\rho_M(\Psi) = \int dx_1\, dx_2\, \psi(x_1)\overline{\psi'(x_2)} \cdot \int_{\substack{x(t_1)=x_1 \\ x(t_2)=x_2}} \mathcal{D}x\, e^{i/\hbar\, S[x]}$$

NON-RELATIVISTIC QM II

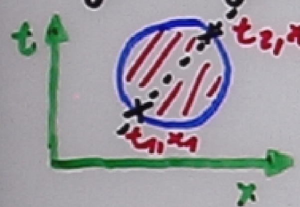
- for n particles

  $K_1 \simeq K_2 \simeq \underbrace{\mathbb{R}^3 \times \mathbb{R}^3 \times \cdots \times \mathbb{R}^3}_{n \text{ times}}$

  $\mathcal{H}_\Sigma = C(K_\Sigma) = C(K_1) \otimes C(K_2)$

  $C(K_1) \simeq C(K_2) \simeq C((\mathbb{R}^3)^n)$ n-particle wave fn.

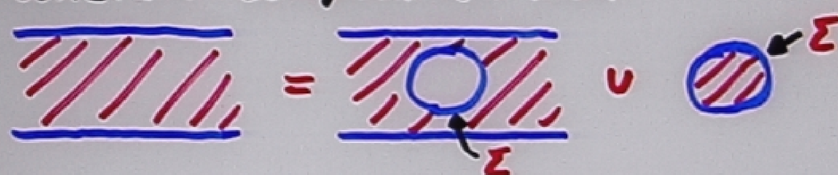
- go to general boundaries, e.g. $M = B^4$

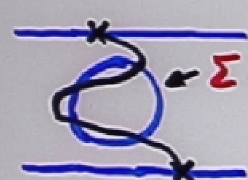


configuration space K_Σ is space of pairs of points $((t_1, x_1), (t_2, x_2))$ on $\Sigma = S^3$ with $t_1 < t_2$

$\mathcal{H}_\Sigma = C(K_\Sigma)$ space of generalized wave functions — does not factorize!

- consider composition rule





path integral contains contributions with particle path intersecting Σ arbitrarily many times.

$\Rightarrow$ 1-particle state space on Σ would violate composition rule

$\Rightarrow$ need n-particle state spaces

$\mathcal{H}_\Sigma = \bigoplus_{n=0}^{\infty} \mathcal{H}_\Sigma^n$ for consistency!

KLEIN-GORDON QFT I

- the description of QFT naturally compatible the discussed quantization is the Schrödinger representation
- $S[\phi] = \frac{1}{2}\int dt\, dx \left(\partial_0\phi\partial_0\phi - \sum_i \partial_i\phi\partial_i\phi - m^2\phi^2\right)$

  classical solutions

  $$\phi(t,x) = \int \frac{dp}{(2\pi)^d 2E}\left(a(p)e^{-i(Et-px)} + \overline{a(p)}e^{i(Et-px)}\right)$$

  for e.o.m. $\left(\partial_0^2 - \sum_i \partial_i^2 + m^2\right)\phi = 0$
- standard setup with t_1, t_2

  $K_\Sigma = K_1 \times K_2$ ← space of field configurations at t_1

  $\mathcal{C}(K_\Sigma) = \mathcal{C}(K_1) \otimes \mathcal{C}(K_2)$ ← space of wave functions at t_1 (functionals)

  $$\langle \psi' | U(t_2 - t_1) | \psi \rangle$$

  $$= \rho(\psi \otimes \psi') = \int d\phi_1\, d\phi_2\, \psi(\phi_1)\psi'(\phi_2) \cdot \int_{\substack{\hat\phi|_{t_1} = \phi_1 \\ \hat\phi|_{t_2} = \phi_2}} d\hat\phi\; e^{iS[\hat\phi]}$$
- use trick to evaluate path integral

  $$\int_{\hat\phi|_{\partial M} = \phi} \mathcal{D}\hat\phi\; e^{iS[\hat\phi]} = \mathcal{N}(M)\cdot e^{iS[\phi_{cl}]}$$

  $\mathcal{N}(M)$: normalization factor depending on manifold M

  ϕ_{cl}: classical solution matching ϕ on ∂M

  works since S is quadratic

OUTLOOK

- perturbative QFT
  - derivation of the S-matrix (more satisfactory than the standard one)
  - extend conceptual understanding of particle states ⟶ local concept?
- non-perturbative QFT
  - lattice gauge theory – renormalization without compactification
- QFT on curved space-time
  - local description
  - rederive Unruh effect "locally"
- quantization
  - extend canonical quantization
- quantum gravity
  - "boundary LQG"
  - 3d quantum gravity to test interpretation
  - spin foam models with space + timelike boundaries