

Title: Entropy and Entanglement for LQG Black Holes

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Abstract:

# Entropy and entanglement on the horizon

Etera Livine and  
Danny Terno



# Entropy and entanglement on the horizon

Etera Livine and  
Danny Terno



# Outline

• ~~Wren Hollow~~  
• ~~Lower Middle & other~~  
• ~~Mar. Mt Boys~~  
• ~~Big Valley~~  
• ~~Virginia Hickories & Mad  
 Sulphur~~  
• ~~3rd & 4th floor Plans~~  
• ~~1st floor~~

# Outline

- Ruler argument, renormalization and spin  $\frac{1}{2}$

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- Entropy and entanglement for spin  $\frac{1}{2}$
- Evaporation and entanglement
- Other scales and spin 1 as an example
- (Other) speculations

## General



# General

The probing scale

$$j = \frac{1}{2}, 1, \frac{3}{2}, \dots$$

Area spectrum

$$a_j \propto \begin{cases} \sqrt{j(j+1)} \\ j + \frac{1}{2} \end{cases}$$



# General

The probing scale

$$j = \frac{1}{2}, 1, \frac{3}{2}, \dots$$

We work at fixed  $j$

Reasons:

- Fundamental scale
- The only spins involved?

The flow: scaling and invariance  
of physical quantities

Area spectrum

$$a_j \propto \begin{cases} \sqrt{j(j+1)} \\ j + \frac{1}{2} \end{cases}$$



# Model

Object: static black hole

Requirement: SU(2) invariance of the horizon states

$$J = 0$$

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Density matrix

$$\rho = \frac{1}{N} \sum_{k=1}^N |\Psi_k\rangle \langle \Psi_k| \quad S = -\text{tr} \rho \log \rho = \log N$$

$$N \sim \frac{2^{2n}}{\sqrt{\pi n^3}}$$

$$N = \frac{C_n^{2n}}{n+1}$$

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$$N \sim \frac{2^{2n}}{\sqrt{\pi n^3}}$$

$$S \sim 2n \log 2 - \frac{3}{2} \log n$$

$$N = \frac{C_n^{2n}}{n+1}$$

# Entanglement

From left to right:  
Lawrence Smith & others  
John May, Mt. Boys  
Big Valley  
Tennessee Hickabees & Med  
McKinbarrie  
3rd Author - Blame  
John D. T.

# Entanglement

Why?

- Lawrence Smith + others
- Martini Boys
- Big Valley
- Oklahoma Hickabees + Med
- Hickabees
- Brothers from another planet

# Entanglement

Why?

How to quantif

in valley

2010 Alaska flicker + Med

reliability

3' > Northern Flicker

2010 - 2011

# Entanglement

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How to quantify?

Pure states

$$|\Psi\rangle = \alpha|+-\rangle + \beta|--\rangle$$

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$$|\Psi\rangle = \alpha|+-\rangle + \beta|--\rangle$$

$$|\Phi\rangle \rightarrow \rho_{A,B} \rightarrow S(\rho_*)$$

$$\rho = \begin{pmatrix} |\alpha|^2 & 0 \\ 0 & 1 - |\alpha|^2 \end{pmatrix}$$

# Entanglement

Why?

How to quantify?

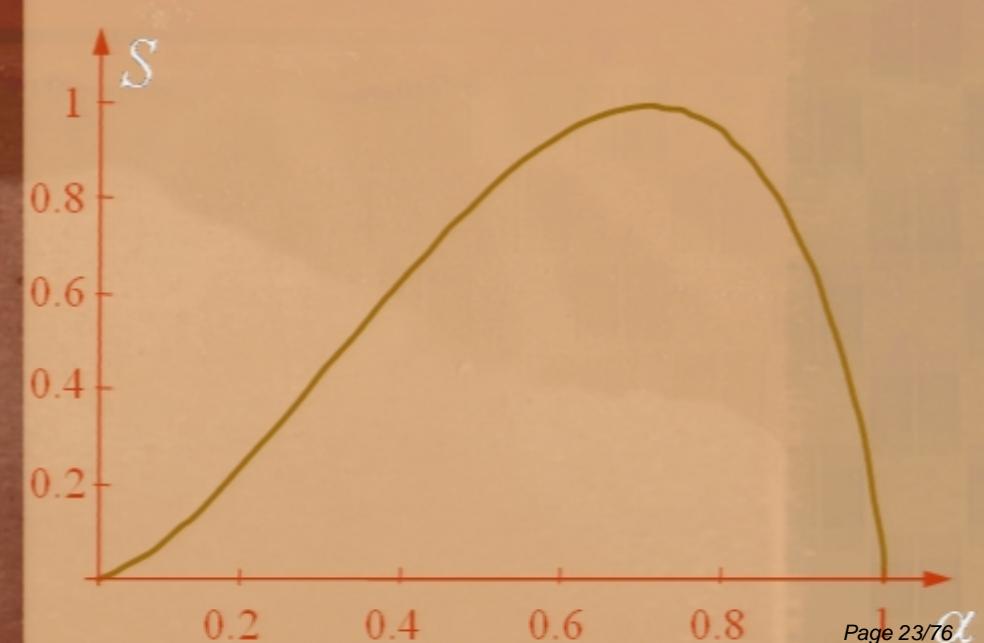
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# Mixed states hierarchy

1. ~~Winnipeg~~  
Lawrence Smith + others  
2. ~~Mar. 2015 Boys~~  
Big Valley  
3. ~~Virginia Huckabee + Med~~  
McCurdy  
3. ~~Northern Plains~~  
~~2015 - 2016~~



# Mixed states hierarchy

Direct product

$$\rho = \rho_A \otimes \rho_B$$

Lawler and other  
Man and Boys  
in valley  
Gorjana Hickabee + Med  
Robbarde  
Northern Plains  
and others



## Mixed states hierarchy

Direct product       $\rho = \rho_A \otimes \rho_B$

Separable       $\rho = \sum_i w_i \rho_A^i \otimes \rho_B^i, \quad \forall w_i > 0, \sum_i w_i = 1$



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## Measures of entanglement

### Entanglement of formation

$$\rho = \sum_i w_i |\Psi_i\rangle\langle\Psi_i|$$

$$S(\{\Psi\}) = \sum_i w_i S(\text{tr}_i |\Psi_i\rangle\langle\Psi_i|)$$

$$S_E \rho = \min_{\{\Psi\}} S(\{\Psi\})$$



## Mixed states hierarchy

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~~$$\rho = \sum_i w_i \rho_A^i \otimes \rho_B^i, \quad \forall w_i > 0, \sum_i w_i = 1$$~~

