Title: Consequences of Space-Time Discreteness on Wave Propagation

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Abstract:

Consequences of spacetime discreteness for propagation of waves

(Fay Dowker, JJH, Rafael Sorkin. One paper out, two more to come.)

Some questions:

1) Does spacetime discreteness imply "Lorentz violation"?
NO.

2) How does the causal set structure avoid Lorentz violation?

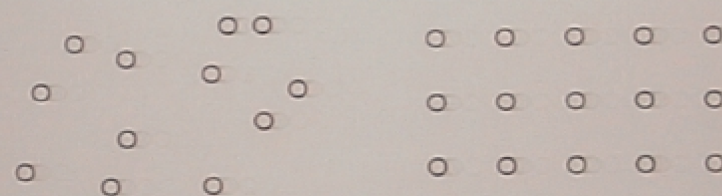
3) What are the phenomenological consequences of causal set discreteness?

4) Can the non-local aspects of the causal set structure be reconciled with the level of locality that we observe?

Symmetry and discreteness

Consider rotational symmetry in condensed matter physics. In what sense is a gas rotationally invariant?

"Whenever a continuum is a good approximation to the underlying structure, the underlying discreteness must not, in and of itself, suffice to distinguish a direction."



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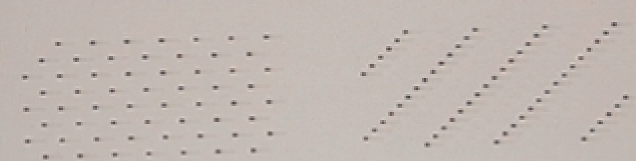
A gas is rotationally invariant. It can be so because its microscopic structure is random.

For spacetime, Lorentz invariance of a discrete underlying structure can be understood by analogy to this.

"Whenever a continuum Lorentzian manifold is a good approximation to the underlying structure, the underlying discreteness must not, in and of itself, suffice to distinguish a local Lorentz frame at any point."

"The approximation, if good in one frame, should be good in all frames."

EXAMPLE 1:
Lattice in 2D Minkowski space



Consequences: Lorentz violating ultraviolet cutoff for waves propagating on the lattice (a "trans-Plankian problem"), maybe altered dispersion relations...

Similar situation for Regge triangulations, spin foams, etc.

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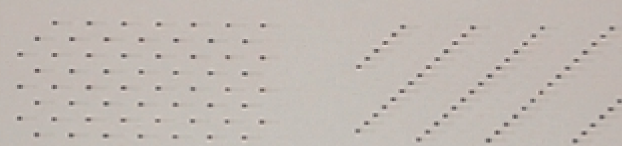

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EXAMPLE 2:
Causal set

A locally finite partial order for which

Order $\leftrightarrow$ Causal order
Number $\leftrightarrow$ Volume

The causal set is taken to be the fundamental structure. For some causets, a Lorentzian manifold is a good approximation.

A manifold is a good approximation to a causet if the causet can be obtained by "sprinkling" the manifold with points at Plankian density.

Sprinkling is a Poisson process:

$$P(n) = \frac{V^n e^{-V}}{n!},$$

is the probability to find n elements sprinkled into a region of volume V.
Volume is a Lorentz invariant quantity, so

The probability distribution for sprinkling is locally Lorentz invariant.

When given a manifold, we can model the microscopic structure by a random sprinkling.

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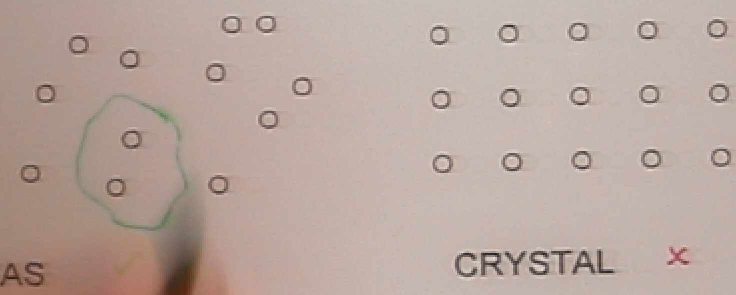

