Title: Background Independent Perturbative Quantum Gravity

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Abstract:

General covariance and non-renormalizibility

"2+1d gravity as an exactly soluble theory"
Witten 1988

- $g_{ab} \to \eta_{ab} + \hat{h}_{ab}$ $\Rightarrow$ uncontrolable divergences
- $g_{ab} \Rightarrow \hat{g}_{ab}$ $\Rightarrow$ renormalizible theory

Can do the same in 4d?

- 3d $\quad S = \int (\underbrace{e \wedge dA}_{\text{quadratic}} + e \wedge A \wedge A)$
- 4d $\quad S = \int (\underbrace{e \wedge e \wedge dA}_{\text{cubic}} + e \wedge e \wedge A \wedge A)$

no natural linear approximation

to do perturbation theory: $g_{ab} \Rightarrow \eta_{ab} + \hat{h}_{ab}$

4d quantum gravity is
non-renormalizible
because it is too non-linear

Mc Dowele - Mansouri BF - theory

A^{AB} - $SO(4,1)$ - connection $\quad F^{AB} = dA^{AB} + A^{AC} \wedge A_C^{\ B}$

$AB = 0 \cdots 4$

$$\boxed{S_{MM} = \frac{1}{\alpha} \int F^{AB} \wedge F^{CD} \varepsilon_{ABCD4}}$$

- decompose: $\mu, \nu = 0 \cdots 3$; $A^{\mu\nu} = \omega^{\mu\nu}$, $A^{\mu 4} = \frac{1}{\ell} e^{\mu}$

$$F_{\mu\nu}(A) = F_{\mu\nu}(\omega) + \frac{1}{\ell^2} e_\mu \wedge e_\nu \ ; \quad F_{\mu 4} = d_\omega e_\mu$$

$$S_{MM} = \underbrace{\frac{2}{\alpha \ell^2} \int F(\omega)^{\mu\nu} \wedge e^\alpha \wedge e^\beta \varepsilon_{\mu\nu\alpha\beta} + \frac{1}{\alpha \ell^4} \int e^\mu \wedge e^\nu \wedge e^\alpha \wedge e^\beta \varepsilon_{\mu\nu\alpha\beta}}_{\text{Palatini action for GR with non-zero } \Lambda}$$

$$+ \underbrace{\frac{1}{\alpha} \int F^{\mu\nu}_{(\omega)} \wedge F^{\alpha\beta}_{(\omega)} \varepsilon_{\mu\nu\alpha\beta}}_{\text{topological } F \wedge F}$$

$$\boxed{\alpha = G\Lambda = 10^{-120}}$$

- $F \wedge F \to BF$

$$\boxed{S_{MM} = \int \left(B^{AB} \wedge F_{AB} + \alpha B^{AB} \wedge B^{CD} \varepsilon_{ABCD5} \right)}$$

No more nonlinear than 2+1 gravity

- S_{MM} = (exactly soluble) + (small)

- [illegible] $\to \int B \wedge F$

Perturbation theory

- Ordinary FT : $\int \mathcal{D}\Phi \, e^{\int \Phi(\Box + m^2)\Phi + \lambda \Phi^m}$

  Gaussian — expand.

$\int \mathcal{D}\Phi \, e^{\int \Phi(\Box + m)\Phi}, \ldots, \lambda^{m_k} \int \mathcal{D}\Phi \, e^{\int \Phi(\Box + m^2)\Phi} \Phi^{m_1}(x_1) \cdots \Phi^{m_k}(x_k)$

All gaussian.

- Quantum Gravity- $\int \mathcal{D}A\mathcal{D}B \, e^{\int B \wedge F + \lambda \Phi(B)}$

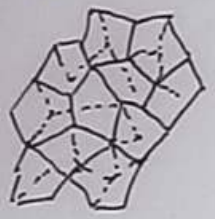
  Topological — Expand

$\int \mathcal{D}A\mathcal{D}B \, e^{\int B \wedge F}, \ldots, \lambda^k \int \mathcal{D}A\mathcal{D}B \, e^{\int B \wedge F} \Phi(B(x_1)) \cdots \Phi(B(x_k))$

All topological with different moduli spaces.