## Measures of entanglement

### Entanglement of formation

Minimal weighted average  
entanglement of constituents



$$\rho = \sum_i w_i |\Psi_i\rangle\langle\Psi_i|$$

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# BH entanglement

2 vs  $2n-2$

Lawrence Dahl + others

John Hart Bous

Big Valley

Zoë Jaszka Flickaboe + Med

Reinhardt

3D Northern Plans

... and many more



# BH entanglement

2 vs  $2n-2$

States of the minimal decomposition

$$|0,0,a_0\rangle \otimes |0,0,b_0\rangle$$

degeneracy indices

$$\frac{1}{\sqrt{3}}(|1,-1,a_1\rangle \otimes |1,1,b_1\rangle - |1,0,a_1\rangle \otimes |1,0,b_1\rangle + |1,1,a_1\rangle \otimes |1,-1,b_1\rangle)$$



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Unentangled fraction  $f_0 \sim \frac{1}{4}$

Entanglement  $S(\rho|2) \sim \frac{3}{4} \log 3$



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Other block sizes

Entropy can be  
generalized

$$S(\rho | m) \sim \frac{1}{2} \log(3 + 4m)$$

$$n \propto n \quad S_E(\rho) \sim \frac{1}{2} \log n$$

Prairie Falcon  
Burrowing Owl + others  
Marmot Boys  
Big Valley  
Gavia stellata  
Mallard  
Burrowing Owl



$$n \text{ vs } n \quad S_E(\rho) \sim \frac{1}{2} \log n$$

Entropy of the whole vs. sum of its parts

$$S(\rho) \leq S(\rho_A) + S(\rho_B)$$

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BH is not made  
from independent qubits,  
but...

$$S(\rho) \sim 2S(\rho_{\text{half}}) - 3S_E(\rho)$$

# Evaporation

A model for Bekenstein-Mukhanov  
spectroscopy (1995)



• May 2005  
• Valley  
• Gorjanci Hickories + Mad  
• McFarlane  
• Brother Blaw  
• C. D. P. T.

# Evaporation

A model for Bekenstein-Mukhanov  
spectroscopy (1995)



Minimal frequency  $\leq$  fundamental  $j$

7.01.1995a Hickman + Med  
McMurtry  
3.20 Southern Plains  
1000 ft. - 1000 ft.

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$$P_{\Delta t}(k_1 | 1) = 2^{-k_1}$$

$f_0 (= \frac{1}{4})$  unentangled fraction  
(of 2-spin blocks)

$$P_{\Delta t}(2m_1 | 1) \propto (f_0)^{m_1}$$

$m_1$  number of blocks

# Other scales

A question: what features persist at different scales?

- Lawrence Smith + others
- May, Mt. Bays
- Big Valley
- Virginia Huckabee + Med
- Hubbard
- Northern Plains
- ...

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$$S(\rho) \sim 2n_j \log(2j+1) - \frac{3}{2} \log n_j$$

$$\frac{3}{2} \log$$

Explanation: a random walk with a mirror

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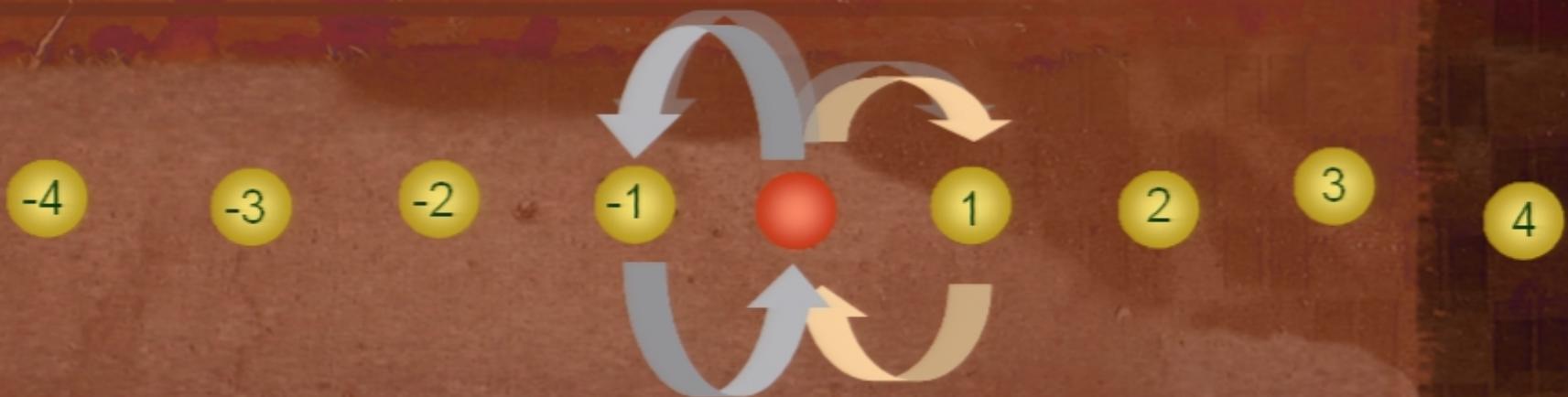
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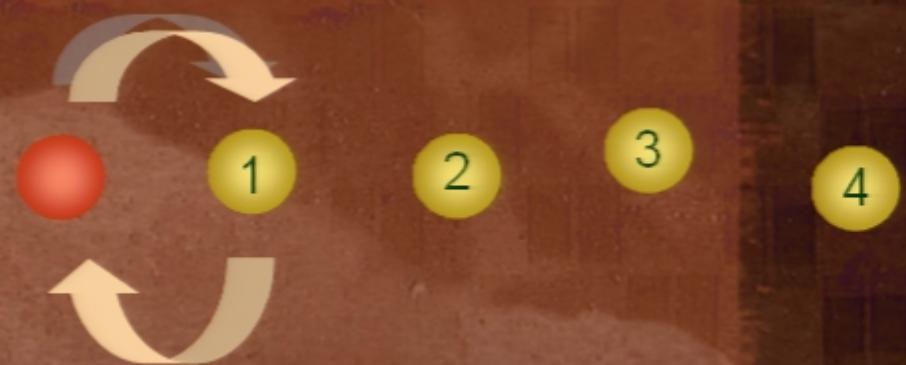
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$$N \sim \frac{(2j+1)^{2n}}{\sqrt{n}}$$



