Title: Dynamics and Spin Foams in Non Perturbative Quantum Gravity

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Abstract:

TOWARD A NON-TRIVIAL GENERALIZATION OF 2+1 GRAVITY TO 3+1 DIMENSIONS
(THE RIEMANNIAN SECTOR)

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1) MOTIVATION

2) THE INTUITIVE IDEA

3) THE MODEL

4) PROPERTIES

Acknowledgments: for discussion Rovelli, Starodubtsev, Noui, Sahlmann, Fleischhack, Conrady, Ashtekar

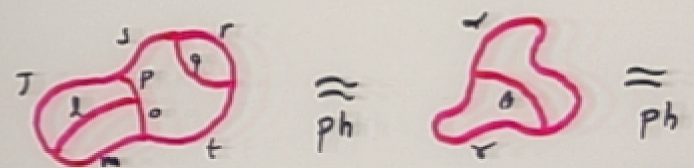
MOTIVATION:

1) SIMPLICITY: if the variables of LQG capture the fundamental degrees of freedom of gravity DYNAMICS should be SIMPLE.

Physical inner Product $\longleftrightarrow$ Simple and combinatorial

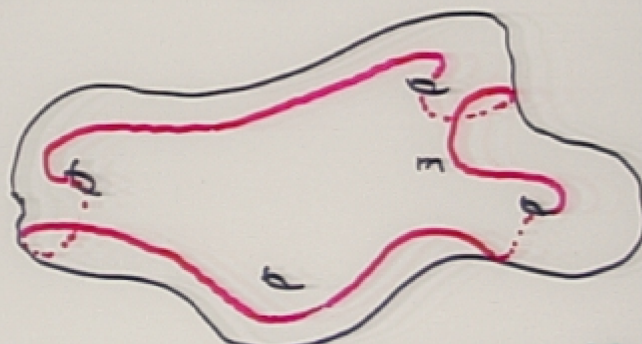
2) 2+1 Gravity satisfies these EXPECTATIONS.
DYNAMICS IS SIMPLE AND COMBINATORIAL BUT THE THEORY IS TRIVIAL: TOPOLOGICAL OR NO LOCAL D.O.F.

relevant constraint $\rightarrow F = 0$



Trivial "excitations"

Non Trivial excitations



HYPOTHESIS: WE CAN LEARN SOMETHING ABOUT 3+1 GRAV. FROM 2+1 GRAVITY

LOCAL DEGREES OF FREEDOM OF 3+1 GRAVITY ARE PERHAPS NO SO DIFFERENT FROM THE TOPOLOGI_CAL ONES IN 2+1 GRAVITY.

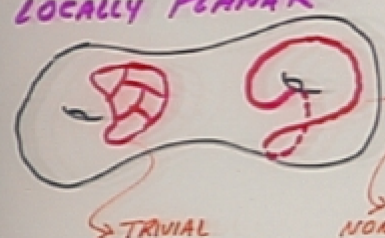
3) Classical 2+1 general relativity is trivial because GRAVITONS JUST "DON'T FIT" in 3D

$$R_{abcd} = C_{abcd} - \frac{2}{d-2}\left(g_{a[c}R_{d]b} - g_{b[c}R_{d]a}\right) - \frac{2R}{(d-1)(d-2)}g_{a[c}g_{d]b}$$

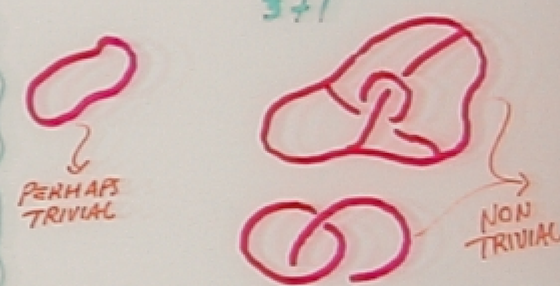
$$\left.\frac{d^2(d^2-1)}{12}\right|_{d=3} = 6 \rightarrow \text{The 6 indep. components of } R_{ab} \Rightarrow C_{abcd} = 0 \text{ for } d=3$$

4) IS THERE AN ANALOG IN QUANTUM GEOMETRY? If YES this can be used as insight for the problem of identifying the physical degrees of freedom of 3+1 Gravity

2+1 "GRAPHS ARE LOCALLY "PLANAR""



3+1



GRAPHS CAN BE KNOTTED!

