Title: Space-Time Granularity and Lorentz Invariance Violation

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Abstract:

The Emperor's New Universe

or the story of the origin of the primordial cosmic inhomogeneities

D. Sudarsky (ICN-UNAM)

work in collaboration with

H. Sahlmann Penn-State
A Perez Penn-State

last decade A big success in cosmology

1) The theoretical prediction of the power spectrum of anisotropies in the CMB

2) A series of observations (COBE, MAXIMA, BOOMERANG, W-MAP) matching these predictions.

So understandably the comunity has applauded the results as a remarkable success.

There is however something very unsettling about this picture:

The universe starts as a "perfectly" homogeneous & isotropic spacetime with a quantum field the inflaton in its "vacuum state" which is also h. & i. → How does it end in a state with certain inhomogeneities and anisotropies?

Is this the Standard QM Measurement Problem?

- When does the measurement occur?
- What is the measured quantity?
- What physical device induces the measurement?

What is what we see directly?

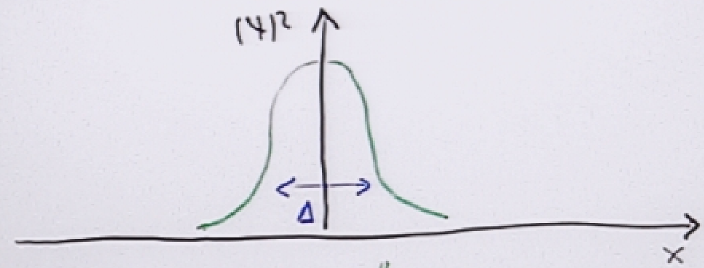
$\Psi(x_D, t_D)$ not $\Psi^2(\ldots)$

$\langle 0| \quad |0\rangle = 0$ $\qquad$ $\langle 0| \quad |0\rangle \neq 0$

Why do the predictions work so well?

Can we expect deviations? (even without resorting to models of QG)?

ets consider forinstance the case of an harmonic oscillator in the ground state



we have "fluctuations" of x with magnitude Δ

but this means that if we measure "$\hat{x}$" on an ensemble of identical systems the resulting values will have a statistical spread Δ

- what can be said about a single system?
- what if we do not measure anything or if we measure the energy E?
- Can Δ be interpreted as indication that x is jumping around? ... in time?
- Most likely value of x is 0!

2 A look to proposed ideas on the transition to classicality

Standard Decoherence

Decoherence without Decoherence

Squeezed States

Alternative to Inflation

Issues

1) Is there a measurement involved in the transition from a field characterized by a quantum state to the ensemble of realizations described by the stochastic field?

2) If so, what is performing the measurement?

3) Precisely, what is the set of quantum observables that is being measured? and what determines them?

4) When is this measurement taking place, and does the answer to this question have any possible observational consequences?

5) Justification for the use of statistics in a single universe?

The standard view, supplemented by the physical collapse hypothesis. Gravity is different.

3 Linearized Einstein's equations and the evolution of small fluctuations

$$ds^2 = a(\eta)^2 \left[-(1+2\Phi)d\eta^2 + (1-2\Psi)\delta_{ij}dx^i dx^j \right] \tag{12}$$

Let us first write down the components of the Einstein tensor ($G_{ab} = R_{ab} - \frac{1}{2}g_{ab}R$) up to first order in the perturbations:

$$\begin{aligned}
G^{(0)}_{00} &= 3\frac{\dot{a}^2}{a^2} \\
G^{(0)}_{ii} &= \frac{\dot{a}^2}{a^2} - 2\frac{\ddot{a}}{a} \\
G^{(1)}_{00} &= 2\nabla^2\Psi - 6\frac{\dot{a}}{a}\dot{\Psi} \\
G^{(1)}_{0i} &= 2\partial_i\dot{\Psi} + 2\frac{\dot{a}}{a}\partial_i\Phi \\
G^{(1)}_{ii} &= (\nabla^2 - \partial_i\partial_i)(\Phi - \Psi) + 2\ddot{\Psi} + 2\left(2\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2}\right)(\Psi + \Phi) + 2\frac{\dot{a}}{a}(\dot{\Psi} + \dot{\Phi}) \\
G^{(1)}_{ij} &= \partial_i\partial_j(\Psi - \Phi) \quad \text{for} \quad i \neq j
\end{aligned} \tag{13}$$

