Title: Coarse Graining in Loop Quantization

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Abstract:

Background indep. Effective gauge theories and coarse graining

Oct. 04

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$\boxed{\text{I Motivation}}$

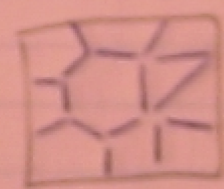
Gauge theory

Finite resolution: The algebra of accessible obs is not "the full one".

This notion should work in the absence of a b.ground metric

Rough example: Loop quantum electromagnetism

Lab with electric flux detectors, C.



$$\downarrow$$

$$\cancel{L^2(A/G)/\sim_C} \quad \text{or} \quad L^2\left(A_L/G_L\right)$$

$$C_1 \leq C_2 \quad \text{/\!\!\leftarrow bigger grant}$$

$$[n]_{C_1} \supset [n]_{C_2}$$

Algebraic description $Cyl(\Lambda_c) \longrightarrow Cyl(c) = Cyl(\Lambda_c)/\sim_c$

Alg. of effect. obs:

where $f_c \sim g_c$ iff $f = g \in Fun(\ldots)$

- The product descends to $Cyl(c)$ making it an algebra.
* For the Hol-Flux alg. one can construct $H\text{-}F(c)$ in the same way after restricting to $VF^{\ast}$ ~~…~~ that preserve $\sim_c$.

Changing the scale:
~~Coarse graining~~
~~Refining~~

$c_1 \le c_2$

$Cyl(c_1) \xleftarrow{Ref} Cyl(c_2)$

$[f_c]_{c_1} \longleftarrow [f]_{c_2}$

$Cyl(c_1) = Cyl(c_2)/\sim_{c_1}$

~~Coarse graining~~

$Cyl(c_1) \xrightarrow{Coarse} Cyl(c_2)$

$[f_c]_{c_1} \longmapsto [f_{c_1} \ldots]_{c_2}$

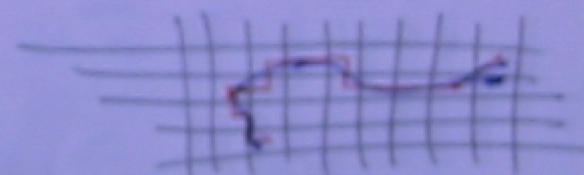
Depends on a choice of $f_c \in [f]_{c_1}$ for each class

$Coarse(Cyl(c_1)) \subseteq Cyl(c_2)$

III Geometric description

$$\overline{\mathcal{A}_\Sigma} \supset \mathfrak{a}_{c,A_0}$$

$\overset{\circ}{A} \Leftrightarrow \Gamma_1 \sim_c \Gamma_2 \Rightarrow h(\Gamma_1, A, A_0) = h(\Gamma_2, A, A_0) \in G$



Theorem:
$\tilde{\imath}(\mathfrak{a}_{c,A_0}) = \mathfrak{a}_{\tilde{c},\tilde{\imath}A_0}$

- parallel transp. using red representative
- the initial and final details can be done with auxiliary A_0

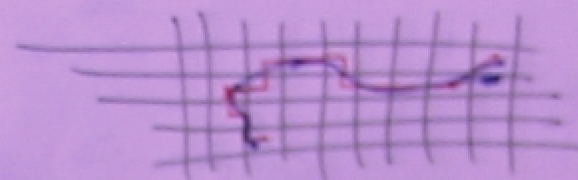
Changing scale:

$$c_1 \leq c_2$$

$$\mathfrak{a}_{c_1,A_0} \xrightarrow{i} \mathfrak{a}_{c_2,A_0}$$

Inclusion $\mathfrak{a}_{c_1,A_0} \subset \mathfrak{a}_{c_2,A_0}$ $\quad i = \mathrm{Res}^*$

$\mathring{A} \Leftrightarrow \Gamma_1 \sim_g \Gamma_2 \Rightarrow h(\Gamma_1, A; A_\infty) = h(\Gamma_2, A, A_\infty) \in G$



Theorem:

$\tilde{\imath}(\mathfrak{a}_{c,A_0}) = \mathfrak{a}_{\Gamma c, \Gamma^* A_0}$

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Changing scale:

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Inclusion $\mathfrak{a}_{C_2,A_0} \subset \mathfrak{a}_{C_1,A_0}$ $\qquad i = \mathrm{Res}^*$

$$\mathfrak{a}_{C_1,A_0} \xleftarrow{\pi} \mathfrak{a}_{C_2,A_0}$$

$$\pi(A) \longleftarrow A$$

$$h(\Gamma, \pi(A)) = h([\Gamma]_{C_1} \mathrm{Coarse}^*(A)) = h(\Gamma_0 \cdot \mathrm{Coarse}[\Gamma]_{C_1}, A)$$

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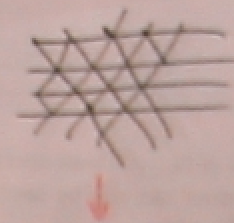
Example:

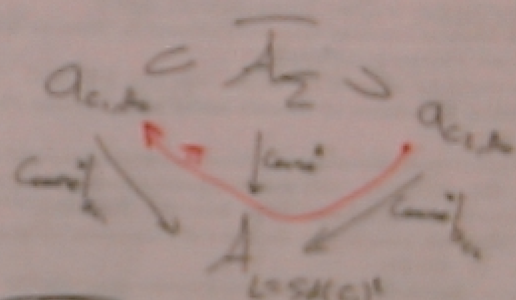
C_1 tiling of Σ.



