Title: Black Hole Geometrodynamics Revisited

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Abstract:

BLACK HOLE GEOMETRODYNAMICS REVISITED

- some results from work in progress with O. Winkler :

gr-qc / "Quantum avoidance of BH singularities" (VH & OW)

Outline

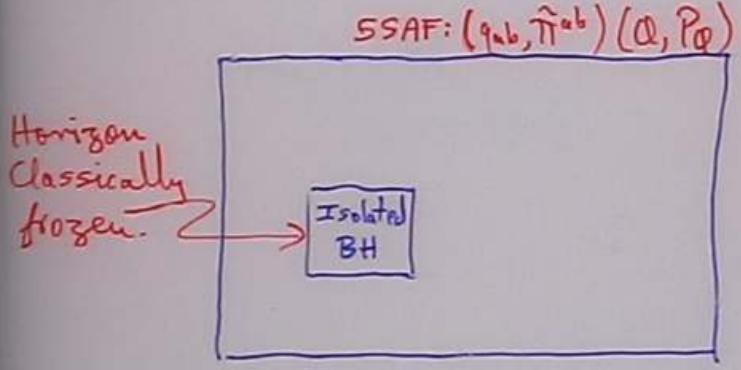
- Introductory remarks
- Classical theory
- Quantization
- Singularity avoidance
- Comments & Outlook.

INTRODUCTION

* Minimal setting for studying BH entropy & Hawking radiation & associated problems (information loss, remnants etc.) is the

  spherically symmetric, asymptotically flat gravity-scalar field midi-superspace. (SSAF)

* An isolated BH is mini- (or possibly midi-) superspace inside this one.



(cf Kuchar, Ryan: "Is mini-superspace quantization valid?")

* A rich model: has black holes, boson stars, dynamical collapse, critical onset of BH formation (Choptuik scaling).

* Full quantization would provide solutions to all BH puzzles, and quantum analog of critical behaviour – 3 classes of quantum states:

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WHEELER-DEWITT

* Define mini- (or midi-) superspace (q_{ab}, π^{ab})

* Schrödinger quantization

$q_{ab} \rightarrow \hat{q}_{ab} \qquad \pi^{ab} \rightarrow \hat{\pi}^{ab}$

OUR APPROACH

* Old variables: (q_{ab}, π^{ab})
* New quantization: representation such that $q_{ab} \rightarrow \hat{q}_{ab}$, but $\hat{\pi}^{ab}$ does not exist. Rather, only an exponentiated form exists.
* Has been applied to cosmology (VH, DW (2003))
  - Used to compute power spectrum (ST, DW (2004))
  - Used for interior Schwarzschild (Modesto (2004))

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CLASSICAL THEORY

* Idea is to write SSAF gravity-scalar field in a parametrization adapted to Painleve-Gullstrand coordinates:

$$ds^2 = -dt^2 + 2\sqrt{\frac{2M}{r}}\, dt\, dr + dr^2 + r^2 d\Omega^2$$

* t=const slices are flat

$$K_{ab} = \sqrt{\frac{2M}{r^3}}\left(e_{ab} - \frac{3}{2} n_a n_b\right)$$

e_{ab} – flat
n_a – unit radial vec.

* This motivates falloff conditions for AF:

$$q_{ab} = e_{ab} + \frac{f_{ab}(\theta,\phi)}{r^{3/2+\epsilon}} + O\left(\frac{1}{r^2}\right)$$

$$\pi^{ab} = \frac{C^{ab}}{r^{3/2}} + \frac{g^{ab}(\theta,\phi)}{r^{3/2+\epsilon}} + O\left(\frac{1}{r^2}\right)$$

- Different from those motivated by Schwarzschild coords.
- Different formulas for mass, BMS obs: mass info. in extrinsic curvature.

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Reduced ADM action:

1) Choose a parametrization for (q_{ab}, π^{ab})

$$q_{ab} = \Lambda^2(r,t)\, n_a n_b + \frac{R^2(r,t)}{r^2}\,(e_{ab} - n_a n_b)$$

$$\pi^{ab} = \frac{P_\Lambda(r,t)}{2\Lambda(r,t)}\, n_a n_b + \frac{r^2 P_R(r,t)}{4R(r,t)}\,(e^{ab} - n^a n^b)$$

2) Plug into ADM action $\frac{1}{8\pi G}\int \pi^{ab}\dot{q}_{ab}$ − constr. ...

$$S_R = \frac{1}{2G}\int \left(P_\Lambda \dot{\Lambda} + P_R \dot{R} + P_\Phi \dot{\Phi} + \cdots\right) dt\,dr$$

3) Get falloff conditions on P_R, R ... by requiring action well-defined − also for N, N^r.