The components of the energy momentum tensor are as follows

$$\begin{aligned}
T^{(0)}_{00} &= \frac{1}{2}(\dot{\phi}_0^2) + a^2V[\phi_0] \\
T^{(0)}_{ii} &= \frac{1}{2}(\dot{\phi}_0^2) - a^2V[\phi_0] \\
T^{(1)}_{00} &= \dot{\phi}_0\delta\dot{\phi} + 2a^2\Phi V[\phi_0] + a^2\partial_\phi V[\phi]\delta\phi \\
T^{(1)}_{0i} &= \dot{\phi}_0\partial_i\delta\phi \\
T^{(1)}_{ii} &= -\Phi\dot{\phi}_0^2 + \dot{\phi}_0\delta\dot{\phi} - \Psi(\dot{\phi}_0^2 - a^2V[\phi_0]) - \frac{1}{2}a^2\partial_\phi V[\phi]\delta\phi \\
T^{(1)}_{ij} &= 0 \quad \text{for} \quad i \neq j
\end{aligned} \tag{14}$$

Finally the scalar field equation yields, to zero order:

$$\ddot{\phi}_0 + 2\frac{\dot{a}}{a}\dot{\phi}_0 + a^2\partial_\phi V[\phi] = 0 \tag{15}$$

The only nontrivial amongst Einstein's equations, to zero order is $G^{(0)}_{00} = 8\pi G T^{(0)}_{00}$ which leads to Friedman's equation

$$3\frac{\dot{a}^2}{a^2} = 4\pi G(\dot{\phi}_0^2 + 2a^2 V[\phi_0]) \tag{16}$$

In the linear order let us start from $G^{(1)}_{ij} = 8\pi G T^{(1)}_{ij}$ which implies the metric perturbation potential to be equal, namely

$$\Psi = \Phi \tag{17}$$

After some algebra we obtain

$$\boxed{\nabla^2\Psi + 4\pi G\dot{\phi}_0^2\Psi = 4\pi G\left[\dot{\phi}_0(\delta\dot{\phi} + \frac{\dot{a}}{a}\delta\phi) - \ddot{\phi}_0\delta\phi\right]} \tag{18}$$

Finally, [redacted] the equation for the scalar field perturbation we obtain

$$\ddot{\delta\phi} + 2\frac{\dot{a}}{a}\dot{\delta\phi} - \nabla^2\delta\phi + a^2\partial^2_{\phi,\phi}V[\phi]\delta\phi - 16\pi G(\dot{\phi}_0)^2\delta\phi - 2\Psi\ddot{\phi}_0 = 0 \tag{19}$$

This differs from the evolution equation of the scalar field perturbations in the background spacetime. However note that the corrections due to the Newtonian potential are suppressed by the factor G.

Our main equations will be then te equation for the scalar field in the background space-time

$$\boxed{\ddot{\delta\phi} + 2\frac{\dot{a}}{a}\dot{\delta\phi} - \nabla^2\delta\phi + a^2\partial^2_{\phi,\phi}V[\phi]\delta\phi = 0} \tag{20}$$

and equation [redacted], which, upon quantization of the scalar field perturbation $\delta\phi$ one can promote the previous equation to semi-classical equation to determine Ψ in terms of $< \dot{\delta\phi} >$.

4 Quantum theory of fluctuations

The inflaton field is a scalar field described by the action

$$S[\phi] = \int \left[-\frac{1}{2}\nabla_a\phi\nabla_b\phi g^{ab} - V[\phi]\right]\sqrt{-g}\,\mathrm{d}^4x \tag{21}$$

Using the form of the FRW line element the previous action becomes

$$S[\phi] = \int \left[\frac{a^2}{2}(-\phi\partial_0\partial_0\phi + \phi\partial_i\partial_j\phi\delta^{ij}) - a^4V[\phi]\right]\mathrm{d}^4x \tag{22}$$

Separate the scalar D.O.F into an homogeneous background component ϕ_0 plus small fluctuation $\delta\phi$, namely

$$\phi = \phi_0 + \delta\phi. \tag{23}$$

The background field ϕ_0 is described in a completely classical fashion while only the fluctuation $\delta\phi$ is quantized.