Calculating topological path integral

- Replace infinite number of variables by a finite number : discretize



$A \longrightarrow g_{e^*} = e^{\int_{e^*} A}$

$B \longrightarrow X_f = \int_{f^*} B$

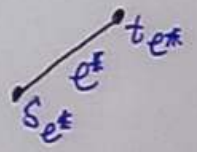
Evaluate a path integral using this variables: The result is independent

Gauge fixing (0-th order)

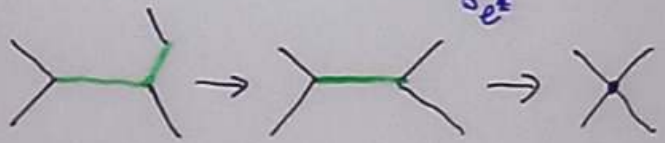
Exploit discretization independence of BF-theory

Gauge symmetries of discretized action:

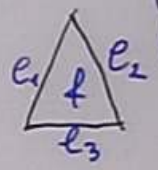
$g_{e^*} \to K_{s e^*}^{-1} \, g_{e^*} \, K_{t e^*}$



fixing

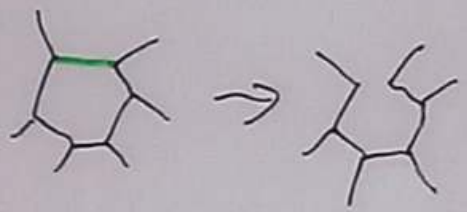


$X_f \to \sum_{e \ni \partial f} (\Phi_e + [\Phi_e, \Omega_e])$



Freidel Louapre

fixing



0th order approximation, quantum dS spacetime.

collapses to:

dS - spinfoam.

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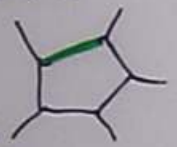
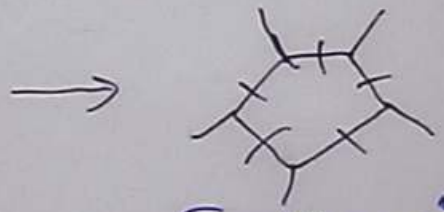
Perturbations

- bubble

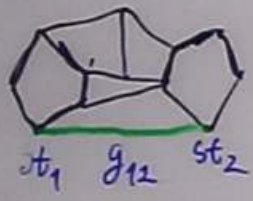


$\delta(0)$ – removed by gauge fixing

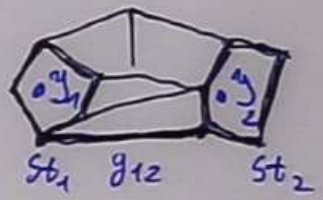


$\delta(g_1^{-1} e^{y_1} g_1 g_2 e^{y_2} g_2 \cdots)$ not removed by gauge fix

- dual faces



$g_{12} = g_1 g_2^{-1} = 1$ removed by gauge fixing



$g_{12} = g_1 g_2^{-1} \neq 1 \Rightarrow \int dg_{12}$ not removed by gauge fixing

- $\frac{\delta}{\delta y^{AB}} \rho(e^y)\big|_{y=0} = \rho(T^{AB})$ – derivatives

- For each $\frac{\delta}{\delta y^{AB}}$ we have extra $\delta(\cdots)$ and $\int dg$

thus $\sum_n \left(\sum_{tt'} \frac{\delta}{\delta y^t_{AB}} \frac{\delta}{\delta y^{t'}_{CD}} \varepsilon_{ABCD5} \right)^n$

generates sum over

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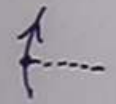
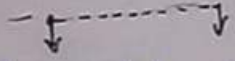
Chord diagrams (with arrows)

- – 2 current $\mathcal{J}$ (must be closed) as current is conserved
-  generator T^{AB}, arrow point to the direction where the derivative acts
- $= 1$ (trivial bubble
- $= 0$ (due to current conservation)
- $= \delta^{(AB)(CD)}$ – intertwining bubble
-  $\alpha \mathcal{E}_{ABCD5}$ – interaction term.

