Title: String Theory With LQG Methods

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Abstract:

STRING THEORY WITH LQG METHODS

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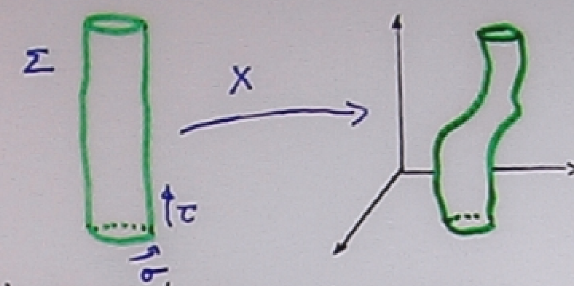
BEFORE WE REALLY START:

STRING THEORY: MEANING THEORY OF FREE CLOSED BOSONIC STRING IN D-DIM FLAT TARGET SPACE

ALSO: MOSTLY OVERVIEW/REVIEW, ONLY FEW NEW IDEAS

CLASSICAL STRING

$$S_{NG}[X] = \int_{\Sigma} d^2x \sqrt{\det G}$$



$G_{ab} = \partial_a X^\mu \partial_b X^\nu \eta_{\mu\nu}$ • INDUCED METRIC ON Σ ! MINKOWSKI SIGNATURE

CONF. VAR.: $X^\mu(\sigma)$

MOMENTUM: $\Pi^\mu(\sigma) = \frac{1}{\sqrt{\det G}} \left(X'^2 \dot{X}^\mu - (X' \cdot \dot{X}) X'^\mu \right)$

CONSTRAINTS: $D = \Pi \cdot X' \qquad H = \frac{1}{2}(\Pi^2 + X'^2)$

$$\{D,D\} \sim D \qquad \{D,H\} \sim H \qquad \{H,H\} \sim D$$

CONVENIENT OTHER VARS: $Y_\pm := \Pi \pm X'$ $\rightsquigarrow$ CONSTRAINTS $L^\pm := (Y_\pm)^2$

$$\{L^+, L^-\} = 0 \qquad \{L^\pm(f), L^\pm(g)\} = L^\pm(f'g - fg')$$

EQUIVALENT FORMULATION:

$$S_P[X,h] = -\frac{1}{2}\int_\Sigma d^2x\,\sqrt{\det h}\,h^{ab}G_{ab} \qquad h: \text{METRIC ON } \Sigma$$

CONF. VAR: $X^\mu(\sigma)$

MOMENTUM: $\Pi^\mu(\sigma) = -\sqrt{\det h}\,(h^{00}\dot{X}^\mu + h^{01}X'^\mu)$

CONSTRAINTS: (EOM OF h!)

$$T_{ab} = -2\frac{1}{\sqrt{h}}\frac{\delta S}{\delta h^{ab}} = G_{ab} - \frac{1}{2}h_{ab}h^{cd}G_{cd}$$

$$T \overset{*}{=} \begin{pmatrix} \frac{1}{2}(\dot{X}^2 + X'^2) & \dot{X}\cdot X' \\ \dot{X}\cdot X' & \frac{1}{2}(\dot{X}^2 + X'^2) \end{pmatrix} = \begin{pmatrix} \frac{1}{2}(\Pi^2 + X'^2) & \Pi\cdot X' \\ \Pi\cdot X' & \frac{1}{2}(\Pi^2 + X'^2) \end{pmatrix}$$

$$\overset{**}{=} \begin{pmatrix} (\Pi + X')^2 & 0 \\ 0 & (\Pi - X')^2 \end{pmatrix}$$

$*$: $h_{ab} = \Omega^2\eta_{ab}$

$**$: AS ABOVE, THEN GO TO LIGHTCONE COORD.

COMPARISON WITH LQG: CLASSICAL

SIMILARITIES:

- BACKGROUND INDEPENDENCE
- SIMILAR CONSTRAINT ALGEBRAS
- CAN IN PRINCIPLE INTERPRETE S_p AS COMING FROM 2D GRAV AND BUNCH OF SCALAR FIELDS

DIFFERENCES:

- 2D VS. 4D
- USUAL PHYSICAL INTERPRET: STRINGS ON FLAT SPACETIME (NO GRAVITY (APRIORI) NO PLANCK LENGTH)

(STRING MUCH SIMPLER)

(STANDARD-) QUANTIZATION OF STRING

DIFFERENT WAYS: OLD CANONICAL, LIGHTCONE, PATH-INTEGRAL, ...
ALWAYS SEE OBSTRUCTIONS TO QUANTIZATION FOR ARBITRARY TARGET SPACE DIMENSION!