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## Area rescaling

General surface: SU(2) invariance is not required

It is made from

•  $\sqrt{g} \partial_\mu \phi \partial^\mu \phi + \frac{1}{2} g_{\mu\nu} \partial^\mu \phi \partial^\nu \phi$   
•  $\lambda \phi^4$   
•  $\lambda' \phi^2$   
•  $\lambda'' \phi^3$

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General surface: SU(2) invariance is not required

It is made from

**Spin-1/2**

Number of punctures

$N_{1/2}$

Most probable spin

$$J \sim \sqrt{N_{1/2}} / 2$$

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**Spin-1**

Number of punctures

$$N_1$$

Most probable spin

$$J \sim \sqrt{2N_1 / 3}$$

## Spin-J

Number of punctures

v

= number of blocks

$$\frac{N_{1/2}}{N_1} = \frac{8}{3}$$



## Spin-J

Number of punctures

$v$

= number of blocks

$$\frac{N_{1/2}}{N_1} = \left( \frac{a_1}{a_{1/2}} \right)^2$$

$$\frac{1}{2} \otimes \frac{1}{2} = 0 \oplus 1$$
  
$$2 \rightarrow 3/4$$

$$\frac{N_{1/2}}{N_1} = \frac{8}{3}$$

$j=1$   
is the most probable  
 $2 < \text{block size} < 4$



## Spin-J

Number of punctures

v

= number of blocks

$$\frac{N_{1/2}}{N_1} = \frac{8}{3}$$



Lawrence Brink + others  
M. Petrini  
J. Polchinski  
R. van der�  
H. Verlinde  
E. Witten

INTERACTIONAL MODEL

## Spin-J

Number of punctures

$v$

= number of blocks

$$\frac{N_{1/2}}{N_1} = \frac{8}{3}$$



## Numerology

Entanglement

## Spin-1

$$S(\rho) \sim 2S(\rho_{\text{half}}) - 3S_E(\rho)$$

$$S_E(\rho) \sim \frac{1}{2} \log n_1$$

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# Open questions



• What if we open  
Lawrence Smith + others  
• Mar. 2015 Boys  
• Valley  
• Virginia Hückelheim + Med  
• McMurtry  
• 3'2" Southern Blues  
• 2015 - 2016

# Open questions

*Dynamics: evolution of entanglement  
dynamical evolution of evaporation  
"H=0" section & the number of states*



# Open questions



*Dynamics:* evolution of entanglement  
dynamical evolution of evaporation  
" $H=0$ " section & the number of states

*Semi-classicality:* requiring states to represent  
semi-classical BH  
rotating BH

