Title: Vacuum State for LQG

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Abstract:

Free vacuum for loop quantum gravity

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Workshop on Quantum Gravity in the Americas
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[gr-qc/0409036]

Plan of talk

1. Motivation
2. Construction of free vacuum state
3. Semiclassical properties

[gr-qc/0409036]

Central problem of loop quantum gravity

What is the semiclassical and low-energy limit?

→ Does the theory contain semiclassical states that reproduce observed Einstein gravity?

→ Can we obtain an effective action for computing graviton scattering?

What we have so far:

- proposals for vacuum states
- tentative results on
  * perturbation theory [Smolin, Starodubtsev . . .]
  * modified dispersions relations [Gambini, Pullin, Thiemann . . .]
  * renormalization [Markopoulou,Oeckl]

Perturbation theory in LQG?

Motivating question for present work:

> Can one translate the perturbative approach to field theory to loop quantum gravity?

Problem: how to translate field-theoretic concepts to a theory based on abstract labelled graphs?

What is the analogue of a field fluctuation in a space of networks?

How to separate operator on spin networks into free and interaction part?

What are the free states?

Scalar field theory

ϕ^4-theory:

$$H = \int \mathrm{d}^3x \left[\frac{1}{2}\pi^2 + \frac{1}{2}(\nabla\phi)^2 + \frac{1}{2}m^2\phi^2 + \lambda\phi^4\right]$$

Perturbation theory around trivial solution $\phi \equiv 0$.

Free part of Hamiltonian:

$$\begin{aligned} H_0 &= \int \mathrm{d}^3x \left[\frac{1}{2}\pi^2 + \frac{1}{2}(\nabla\phi)^2 + \frac{1}{2}m^2\phi^2\right] \\ &= \int \mathrm{d}^3k \left[\frac{1}{2}|\pi(\underline{k})|^2 + \frac{1}{2}(k^2+m^2)|\phi(\underline{k})|^2\right] \end{aligned}$$

Associated free vacuum:

$$\Psi_0[\phi(\underline{k})] = \mathcal{N} \exp\left(-\int \mathrm{d}^3k \, \frac{\sqrt{k^2+m^2}}{2\hbar} \, |\phi(\underline{k})|^2\right)$$

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Traditional quantum gravity

Extended ADM variables (E^a_i, K^j_b).

Constraints:

$$
\begin{aligned}
G_{jk} &= K_{a[j}E^a_{k]} \\
V_a &= -D_b[K^j_a E^b_j - \delta^b_a K^j_c E^c_j] \\
C &= \frac{1}{\sqrt{|E|}}\left(K^l_a K^j_b - K^j_a K^l_b\right) E^a_j E^b_l - \sqrt{|E|}\, R(E) \\
&\quad + \ldots
\end{aligned}
$$

Perturbation theory around flat torus T^3 of macroscopic size L. Introduce fluctuation variables

$$e^a_k := E^a_k - \delta^a_k\,, \qquad K^k_a = K^k_a - 0\,,$$

and impose linearized constraints and gauge conditions.

→ reduced phase space variables

$$(e^{\mathrm{red}}_{ab}, K^{cd}_{\mathrm{red}})\,.$$

$$\hat{H}_0 = \frac{1}{\kappa} \int \mathrm{d}^3x \ \left[\hat{K}^{ab}_{\text{red}}(\underline{x}) \hat{K}^{ab}_{\text{red}}(\underline{x}) + \partial_c \hat{e}^{\text{red}}_{ab}(\underline{x}) \partial_c \hat{e}^{\text{red}}_{ab}(\underline{x}) \right]$$

Associated "free" vacuum:

$$\Psi_0[e^{\text{red}}_{ab}(\underline{k})] = \mathcal{N} \exp \left[-\frac{1}{\hbar\kappa} \sum_{k<\Lambda} k \, e^{\text{red}*}_{ab}(\underline{k}) e^{\text{red}}_{ab}(\underline{k}) \right]$$

(To make state well-defined, we introduced UV cutoff Λ!)

In position space:

$$\Psi_0[e^{\text{red}}_{ab}(\underline{k})] = \mathcal{N} \exp \left[-\frac{1}{\hbar\kappa} \left(\int \mathrm{d}^3x \int \mathrm{d}^3y \ W_\Lambda(\underline{x}, \underline{y}) \, e^{\text{red}}_{ab}(\underline{x}) e^{\text{red}}_{ab}(\underline{y}) \right) \right]$$

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Our approach

Since we do not know how to linearize LQG itself, we take an indirect strategy:

- know how to linearize ADM gravity.
- know that fields and loops are physically related.

