Title: Asymptotic Safety for Quantum Gravity

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Abstract:

"Quantum Gravity in the Americas"
Perimeter Institute, Canada, 28-31 October 2004

Asymptotic Safety for Gravity Coupled to Matter

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(Penn State University)

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Outline

- Asymptotic Safety
- Average Effective Action and ERGE
- Pure Gravity
- Gravity Coupled to Matter
- Conclusions

Asymptotic Safety

[S. Weinberg, '79]

- Criterion of consistency for a QFT that generalizes perturbative renormalisability
- Requirement that observable quantities (like S-matrix elements) be finite in UV regime and theory be fixed by a finite number of parameters

1) $\boxed{\lim_{k\to\infty} g_i(k) = g_i^*}$ $\longleftrightarrow$ no UV divergences

$\longrightarrow$ fixed point (FP) in the Renormalisation Group flow

2) $\boxed{\dim(\mathcal{S}_{UV}) < \infty}$ $\longleftrightarrow$ predictivity

$\mathcal{S}_{UV}$: space spanned by trajectories hitting the fixed point

k is the energy scale at which the theory is described by an effective action Γ_k

- Perturbative renormalisability + asymptotic freedom is equivalent to asymptotic safety at the Gaussian FP, $g_i^* = 0$

Average Effective Action

[Wetterich, '91]

- Γ_k: quantum action coarse-grained a la Wilson
  Describes physics at the energy scale k
  All quantum effects contained in running couplings

$$Z_k[J] = \exp\left(W_k[J]\right) = \int \mathcal{D}\phi \exp\left(-S[\phi] + \int J\cdot\phi - \Delta_k S[\phi]\right)$$

- IR cutoff term $\quad \Delta_k S[\phi] = \frac{1}{2}\int \phi R_k(\Box)\phi$

$$\boxed{\Gamma_k[\phi_{cl}] = \int J\cdot\phi_{cl} - W_k[J] - \Delta_k S[\phi_{cl}]} \qquad \phi_{cl} = \frac{\delta W_k[J]}{\delta J}$$

- Exact RG Equation (scalar theory)

$$\boxed{\partial_t \Gamma_k[\phi] = \frac{1}{2}\mathrm{Tr}\left[\left(\frac{\delta^2\Gamma_k}{\delta\phi\delta\phi} + R_k\right)^{-1}\partial_t R_k\right]}$$

[Wetterich '93]

ERGE for Gravity

$$Z_k[J] = \int \mathcal{D}\phi \exp\left(-S[\phi] + \int J\cdot\phi - \Delta_k S[\phi]\right)$$

$$\partial_t \Gamma_k[\phi_{cl}] = \frac{1}{2}\mathrm{Tr}\left[\left(\frac{\delta^2\Gamma_k}{\delta\phi\delta\phi} + R_k\right)^{-1}\partial_t R_k\right]$$

- Same construction for $\phi \longrightarrow g_{\mu\nu}$
  Euclidean signature
- Diffeo invariance $\longrightarrow$ gauge fixing and ghosts
  (background technique with metric $\bar{g}$)

$$\partial_t \Gamma_k[g, v, \bar{v}, \bar{g}] = \frac{1}{2}\mathrm{Tr}\left[\sum_{\phi_i^{grav}}\left(\Gamma_k^{(2)} + \mathcal{R}_k\right)^{-1}_{\phi_i\phi_j}\partial_t(\mathcal{R}_k)_{\phi_i\phi_j}\right] - \frac{1}{2}\mathrm{Tr}\left[\sum_{\phi_i^{ghost}}\left(\Gamma_k^{(2)} + \mathcal{R}_k\right)^{-1}_{\phi_i\phi_j}\partial_t(\mathcal{R}_k)_{\phi_i\phi_j}\right]$$

[M. Reuter '96
O. Lauscher and M. Reuter '02]

- Diffeomorphism invariance

$$\Gamma_k[g + \mathcal{L}_\epsilon g, v + \mathcal{L}_\epsilon v, \bar{v} + \mathcal{L}_\epsilon \bar{v}, \bar{g} + \mathcal{L}_\epsilon \bar{g}] = \Gamma_k[g, v, \bar{v}, \bar{g}]$$

