Title: Some Obstructions to Spin Networks for Non-Compact Gauge Groups

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Abstract:

Obstructions to Spin-Networks for Some Non-Compact Groups

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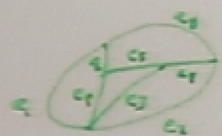
Plan of Talk

- <u>Brief</u> review of compact case & previous work on non-compact case
- Successful strategy for 'non-compact Abelian: Compactify the Group
- Not so successful for $SL(2,\mathbb{R})$ or $SL(2,\mathbb{C})$; a partial 'no-go' theorem
- Conclusions & outlook

THE COMPACT CASE



Graph γ

Start from cylindrical functions: depend on connection only through holonomies along edges of graph

For single graph, inner product defined with Haar measure on G^n

$$\langle f_\gamma | h_\gamma \rangle = \mu(\bar{f} h) = \int_{G^n} \bar{f} \, h \; d\mu_{Haar}$$

Flux operators come from left/right invariant vector fields on each copy of G

Patch together different graphs

$$\langle f_{\gamma_1} | h_{\gamma_2} \rangle = \langle f_{\gamma'} | h_{\gamma'} \rangle \qquad \gamma_1, \gamma_2 \leq \gamma'$$

Possible because constant functions are integrable on G

So can't extend measure in straightforward way if G non-compact.

Previous Work
(Non-Compact Groups)

- Freidel & Livine JMP 44 (2003) 1322–1356

  Constructed Hilbert space, but representation of algebra is highly reducible

- General arguments exclude several plausible strategies: non-normalizability of measure is crucial to obstruction (Ashtekar, Fleischhack, Lewandowski, Okołów, Sahlmann 2002 (unpublished))

- Baez LMP 31 (1994) 213–223
  Implicitly treats amenable groups $\Leftrightarrow$ working with compactification of the group (Stone–Čech)

In each case:

- Absence of constant functions from integrable $\Rightarrow$ problems

Suggests we try compactifying group

The Case of $\mathbb{R}$
(also Okołów)

- Consider algebra of almost periodic functions on $\mathbb{R}$

$$f(x) = \sum_j f_j \, e^{i\lambda_j x}$$

$$\mu(f) := \sum_j f_j \, \delta_{\lambda_j, 0} \qquad \text{(invariant)}$$

$$\langle f | h \rangle = \mu(\bar{f} h) = \sum_j \bar{f}_j h_j$$

- Underlying compact space $b\mathbb{R}$ is in fact a compact topological group. $\mathbb{R}$ is densely embedded

- Derivation operators

$$\hat{E} f := \sum_j f_j \lambda_j \, e^{i\lambda_j x}$$

- Now constant functions are integrable!

- Patch together different graphs just as in compact case: use a copy of $b\mathbb{R}$ for each edge in graph.

- Flux operators built out of $\hat{E}$ operators

How GENERAL?

Almost periodic compactification works for

$$G = \text{compact} \times \mathbb{R}^n$$

but no others

Can try other 'standard' compactifications, but for $G = SL(2,\mathbb{R})$ or $G = SL(2,\mathbb{C})$ none found where algebra all hold:

- Separates points of G
- Possesses invariant measure μ
- Resulting Hilbert space is infinite-dimensional

Not encouraging, but also far from conclusive

Can we say anything more?

MAIN THEOREM

WHAT functions would we like to have in our algebra? Matrix elements

$D^{\lambda}_{pq}(g)$ — λ labels irrep of G; p, q label basis of irrep

Corresponds to Peter-Weyl decomposition in compact case.

But ...

THEOREM: If $G = SL(2,\mathbb{R})$ or $SL(2,\mathbb{C})$, in any C^* algebra containing the matrix elements and possessing an invariant measure, the matrix elements have zero norm in the Hilbert space

So, if there is a compactification that works, the matrix elements are not in the algebra

$\Rightarrow$ they are non-normalizable

Conclusions & Outlook

- Compactifying a non-compact structure group works for G= compact × Abelian but so far not otherwise

- Most natural candidates for 'spin networks' for $SL(2,\mathbb{R})$ or $SL(2,\mathbb{C})$ are not normalizable states (if they exist at all)

- Still lack complete 'no-go' theorem:

  - Is compactification absolutely necessary?

  - If so, is there some compactification for which non-normalizable spin network states exist, or is that also impossible?