→ employ this relation to translate vacuum and graviton states of linearized ADM gravity into states of the loop representation. [Ashtekar, Rovelli, Smolin, Varadarajan]

Purpose: obtain information on

- how semiclassical properties manifest themselves in the loop framework,
- and how this could be exploited to do perturbation theory.

Novel features:

- We arrive at states in the kinematic Hilbert space of *full non-linearized* LQG.
- Momentum cutoff of ADM states is translated into *cutoff graph* for LQG states.

$\downarrow$ canonical transformation $(E_i^a, K_b^j) \to (E_i^a, A_b^j)$

$$\Psi_0[e_{ab}^{\mathrm{red}}(\underline{k})] = \mathcal{N} \exp\left[-\frac{1}{\hbar\kappa}\left(\int \mathrm{d}^3x \int \mathrm{d}^3y \, W_\Lambda(\underline{x},\underline{y}) \, e_{ab}^{\mathrm{red}}(\underline{x}) e_{ab}^{\mathrm{red}}(\underline{y}) - \frac{\mathrm{i}}{\beta} \int \mathrm{d}^3x \, e_{ab}^{\mathrm{red}}(\underline{x}) \, \varepsilon_{acd} \partial_c e_{db}^{\mathrm{red}}(\underline{x}) \right)\right]$$

$\downarrow$ extension to full configuration space

$$\Psi_0[e_{ab}(\underline{k})] = \mathcal{N} \exp\left[-\frac{1}{\hbar\kappa}\left(\int \mathrm{d}^3x \int \mathrm{d}^3y \, W_\Lambda(\underline{x},\underline{y}) \, e_{ab}(\underline{x}) e_{ab}(\underline{y}) - \cdots \right)\right]$$

$\downarrow$ replace $e_{ia} = E^a_i - \delta^a_i$ by $\frac{1}{2}\left(E^a_i(\underline{x})E^b_i(\underline{x}) - \delta^{ab}\right) \equiv \frac{1}{2}\left(g^{ab}(\underline{x}) - \delta^{ab}\right)$

$$\Psi_0[E^a_i(\underline{k})] = \mathcal{N} \exp\left[-\frac{1}{4\hbar\kappa}\left(\int d^3x \int d^3y \, W_\Lambda(\underline{x},\underline{y}) \, \left(g^{ab}(\underline{x}) - \delta^{ab}\right)\left(g^{ab}(\underline{y}) - \delta^{ab}\right)\right)\right].$$

$\downarrow$ bring state into "complexifier" form

$$\Psi_0(A) = \mathcal{N} \exp\left(-\hat{C}\right) \delta(A)$$

where

$$\hat{C} := \frac{1}{4\hbar\kappa} \int d^3x \int d^3y \, W_\Lambda(\underline{x},\underline{y}) \left(\hat{g}^{ab}(\underline{x}) - \delta^{ab}\right)\left(\hat{g}^{ab}(\underline{y}) - \delta^{ab}\right).$$

From momentum cutoff to triangulation

Choose triangulation $\mathcal{T}_\Lambda$ and dual complex $\mathcal{T}^*_\Lambda$ that mimic UV cutoff:

edges of $\mathcal{T}^*_\Lambda$ straight (w.r.t. background) and edge length close to $l_\Lambda := \pi/\Lambda$.

Connection is replaced by generalized connection on $\mathcal{T}^*_\Lambda$.

Delta function:

$$\delta(A) \quad \rightarrow \quad \delta_{\mathcal{T}^*_\Lambda}(\overline{A}) = \sum_{\tilde{S}\subset\mathcal{T}^*_\Lambda} \tilde{S}^*(0)\,\tilde{S}(\overline{A})$$

Metric operator:

$$\hat{g}^{ab}(\underline{x})\,\tilde{S} = g^{ab}_S(\underline{x})\,\tilde{S}$$

Final formula

After gauge-averaging, final form of state is

$$\Psi_0 = \mathcal{N} \sum_{S \subset \mathcal{T}_\Lambda^*} \Psi_0(S)\, S\,,$$

where

$$\Psi_0(S) := S^*(0)\, \exp\left[-\frac{1}{4\hbar\kappa} \int d^3x \int d^3y\, W_\Lambda(\underline{x}-\underline{y})\, \left(g_S^{ab}(\underline{x}) - \delta^{ab}\right) \left(g_S^{ab}(\underline{y}) - \delta^{ab}\right) \right].$$