*End of slide show, click to exit.*



Century Gothic

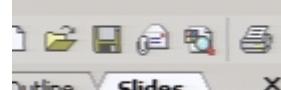
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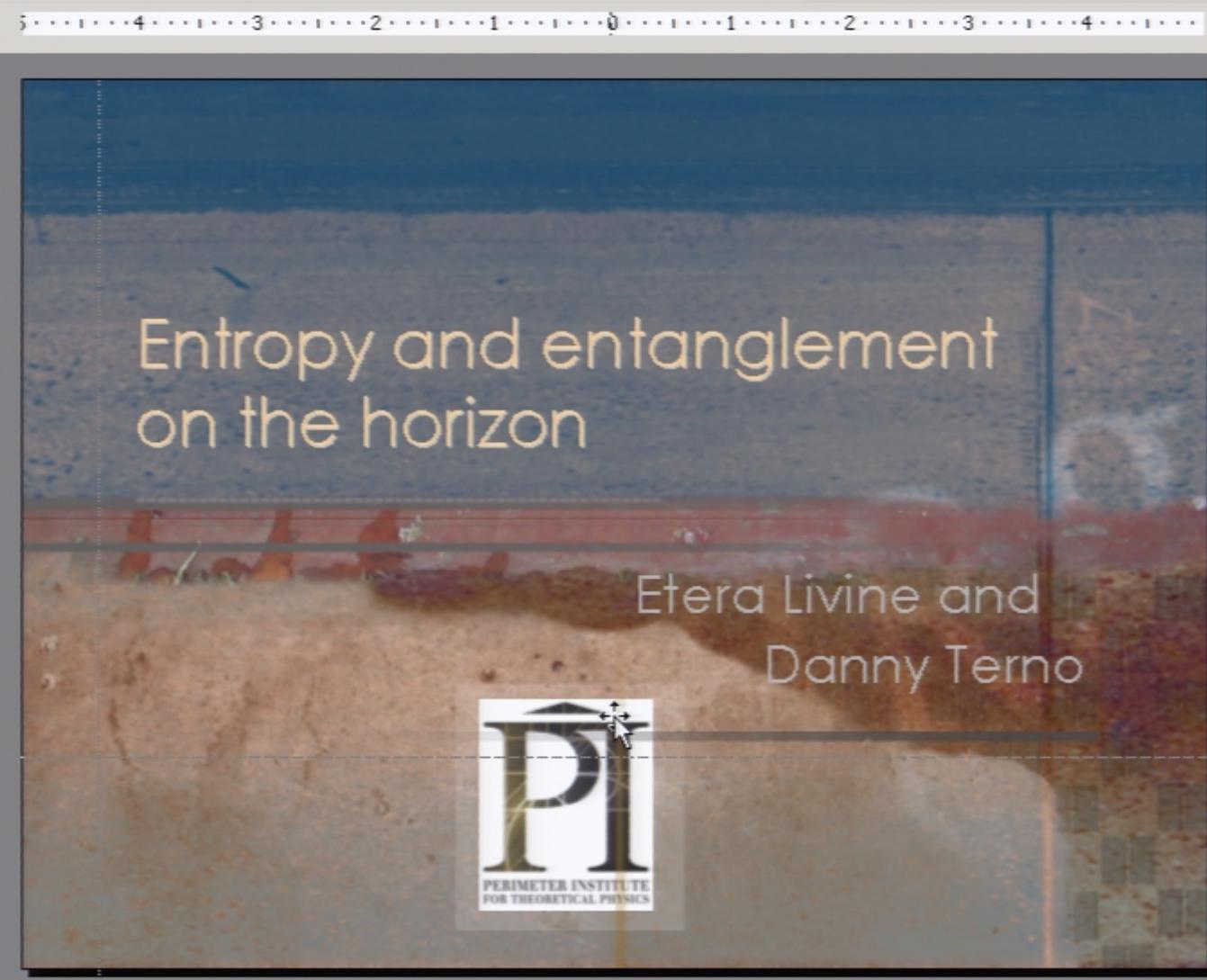
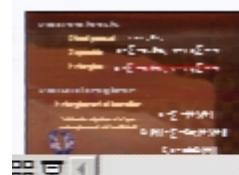
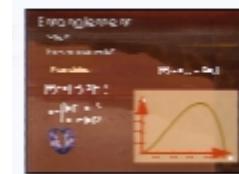
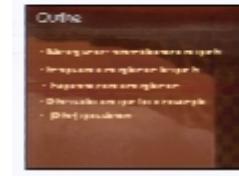
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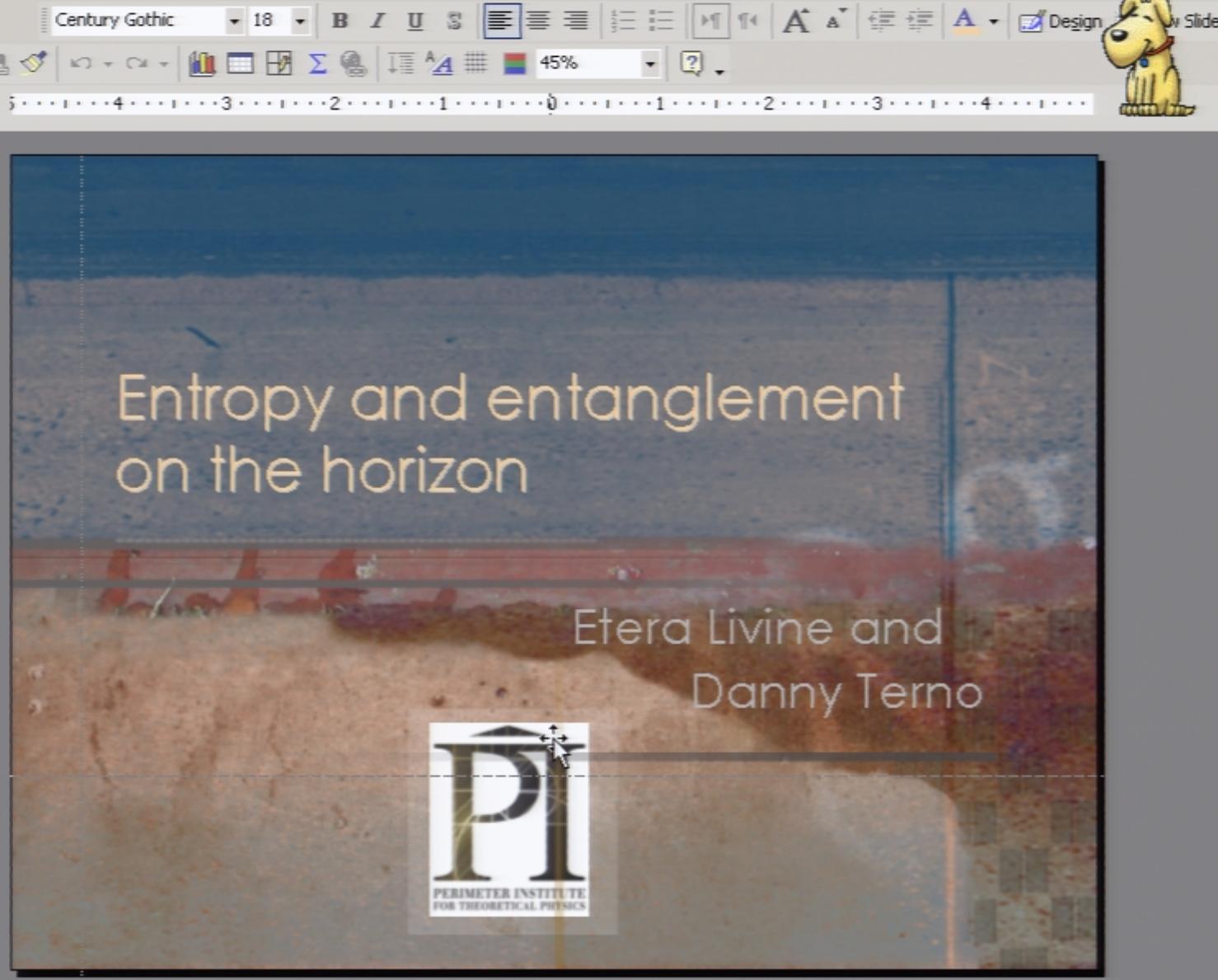
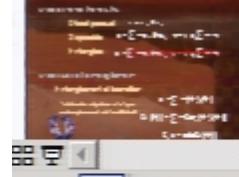
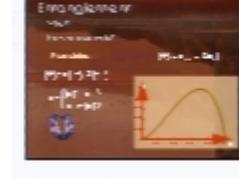
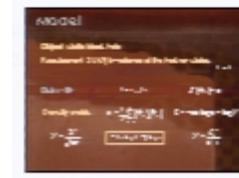
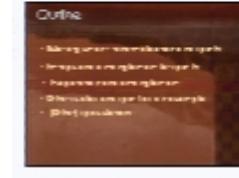
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Outline Slides



