Title: Synchronization Radiation in Myers-Pospelov Electrodynamics

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Abstract:

PLAN OF THE TALK

- MOTIVATION
- THE MYERS-POSPELOV LORENTZ-VIOLATING (LIV) EFFECTIVE MODEL
- SYNCHROTRON RADIATION: spectral angular distribution, total spectral distribution, polarization parameters,... [J. Schwinger, PRD**75**(1949)1912; J. Schwinger et. al. Ann. Phys. (N.Y.)**96**(1976)303; J. Schwinger et. al, Classical Electrodynamics, Westview Press, 1998.]
- THE HIGH m LIMIT: average degree of circular polarization
- (THE DOMINANT AMPLIFYING FACTOR)
- THE FAR-FIELD APPROXIMATION
- SUMMARY AND OUTLOOK

MOTIVATION

- **Recent interest in quantum gravity as a source of tiny modifications to dynamics in flat space. In particular, modified dispersion relations would arise** [Amelino-Camelia et. al., Nature **393**,(1998)763]

$$\omega^2(k) = k^2 \pm \xi \frac{k^3}{M} : \textit{photons}$$

$$E^2(p) = p^2 + m^2 + \eta_{R,L} \frac{p^3}{M} : \textit{fermions}$$

[Gambini, Pullin (1999); Alfaro, Morales, Urrutia (2000); Thiemann, Salhmann, Winkler (2001); Ellis et. al. (2000); Myers, Pospelov (2003),.....], SMOLIN ET. AL., MAJOR,...

- **Very stringent bounds upon the parameters** ξ, η, Θ

  - **Cosmic ray spectra:** [Alfaro and Palma (2002, 2003)]. Details in Alfaro's talk.

  - **Atomic Physics:** $|\Theta_2 + \Theta_4/2| < 10^{-9}$, [Sudarsky, Urrutia, Vucetich (2002)]

- Very stringent bounds upon the parameters ξ, η, Θ

  - Polarization measurements from astrophysical sources: $\xi < 10^{-4}$, [Gleiser, Kozameh (2001)]; $\xi < 10^{-16}/d_{0.5}$, [Jacobson, Liberati, Mattingly, Stecker (2003)] ¿

  - Synchrotron radiation (SR) from CRAB nebulae: either one of $\eta_{R,L} > -7 \times 10^{-8}$, [Jacobson, Liberati, Mattingly (2003)]. Based on reasonable extrapolation of standard SR to LIV case.

- Measure of linear polarization $\Pi = 80 \pm 20\%$ in GRB021206 ¿ $\Longrightarrow$ SR models for emission [Coburn, Boghs (2003)]. Also SR models for BL Lac objects: Markarian 421,501 with electrons [Konopelko et. al. (2003)] or with protons [Aharonian (2000) ; Mucke, Protheroe (2000)]

  Results challenged by: Rutledge and Fox (2004) and Wigger et. al. (2004)

- Possibility that SR, for other astrophysical objects , would impose constraints upon the photon LIV parameter ξ.

DOES QUANTUM GRAVITY INDUCES MODIFICATIONS TO PARTICLE DYNAMICS (IN FLAT SPACE) ???

PHYSICAL SEMICLASSICAL APPROX.
(LQG: PHYSICAL SEMICLASSICAL STATE)

a) NO CORRECTIONS ARISE : LI

b) CORRECTIONS DO ARISE :

(b-1) LIV : PREFERRED FRAME + PHYSICAL UPPER BOUND IN $|\vec{p}|$ → EXTREME FINE TUNING PROBLEMS (COLLINS, PEREZ SUDARSKY, LU, URRUTIA)

(b-2) LIV : EXTENDED RELATIVITY PRINCIPLE (DSR) (AMELINO-CAMELIA, MAGUEIJO-SMOLIN, ...)

(b-3) SPONTANEOUSLY BROKEN LI.
$\langle V^\mu \rangle \to W^\mu$ ... PREFERRED FRAME, NO UPPER PHYSICAL BOUND ON $|\vec{p}|$ → EFFECTIVE F.T.

(b-4) ONLY ZERO CHARGED PARTICLES RECEIVE CORRECTIONS (ELLIS ET. AL)

⋮ ⋮ ⋮ ⋮

MYERS AND POSPELOV EFFECTIVE THEORIES

(Phys. Rev. Lett. **90**(2004)211601)

- Actions

$$S_{scalar} = \int d^4x \left[\partial_\mu \varphi^* \partial^\mu \varphi - \mu^2 \varphi^* \varphi + i\frac{\eta}{M}\varphi^* \left(V^\nu \partial_\nu\right)^3 \varphi\right],$$

$$\begin{aligned} S_{photon} &= \int d^4x \left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - 4\pi J^\mu A_\mu \right. \\ &\left. + \frac{\xi}{M}\left(V^\alpha F_{\alpha\delta}\right)\left(V^\nu \partial_\nu\right)\left(V_\beta \tilde{F}^{\beta\delta}\right)\right]. \end{aligned}$$

- Work in coordinate system where $V^\mu = (1, \vec{0})$.

