

Title: Some Obstructions to Spin Networks for Non-Compact Gauge Groups

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Abstract:

Obstructions to Sprn-Networks
for Some Non-Compact
Groups

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Plan of Talk

- Brief review of compact case & previous work on non-compact case
- Successful strategy for non-compact Abelian: Compactify the Group
- Not so successful for $SL(2, \mathbb{R})$ or $SL(2, \mathbb{C})$; a partial 'no-go' theorem
- Conclusions & outlook

THE COMPACT CASE



Graph γ

Start from cylindrical functions:
depend on connection only
through holonomies along edges
of graph

For single graph, inner product defined
with Haar measure on G^n

$$\langle f_1 | f_2 \rangle = \int_{G^n} \bar{f}_1 f_2 d\mu_{\text{Haar}}$$

Flux operators come from left/right
invariant vector fields on each copy of G

Patch together different graphs

$$\langle f_{\gamma_1} | f_{\gamma_2} \rangle = \langle f_{\gamma_1} | f_{\gamma_1} \rangle \quad \gamma_1, \gamma_2 \subseteq \gamma$$

Possible because constant functions are
integrable on G

So can't extend measure in straightforward
way if G non-compact.

Previous Work (Non-compact Groups)

- Freidel & Livine JMP 44 (2003) 1322-1356

Constructed Hilbert space, but representation of algebra is highly reducible

- General arguments exclude several plausible strategies: non-normalizability of measure is crucial to obstruction (Ashtekar, Fleischhack, Lewandowski, Okolow, Sahlmann 2002 (unpublished))

- Baez LMP 31 (2004) 213-223

Implicitly treats amenable groups
↔ working with compactification of the group (Stone-Čech)

In each case:

- Absence of constant functions from integrable \Rightarrow problems

Suggests we try compactifying group

THE CASE OF \mathbb{R} (also \mathbb{Q} or \mathbb{Z})

- Consider algebra of almost periodic functions on \mathbb{R}

$$f(x) = \sum_j f_j e^{i\lambda_j x}$$

$$\mu(f) := \sum_j f_j \delta_{\lambda_j, 0} \quad (\text{invariant})$$

$$\langle f | h \rangle = \mu(\bar{f}h) = \sum_j \bar{f}_j h_j$$

- Underlying compact space $\widehat{\mathbb{R}}$ is in fact a compact topological group. \mathbb{R} is densely embedded

- Derivation operators

$$\hat{L}f := \sum_j f_j \lambda_j e^{i\lambda_j x}$$

- Now constant functions are integrable!
- Patch together different graphs just as in compact case: use a copy of $\widehat{\mathbb{R}}$ for each edge in graph.
- Flux operators built out of \hat{L} operators

How GENERAL?

Almost periodic compactification works for

$$G = \text{compact} \times \mathbb{R}^n$$

but no others

Can try other 'standard' compactifications, but for $G = \text{SL}(2, \mathbb{R})$ or $G = \text{SL}(2, \mathbb{C})$ none found where algebra all hold:

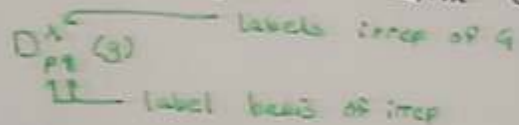
- Separates points of G
- Possesses invariant measure μ
- Resulting Hilbert space is infinite-dimensional

Not encouraging, but also far from conclusive

Can we say anything more?

MAIN THEOREM

WHAT functions would we like to have in our algebra? Matrix elements



Corresponds to Peter-Weyl decomposition in compact case.

BUT...

THEOREM: If $G = SL(2, \mathbb{R})$ or $SL(2, \mathbb{C})$, in any C^* algebra containing the matrix elements and possessing an invariant measure, the matrix elements have zero norm in the Hilbert space

So, if there is a compactification that works, the matrix elements are not in the algebra

\Rightarrow they are non-normalizable

Conclusions & Outlook

- Compactifying a non-compact structure group works for $G = \text{compact} \times \text{Abelian}$ but so far not otherwise
- Most natural candidates for 'spin networks' for $SL(2, \mathbb{R})$ or $SL(2, \mathbb{C})$ are not normalizable states (if they exist at all)
- Still lack complete 'no-go' theorem:
 - Is compactification absolutely necessary?
 - If so, is there some compactification for which non-normalizable spin network states exist, or is that also impossible?