Title: Black Hole Geometrodynamics Revisited

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Abstract:

BLACK HOLE GEOMETRODYNAMICS REVISITED

– some results from work in progress with O. Winkler:

gr-qc / "Quantum avoidance of BH singularities" (VH & OW)

Outline

- Introductory remarks
- Classical theory
- Quantization
- Singularity avoidance
- Comments & Outlook.

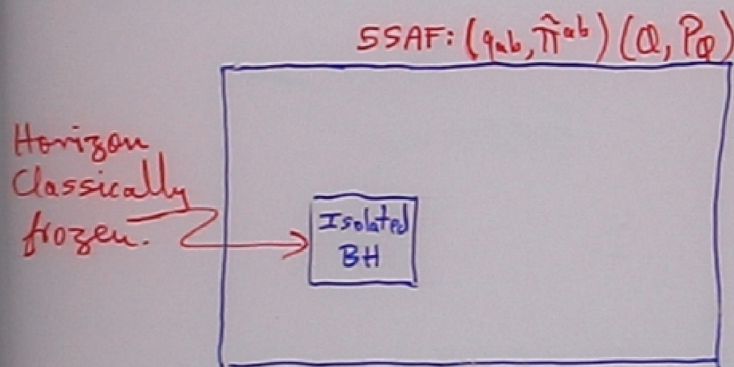
INTRODUCTION

* **Minimal setting** for studying BH entropy & Hawking radiation & associated problems (information loss, remnants etc.) is the

  spherically symmetric, asymptotically flat gravity - scalar field midi-superspace.
  (SSAF)

* An isolated BH is mini- (or possibly midi-) superspace inside this one.

(cf Kuchar, Ryan: "Is mini-superspace quantization valid?")



* A rich model: has black holes, boson stars, dynamical collapse, critical onset of BH formation (Choptuik scaling).

* Full quantization would provide solutions to all BH puzzles, and quantum analog of critical behaviour - 3 classes of quantum states:

WHEELER-DEWITT

* Define mini- (or midi-) superspace
(q_{ab}, π^{ab})

* Schrödinger quantization
$q_{ab} \to \hat{q}_{ab} \qquad \pi^{ab} \to \hat{\pi}^{ab}$

OUR APPROACH

* Old variables : (q_{ab}, π^{ab})
* New quantization: representation such that $q_{ab} \to \hat{q}_{ab}$, but $\hat{\pi}^{ab}$ does not exist. Rather, only an exponentiated form exists.
* Has been applied to cosmology (VH, OW (2003))
  - Used to compute power spectrum (ST, OW (2004))
  - Used for interior Schwarzschild (Modesto (2004))

③

CLASSICAL THEORY

* Idea is to write SSAF gravity-scalar field in a parametrization adapted to Painleve-Gullstrand coordinates:

$$ds^2 = -dt^2 + 2\sqrt{\frac{2M}{r}}\, dt\, dr + dr^2 + r^2 d\Omega^2$$

* $t =$ const slices are flat

$$K_{ab} = \sqrt{\frac{2M}{r^3}}\left(e_{ab} - \frac{3}{2} n_a n_b\right)$$

e_{ab} – flat
n_a – unit radial vec.

* This motivates falloff conditions for AF:

$$q_{ab} = e_{ab} + \frac{f_{ab}(\theta,\varphi)}{r^{3/2+\epsilon}} + O\left(\frac{1}{r^2}\right)$$

$$\pi^{ab} = \frac{c^{ab}}{r^{3/2}} + \frac{g^{ab}(\theta,\varphi)}{r^{3/2+\epsilon}} + O\left(\frac{1}{r^2}\right)$$

– Different from those motivated by Schwarzschild coords.

– Different formulas for mass, BMS obs: mass info. in extrinsic curvature.

Reduced ADM action:

1) Choose a parametrization for (q_{ab}, Π^{ab})

$$q_{ab} = \Lambda^2(r,t)\, n_a n_b + \frac{R^2(r,t)}{r^2}\left(e_{ab} - n_a n_b\right)$$

$$\Pi^{ab} = \frac{P_\Lambda(r,t)}{2\Lambda(r,t)}\, n_a n_b + \frac{r^2 P_R(r,t)}{4R(r,t)}\left(e^{ab} - n^a n^b\right)$$

2) Plug into ADM action $\frac{1}{8\pi G}\int \Pi^{ab}\dot{q}_{ab}$ − constr. ...

$$S_R = \frac{1}{2G}\int \left(P_\Lambda \dot{\Lambda} + P_R \dot{R} + P_Q \dot{Q} + \cdots\right) dt\, dr$$

3) Get falloff conditions on P_R, R ... by requiring action well-defined – also for N, N^r.