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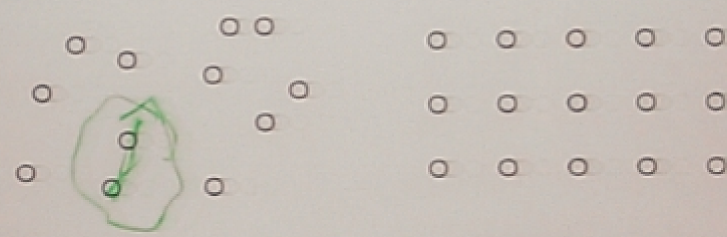

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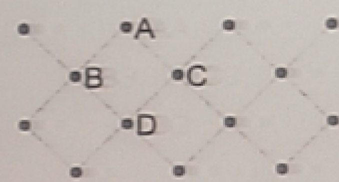
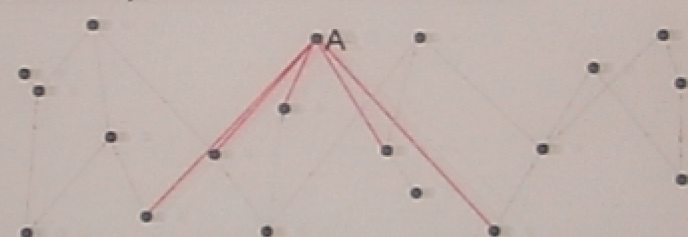
Locality, Lorentz Invariance: a tradeoff?

In the 2D lattice case:



B and C are A's "past neighbors". This residual locality allows us easily to define operators that are local in the continuum limit.

In a causet sprinkled into 2D Minkowski:



What are A's past neighbors? Elements *linked* to A have the least timelike separation from A in the original spacetime. But there are infinitely many of them, spread along the light cone (because there is infinite volume to the past of A and spacelike to any finite set of elements linked to A).

Why choose LLI over locality? LLI is better tested. Causal structure is easily preserved. No need for other data to reconstruct the manifold.

"Could a quantum 'sum over lattices' recover LLI?"

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Phenomenological Consequences

The causal set hypothesis implies no violation of LLI.

Are there consequences of Plank scale spacetime structure for wave propagation over long distances (Lieu and Hillman, 2003)?

More specifically, will spacetime discreteness conflict with observation here?
Not causal set discreteness.

EXAMPLE: Scalar field on 4D Minkowski space. Dynamics are expressed by the Green's function

$$G(x,y) = \begin{cases} \frac{1}{2\pi}\delta(|x-y|^2) & \text{if } y \geq x \\ 0 & \text{otherwise} \end{cases}$$

, a δ-function on the past light cone. For the causet discretisation, there is a simple function that goes to the above in the continuum limit:

$$G(x,y) \approx L(e_x, e_y) \times \text{const.},$$

For a causal set sprinkled into Minkowski at Plank density, there is an excellent level of correspondence between the continuum and discrete models of propagation from source to receiver. No "trans-Plankian" problem.

Positive consequences? Dispersion in phase space ("swerves")? Explanation for HECR's? Effects of non-locality? Need more detailed models.

