Title: Semi-Discrete Solution to the Dynamics of Loop Quantum Gravity

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Abstract:

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LSU Relativity

Consistent discretization as a dynamics for loop quantum geometry

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With Rodolfo Gambini

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The mechanism can eliminate the black hole information paradox

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"*While being a fascinating possibility, such a procedure would be a rather drastic step in the sense that it would render most results of LQG obtained so far obsolete*" T. Thiemann "*The Phoenix Project*" gr-qc/0305080

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- Therefore it can be consistently imposed as a constraint.
- The resulting theory, with this extra constraint, has the exact same kinematics as LQG but an explicit dynamics (no Hamiltonian constraint).

We start with the usual action,

$$S = \int dt d^3x \left(\tilde{P}^a_i F^i_{0a} - N^a C_a - NC \right)$$

$$C^a = \tilde{P}^a_i F^i_{ab}$$

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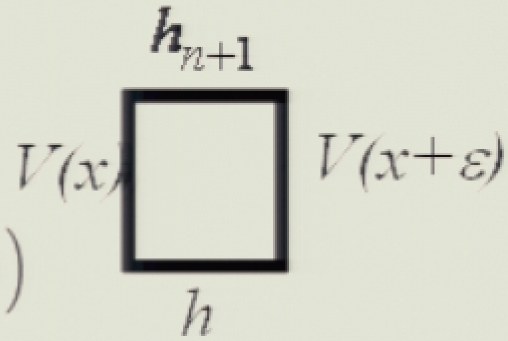

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Where V is a "vertical" parallel transport (in the time direction). To get this expression one first discretizes in time and space, replaces the F_{ab} with a holonomy and then takes the limit in which the spatial links go to zero. The last term just enforces that the V's be in SU(2).

SU(2) gauge invariance is exactly preserved.

If now one performs an infinitesimal spatial transformation $x'^a = x^a + v^a$ one immediately sees the action is invariant. The associated conserved quantity through Noether's theorem is,

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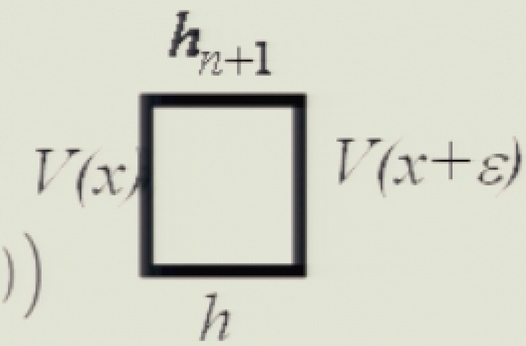

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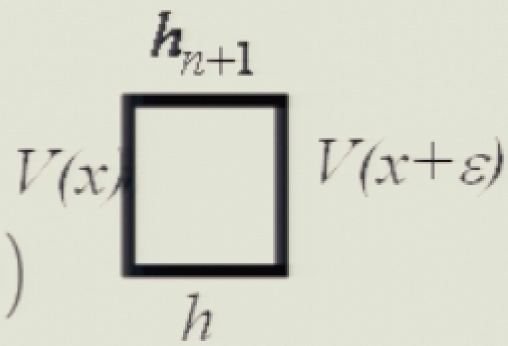

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Quantization is straightforward! (no constraints) but computationally complex for the case of full GR. The quantum kinematics is the usual of LQG.

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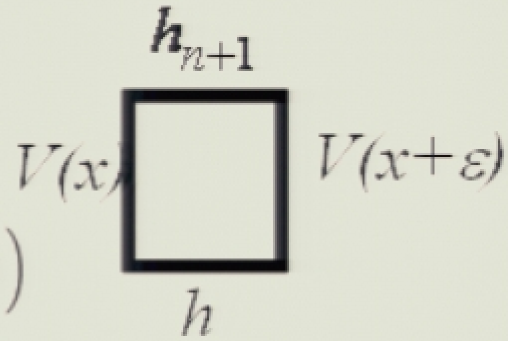

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An explicit example: BF theory

$$L(n,n+1) = \mathrm{Tr}\left\{B_0(x)F_{12}(x) + B_1(x)\left(A_2(x) - V(x)A_{n+1,2}(x)V^{-1}(x) + \partial_2(V(x))V^{-1}(x)\right)\right.$$
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And the definition of P^{B0} yields F_{12}=0.

The momentum of V also vanishes and yields Gauss' law.

Quantization is achieved by implementing the evolution equations as a unitary transformation. This can be immediately done:

$$U(A', A) = \langle A', n+1 | A, n \rangle$$

$$= \delta\left(A_1' - V^{-1} A_1 V + \partial_1(V^{-1})V\right) \delta\left(A_2' - V^{-1} A_2 V + \partial_2(V^{-1})V\right) \exp\left(\mathrm{Tr}\int B_0 F\right),$$

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One has to impose the constraint $F_{12}=0$, which yields the usual space of states of BF theory (Ooguri, Noui-Perez).

The unitary evolution depends on two free functions B_0 and V, but it is clear that on the space of physical states the evolution is independent on them.

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- This opens a concrete avenue for numerical quantum gravity.