4) Get surface terms $\int_\infty$ from functional diff. of S_R

− completes classical definition of the model:

* 1+1 field theory
* 2 first class constraints $H(r)$, $C^r(r)$
* Canonical variables: (R, P_R), (Λ, P_Λ), (Φ, P_Φ)

(cf Kuchař − Schwarzschild words)

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QUANTIZATION

Basic idea is to represent classical observables

$R_f = \int_0^\infty R(r,t) f(r)\, dr$ and $e^{i\lambda P_R(r,t)}$ etc.

and the bracket

$$\{ R_f, e^{i\lambda P_R} \} = i 2G\lambda f(r)\, e^{i\lambda P_R} \quad \text{etc.}$$

* States: $|a_1 \cdots a_{N_1} ; b_1 \cdots b_{N_2} ; c_1 \cdots c_{N_3}\rangle$

$a_i = R(x_i)$ $\quad b_i = \Lambda(y_i)$ $\quad c_i = \Phi(z_i)$

$\{x_i\}, \{y_i\}, \{z_i\} \in [0, \infty)$

* Operators:

$$\hat{R}_f\, |a_1 \cdots a_{N_1} ; b_1 \cdots b_{N_2} ; c_1 \cdots c_{N_3}\rangle$$

$$= L^2 \sum_K a_K f(x_K)\, |a_1 \cdots a_{N_1} ; b_1 \cdots b_{N_2} ; c_1 \cdots c_{N_3}\rangle \,.$$

$$\widehat{e^{i\lambda_\ell P(x_\ell)}}\, |a_1 \cdots a_{N_1} ; b_1 \cdots b_{N_2} ; c_1 \cdots c_{N_3}\rangle$$

$$= |a_1 \cdots, a_\ell - \lambda_\ell \cdots a_{N_1} ; \ldots \ldots \rangle$$

(cf. Thiemann (1998); Ashtekar, Lewandowski, Sahlmann (2003))

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SINGULARITY AVOIDANCE

* Classical curvatures $\sim \left(\frac{1}{r}\right)^{\alpha}$ $\alpha > 0$ in gauge $R(r,t) = r$.

Also $R^2 \sim$ areas of spheres in the parametrization

* Is there a "$\frac{1}{R}$" operator a'la Thiemann?

– Or equivalently, an inverse area operator?
(Full theory?)

We can use the identity

$$\frac{1}{R_f} = \left[\frac{1}{iG_f}\, e^{-iP_R}\left\{\sqrt{|R_f|}\,,\, e^{iP_R}\right\}\right]^2$$

to define an operator

* Test functions f:

Consider the state $|a_0\rangle$ such that

$$\hat{R}_f\,|a_0\rangle = 2\ell_p^2\, f(0)\, a_0\, |a_0\rangle = 2\ell_p^2 a_0\, |a_0\rangle$$

It represents an excitation a_0 of the field R at $r=0$. Then

$$\hat{\frac{1}{R}}\,|a_0\rangle = \frac{2}{\ell_p}\left[|a_0|^{\frac{1}{2}} - |a_0-1|^{\frac{1}{2}}\right]^2$$

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COMMENTS & OUTLOOK

* Kinematic Hilbert space for SSAF gravity-scalar model.
* $\widehat{\frac{1}{R_I}}$ exists & is bounded above — "inverse radius of smallest sphere is bounded above."
  OR: $\widehat{AREA}|\rangle = 0$, $\widehat{\frac{1}{AREA}}|\rangle$ bounded above.

Many open questions:

* Hamiltonian constraint (in progress VH, D. Terno o. winkler)
* How are BH states identified?

  Apparent Horizon condition is a phase space eqn: "$\widehat{AH}\,|BH\rangle = 0$" ?
* BH entropy
* Hawking radiation } Need $\hat{H}$.
* Information loss }
* Reduced phase space vs Dirac: Gauge R=r freezes singularity into place classically. (observed for Gowdy models VH (1989))

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