In these coordinates, the field equation becomes

$$\boxed{\delta\ddot{\phi} - \nabla^2\delta\phi + 2\frac{\dot{a}}{a}\delta\dot{\phi} = 0} \tag{24}$$

where dots denote derivatives with respect to η and Δ is the Laplacian on flat Euclidean three space.

Note that we have neglected a terms proportional to $\partial^2_{\phi^2}V[\phi]$ using the slow rolling approximation.

If we expand the fluctuation in its Fourier components the equations of motion for the mode $\delta\phi_k$ becomes

$$\delta\ddot{\phi}_k + 2\,\frac{\dot{a}}{a}\,\delta\dot{\phi}_k - k^2\delta\phi_k = 0 \tag{25}$$

Introduce an auxiliary field $y = a\delta\phi$.

$$\ddot{y} - \left(\nabla^2 + \frac{\ddot{a}}{a}\right)y = 0, \tag{26}$$

We will now proceed to quantize y.

Let us consider the field in a Box of side L, decompose a real classical field y satisfying (??) into plane waves

$$\boxed{\hat{y}(\eta,\vec{x}) = \frac{1}{(L)^3}\Sigma_{\vec{k}}\left(a_k(\eta)e^{i\vec{k}\cdot\vec{x}} + \bar{a}_k(\eta)e^{-i\vec{k}\cdot\vec{x}}\right),} \tag{27}$$

the sum is over $\vec{k}$ satisfying $k_i L = 2\pi n_i$ for $i = 1, 2, 3$ with n_i integers.

We impose standard conmutation relations between the field y and its canonical conjugate momentum $\pi^{(y)}$. Thus we write

$$\hat{y}(\eta, \vec{x}) = \frac{1}{(L)^3}\Sigma_{\vec{k}}\left(\hat{a}_k(\eta)e^{i\vec{k}\cdot\vec{x}} + \hat{a}_k^\dagger(\eta)e^{-i\vec{k}\cdot\vec{x}}\right), \qquad \hat{a}_k(\eta) = y_k(\eta)\hat{a}_k, \tag{28}$$

where $y_k(\eta)$ is a solution of (??) and $\hat{a}_k$ is the usual annihilation operator on the one particle space $\mathcal{H} = \mathcal{L}^2(L^3, d^3x)$. Upon choosing the solutions $y_k(\eta)$, $\hat{y}$ thus becomes an operator on the Fock space over $\mathcal{H}$. Similarly the canonical conjugate to y;

$$\hat{\pi}^{(y)}(\eta, \vec{x}) = \dot{y}(\eta, \vec{x}) - \frac{\dot{a}}{a}\,\hat{y}(\eta, \vec{x} = \tag{29}$$

$$\frac{1}{(L)^3}\Sigma_{\vec{k}'}\left(\hat{a}_k g_k(\eta)e^{i\vec{k}\cdot\vec{x}} + \hat{a}_k^\dagger g_k(\eta)e^{-i\vec{k}\cdot\vec{x}}\right), \tag{30}$$

where

$$g_k = \dot{y}_k - \frac{\dot{a}}{a}y_k. \tag{31}$$

To complete the quantization, we have to specify the classical solutions $y_k(\eta)$. To insure $[\hat{a}_k, \hat{a}_{k'}^\dagger] = \hbar L^3\delta_{k,k'}$, we need

$$y_k(\eta)\overline{g_k}(\eta) - \overline{y_k}(\eta)g_k(\eta) = i \tag{32}$$

for all k at some (and hence any) time η.