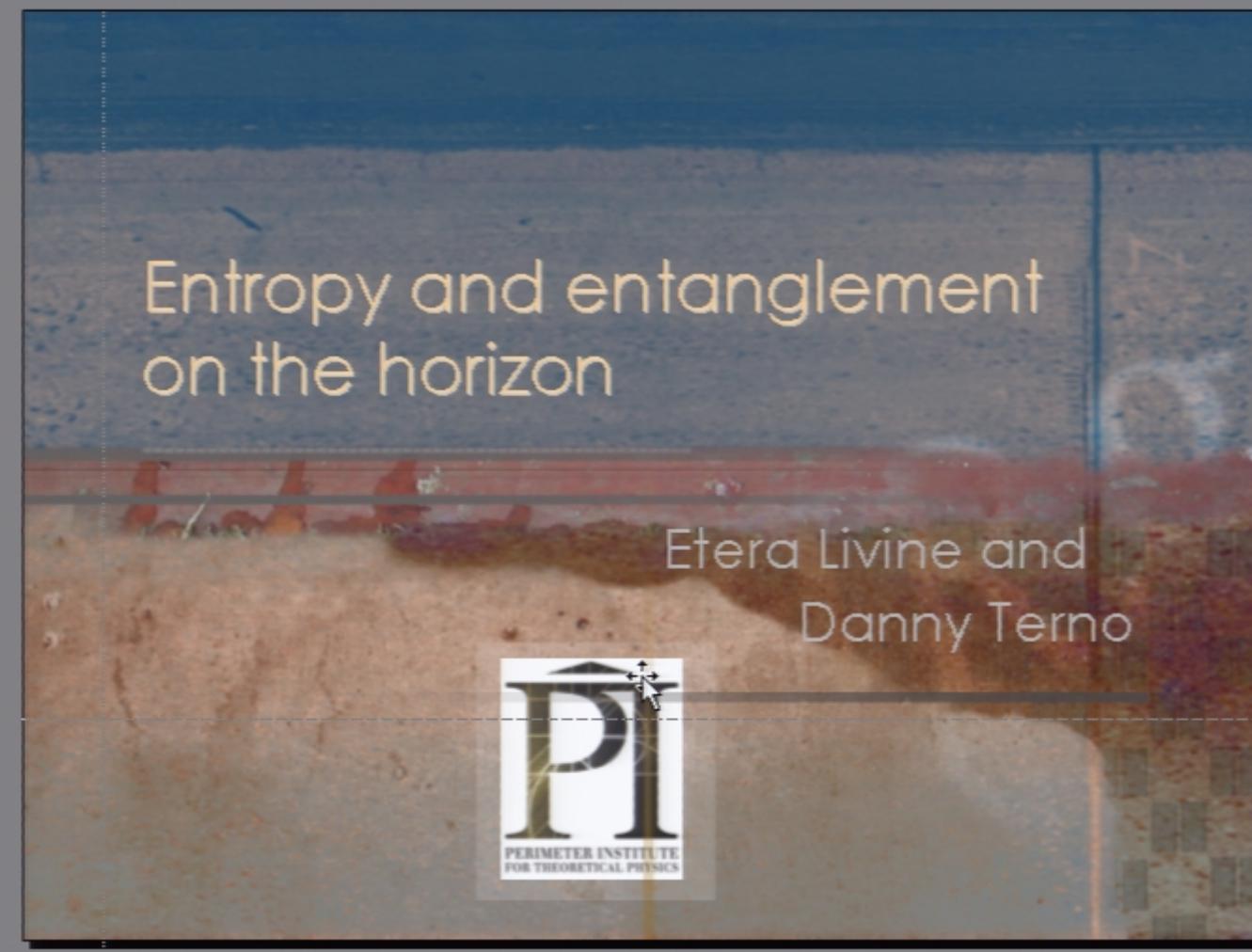
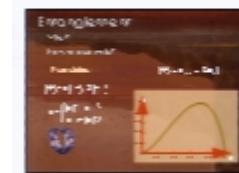
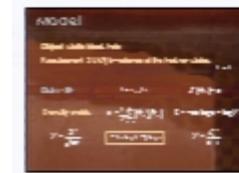
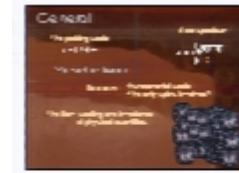
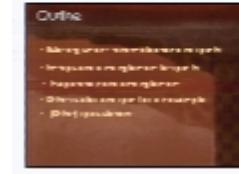
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## Outline Slides



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Design

Outline

Slides



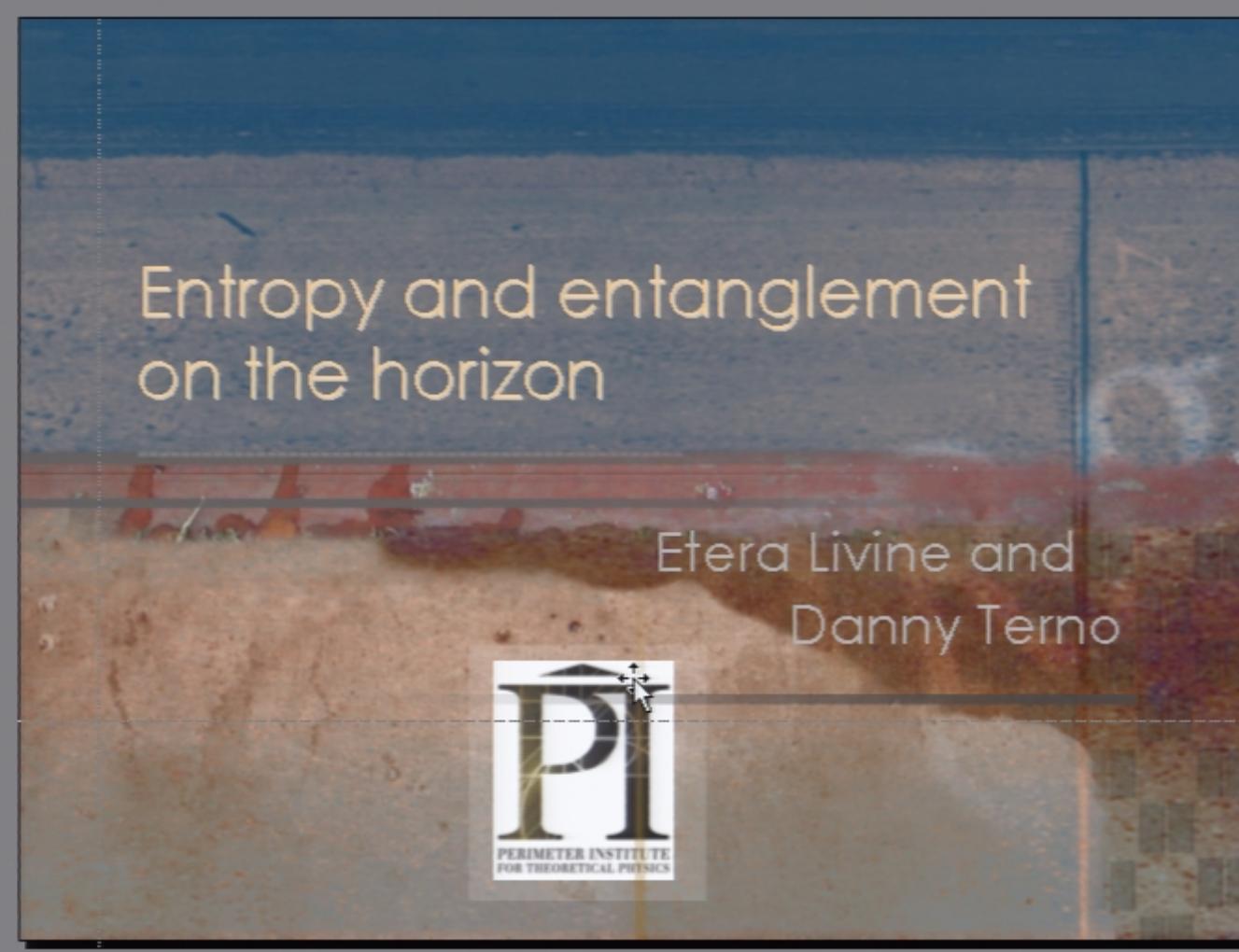
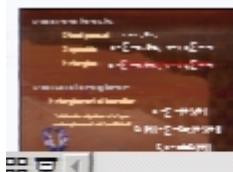
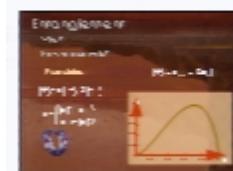
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- Entropy and entanglement on the horizon
- Entanglement length scale
- Properties of entanglement
- Different ways to measure it
- Open problems



