Title: Canonical GR on Null Hypersurfaces

Date: Oct 31, 2004 02:00 PM

URL: http://pirsa.org/04100049

Abstract:

1

CANONICAL GR ON NULL HYPERSURFACES

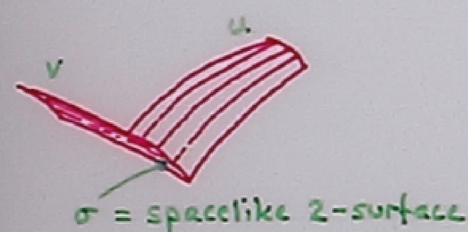
MICHAEL REISENBERGER

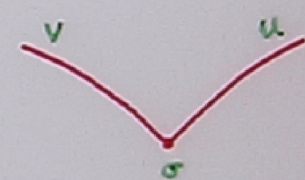
UNIVERSIDAD de la REPÚBLICA, MONTEVIDEO, URUGUAY

WHAT IS IT ?

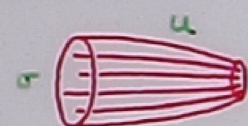
- Usually canonical GR formulated in terms of initial data on a <u>spacelike</u> hypersurface
- here we consider data on a piecewise null hypersurface (normal vector is lightlike)

  – specifically we consider two intersecting null hypersurfaces

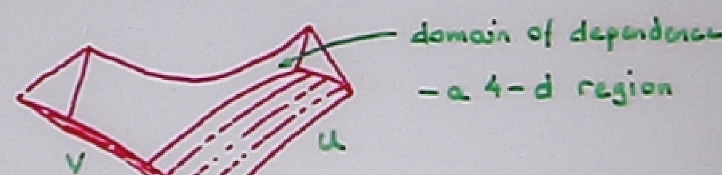




u, v = null 3-surfaces swept out by null rays emerging normally from two sides of σ



– initial data specifies maximal Cauchy development = metric on domain of dependence



③

WHY STUDY THIS?

① Technical simplification:

1962 Penrose, Sachs, Bondi, Van der Burgh, Dautcourt

identify <u>free</u> initial data (no constraints!)

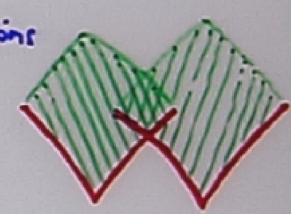
② Observables for mostly vacuum universe – like ours (?)

Ellis et. al. 1985 → similar data on lightcone = results of idealized astronomical observations

③ Quasi-local formulation of GR similar to algebraic field theory

– any exitation of gravitational field causes null rays in $\mathcal{U}, \mathcal{V}$ to cross (form caustics) ⇒ $\mathcal{U}, \mathcal{V}$ must be finite

– dynamics of GR encoded in relations between data for hypersurfaces with overlapping domains of dependence



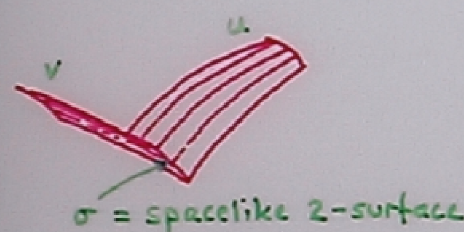
④ Entropy bounds – holographic principle

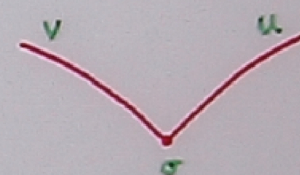
– possibility of a proof from first principles

WHAT IS IT ?

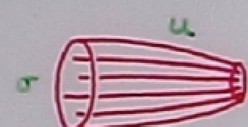
- Usually canonical GR formulated in terms of initial data on a <u>spacelike</u> hypersurface
- here we consider data on a piecewise null hypersurface (normal vector is lightlike)

  – specifically we consider two intersecting null hypersurfaces

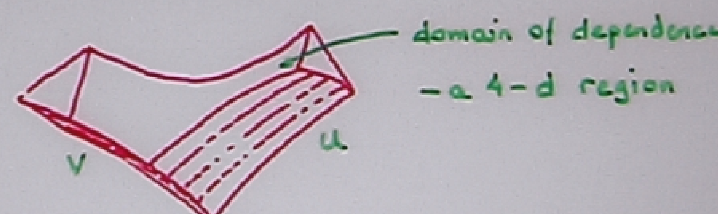




u, v = null 3-surfaces swept out by null rays emerging normally from two sides of σ



– initial data specifies maximal Cauchy development
= metric on domain of dependence



③

WHY STUDY THIS ?