A pair of independent solutions is given by

$$y_k^{(\pm)}(\eta) = \frac{1}{\sqrt{2k}}\left(1 \pm \frac{i}{\eta k}\right)\exp(\pm ik\eta) \tag{33}$$

Note Could this be the trigger of the collapse?

Also

$$g_k^{\pm}(\eta) = \pm i\sqrt{\frac{k}{2}}\exp(\pm ik\eta) \tag{34}$$

Note that the dimensions implied for $\hat{a}_k$ is $(Mass)^{1/2}(Lenght)^2$ which is compatible with the dimentionalized conmutator $[\hat{a}_k, \hat{a}_{k'}^\dagger] = \hbar L^3\delta_{k,k'}$.

5 Evolution of the fluctuations through collapse

The collapsing modes

In the present paper, we choose the following modes:

$$\widehat{y}_k(\eta) := y_k(\eta)\widehat{a}_k + \overline{y_k}(\eta)\widehat{a}_k^\dagger. \tag{35}$$

These modes are mutually independent (i.e. commuting), they afford a decomposition of the quantum field into a collection of harmonic oscillators, and, together with their canonical conjugates

$$\widehat{\pi}_k^{(y)}(\eta) := g_k(\eta)\widehat{a}_k + \overline{g_k}(\eta)\widehat{a}_k^\dagger \tag{36}$$

contain complete information about the field.

As we have to follow the evolution of these modes during inflation, let us collect some formulas for the evolution of their lowest moments: Let $|\Xi\rangle$ be any state in the Fock space of $\hat{y}$. Let us introduce the following constants:

$$d_k = <\widehat{a}_k>_\Xi, \qquad c_k = <\widehat{a}_k^2>_\Xi, \qquad e_k = <\widehat{a}_k^\dagger\widehat{a}_k>_\Xi . \tag{37}$$

In terms of these, the expectation values of the modes are expressible as

$$<\widehat{y}_k>_\Xi = 2Re(y_k d_k) \qquad , <\widehat{\pi}_k^{(y)}>_\Xi = 2Re(g_k d_k) \tag{38}$$

while their corresponding dispersions are

$$(\Delta\widehat{y}_k)^2_\Xi = 2Re(y_k^2 c_k) + |y_k|^2(\hbar L^3 + 2e_k) - 4Re(y_k d_k)^2 \tag{39}$$

and

$$(\Delta\widehat{\pi}_k)^2_\Xi = 2Re(g_k^2 c_k) + |g_k|^2(\hbar L^3 + 2e_k) - 4Re(g_k d_k)^2 \tag{40}$$

For the vacuum state $|0\rangle$ we certainly have $d_k = c_k = e_k = 0$, and thus

$$<\widehat{y}_k>_0 = 0, \qquad <\widehat{\pi}_k^{(y)}>_0 = 0, \tag{41}$$

while their corresponding dispersions are

$$(\Delta\widehat{y}_k)^2_0 = |y_k|^2(\hbar L^3), \qquad (\Delta\widehat{\pi}_k)^2_0 = |g_k|^2(\hbar L^3). \tag{42}$$

The collapse

What we have to describe is the state $|\Theta\rangle$ after the collapse. At this point we will not specify the state completely, but only the expectation values

$$d_k^c = < \hat{a}_k >_\Theta, \qquad c_k^c = < \hat{a}_k^2 >_\Theta, \qquad e_k^c = < \hat{a}_k^\dagger \hat{a}_k >_\Theta . \tag{43}$$

At this point a few remarks on our statistical treatment are in order.

We view the collapsed state of the field corresponding to our universe to be a single state $|\Theta\rangle$ and not in any way an ensemble of states.

The way statistics enters our picture is related to fact that we do not measure directly and separately the modes with specific values of k, but rather an aggregate contribution of all such modes to the spherical harmonic decomposition of the temperature fluctuations on the celestial sphere. In order to proceed we we construct an imaginary ensemble of universes. Thus we have an ensemble of universes characterized by the after-collapse state $|\Theta\rangle_i$ where the label i identifies the specific element in the ensemble. Then we will have an independent random series of numbers $q_k^{(i)}$ pertaining to value of physical quantities in the collapsed state in each element i in the ensemble for every single $\vec{k}$ (we will be assuming there are no correlations among the various harmonic oscillators). Our universe however, corresponds to a single element i_0 in the emnsemble, so for each $\vec{k}$ we have a number $q_k^{(i_0)}$. The point is then, that sequence $q_k^{(i_0)}$ for fixed i_0 but for the full set of $\vec{k}$ will, as a result, also a random sequence.