4) Get surface terms $\int_\infty$ from functional diff. of S_R

– completes classical definition of the model:

* 1+1 field theory
* 2 first class constraints $H(r)$, $C^r(r)$
* Canonical variables: (R, P_R), (Λ, P_Λ), (Q, P_Q)

(cf Kuchař – Schwarzschild geod.)

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QUANTIZATION

Basic idea is to represent classical observables

$$R_f = \int_0^\infty R(r,t) f(r)\, dr \quad \text{and} \quad e^{i\lambda P_R(r,t)} \quad \text{etc.}$$

and the bracket

$$\{ R_f, e^{i\lambda P_R} \} = i\, 2G\lambda f(r)\, e^{i\lambda P_R} \quad \text{etc.}$$

* States: $|a_1 \cdots a_{N_1} ; b_1 \cdots b_{N_2} ; c_1 \cdots c_{N_3} \rangle$

$$a_i = R(x_i) \qquad b_i = \Lambda(y_i) \qquad c_i = Q(z_i)$$

$$\{x_i\}, \{y_i\}, \{z_i\} \in [0, \infty)$$

* Operators:

$$\hat{R}_f\, |a_1 \cdots a_{N_1} ; b_1 \cdots b_{N_2} ; c_1 \cdots c_{N_3} \rangle$$

$$= L^2 \sum_k a_k f(x_k)\, |a_1 \cdots a_{N_1} ; b_1 \cdots b_{N_2} ; c_1 \cdots c_{N_3} \rangle .$$

$$\widehat{e^{i\lambda_\ell P(x_\ell)}}\, |a_1 \cdots a_{N_1} ; b_1 \cdots b_{N_2} ; c_1 \cdots c_{N_3} \rangle$$

$$= |a_1 \cdots, a_\ell - \lambda_\ell, \cdots a_{N_1} ; \cdots \rangle$$

(cf. Thiemann (1998); Ashtekar, Lewandowski, Sahlmann (2003))

SINGULARITY AVOIDANCE

* Classical curvatures $\sim \left(\frac{1}{r}\right)^{\alpha}$ $\alpha > 0$ in gauge $R(r,t) = r$.

Also $R^2 \sim$ areas of spheres in the parametrization

* Is there a "$\widehat{\frac{1}{R}}$" operator a'la Thiemann?

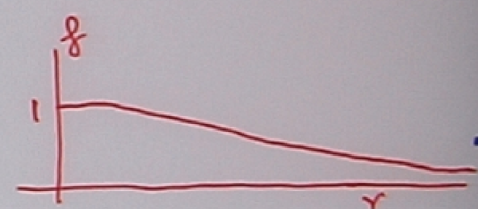
– Or equivalently, an inverse area operator?
(Full theory?)

We can use the identity

$$\frac{1}{R_f} = \left[\frac{1}{iGf}\, e^{-iP_R}\left\{\sqrt{|R_f|},\, e^{iP_R}\right\}\right]^2$$

to define an operator

* Test functions f :



consider the state $|a_0\rangle$ such that

$$\hat{R}_f\,|a_0\rangle = 2\ell_P^2\, f(0)\, a_0\, |a_0\rangle = 2\ell_P^2\, a_0\, |a_0\rangle$$

It represents an excitation a_0 of the field R at $r=0$. Then

$$\hat{\ } \ldots = \frac{2}{\ldots}\left[|a_0|^{\frac{1}{2}} - |a_0 - 1|^{\frac{1}{2}}\right]^2$$

(7)

COMMENTS & OUTLOOK

* Kinematic Hilbert space for SSAF gravity-scalar model.

* $\widehat{\frac{1}{R_f}}$ exists & is bounded above — "inverse radius of smallest sphere is bounded above."

  OR: $\widehat{AREA}|\rangle \neq 0$, $\widehat{\frac{1}{AREA}}|\rangle$ bounded above.

Many open questions:

* Hamiltonian constraint (in progress VH, D. Terno O. Winkler)

* How are BH states identified?

  Apparent Horizon condition is a phase space eqn: "$\widehat{AH}\,|BH\rangle$" $= 0$?

* BH entropy

* Hawking radiation

* Information loss

$\Big\}$ Need $\hat{H}$.

* Reduced phase space vs Dirac: Gauge $R=r$ freezes singularity into place classically. (Observed for Gowdy models VH (1989))