① Technical simplification :
1962 Penrose, Sachs, Bondi, Van der Burgh, Dautcourt
identify <u>free</u> initial data (no constraints!)

② Observables for mostly vacuum universe – like ours (?)
Ellis et. al. 1985 → similar data on lightcone = results of idealized astronomical observations

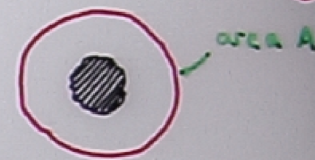
③ Quasi-local formulation of GR similar to algebraic field theory

– any exitation of gravitational field causes null rays in u, v to cross (form caustics) ⇒ u, v must be finite

– dynamics of GR encoded in relations between data for hypersurfaces with overlapping domains of dependence

④ Entropy bounds – holographic principle
– possibility of a proof from first principles

BECKENSTEIN BOUND



Object inside sphere of area A in stationary asymptotically flat spacetime

<u>Claim</u>: Entropy $S \leq$ entropy of black hole of area A

$$= \frac{A}{4} \qquad \text{(Planck units)}$$

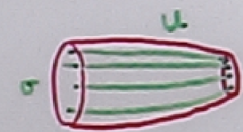
<u>Argument</u>: Can collapse a spherical shell of matter around object so it becomes a black hole of area A without increasing entropy of rest of universe.

2^{nd} law thermodynamics $\Rightarrow S_{initial} \leq S_{final} = \frac{A}{4}$

5

BOUSSO BOUND — more general

- null hypersurface $\mathcal{U}$ swept out by null rays emerging normally from 2-surface σ of area A



- null rays <u>non-diverging</u> at σ (σ not convex)
- $\mathcal{U}$ truncated before rays cross

<u>Claim</u>: "Entropy on $\mathcal{U}$" $\leq \frac{A}{4}$

↑ flux of entropy current — defined for matter in local equilibrium

Flanagan et. al. proved Bousso bound assuming inequality between $T^{\mu\nu}$ and entropy current satisfied by known matter models

- Not fundamental law

BUT

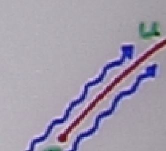
$S = \log \dim \mathcal{H}'$ $\mathcal{H}'$ subspace of Hilbert space $\mathcal{H}$ compatible with macroscopic state

$\leq \log \dim \mathcal{H}$

<u>Conjecture</u>: $\dim \mathcal{H} \leq e^{A/4}$

- but what is Hilbert space of states on $\mathcal{U}$?

$\mathcal{U}$ by itself has no domain of dependence



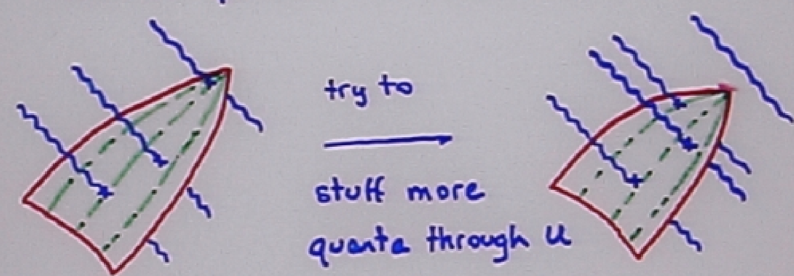
HOW COULD THIS CONJECTURE BE TRUE ?

Vacuum GR – infinity of Fourier modes of data on $\mathcal{U}$

$\Rightarrow$ quantization of linearized theory has $\dim \mathcal{H} = \infty$

But if you include back reaction ...

- remember quantum for each mode has finite energy



$\rightarrow$ null generators focus more

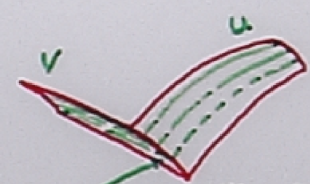
$\Rightarrow$ same number of quanta registered on $\mathcal{U}$

Can one make a proof ?