In our specific calculation this approach will be taken with the quantities , d_k^c : In the vacuum state, $\hat{y}_k$ and $\hat{\pi}_k^{(y)}$ individually are distributed according to Gaussian distributions centered at 0 with spread $(\Delta\hat{y}_k)_0^2$ and $(\Delta\hat{\pi}_k^{(y)})_0^2$ respectively. However, since they are mutually non-commuting, their distributions are certainly not independent. In our collapse model, we do not want to distinguish one over the other, so we will ignore the non-conmutativity and make the following assumption about the (distribution of) state(s) $|\Theta\rangle$ after collapse:

$$< \hat{y}_k(\eta_k^c) >_\Theta = X_k', \qquad < \hat{\pi}_k^{(y)}(\eta_k^c) >_\Theta = X_k'' \tag{44}$$

where X', X'' are random variables, distributed according to a Gaussian distribution centered at zero with spread $(\Delta\hat{y}_k)_0^2$, $(\Delta\hat{\pi}_k^{(y)})_0^2$, respectively. Another way to express this is

$$< \hat{y}_k(\eta_k^c) >_\Theta \; = \; x_k' \sqrt{(\Delta\hat{y}_k)_0^2} = x_k' |y_k(\eta_k^c)| \sqrt{\hbar L^3}, \tag{45}$$

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$$< \hat{\pi}_k^{(y)}(\eta_k^c) >_\Theta \; = \; x_k'' \sqrt{(\Delta \hat{\pi}_k^{(y)})_0^2} = x_k'' |g_k(\eta_k^c)| \sqrt{\hbar L^3}, \tag{46}$$

where x_k', x_k'' are now distributed according to a Gaussian distribution centered at zero with spread one.

We now take these equations and solve for d_k^c. Defining the angles α, β, γ as $\alpha_k = \arg(d_k), \beta_k = \arg(y_k)$, $\gamma_k = \arg(g_k)$, the above equations can be written

$$|d_k^c| \cos(\alpha_k + \beta_k) = \frac{1}{2} x_k' \sqrt{\hbar L^3}, \qquad |d_k^c| \cos(\alpha_k + \gamma_k) = \frac{1}{2} x_k'' \sqrt{\hbar L^3}. \tag{47}$$

While we could solve these equations in full generality, let us make the simplifying assumption that the collapse happens early, i.e. that $|k\eta_k^c|$ is large. Under this assumption we find that $\gamma_k \approx \beta_k + \pi/2$ whence we get

$$|d_k^c| \approx \frac{1}{2} \sqrt{\hbar L^3} x_k, \qquad \tan(\alpha) \approx \frac{x_k''}{x_k'}. \tag{48}$$

where $x_k = \sqrt{x_k'^2 + x_k''^2}$. Using the distribution of x_k' and x_k'', we find the uniform distribution for γ_k and for x_k a one-sided Gaussian distribution with spread one.

We need to concentrate on the expectation value of the quantum operator $\delta\dot{\phi}$ which appears in our basic formula

$$\nabla^2 \Psi + m^2 \Psi = s\Gamma \tag{49}$$

(where we introduced the abbreviations $m^2 = 4\pi G \dot{\phi}_0^2$ and $s = 4\pi G \dot{\phi}_0$) and $\Gamma = \delta\dot{\phi} + (\frac{\dot{a}}{a} - \frac{\ddot{\phi}_0}{\dot{\phi}_0})\delta\phi$, while in the slow roll approximation we have $\Gamma = \delta\dot{\phi} = a^{-1}\pi^y$. We want to say that, upon quantization, the above equation turns into

$$\nabla^2 \Psi + m^2 \Psi = s\langle \hat{\Gamma} \rangle. \tag{50}$$

Before the collapse occurs, the expectation value on the right hand side is zero. Let us now determine what happens after the collapse: To this end, take the Fourier transform of (50) and rewrite it as

$$\Psi_k(\eta) = \frac{s}{k^2 - m^2} \langle \hat{\Gamma}_k \rangle_\Theta. \tag{51}$$

Its most likely value is 0 !