5) The closest relative: 3+1 BF theory ⇒ BUT IT IS TRIVIAL

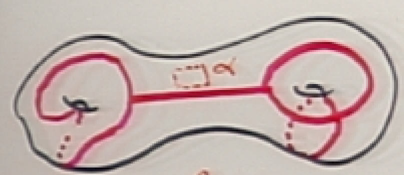
RELEVANT CONSTRAINT → $F=0$

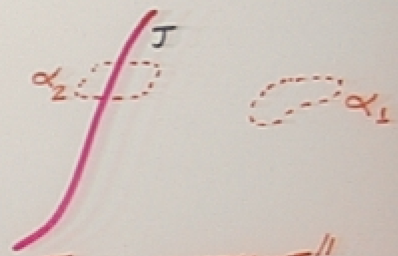
WE NEED TO RELAXE THE CONDITION $F=0$ EVERYWHERE!

THE MODEL: INTUITIVE IDEA

What is the simplest way to relax the constraint $F=0$ compatible with the previous motivation?

2+1 | 3+1





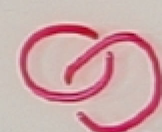
"$F=0$" $\hat{h}_\alpha - 1 = 0$ $\forall \alpha$ contractible!

"RELAXED $F=0$ CONSTRAINT"

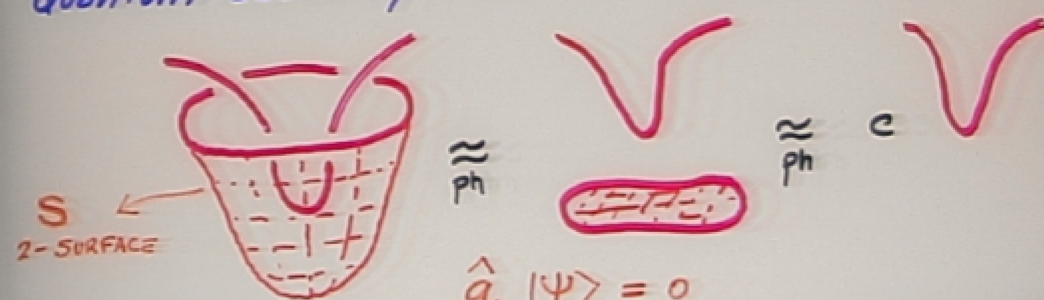
$$\begin{cases} \hat{h}_{\alpha_1} - 1 = 0 \quad \forall \alpha_1 \text{ contractible} \\ \hat{h}_{\alpha_2} = \text{LEFT UN-CONSTRAINED} \quad \forall \alpha_2 \text{ "non contractible"} \end{cases}$$

IF D=3 THE NEW RELAXED CONST. REDUCES TO THE STANDARD $F=0$.

NOTICE THAT THE "PHYSICAL INNER PRODUCT" CANNOT UNTIE knots anylonger.

 $\not\approx_{ph}$ 1

Quantum Geometry and the relaxed constraint:



$$\hat{a}_S |\Psi\rangle = 0$$

THE PARALLEL TRANSPORT AROUN A "ZERO AREA LOOP" IS TRIVIAL = "RELAXED $F=0$ CONSTRAINT"

COUNTING OF D.O.F.

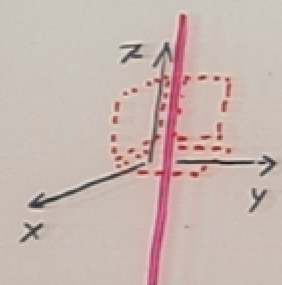
F_{ab}^i has 9 components. However in 3D only 6 are independent due to the Bianchi Identity

$$B^i = \epsilon^{abc} D_a F_{bc} = 0$$

We can take

$$F_{ab}^{1,2}$$

as independent components.