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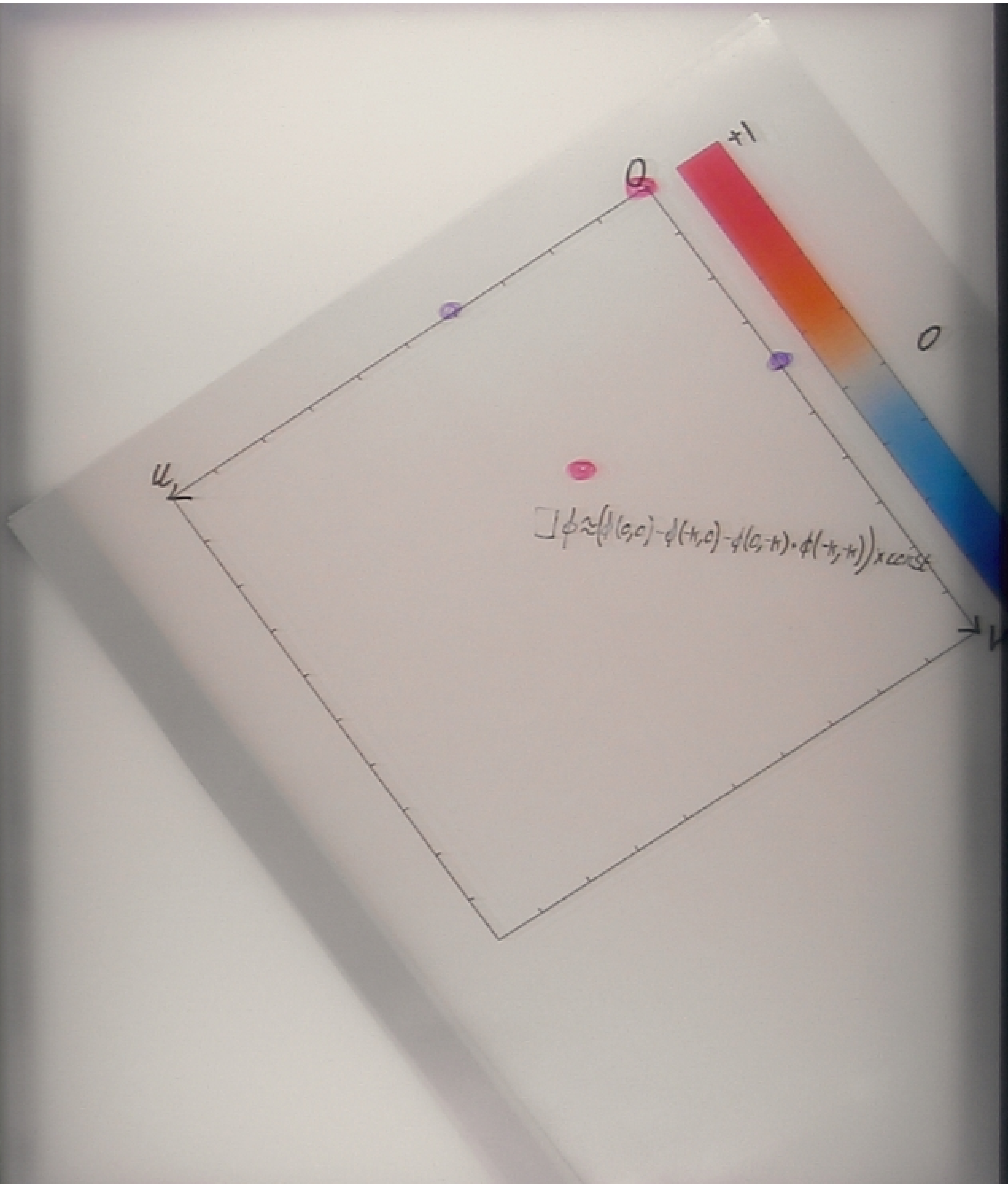