Let us focus now on the slow roll approximation and compute the right hand side, we note that $\delta\dot{\phi} = a^{-1}\hat{\pi}^{(y)}$ and hence

$$\delta\dot{\phi}_k = \frac{1}{a}(g_k \hat{a}_k + \overline{g_k}\hat{a}^{\dagger}_{-k}). \tag{52}$$

For the expectation value we find

$$\begin{aligned} \langle \Gamma_k \rangle_\Theta &= \frac{1}{a}\left(d_k g_k + \overline{d}_{-k}\overline{g}_k\right) = i\sqrt{\frac{\hbar L^3 k}{2}}\frac{1}{a}F(k) && (53)\\ &= i\sqrt{\frac{\hbar L^3 k}{2}}\frac{1}{a}\left(x_k e^{i(\alpha_k+\gamma_k)} - x_{-k}e^{-i(\alpha_{-k}+\gamma_k)}\right) && (54)\\ & && (55) \end{aligned}$$

6 Recovering the observational quantities.

Here a crucial observation is to recognize the fact that we can not measure Ψ_k for each individual value of k. What we measure in fact is the "Newtonian potential" on the surface of last scattering: $\Psi(\eta_D, \vec{x}_D)$ which is a function of the coordinates on the celestial two-sphere, i.e a function of two angles. From this we extract

$$\alpha_{lm} = \int \Psi(\eta_D, \vec{x}_D) Y_{lm} d^2\Omega \tag{56}$$

In fact the quantity that is measured is $\frac{\Delta T}{T}(\theta, \varphi)$ which is expressed as $\Sigma_{lm} a_{lm} Y_{l,m}(\theta, \varphi)$.

The angular variations of the the temperature is then identify with the corresponding variations in the "Newtonian Potential" Ψ,

Thus we identify the theoretical expectation a_{lm} with the observed quantity α_{lm} The quantity that is presented as the result of observations is $OB_l = l(l+1)C_l$ where $C_l = (2l+1)^{-1}\Sigma_m |a_{lm}|^2$. The observations indicate that the quantity OB_l is independent of l and this is interpreted as a reflection of the "scale invariance" of the primordial spectrum of fluctuations.

To evaluate the quantity of interest we write first

Justification for the use of statistics

$$\Psi(\eta, \vec{x}) = \frac{is}{k^2 - m^2}\sqrt{\frac{\hbar k}{2L^3}}\frac{1}{a}\Sigma_{\vec{k}} F(\vec{k}) e^{i\vec{k}\cdot\vec{x}}, \tag{57}$$

Random walk in 2 dimensions

Thus using $\vec{x} = R_D(\sin(\theta)\sin(\varphi), \sin(\theta)\cos(\varphi), \cos(\theta))$ and standard results we obtain

$$\boxed{\begin{aligned} a_{lm} &= is\sqrt{\frac{\hbar}{2L^3}}\frac{1}{a}\Sigma_{\vec{k}}\frac{\sqrt{k}}{k^2 - m^2}\int F(\vec{k}) e^{i\vec{k}\cdot\vec{x}} Y_{lm}(\theta, \varphi) d^2\Omega && (58) \\ &= is\sqrt{\frac{\hbar}{2L^3}}\frac{1}{a}\Sigma_{\vec{k}}\frac{\sqrt{k}}{k^2 - m^2} F(\vec{k}) 4\pi i^l j_l((|\vec{k}| R_D) Y_{lm}(\hat{k}) && (59) \end{aligned}}$$

where $\hat{k}$ indicates the direction of the vector $\vec{k}$. Now we compute the expected magnitude of this quantity. As a first step

Most likely value.