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Click to add notes



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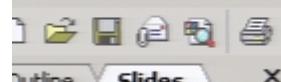
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Design



Slides



Outline

- Entropy and entanglement on the horizon
- Temperature and entropy
- Properties of entanglement
- Other applications of entanglement
- Open problems



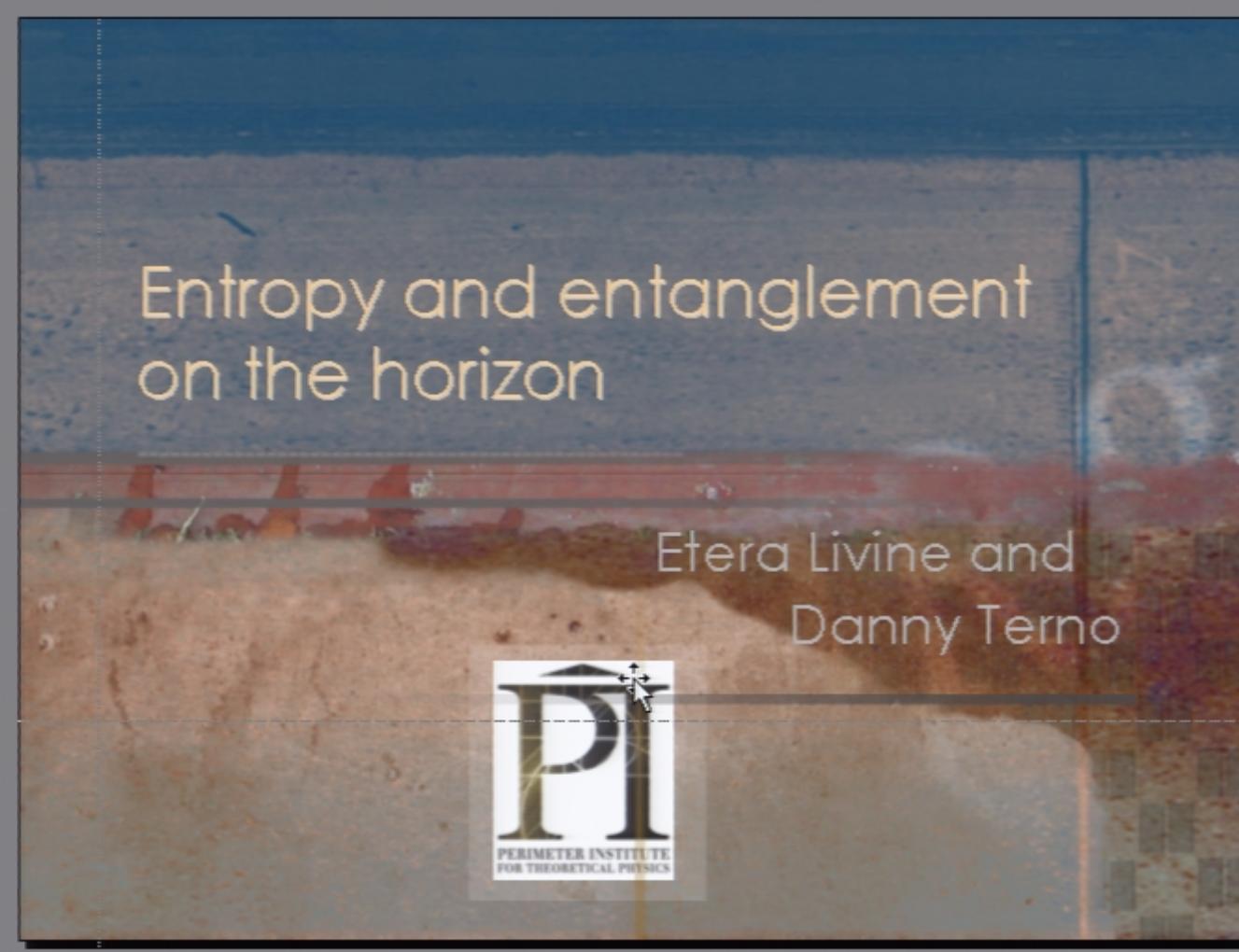
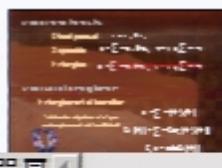
Content

- The entropy of a black hole
- Entanglement entropy
- Properties of entanglement
- Other applications of entanglement
- Open problems



Entropy and entanglement

- Entropy and entanglement
- Properties of entanglement
- Other applications of entanglement
- Open problems