OLD CANONICAL:

- GAUGE FIX: $h_{ab} = \Omega^2 \eta_{ab}$
- MODE DECOMPOSITION

$$X(\tau,\sigma) = x + p\tau + \frac{i}{2} \sum_{n\neq 0} \frac{1}{\sqrt{n}} a_n e^{-in(\tau-\sigma)} + \frac{i}{2} \sum_{n\neq 0} \frac{1}{\sqrt{n}} \tilde{a}_n e^{-in(\tau+\sigma)}$$

- FOCK QUANTIZATION: $[\hat{a}^\mu_m, \hat{a}^\nu_n] = \delta_{m+n} \cdot \mathrm{sign}(m)\, \eta^{\mu\nu}$, $\hat{a}^\mu_n |0\rangle = 0 \quad n>0$

LEADS TO SPACE WITH INDEFINITE INNER PROD.

— 6 —

CONSTRAINTS: WORK IN TERMS OF

$$L_m^{\pm} = \int_0^{2\pi} e^{-im\sigma} L^{\pm}(\sigma)\, d\sigma$$

- NO ORDERING AMBIGUITY IN QUANTIZATION EXCEPT FOR L_0
  $\longrightarrow$ PARAMETER a

- PHYSICAL STATES: $L_n^{\pm} \psi = 0 \quad \forall n > 0$ (MORE NOT POSSIBLE DUE TO CENTRAL EXTENSION)

- SHOW THAT NO PHYSICAL NEGATIVE NORM STATES ONLY

  FOR $D=26$, $a=1$ OR $D \leq 25$, $a \leq 1$

COMPARISON WITH LQG – QUANTUM

STRING

- KIN. QUANTIZATION NON-GAUGE-INVARIANT
- OPERATE WITH GENERATORS OF GAUGE TRANSFORMATIONS
- KIN. SPACE HAS INDEFINITE INNER PRODUCT

LQG

- START WITH REP THAT IS ALR. PARTIALLY INVARIANT UNDER GAUGE TRAFOS
- LOOK AT GROUPS THEMSELVES
- KIN. SPACE IS HILBERT SPACE

NON-STANDARD QUANTIZATIONS

A. STARODUBTSEV (gr-qc/0201089):

$$\text{QUANTIZE} \begin{cases} e^{ik_\mu X^\mu(\sigma)} \\ \int \Pi_\mu(\sigma) f^\mu(\sigma)\, d\sigma \end{cases}$$

ON AL-LIKE HILBERT SPACE GENERATED BY STATE

$$\omega\left(\widehat{e^{ik_\mu X^\mu(\sigma)}}\right) = \delta_{0,k} \qquad \omega\left(\widehat{\int \Pi_\mu(\sigma) f^\mu(\sigma)\, d\sigma}\right) = 0$$

GROUP-AVERAGING GIVES HILBERT-SPACE OF STATES THAT SOLVE THE DIFFEO CONSTRAINT.
HAMILTON CONSTRAINT HARD TO QUANTIZE, NO RESULTS ON SOL. SPACE.

T. THIEMANN (hep-th/0401172):

REMEMBER: $y_\pm(f) := \int (\pi^\mu(\sigma) \pm X'^\mu(\sigma)) f_\mu(\sigma) \, d\sigma$

QUANTIZE: $\exp i \, y_\pm(f)$

REPRESENTATION: GIVEN BY STATE

$$\omega\left(e^{i y_\pm(f)}\right) = \delta_{0,f}$$

INVARIANT UNDER ALL GAUGE TRANSFORMATIONS.

GROUP-AVERAGING GIVES HILBERT SPACE OF PHYSICAL STATES!

(THOMAS DOES EVEN MORE IN HIS PAPER...)

REMARKS: HAVE SEEN THERE ARE ALREADY VERY INTERESTING APPROACHES TO QUANTIZING THE STRING WITH LOOPY METHODS.
BUT THERE IS MORE TO THINK ABOUT...

① CAN THE TWO WAYS OF QUANTIZING BE BROUGHT CLOSER TOGETHER?

- START FROM FOCK-REP, IMPLEMENT DIFFEO-CONSTRAINT

$$D_u = L^+_u - L^-_{-u}$$

→ NO ANOMALY
→ MIXING OF +/- SECTOR, OF POS/NEG FREQUENCY PART
→ ?

- START WITH ANOTHER STATE FOR THE PROPOSED ALGEBRAS.
(EX. $\omega\left(\widehat{\exp i k_\mu X^\mu(\sigma)}\right) = \exp(-k^2)$. IS NOT POS. BUT...)

- UNDERSTAND HOW TO COMPARE TWO QUANTIZATIONS
→ OBSERVABLES FOR STRING (DDF, POHLMEYER, ?)

• UNDERSTAND RELATION

NON-REGULAR STATE $\longleftrightarrow$ NO NEED FOR INDEF. INNER PROD.

(2) COULD THE STANDARD QUANTIZATIONS FOR THE STRING BE OF INTEREST FOR LOOP-QG ?

- DISENTANGLE 2d FEATURES FROM GENERAL ONES
- CENTRAL EXT. FOR 4d DIFFEOS ¿

(3) CAN CONTACT BE MADE ON THE LEVEL OF PHYSICS ?

- STRINGS ON LQG BACKGROUND ?
- ¿??