$$|a_{lm}|^2 = s^2 \frac{4\pi^2\hbar}{L^3}\frac{1}{a^2}\Sigma_{\vec{k},\vec{k}'}\frac{\sqrt{k}}{k^2 - m^2}\frac{\sqrt{k'}}{k'^2 - m^2} F(\vec{k})\overline{F(\vec{k}')} j_l(kR_D) j_l(k'R_D) Y_{lm}(\hat{k}) Y_{lm}(\hat{k}'). \tag{60}$$

It is not 0

Now we simplify by taking the expectation value with respect to the random phases α_k. To do this, note that

$$F(\vec{k})\overline{F(\vec{k'})} \propto (x_k e^{i(\alpha_k+\gamma_k)} - x_{-k}e^{-i(\alpha_{-k}+\gamma_k)})(x_{k'}e^{-i(\alpha_{k'}+\gamma_{k'})} - x_{-k'}e^{i(\alpha_{-k'}+\gamma_{k'})}) \tag{61}$$

The randomness of the phases will make the off diagonal elements to cancel. Next write the sum as an integral by noting that the values of the components of $\vec{k}$ are separated by $\Delta k_i = 2\pi/L$, thus

$$< |a_{lm}|^2 >= (\frac{s4\pi\hbar^{1/2}}{a(L)^{3/2}})^2(L/2\pi)^3\Sigma_{\vec{k}}\frac{k}{(k^2-m^2)^2}j_l^2((|\vec{k}|R_D)|Y_{lm}(\hat{k})|^2(\Delta k_i)^3 \tag{62}$$

$$= (s\hbar^{1/2}/a)^2(2/\pi)\int\frac{k}{(k^2-m^2)^2}j_l^2(|\vec{k}|R_D)|Y_{lm}(\hat{k})|^2d^3k \tag{63}$$

The last expression can be made more useful by changing the variables of integration to $x = kR_D$ leading to

$$< |a_{lm}|^2 >= (s\hbar^{1/2}/a)^2(1/\pi)\int|\frac{1}{|(x^2-(mR_D)^2)}|^2x^3j_l^2(x)dx \tag{64}$$

which in the regime where m is negligible we find:

$$< |a_{lm}|^2 >= (s\hbar^{1/2}/a)^2(1/2\pi)I_1(l) = (s\hbar^{1/2}/a)^2\frac{1}{2l(l+1)}. \tag{65}$$

Now, since this does not depend on m it is clear that the expectation of $C_l = (2l+1)^{-1}\Sigma_m|a_{lm}|^2$ is just $< |a_{lm}|^2 >$ and thus the observational quantity $OB_l = l(l+1)C_l = (1/2)(s\hbar^{1/2}/a)^2$ independent of l and in agreement with the scale invariant spectrum obtained in ordinary treatments and in the observational studies. Now lets look at the predicted value for the observational quantity OB_l. Using the equation of motion for the scalar field in the backgroound in the slow roll approximation we have $\dot{\phi} = -\frac{a^3}{3\dot{a}}V'$ where $V' = \frac{\partial V}{\partial\phi}$, and the first of Einstein's equations, in the background which gives $3(\dot{a})^2 = 8\pi Ga^4V(\phi_0)$, we find,

small.