Click to add notes



Outline

Slides



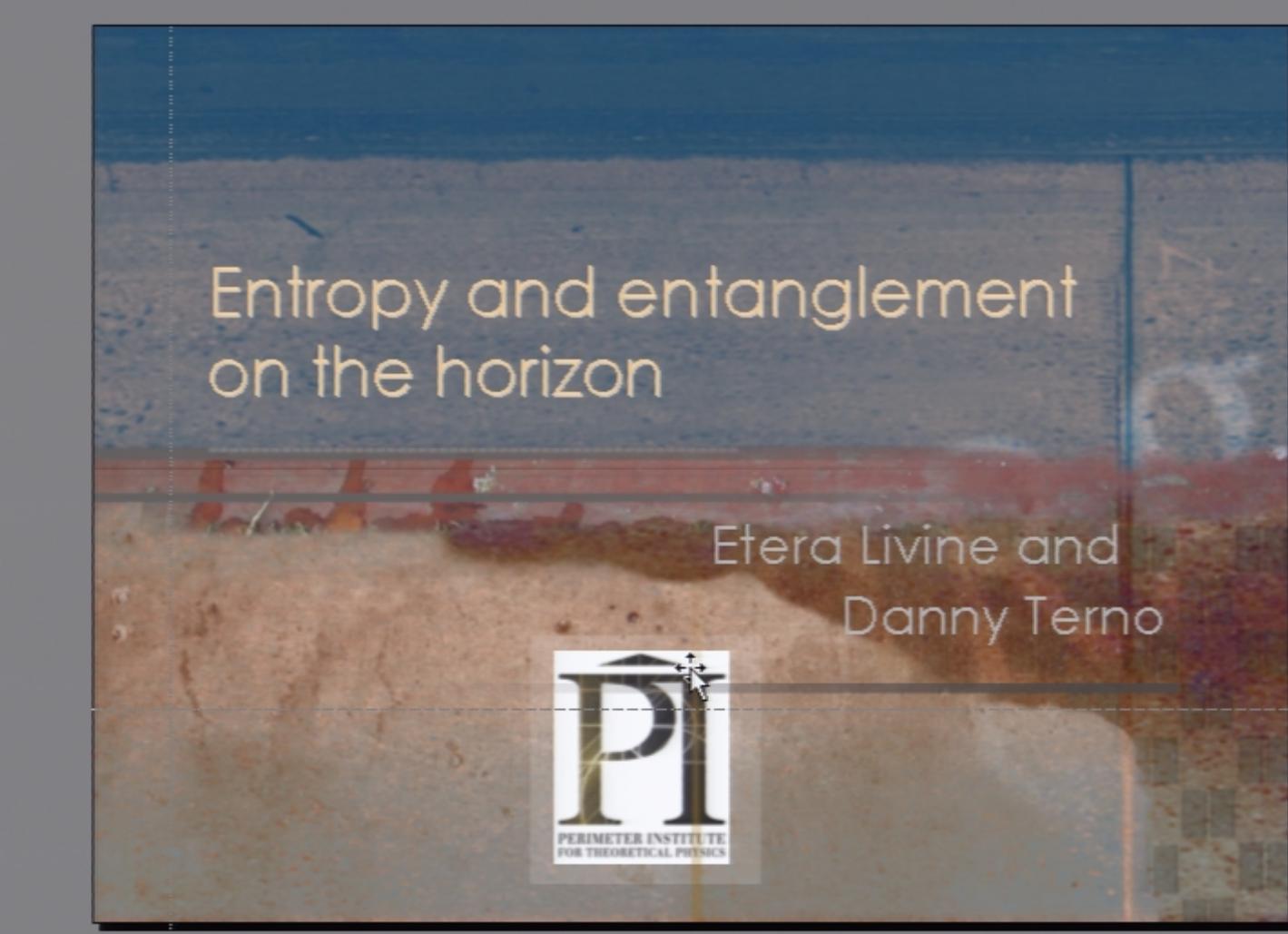
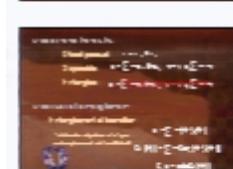
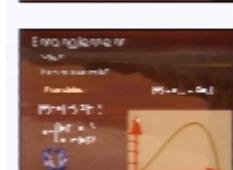
## Outline

- Entropy and entanglement on the horizon
- Entanglement length scale
- Properties of entanglement
- Different entropy functionals
- Other possibilities



## Content

- The entropy scale and the entanglement length scale
- What can we learn from it?
- Entanglement length scale
- Other possibilities



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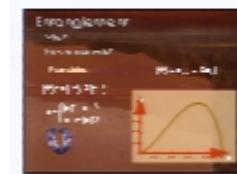
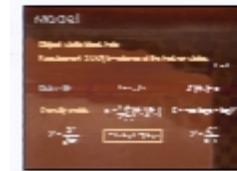
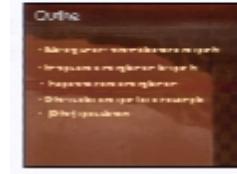


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## Outline Slides



# Entropy and entanglement on the horizon

Etera Livine and  
Danny Terno



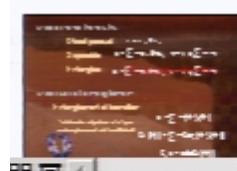
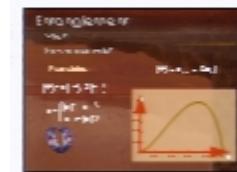
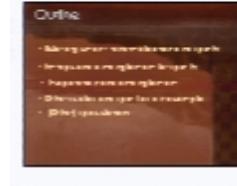
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## Outline Slides



# Entropy and entanglement on the horizon

Etera Livine and  
Danny Terno



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Design New Slide



Outline



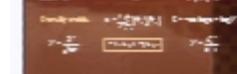
Outline

- Entropy and entanglement on the horizon
- Entanglement length scale
- Properties of entanglement
- Different ways to measure it
- Other examples



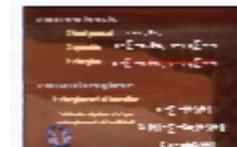
Concept

- The entropy scale is  $\sim 10^{100}$
- The entanglement length scale is  $\sim 10^{-30} \text{ cm}$
- The properties of entanglement are  $\sim 10^{-30} \text{ cm}$
- The different ways to measure it

