Title: Particles, Cosmology and Spinfoams

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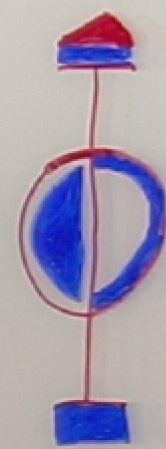
Abstract:



IN THE SKY

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PLAN

- Orthodox introduction of $\mathbb{P}_\Phi$
  the superstar in cosmology
- Planckian imprints in $\mathbb{P}_\Phi$ so far
- A Quantum Cosmology set-up
- The Q_{Bohr}FRW model
- $\mathbb{P}_\Phi$ in $\langle Q_{\text{Bohr}}\text{FRW}\rangle$
- Outlook

P_ΦOWER SPECTRUM

¢ FRW background:

$$ds^2 = dt^2 - a_{cl}^2(t)\, d\vec{x}^2$$

cosmic time — classical scale-factor — comoving coordinates

Φ on ¢ FRW background:

$$H_s[\Phi, \pi, a_{cl}] = \int \mathrm{vol}\; \rho_s(\Phi, \pi, a_{cl})$$

energy density

Define vacuum at "initial time":

$$A(\Phi, \pi; k, t_i)\,|0, t_i\rangle = 0 \quad \forall k$$

Solution (Schrödinger picture in Φ-representation):

$$|\Psi[\Phi(k,t_i), t]|^2 \sim \exp\left[-\frac{(Lk)^3}{2\pi^2}\,\frac{|\Phi(k,t_i)|^2}{P_\Phi(k,t)}\right]$$

minimal uncertainty wavefunction

RELEVANCE OF $\mathcal{P}_{\Phi}$

For a generic fluctuation $F(k,t)$ in the linea regime:

$$\mathcal{P}_F(k,t) = \mathcal{P}_{\Phi}(k,t_*)\, T_F(k,t)\, D_F^2(k)$$

$\uparrow$	$\uparrow$	$\uparrow$
initial spectrum	gravitational evolution	damping factor
$\uparrow$	$\uparrow$	$\uparrow$
model of inflation	cosmological perturbation theory	nonequilibrium physics
$\uparrow$	$\uparrow$	$\uparrow$
QFT of Φ	General Relativity	QFT of F

$\mathcal{P}_{\Phi}$ is universal

T_F is fixed by the cosmological model

D_F: depends strongly on F

Φ IN $\langle QFRW \rangle$ - COSMOLOGY

Hilbertspace: $H_{QFRW} \otimes H_{\Phi}$

Idea: $a \longrightarrow \langle \mu | a | \mu \rangle \equiv \langle a \rangle \, , \dots$

$$H_s\,[\Phi, \pi ; a, a^{-1}; p] \longrightarrow H_s[\Phi, \pi ; \langle a \rangle, \langle a^{-1} \rangle, \langle p \rangle]$$

$$H_s = \int \langle vol\, \rho_s \rangle$$

$$\langle \rho_s \rangle = \frac{1}{2}\left(\langle a^{-3} \rangle \pi\right)^2 + \frac{1}{2}\left(\langle a^{-1} \rangle \nabla \Phi\right)^2 + V(\Phi)$$

Equation of motion for Φ:

$$\ddot{\Phi} - \left[3\,\frac{\langle \dot{a} \rangle}{\langle a \rangle} + 2\,\frac{\langle \dot{a}^{-3} \rangle}{\langle a^{-3} \rangle}\right] \dot{\Phi}$$

$$- \left(\langle a^{3} \rangle \langle a^{-3} \rangle\right)^2 \left[\left(\langle a^{-1} \rangle \nabla\right)^2 \Phi - \frac{dV}{d\Phi}\right]$$

$$= 0$$

closed

Equation of motion for $\langle a \rangle$:

$$\frac{\langle \dot{a} \rangle}{\langle a \rangle} = \frac{8\pi}{3} G_N \langle \rho_s \rangle$$

QFRW CLOSE TO CFRW

Close to CFRW:

$$\langle a \rangle = a_{cl}$$

$$\langle a^{-1} \rangle = a_{cl}^{-1}\left[1 + \langle a^{-1}\rangle_q\right]$$

and $\langle a^{-1}\rangle_q \ll 1$ (Naturalness)