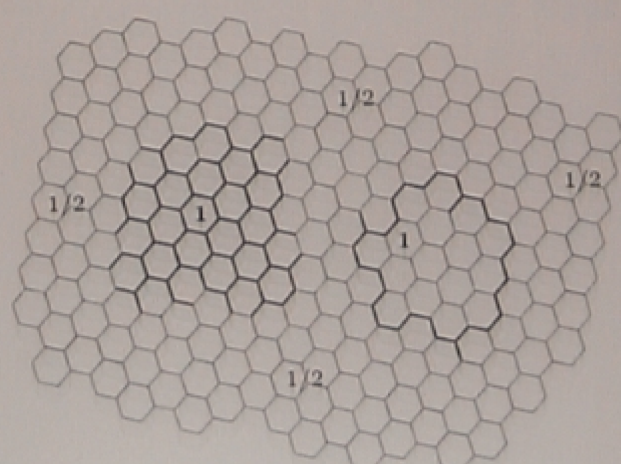
Semiclassical properties of vacuum state

S-dependence of "wavefunction" $\Psi_0(S)$ is of Gaussian type:

- peak given by spin networks S for which eigenvalue $g_S^{ab} = \delta^{ab}$.

→ peak spin networks similar to weaves! [Ashtekar, Rovelli, Smolin]

- exponential dampening in the deviation $g_S^{ab} - \delta^{ab}$.



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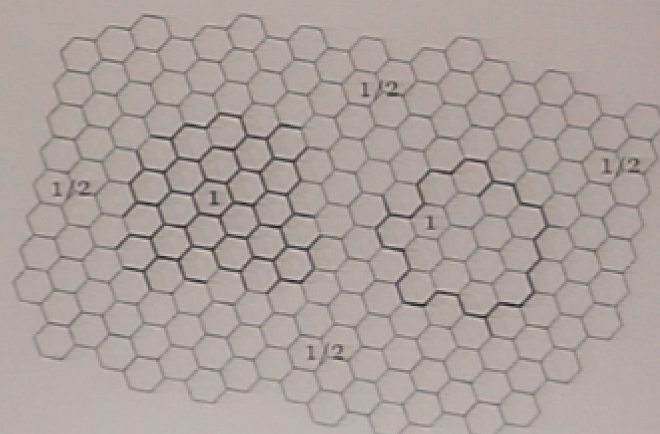
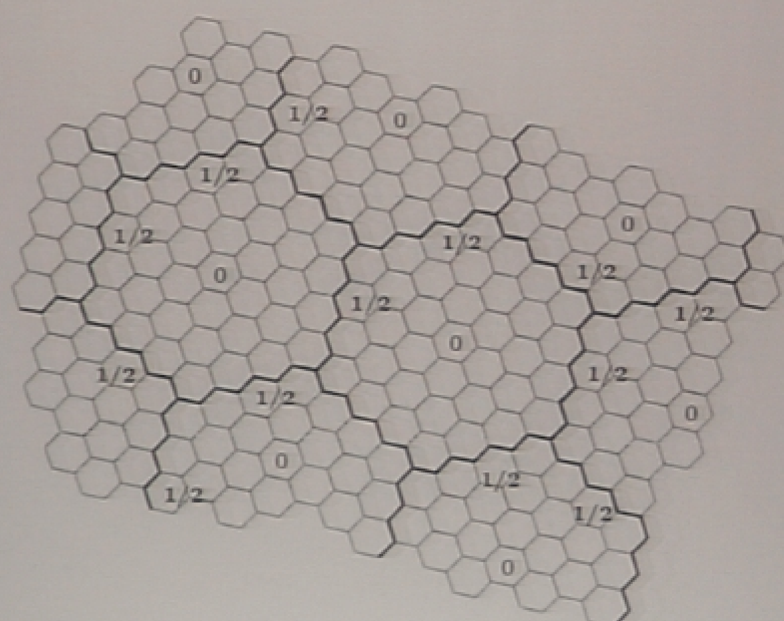


Limit $l_\Lambda \ll l_p$

Our analysis indicates that for $l_\Lambda \to 0$

- the peak weaves becomes independent of the cutoff,
- have spin label 1/2,
- and graphs of a length scale close to the Planck scale.



Summary of results

- We have constructed states that could play the role of free vacuum and graviton states in LQG.
- Properties of these states:
  * elements of the kinematic Hilbert space of the full theory.
  * built-in cutoff graph, as a natural consequence of momentum cutoff of original ADM states.
  * Gaussian superposition of spin networks.
- We have analyzed the peak of the Gaussian:
  * constituted by weave states.
  * indication that weaves maintain effective Planck scale discreteness when $l_\Lambda \to 0$.

$\leadsto$ unification of coherent state and weave approach.
$\leadsto$ inherent Planck scale cutoff in quantum states.

[gr-qc/0409036]

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