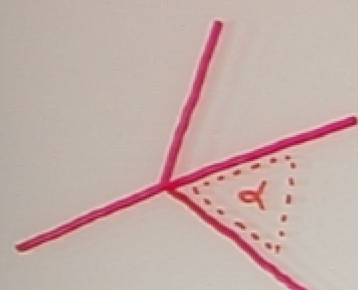
$$F_{zy}^{1,2} = F_{zx}^{1,2} = 0$$

4 INDEPENDENT CONSTRAINTS

+ 3 GAUSS CONST. $D_a E^a_i = 0$

2 LOCAL D.O.F. $\Leftarrow$ FOR 9 comp. of A_a^i $\Leftarrow$ 7 CONSTRAINTS

THIEMANN's Quantum Constraint annihilates the PHYSICAL STATES OF THE MODEL



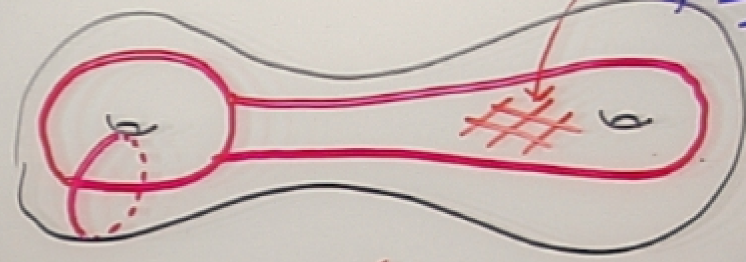
$$S = FEE$$

$$\hat{S} = \underbrace{\hat{F}}_{\hat{F} \sim \hat{h}_\alpha - \hat{h}_\alpha^{-1} = 0} \hat{E}\hat{E}$$

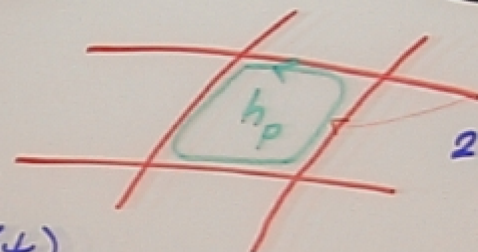
By const. in the model

REVIEW OF 2+1 GRAVITY

Introduce a cellular decomp Δ of Σ adapted to the state ψ



Holonomy around one elementary 2-cell in Δ



DEFINE $P(\psi)$

$$P(\psi) = \langle 0 \prod_{p\in\Delta} \delta(h_p)\, \psi \rangle$$

1) $P(\psi^* \psi) \geq 0$

2) Normalizable $P(\psi) \leq c\, P(1) = c\, Z^{BF}_{2D}(\Sigma) =$

$$= c \sum_J (2J+1)^{2-2g}$$

g: genus of Σ

GNS CONSTRUCTION $\Longleftrightarrow$ Physical Hilbert Space isomorphic to the one obtained by other methods

DEFINITION OF THE MODEL IN 3+1

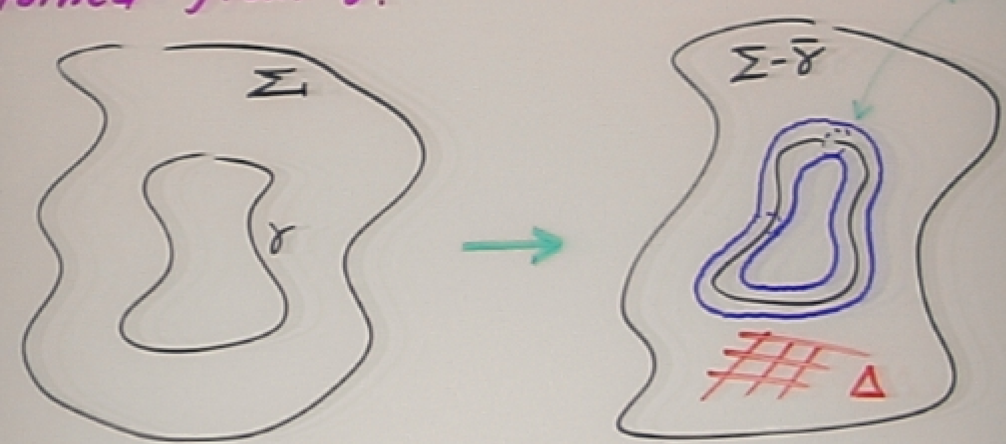
$$\psi = \sum_{\gamma} \sum_{\{J\}} \sum_{\{i\}} c_{\{J\}\{i\}\gamma} \, T_{\gamma,\{J\}\{i\}}$$

↳ "IRREDUCIBLE SPIN NETWORK STATES", i.e., each edge is labeled by a non trivial spin