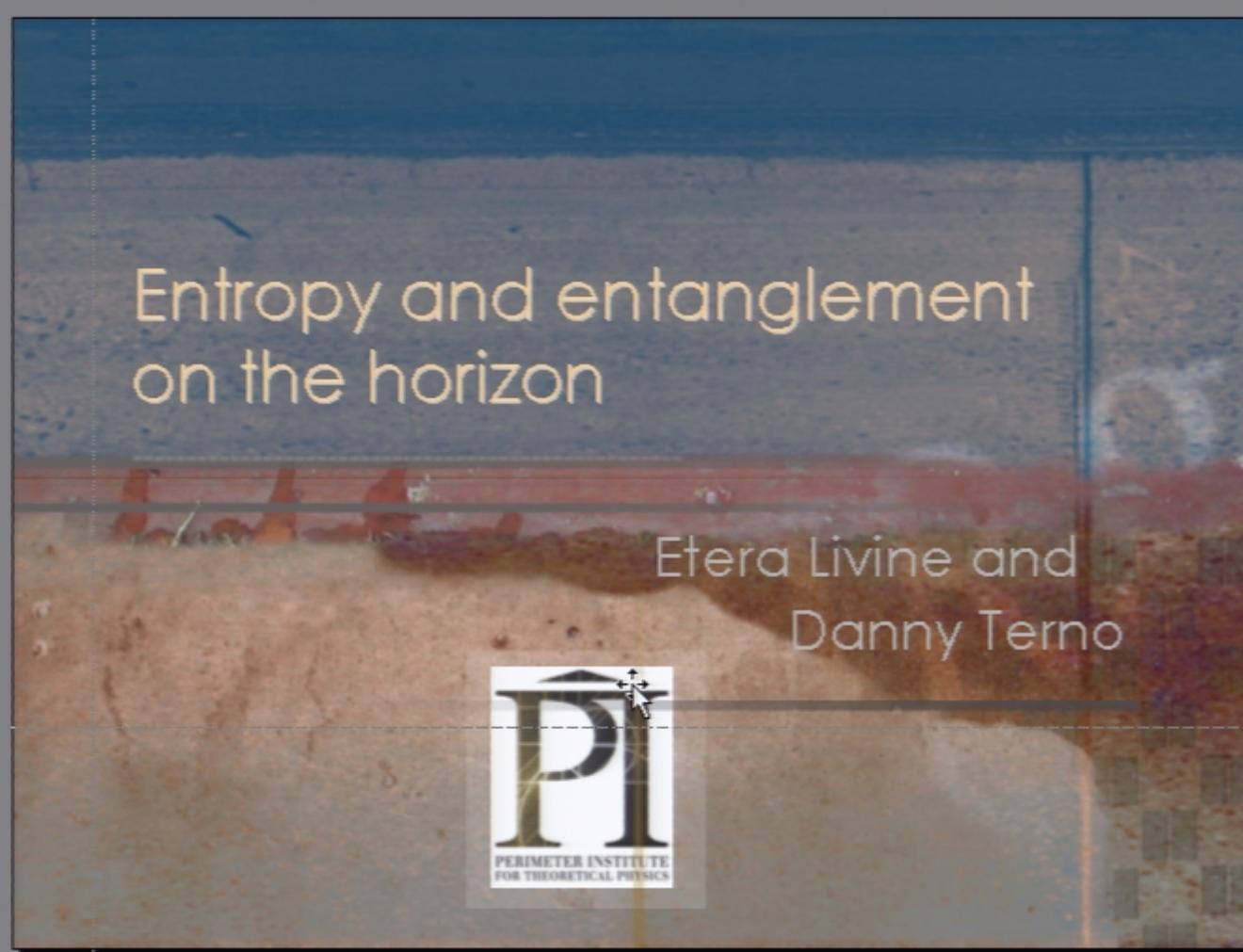


Model

- Entropy and entanglement
- Entanglement length scale
- Properties of entanglement
- Different ways to measure it



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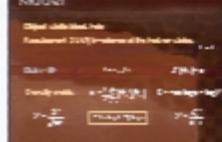
Outline



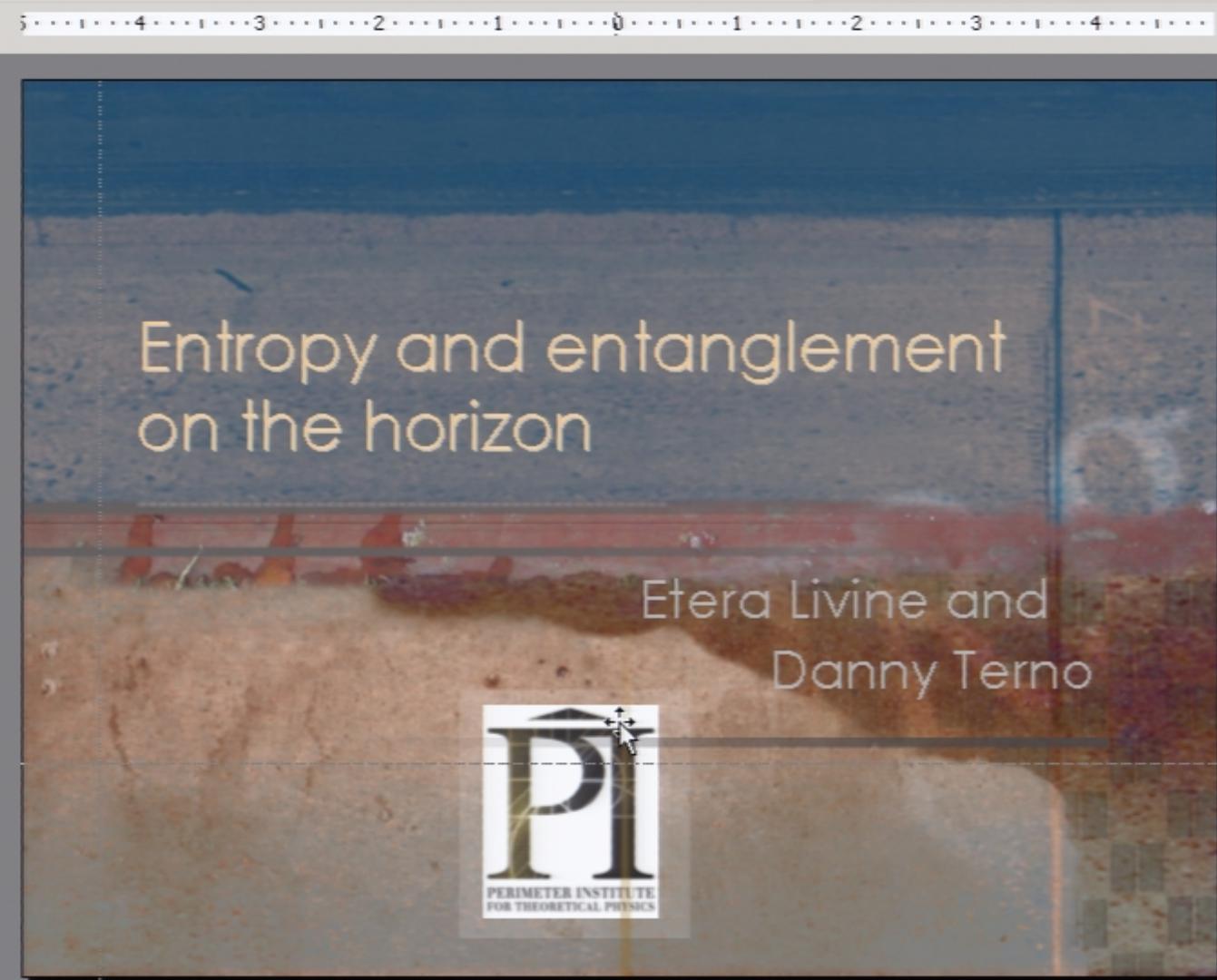
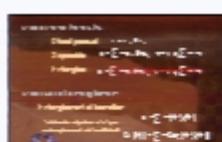
Outline



Model



Entanglement



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