- Maxwell's equations

$$\nabla \cdot \mathbf{E} = 4\pi\rho,$$

$$-\frac{\partial \mathbf{E}}{\partial t} + \nabla \times \mathbf{B} + \frac{\xi}{M}\frac{\partial}{\partial t}\left(-\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t}\right) = 4\pi \mathbf{J}.$$

- Standard definitions

$$\beta = \frac{\mathbf{v}}{c}, \quad c = 1, \quad \gamma^2 = \frac{1}{1-\beta^2}$$

- Particle in constant magnetic field ($\mathbf{v} \perp \mathbf{B}$)
  - From the modified dispersion relations indentify $E = H$

$$\ddot{\mathbf{r}} = \frac{q}{E}\left(1 - \frac{3}{2}\frac{\eta}{M}E + \frac{9}{4}E^2\left(\frac{\eta}{M}\right)^2\right)(\mathbf{v}\times\mathbf{B})\,.$$

  - Modified Larmor frequency

$$\omega_0 = \frac{|q|B}{E}\left(1 - \frac{3}{2}\frac{\eta}{M}E + \frac{9}{4}E^2\left(\frac{\eta}{M}\right)^2\right), \quad R = \frac{\beta}{\omega_0}$$

  - Modified relation $\beta = \beta(E)$

$$1-\beta^2 = \frac{\mu^2}{E^2}\left[1 + 2\frac{\kappa E^3}{\mu^2} - \frac{15}{4}\frac{\kappa^2 E^4}{\mu^2} + O(\kappa^3)\right].$$

$\kappa = \frac{\eta}{M}$

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MP ELECTRODYNAMICS

- Energy-momentum tensor

$$T^0_0 = \frac{1}{4\pi}\left(\frac{1}{2}(\mathbf{E}^2+\mathbf{B}^2) - \frac{\xi}{M}\mathbf{E}\cdot\frac{\partial \mathbf{B}}{\partial t}\right),$$

$$\mathbf{S} = \frac{1}{4\pi}\left(\mathbf{E}\times\mathbf{B} - \frac{\xi}{M}\mathbf{E}\times\frac{\partial \mathbf{E}}{\partial t}\right).$$

- Work with usual potentials in the standard radiation gauge
- Equation for $\vec{A}$

$$\left(-\omega^2 + k^2 - 2i\frac{\xi}{M}\omega^2\,\mathbf{k}\times\right)\mathbf{A}(\mathbf{k},\omega) = 4\pi\,\mathbf{J}_T(\mathbf{k},\omega).$$

- Can be diagonalized in the circular polarizacion basis (birrefringence)

$$\left(-\omega^2 + k^2 \pm 2\frac{\xi}{M}\omega^2 k\right)\mathbf{A}^{\pm} = 4\pi\mathbf{J}^{\pm}_T.$$

- Each mode propagates with velocity ($c = 1$)

$$v_\lambda = \frac{1}{n(\lambda z)}, \quad \lambda = \pm, \quad z = \frac{\xi}{M}\omega, \quad n(\lambda z) = \sqrt{1+z^2} + \lambda z$$

- We call $\tilde{\xi} = \xi/M$, $\tilde{\eta} = \eta/M$ in the sequel

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SYNCHROTRON RADIATION

- General power spectrum

$$\frac{d^2P(T)}{d\omega d\Omega} = \frac{1}{4\pi^2}\frac{\omega^2}{\sqrt{1+z^2}}\int_{-\infty}^{\infty} d\tau\, e^{-i\omega\tau}\sum_{\lambda=\pm}$$

$$\times\left[n^2(\lambda z)J_i^*\left(T+\tau/2,\mathbf{k}_\lambda\right)\; P_{ik}^\lambda\; J_k\left(T-\tau/2,\mathbf{k}_\lambda\right)\right],$$

$$P_{ik}^\lambda = \frac{1}{2}\left(\delta_{ik}-\hat{k}_i\hat{k}_k+\lambda i\epsilon_{ijk}\hat{k}_j\right).$$

- Circular orbit

$$J_k(t,\mathbf{k}) = q\mathbf{v}(t)e^{-i\mathbf{k}\cdot\mathbf{r}(t)}, \quad \mathbf{v}(t) = (-\beta\sin\omega_0 t,\ \beta\cos\omega_0 t,\ 0)$$