0-th order

1-st order $= \delta^{(AB)(CD)} \mathcal{E}_{ABCD5} = 0$

2nd order $= \mathcal{E}^{ABCD5} \mathcal{E}_{ABCD5} = 4!$

Point Charges and matter particles

- action with point sources:

$$S' = \int tr(B \wedge F(A)) + \int_L tr(jA)$$

j – 1-current

$$A \to \bar{A} = g^{-1} A g + g^{-1} dg$$

- gauge invariance:

$$\int_L tr(jA) = \underbrace{\int_L tr(j_0 A)}_{S_{j_0}\text{ - fixed charge}} + \underbrace{\int_L (g^{-1}\dot{g} j_0)}_{S_p\text{ - particle action}}$$

split in 2 "Partical observables" C. Rovelli

- $S_j = \int_L e^\mu p_\mu + \int \omega^{\mu\nu} y_{\mu\nu}$ – source term

casimirs:

$$C_1 = T^{AB} T_{AB} = \frac{1}{\Lambda} p^\mu p_\mu \,;\; C_2 = \left(T^{AB} T^{CD} \varepsilon_{ABCDE}\right)^2 = C_1 (S^i)^2$$

S_p – the action of spinning particle

$$S_p = 2 \int_e dt\, \dot{h}_\mu p^\mu + \int_L tr\left(g_{rot}^{-1} \dot{g}_{rot} T^{\mu\nu}\right) y_{\mu\nu}$$

With $m^2 = \Lambda C_1$ $\quad s^2 = \frac{C_2}{C_1}$

the coupling constant is $\alpha = G\Lambda$ because it is the coupling constant for particles of $m \sim \sqrt{\Lambda}$

Interaction between particles and gravity

- 't Hooft Deser Jackiw particles in spinfoam

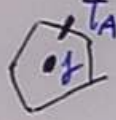
  Freidel Louapre

Worldline - sequence of bubbles with charges

- Empty bubble

$T_{AB} = 0$ (conservation of 2-current)

- Charged bubble

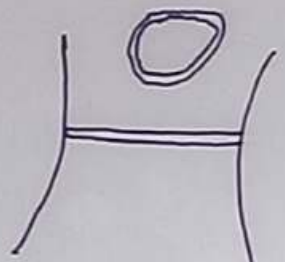


$T_{AB} \neq 0$ (Now 2-current has a source)

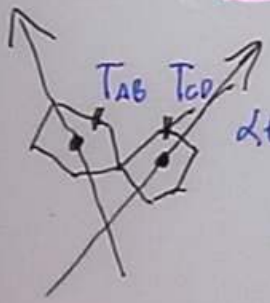
- diagram

2-current

1- current

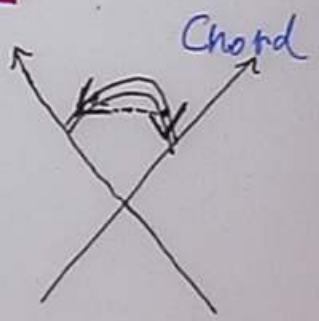


Scattering



$\alpha\, tr(T_{AB} T_{CD} \gamma_5)$

1-st order effect.



- Classicaly

$$\int B \wedge F + \alpha\, B \wedge B\, \gamma_5 + \int \gamma_i\, \delta(x - x_i) + \int \dot{h}_{i\mu} p^{i\mu}$$

- Solve at $\alpha = 0$: $B_0 = g^{-1} d\varphi\, g + B_p$
- Plug into the interaction term: $S_I = \alpha B_0 \wedge B_0 \gamma_5$

$$\frac{\delta}{\delta \varphi} S_I(B_0) = 0 \Rightarrow \varphi_0$$

$$\frac{\delta}{\delta h}\left(S_I(B_0, \varphi_0) + \int \dot{h}_\mu p^\mu \right) = 0$$

take initial data $p^\mu = \left(\frac{m}{\sqrt{\Lambda}}, 0, 0, 0\right)$

gauge $h_0 \equiv t$

$$\boxed{\dot{p}_{ia} = \frac{(h_{ia} - h_{ja})}{(h_i - h_j)^3}\, m_1 m_2 \frac{\alpha}{\Lambda} + \cdots}$$

Newtonian force in physical coordinates

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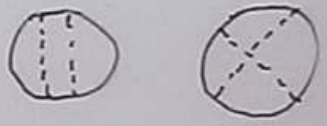
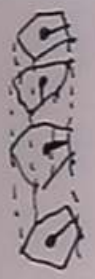
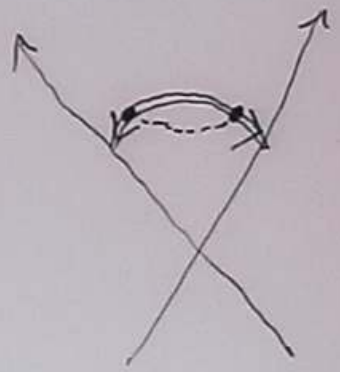
Conclusions

- dS - spacetime

- Vacuum loops



- Propagating matter particles



- Scattering, Newtonian potential.