Then,

$$\ddot{\Phi} + 3\left[H_{cl} - 2\,\frac{\langle \dot{a}^{-1}\rangle_q}{1+\langle a^{-1}\rangle_q}\right]\dot{\Phi}$$

$$- \left(1+\langle a^{-1}\rangle_q\right)^6\left[\left(1+\langle a^{-1}\rangle_q\right)^2\left(\frac{\nabla}{a_{cl}}\right)^2\Phi - \frac{dV}{d\Phi}\right]$$

$$= 0$$

correct limit for QFRW $\longrightarrow$ CFRW

Φ IN $\langle Q_{Bohr}$ FRW$\rangle$ COSMOLOGY

Equation of motion for Φ:

$$\ddot{\Phi} =$$

$$- 3H \left[\frac{4\mu}{(|\mu|\,|\mu-1|)^{1/2}} \frac{\operatorname{sgn}(\mu-1)\,|\mu|^{1/2} - \operatorname{sgn}(\mu)\,|\mu-1|^{1/2}}{d(\mu)} - 1 \right] \dot{\Phi}$$

$$+ (\mu d^2(\mu))^6 \left[\left(\frac{d^2(\mu)\,\nabla}{(8\pi)^{1/2} L_p} \right)^2 \Phi - \frac{dV}{d\Phi} \right]$$

Goal:

Calculate vacuum fluctuations of Φ

in a Q_{Bohr}FRW – Universe close to the

CFRW-Universe during inflation

SΦLUTI☺N

Definitions:

$$z \overset{\mathrm{Df}}{=} k \Big/ \left(\sqrt{8\pi}\, L_P \mu \cdot H\right) \sim \frac{R_H}{\lambda_{phys}(k)}$$

$$\Delta \overset{\mathrm{Df}}{=} 1 - \left[3/(2\mu z)\right]^2$$

Φ in the sky:

$$W(k,\mu) = \exp\left(\frac{3}{2\mu}\right) \exp\left(-i\Delta^{1/2}\cdot z\right) \cdot$$

$$\cdot \left\{ \frac{C_1(k)}{\mu^3}\, \mathcal{U}\left[2 + \frac{3}{2i\Delta^{1/2}\mu z},\, 4,\, 2i\Delta^{1/2} z\right] + \frac{C_2(k)}{\mu^3}\, {}_1F_1\left[2 + \frac{3}{2i\Delta^{1/2}\mu z},\, 4,\, 2i\Delta^{1/2} z\right] \right\}$$

This solution has the correct behaviour

- at late times
- deep inside the horizon

SOLUTION

In the CFRW - limit :

$$W(k,\mu) = \underbrace{\frac{H}{(2L^3k^3)^{1/2}}\,(i-z)\,\exp(-iz)}_{\text{orthodox solution}}$$

Deep inside the horizon :

$$W(k,\mu) = \underbrace{\frac{k/a}{(2L^3k^3)^{1/2}}\,\exp\left(i(k/a)t\right)}_{\substack{\text{orthodox solution}\\ \text{in flat space-time}}}$$

Note :

$$\mathcal{P}_{\Phi}(k,\mu)\Big|_{\substack{\mu\gg 1\\ z=1}} = \left(\frac{H}{2\pi}\right)^2\Big|_{z=1}$$

$$\mathcal{P}_{\Phi}(k,\mu)\Big|_{z\ll 1} = \left(\frac{k/a}{2\pi}\right)^2$$

$\mathcal{P}_\Phi$ IN ⟨QFRW⟩-COSMOLOGY

$$\mathcal{P}_\Phi(k,\mu)\Big|_{z=1} =$$

$$\left(\frac{H}{2\pi}\right)^2\Big|_{z=1}\left[1+\left(\frac{3}{2}\right)^2(1-\gamma_E)^2\left(\sqrt{8\pi}\,L_p\right)^2\left(\frac{H}{k}\right)^2+\cdots\right]$$

$\uparrow$

$\mathcal{P}_\Phi$ in CFRW

$\uparrow$

$\mathcal{O}(L_p^2)$ correction

from Q FRW

Bohr

Σ & OUTLOOK

- First calculation of P_Φ

  the superstar in cosmology

  in Quantum Cosmology

- should be similar in LQC

  (work in progress)

- Great times in Quantum Cosmology:

  we started to predict cosmological observables

- Welcome:

  ambiguities, TeV-black holes, ...

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  P_Φ in LQG