Treatment of the ERGE

- Knowledge of exact form of Γ_k is equivalent to solving the quantum theory
- No path-integral techniques beyond perturbation theory

- Do not specify form of S and functional integrate
- Take ERGE as defining the theory
- Write down most general form of Γ_k, accounting for symmetries
- Make approximations if necessary (e.g. truncations)
- Study attractivity of FP's by linearizing RG equations

Einstein-Hilbert Truncation

- Ansatz ($\tilde{\kappa}_k = 1/16\pi\tilde{G}_k$)

$$\Gamma_k[g] = \tilde{\kappa}_k \int \mathrm{d}^4x\sqrt{g}\left(-R + 2\tilde{\Lambda}_k\right)$$

- There is a non-Gaussian FP

[Souma '01
Lauscher and Reuter, '02]

$$\Lambda_* = 0.339$$

$$G_* = 0.344$$

- Stability matrix $M|_{\mathrm{NGFP}} = \begin{pmatrix} -0.863 & 4.196 \\ -4.205 & -2.863 \end{pmatrix}$

  with eigenvalues $\theta = \theta' \pm i\theta'' = -1.863 \pm 4.078i$

- NGFP satisfies conditions of asymptotic safety

- Inclusion of R^2 term [Lauscher and Reuter '02]

$$\Gamma_k[g,\bar{g}] = \int \mathrm{d}^dx\sqrt{g}\left\{\tilde{\kappa}_k\left(2\tilde{\Lambda}_k - R\right) + \tilde{\beta}_k R^2\right\}$$

  Small modifications w.r.t. previous truncation

Beta Functions

$$\partial_t(\tilde{\kappa}\tilde{\Lambda}) = \frac{1}{64\pi^2}\left\{\tilde{Q}_2\left[\frac{\partial_t\mathcal{P}(2\mathcal{P}+8\tilde{\Lambda})}{\mathcal{P}\left(\mathcal{P}-2\tilde{\Lambda}\right)}\right] + \frac{\partial_t\tilde{\kappa}}{\tilde{\kappa}}\tilde{Q}_2\left[\frac{\mathcal{R}\left(10\mathcal{P}-8\tilde{\Lambda}\right)}{\mathcal{P}\left(\mathcal{P}-2\tilde{\Lambda}\right)}\right]\right\}$$

$$\partial_t\tilde{\kappa} = \frac{1}{384\pi^2}\left\{\tilde{Q}_1\left[\frac{\partial_t\mathcal{P}\left(13\mathcal{P}-10\tilde{\Lambda}\right)}{\mathcal{P}\left(\mathcal{P}-2\tilde{\Lambda}\right)}\right]\right.$$

$$+5\,\tilde{Q}_2\left[\frac{\partial_t\mathcal{P}\left(11\mathcal{P}^2-12\mathcal{P}\tilde{\Lambda}+12\tilde{\Lambda}^2\right)}{\mathcal{P}^2\left(\mathcal{P}-2\tilde{\Lambda}\right)^2}\right]$$

$$+\frac{\partial_t\tilde{\kappa}}{\tilde{\kappa}}\left[\tilde{Q}_1\left[\frac{\mathcal{R}\left(3\mathcal{P}+10\tilde{\Lambda}\right)}{\mathcal{P}\left(\mathcal{P}-2\tilde{\Lambda}\right)}\right]\right.$$

$$\left.\left.+5\,\tilde{Q}_2\left[\frac{\mathcal{R}\left(5\mathcal{P}^2+12\mathcal{P}\tilde{\Lambda}-12\tilde{\Lambda}^2\right)}{\left(\mathcal{P}-2\tilde{\Lambda}\right)^2}\right]\right]\right\}$$

[M. Reuter, D. Dou and R. Percacci]

Einstein-Hilbert Truncation

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Matter Fields

- Assuming asymptotic safety with pure gravity, does this still hold when matter is included?
- It may happen that the NGFP disappears or does not have good properties
- Reproducing the matter content we see in nature (i.e. Standard Model) is very difficult
- Two approximations