- Averaged angular distribution of the m^{th} harmonic

$$\left\langle\frac{d^2P(T)}{d\omega d\Omega}\right\rangle_T = \sum_{\lambda=\pm}\sum_{m=0}^{\infty}\delta(\omega-m\omega_0)\frac{dP_{m,\lambda}}{d\Omega},$$

$$\frac{dP_{m,\lambda}}{d\Omega} = \frac{\omega^2q^2}{4\pi}\frac{1}{\sqrt{1+z_m^2}}\left[\lambda\beta n(\lambda z_m)J'_m(W_{\lambda m})+\cot\theta\, J_m(W_{\lambda m})\right]^2$$

$$W_{\lambda m} = mn(\lambda z_m)\beta\sin\theta, \qquad z_m = \bar{\xi}m\omega_0$$

- Integrated power in the m^{th} harmonic

$$P_{m,\lambda} = \frac{q^2\, m\omega_0^2}{2\sqrt{1+z_m^2}}\beta n(\lambda z_m)\left[2J'_{2m}(2m\,\beta n(\lambda z_m))\right.$$

$$\left.-\left[\frac{1}{[\beta n(\lambda z_m)]^2}-1\right]\int_0^{2m\,\beta n(\lambda z_m)} dx\, J_{2m}(x)\right].$$

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DATA OF SOME RELEVANT OBJECTS

$OBJECT$	$r(l.y)$	γ	$B(Gauss)$	$\omega_{obs}(GeV)$	$\omega_0(GeV)$	m	m/γ
$CRAB$	10^4	10^9	10^{-3}	10^{-1}	10^{-30}	10^{29}	10^{20}
$(MARKARIAN)_e$	10^8	10^{11}	10^2	10^4	10^{-26}	10^{30}	10^{19}

$OBJECT$	$\frac{r'}{r}$	$\tilde{\xi}\omega$	$\tilde{\xi}\omega\frac{r'}{r}$	$\left(\frac{r'}{r}\right)^2$
$CRAB$	10^{-6}	$10^{-20}\xi$	$10^{-26}\xi$	10^{-12}
$(MARKARIAN)_e$	10^{-14}	$10^{-15}\xi$	$10^{-29}\xi$	10^{-28}

HIGH m EXPANSIONS

- We are in the regime $1 - [\beta n]^2 > 0$
- Integrated power in the m^{th} harmonic

$$P_{\lambda m} = \frac{q^2 m \omega_0}{\sqrt{3}\pi R} \frac{1}{1 + n^2(\lambda z_m)} \left\{ \int_{m/\bar{m}_c}^{\infty} dx \left(\frac{3}{2\bar{m}_c}\right)^{2/3} K_{5/3}(x) - 2\left(\frac{3}{2\bar{m}_c}\right)^{4/3} K_{2/3}\left(\frac{m}{\bar{m}_c}\right) \right\}.$$

- The cut-off frequency

$$\bar{m}_c = \frac{3}{2}\left(1 - [\beta n(\lambda z_m)]^2\right)^{-3/2}$$

because for $m > \bar{m}_c$

$$P_{\lambda m} \approx e^{-m/\bar{m}_c}$$

- Integrated total power in the m^{th} harmonic to second order in $\bar{\xi}$

$$P_m = \frac{q^2 m \omega_0}{\sqrt{3}\pi R \gamma^2} \left\{ \frac{m_c}{m} \kappa \left(\frac{m}{m_c}\right) - \frac{2}{\gamma^2} K_{2/3}\left(\frac{m}{m_c}\right) + 2\bar{\xi}^2 (m\omega_0\beta)^2 \left[\left(\frac{m}{\gamma}\right)^2 - \frac{1}{2}\right] K_{2/3}\left(\frac{m}{m_c}\right) \right\},$$

$$m_c = \frac{3}{2}\gamma^3.$$

$\left(\xi \omega \frac{m}{\gamma}\right)^2$

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AVERAGED DEGREE OF CIRCULAR POLARIZATION

- Let us assume that the relativistic electrons have an energy distribution of the type

$$N(E)dE = CE^{-p}dE, \quad 2 < p < 3$$

- Let us define the circular degree of polarization as

$$\Pi_{\odot} = \frac{\langle P_+(\omega) - P_-(\omega)\rangle}{\langle P_+(\omega) + P_-(\omega)\rangle}$$

where $P_{\pm}(\omega)$ is the total power distribution per unit frequency and polarization $\lambda = \pm 1$, so that