+1
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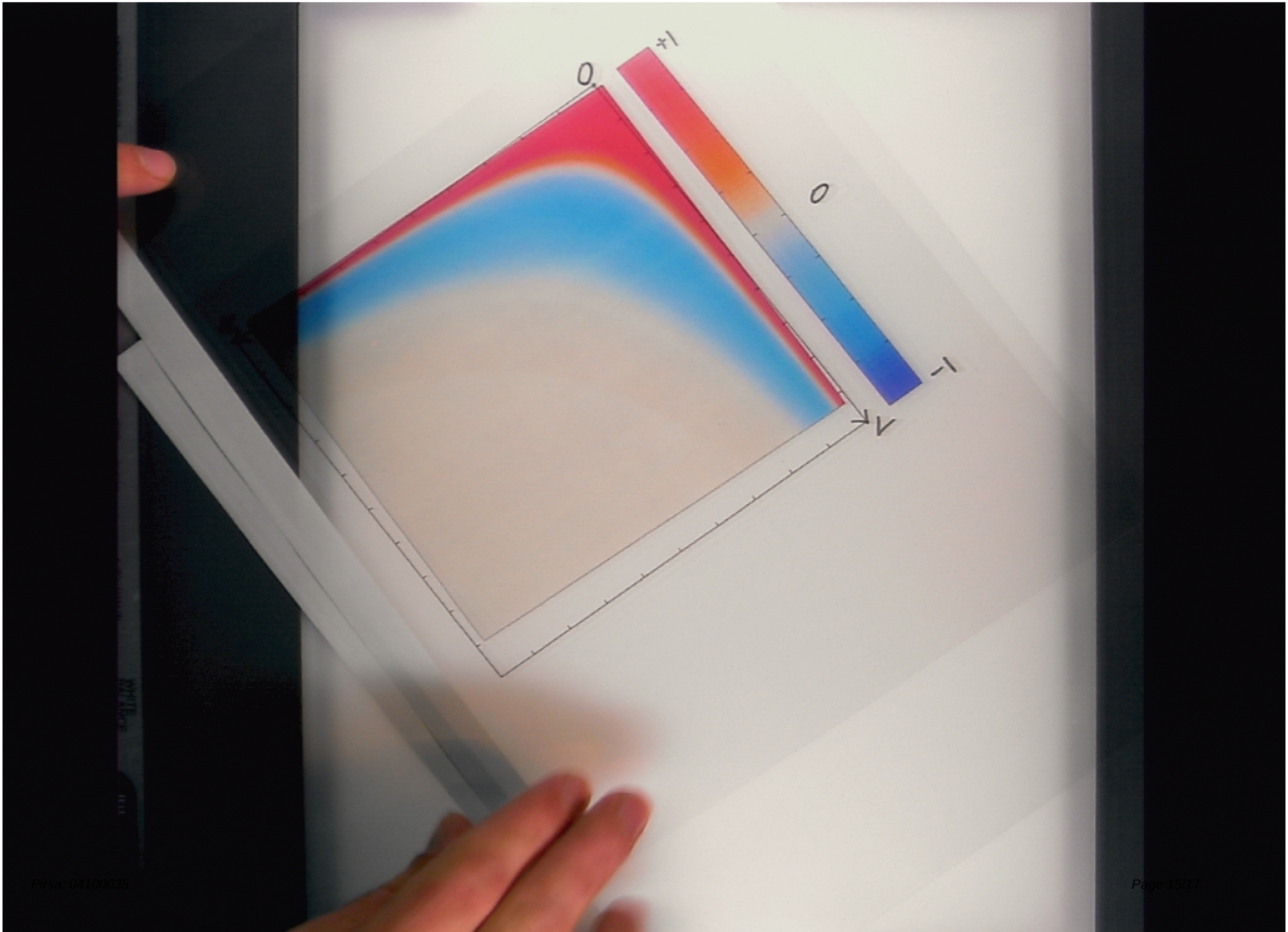
$\Box\phi \approx (\phi(0,0) - \phi(-\pi,0) - \phi(0,-\pi) + \phi(-\pi,\pi)) \times const$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Scalar field dynamics on a causal set

Start with massless field in 2D Minkowski. Wave equation: $\Box\phi(x) = 0$.
How can we approximate $\Box\phi(x)$?

Answer: Discretise this non-local expression, which approximates $\Box\phi(0)$.

$$\Box\phi(0) \approx k\phi(0) - 2k^2 \int_{x \prec 0} d^2x\, \phi(x)\, e^{-kV(x)} \left(1 - 2kV(x) - \frac{1}{2}k^2 V(x)^2\right)$$

Using a sprinkled causet, the integral ≈ a sum with contributions from each element. The causal set structure provides a measure of the "volume" V of the interval between two elements (the number of elements in it).

The scheme can be straightforwardly generalised to other dimensions.

(Spin-off: An expression for the scalar curvature in terms of the causal set structure?)

How does it work? Can we see why the above Lorentz-invariant expression is approximately local? Why isn't it sensitive to values of ϕ "far down the light-cone"?