1) Minimally coupled matter fields

[R. Percacci and D. Perini, '02]

2) Scalar field with arbitrary potential

[R. Percacci and D. Perini, '03]

Minimally Coupled Fields

- Minimally coupled, massless matter fields
  n_S scalars, n_W Weyl fermions, n_M Maxwell, n_{RS} Rarita-Schwinger
- Scalar $\quad \Gamma_k|_{\rm scalar} = \int d^4x \sqrt{g}\, \frac{1}{2}\phi \left(-\nabla^2 + m_S^2\right) \phi$

$$\partial_t \Gamma_k|_{\rm scalar} = \frac{1}{2} {\rm Tr}_0 \, \frac{\partial_t \mathcal{P}}{\mathcal{P}_k^2 + m_S^2}$$

- Spinor $\quad \Gamma_k|_{\rm spinor} = \int d^4x \sqrt{g}\, \bar{\psi}(\gamma^\mu \nabla_\mu + m_W)\psi$

$$\partial_t \Gamma_k|_{\rm spinor} = -\frac{1}{2} {\rm Tr}_{1/2} \, \frac{\partial_t \mathcal{P}}{\mathcal{P}_k + \frac{R}{4} + m_W^2}$$

- Vector $\quad \Gamma_k|_{\rm vector} = \frac{1}{4} \int d^4x \sqrt{g}\, F_{\mu\nu} F^{\mu\nu}$

$$(S_{gf})|_{\rm vector} = \frac{1}{2\lambda} \int d^4x \sqrt{g}\, (\nabla_\mu A^\mu)^2,$$

$$(S_{gh})|_{\rm vector} = \int d^4x \sqrt{g}\, \bar{c}(-\nabla^2)\, c$$

Beta Functions

[R. Percacci and D. Perini, '02]

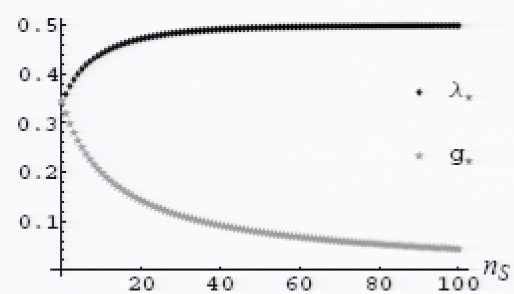
$$\beta_{\tilde{\kappa}\tilde{\Lambda}} = \beta_{\tilde{\kappa}\tilde{\Lambda}}\big|_{\text{pure gravity}} + \frac{1}{64\pi^2} \cdot$$
$$\cdot \left(n_S - 2n_W + 2n_M - 4n_{RS}\right) \tilde{Q}_2\left[\frac{\partial_t \mathcal{P}}{\mathcal{P}}\right]$$

$$\beta_{\tilde{\kappa}} = \beta_{\tilde{\kappa}}\big|_{\text{pure gravity}} + \frac{1}{384\pi^2} \cdot$$
$$\cdot \left\{ \left(-2n_S + 4n_W - n_M\right) \tilde{Q}_1\left[\frac{\partial_t \mathcal{P}}{\mathcal{P}}\right] \right.$$
$$\left. + \left(-6n_W + 9n_M - 16n_{RS}\right) \tilde{Q}_2\left[\frac{\partial_t \mathcal{P}}{\mathcal{P}^2}\right] \right\}$$

- How do the NGFP's properties change when we vary the number of fields?