$$P_{\lambda}(\omega) = \frac{P_{m\lambda}}{\omega_0}$$

- The leading order result is

$$\Pi_{\odot} = \bar{\xi}\omega\left(\frac{\mu\omega}{qB}\right)\Pi(p), \quad p > 7/3.$$

$$\Pi(p) = \frac{(p-3)(3p-1)}{3(3p-7)}\frac{(p+1)}{(p-1)}\frac{\Gamma\left(\frac{p}{4}+\frac{13}{12}\right)\Gamma\left(\frac{p}{4}+\frac{5}{12}\right)}{\Gamma\left(\frac{p}{4}+\frac{19}{12}\right)\Gamma\left(\frac{p}{4}+\frac{11}{12}\right)}$$

- This is the analogous expression for the average of the degree of linear polarization

$$\Pi_{LIN} = \frac{p+1}{p+7/3}$$

THE FAR-FIELD APPROXIMATION

- The phase in the Green function is

$$n(\lambda z)\omega\left|\mathbf{r}-\mathbf{r}'\right| \simeq \omega r\left(1-\frac{\mathbf{n}\cdot\mathbf{r}'}{r}+\lambda\bar{\xi}\omega-\lambda\bar{\xi}\omega\frac{\mathbf{n}\cdot\mathbf{r}'}{r}+\frac{1}{2}\frac{r'^2}{r^2}\right) \tag{13}$$

- If

$$\left|\bar{\xi}\omega\right|\frac{r'}{r} > \left(\frac{r'}{r}\right)^2$$

we can neglect only the term quadratic in $r'$and both $\bar{\xi}$-dependent terms remains in the phase:

$$n(\lambda z)\omega\left|\mathbf{r}-\mathbf{r}'\right| \simeq n(\lambda z)\omega(r-\hat{\mathbf{n}}\cdot\mathbf{r}')$$

- Other possibility is that

$$\left(\frac{r'}{r}\right)^2 < \left|\bar{\xi}\omega\right| < \frac{r'}{r}$$

which leads to

$$n(\lambda z)\omega\left|\mathbf{r}-\mathbf{r}'\right| \simeq n(\lambda z)\omega r-\hat{\mathbf{n}}\cdot\mathbf{r}'$$

- Finally, if

$$\left|\bar{\xi}\omega\right| < \left(\frac{r'}{r}\right)^2 \tag{14}$$

all the dependence on $\bar{\xi}$ is negligible in the phase, which reduces to

$$n(\lambda z)\omega\left|\mathbf{r}-\mathbf{r}'\right| \simeq \omega(r-\hat{\mathbf{n}}\cdot\mathbf{r}') \tag{15}$$

This corresponds to the CRAB nebulae case.

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SUMMARY AND OUTLOOK

- EXACT AND COMPLETE DESCRIPTION OF SYNCHROTRON RADIATION (SR) IN THE MYERS-POSPELOV EFFECTIVE MODEL

- The Schwinger et. al. (1976) result for the opening angle $\delta\theta$ of the radiation

$$\delta\theta \approx m^{-1/3} \approx m_c^{-1/3} = \left(1 - [\beta(E)n(z)]^2\right)^{1/2}$$

is recovered.

- IN THE FULL FAR-FIELD APPROXIMATION WE FIND AMPLIFYING FACTORS

$$(\tilde{\xi}\omega)\left(\frac{m}{\gamma}\right), \qquad (\tilde{\xi}\omega)\left(\frac{\mu\omega}{qB}\right)$$

To zeroth-order approximation ($\tilde{\eta} = \tilde{\xi} = 0$) we can write

$$\left(\frac{\mu\omega}{qB}\right) = \frac{m}{\gamma}$$

Similar results are found in non-commutative SR :

Castorina, Iorio and Zappala: PRD**69**(2004)065008.

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• THE CRAB NEBULAE

n(z)=1 in the phase of the radiation fields

CORRECTIONS APPEAR ONLY VIA $\beta(E)$, $\tilde{\eta}$. IN THIS CASE WE HAVE

$$\delta\theta \approx \gamma^{-1}(E), \qquad \omega_c = \frac{eB}{E}\gamma^3(E).$$

Jacobson et al. results recovered

• REPEAT THE ANALYSIS FOR OTHER ASTROPHYSICAL OBJECTS AND LOOK FOR

- Average circular polarization.
- Corrections to average linear polarization.
- Stokes parameters.

• Gambini-Pullin electrodynamics also presents birrefringence, it is non-local in the potentials A_μ and coincides with MP to first order in ξ.

• Ellis et. al electrodynamics is not birrefringent and the associated SR is currently under investigation.

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