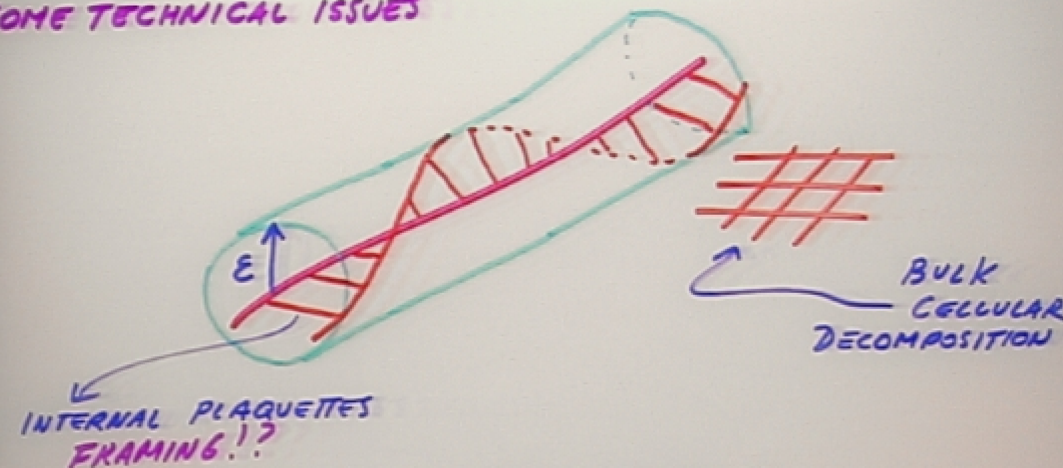
$$P(\psi) = \sum_{\gamma} \sum_{\{J\}} c_{\{J\}\gamma} \, P\left[T_{\gamma,\{J\}}\right]$$

$$P\left[T_{\gamma,\{J\}}\right] = \left\langle 0 \prod_{p \in \Delta(\Sigma - \bar{\gamma})} \delta(\hat{h}_p) \, T_{\gamma\{J\}} \right\rangle$$

where $\Delta(\Sigma - \bar{\gamma})$ is A CELLULAR DECOMPOSITION OF $\Sigma - \bar{\gamma}$ and $\bar{\gamma}$ is a thikened graph obtained from γ.



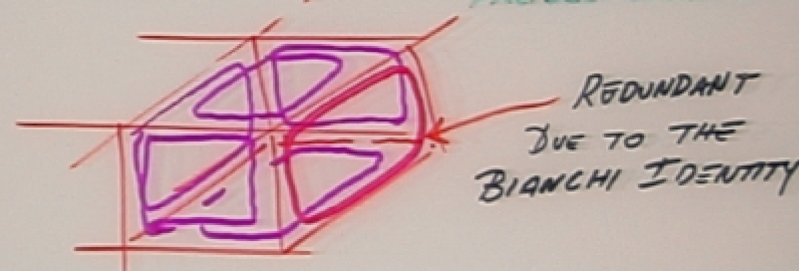
SOME TECHNICAL ISSUES



1) AMPLITUDES DEPEND ON THE FRAMING!

2) Once a framing is given $P(\Gamma)$ does not depend on ε and on Δ

3) FINITENESS? Yes after appropriate gauge fixing
FREIDEL–LOUAPRE



4) Diff INVARIANT

THE FACT THAT AMPLITUDES DEPEND ON THE FRAMING IS NOT COMPLETELY SURPRISING

Framing $\longleftrightarrow$ self linking

ONE WOULD LIKE TO EXTEND THE KINEMATICS TO INCLUDE THE FRAMING AS AN ADDITIONAL D.O.F.

ORBITAL ANGULAR MOMENTUM $\longleftrightarrow$ SPIN

CONCLUSIONS:

1) I have presented the main ingredients of an idea leading to a model for background independent QFT with local D.O.F.

2) Given two "framed" spin networks s_1 and s_2 the model provides a definite answer to the question what is the physical transition amplitude

$$\langle s_1\, s_2 \rangle_P = \langle 0\, P\, s_1^{*} s_2 \rangle \rightarrow \text{SIMPLE AND COMBINATORIAL}$$

3) Physical states are solutions of the quantum constraints of LQG.

4) Naive counting of D.O.F. yields 2 quasi local D.O.F

5) Physical states are highly non-local "Long range correlations"

6) The main issue to be studied in detail is whether the framing ambiguity can be succesfully incoorporated into the kinematics.

I do have ideas Baez - Ashtekar
Smolin - Markopoulou
Crane