Study of the NGFP

- NGFP determined by the zeroes of a function $h(\Lambda)$
- Study numerically bounds on matter content
- Example: Λ_* and G_* as functions of n_S



Study of the NGFP

- More generally, the behaviour of h is determined by the sign of two parameters:

$\Delta' = c(0) = 1/4h'(0)$

$\tau \propto 1/\lim_{\Lambda \to -\infty} h(\Lambda)$

- The parameter space can be divided into four regions

	$\tau < 0$	$\tau > 0$
$\Delta' < 0$	III	IV
$\Delta' > 0$	I	II

I) one NGFP with $\Lambda_* > 0, G_* > 0$ ✓

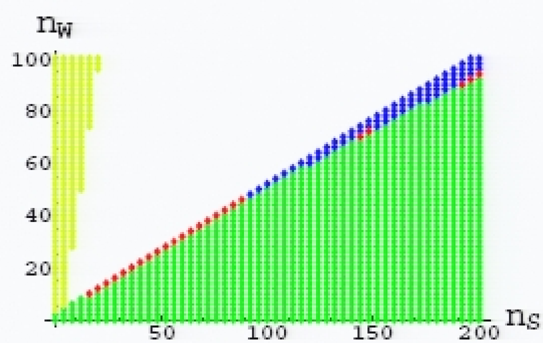
II) one NGFP with $\Lambda_* > 0, G_* > 0$ ✓
one NGFP with $\Lambda_* < 0, G_* < 0$ ×

III) one NGFP with $\Lambda_* < 0, G_* > 0$ ✓

IV) no NGFP's ×

Attractivity

- Only numerical study (point by point)
- For $n_M = 0, n_{RS} = 0$



- When NGFP exists, it is attractive in both directions

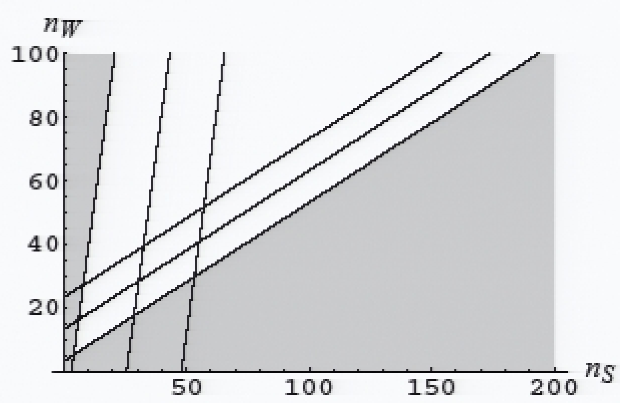
Attractivity

- In general, the matter content must satisfy certain bounds $(a = 1/2)$
- regions with $\Lambda \gtrless 0$, separated by the plane

$$n_S + 2n_M - 2n_W - 4n_{RS} - 4.16 = 0$$

- the other plane has equation

$$6.58n_S - 14.7n_M - 1.6n_W + 32n_{RS} - 21.45 = 0$$



- Popular GUT and SUSY theories are OK
- SM and minimal $SU(5)$ in region I
  $SO(10)$ in region I or III (depends on symmetry breaking)

Scalar Field with Gravity

- Scalar field with arbitrary potential and coupling to R [R. Percacci and D. Perini, '03]
- Scalar field may be though of as a sample of matter fields or part of the gravitational sector (dilaton)

$$\Gamma_k = \int \mathrm{d}^4x\sqrt{g}\left(V(\phi^2) - F(\phi^2)R + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right)$$

$$V(\phi^2) = \sum_{n=0}^{\infty}\tilde{\lambda}_{(2n)}\phi^{2n}, \qquad F(\phi^2) = \sum_{n=0}^{\infty}\tilde{\xi}_{(2n)}\phi^{2n}$$

- Infinite number of couplings may produce an infinite-dimensional $\mathcal{S}_{UV}$
- There is a NGFP for

$\lambda_0 \equiv 2\kappa\Lambda \neq 0$	$\lambda_{2i} = 0 \ (i > 0)$
$\xi_0 \equiv \kappa \neq 0$	$\xi_{2i} = 0 \ (i > 0)$

This is the "Gaussian-Matter FP" (GMFP)
It is attractive in four directions, thereby it satisfies the conditions of asymptotic safety

Conclusions

- Asymptotic safety: non-perturbative approach
- Pure gravity seems to admit a NGFP
- Not all matter contents are compatible. Explicit bounds on type of matter
- Conclusions seem robust w.r.t. addition of extra terms (scalar field)

- Understand better the properties of the FP
- Applications to phenomenology
- Understand relation with other approaches to Quantum Gravity