$$OB_l = (\pi/3)G\hbar\frac{(V')^2}{V} = (2\pi/3)\epsilon(V/M_{Pl}^4) \tag{66}$$

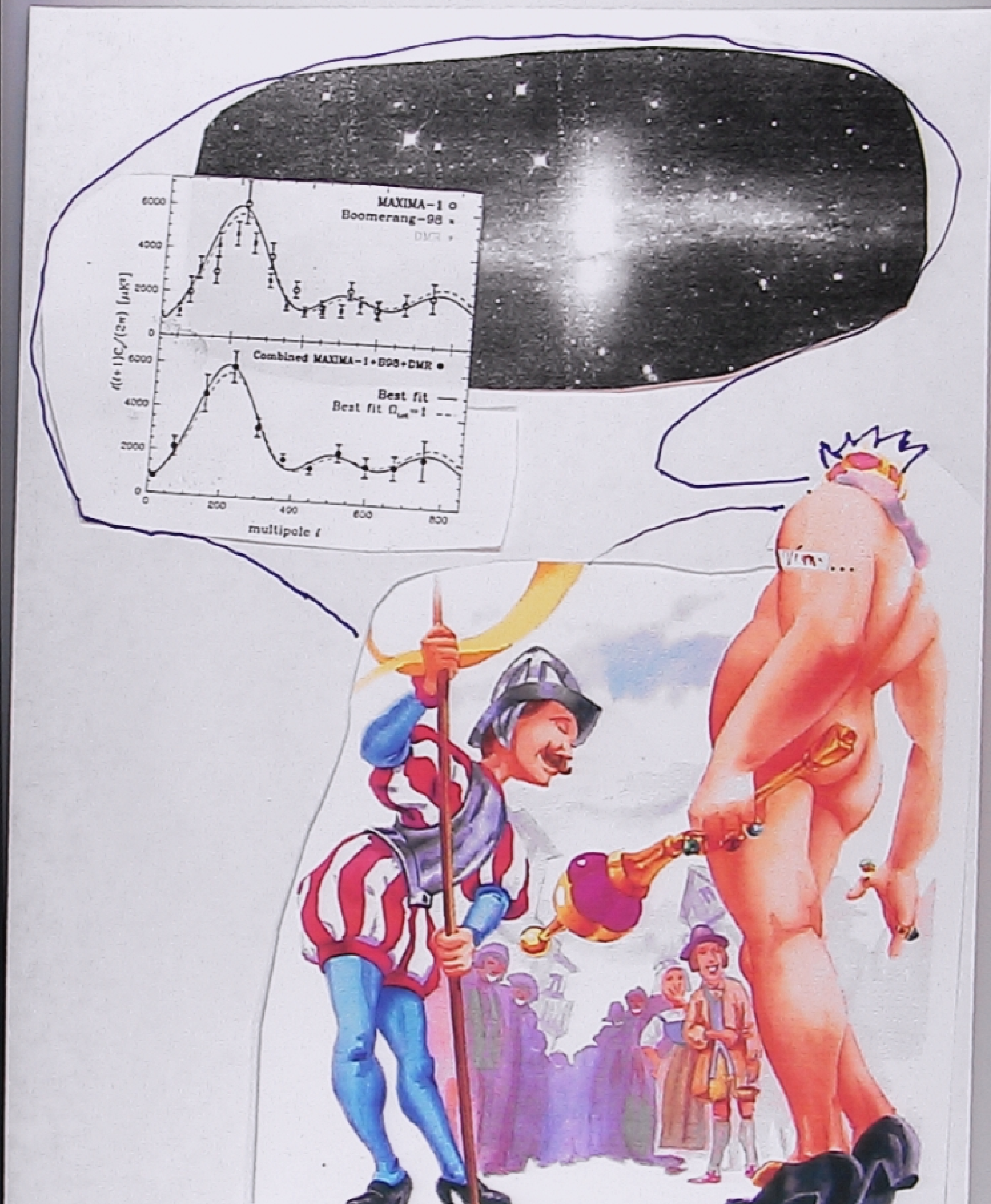
where in the last equality we have used the standard definition of the slow roll parameter $\epsilon = (1/2)M_{Pl}^2(V'/V)^2$, and $G\hbar = M_{Pl}^{-2}$.

Contrast with standard result

Our main point is then, that the standard view regarding the origin of the primordial fluctuations contains a serious nakedness and that instead of ignoring it, we should examine it carefully as it might reveal interesting things indeed:

- Mechanism of colapse   QG?
- effective size of the "universe"
- Details of inflation and/or the "initial state"
- Other things. ?

By the way:
- what transplanckian problem?
- what frame do we have in mind?


MAXIMA-1
Boomerang-98
DMR
Combined MAXIMA-1+B98+DMR
Best fit
Best fit $\Omega_{tot}=1$
$\ell(\ell+1)C_\ell/(2\pi)$ [μK^2]
multipole ℓ