Perhaps with null canonical GR

7

NULL CANONICAL GR



σ = spacelike 2-surface

null direction

hypersurface null $\Rightarrow$ metric of form $\left[\begin{array}{c|cc} 0 & 0 & 0 \\ \hline 0 & & \\ 0 & \multicolumn{2}{c}{g_{ij}} \end{array}\right]$

<u>Sachs data</u>: 1. $e_{ij} = \dfrac{g_{ij}}{\sqrt{\det g}}$ on U and V

2. some data on σ

They are free and complete so they are good coordinates on phase space (set of Cauchy developments)

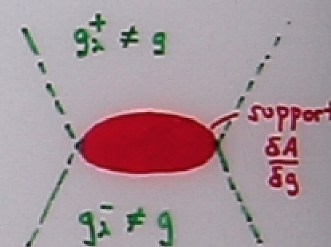
– need Poisson brackets of these coordinates

PEIERLS BRACKET

A, B – diffeo invariant functionals of metric such that

$\frac{\delta A}{\delta g_{\mu\nu}}$, $\frac{\delta B}{\delta g_{\mu\nu}}$ have compact support

g – metric satisfying Einstein's equation



I = action

$I_\lambda = I + \lambda A \quad \lambda \in \mathbb{R}$

g_λ^+ = "retarded modified solution"

= stationary point of I_λ which reduces to g in past

g_λ^- = "advanced modified solution"

Definition

$$\{A, B\} = \frac{d}{d\lambda} B[g_\lambda^+]\Big|_{\lambda=0} - \frac{d}{d\lambda} B[g_\lambda^-]\Big|_{\lambda=0}$$

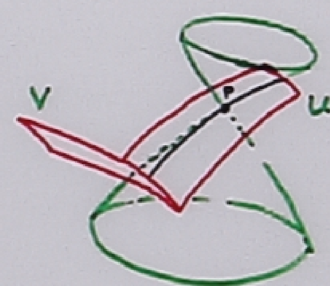
(9)

SOME QUALITATIVE FEATURES OF THE BRACKET

- $\{A, B\}$ non-zero only if A, B function of metric in causally related regions

$\longrightarrow$ fields at spacelike separated points commute

- on $\mathcal{U}$ only points on same null generator are causally related



$\Rightarrow$ 1. variables on $\mathcal{U}$ almost all Poisson commute with those on V

$\overset{?}{\Longrightarrow}$ Hilbert space $\mathcal{H} = \mathcal{H}_{\mathcal{U}} \otimes \mathcal{H}_V$

$\Rightarrow$ 2. if $\dim \mathcal{H}_{\mathcal{U}}$ finite then $\log \dim \mathcal{H}_{\mathcal{U}} \propto A$

A PRELIMINARY RESULT

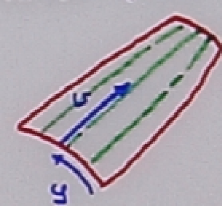
Bracket between e_{ij}'s

– change variables

$$ds^2 = g_{ij}\, dy^i dy^j = \frac{h}{1-|\mu|^2}(dz + \mu\, d\bar{z})(d\bar{z} + \bar{\mu}\, dz)$$

$$z = y^1 + i y^2 \qquad h = \sqrt{\det g}$$

μ is a $\mathbb{C}$ valued field that encodes e_{ij}



Then

$$\{\mu(x), \bar{\mu}(x')\} = \frac{1}{8}\left[\frac{1-|\mu|^2}{\sqrt{h}}\right]_x \left[\frac{1-|\mu|^2}{\sqrt{h}}\right]_{x'} \times e^{\int_v^{v'} \frac{1}{1-|\mu|^2}[\bar{\mu}\,\partial_v \mu - \mu\,\partial_v \bar{\mu}]\, dv}\; \delta^2(y'-y)\, \mathrm{sgn}(v'-v)$$

$$\{\mu(x), \mu(x')\} = 0$$

$\Rightarrow$ μ defines a complex polarization

can represent quantum states by holomorphic functions of μ.