$Sd(C_1)^1$

Coarse: $Sd(C_1)^1 \longrightarrow \Sigma$

choice of embedding



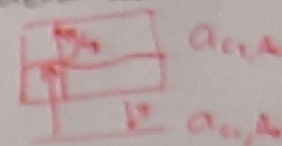
Clarification

Actually π depends only on $[\text{Coarse}]_{\dots}$, which is equiv to

$\widetilde{\text{Coarse}}: \text{Paths}(Sd(C_1)^1) \longrightarrow \text{Paths}(Sd(C_2)^1)$ injective.

There are only "a few" choices for π.

Theorem $\mathcal{A}_{c_1,\Delta} \underset{i}{\overset{\pi}{\rightleftarrows}} \mathcal{A}_{c_2,\Delta}$ is a principal fiber bundle with a preferred section i.



Theorem

$$[\pi^*, (k_g \circ i)_*] : \text{Hol-Flow}(c_1) \longrightarrow \text{Hol-Flow}(c_2)$$

is a ~~[illegible]~~ algebra ~~[illegible]~~ embedding.

Theorem The π-push forward of measures is indep of choice of π and

$$\pi_*(W \, d\mu_{Haar,c_2}) = (i^* W)\, d\mu_{Haar,c_1}$$

"Integrating out extra dof"

We also know that "$i_* d\mu_{Haar,c_1} = d\mu_{Haar,c_2} \cdot \delta(g=1)$"

other connection dof restricted to be flat.

~~In progress: – Renorm [illegible] & study of RG flow~~

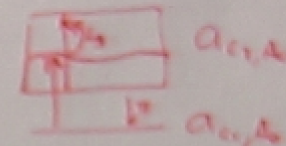
~~[illegible] (comparing with usual LGT)~~

~~– Hamiltonian~~

~~– Gen. bdary~~

Theorem

$$\mathcal{A}_{C_1,A_0} \underset{i}{\overset{\pi}{\leftrightarrows}} \mathcal{A}_{C_2,A_0}$$ is a principal fiber bundle with a preferred section i.



Theorem

$$[\pi^*, (h_g \circ i)_*] : \text{Hol-Flow}(C_1) \longrightarrow \text{Hol-Flow}(C_2)$$

is a ~~…~~ algebra ~~morphism~~ embedding.

Theorem The π-push forward of measures is indep. of choice of π and
$$\pi_*\left(W d\mu_{\mathrm{Haar},C_2}\right) = (i^* W)\, d\mu_{\mathrm{Haar},C_1}$$

"Integrating out extra dof"

We do know that $i_* d\mu_{\mathrm{Haar},C_1} = d\mu_{\mathrm{Haar},C_2}\, \delta_{\tilde{g}}(g=1)$

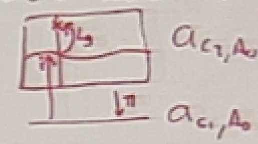
extra connection dof restricted to be flat

~~In progress: – Renorm. procedure – Ren. group & study of RG flow~~
~~– … (comparing with usual LQT)~~
~~· Hamiltonian~~
~~· Gen. bdary~~

~~Examples~~
~~– Examples 2d YM 4D …~~

orem

$\mathcal{A}_{c_1,A_0} \underset{i}{\overset{\pi}{\rightleftarrows}} \mathcal{A}_{c_2,A_0}$ is a principal fiber bundle with a preferred section i.



Theorem

$$[\pi^*, (L_g \circ i)_*] : \mathrm{Hol\text{-}Flux}(c_1) \longrightarrow \mathrm{Hol\text{-}Flux}(c_2)$$

is a ~~[illegible]~~ $*$-algebra ~~morphism~~ embedding.

Theorem The π-push forward of measures is indep of choice of π and.

$$\pi_*(W \, d\mu_{Haar,c_2}) = (i^* W) \, d\mu_{Haar,c_1}$$

"Integrating out extra dof"

We also know that $i_* d\mu_{Haar,c_1} = d\mu_{Haar,c_2} \cdot \delta_F(g=1d)$

extra connection dof restricted to be flat.

~~In progress: - Renorm. procedure { Renorm. prescrip. / - Study of RG flow~~

~~Lagrangian (comparing with usual LGT)~~
~~· Hamiltonian~~
~~· Gen. bdary~~

~~Examples.~~ ~~- Examples { 2d YM 90% / 3+d U(1) Y-M in weak coupl. lim 70%~~

Theorem There are choices of projections such that the π diagr. commutes

$$\begin{tikzcd} & \mathcal{A}_{c_2,A_0} \arrow[dl, shift right, "\pi"'] \arrow[dr, shift left, "i"] & \\ \mathcal{A}_{c_1,A_0} \arrow[ur, shift right, "i"'] \arrow[rr, shift right, "i"'] & & \mathcal{A}_{c_3,A_0} \arrow[ul, shift left, "\pi"] \arrow[ll, shift right, "\pi"'] \end{tikzcd}$$

$\overline{\mathcal{A}}_\Sigma$

Then $\mathcal{A}_{c_1,A_0} \underset{i}{\overset{\pi}{\rightleftarrows}} \mathcal{A}_{c_2,A_0} \underset{i}{\overset{\pi}{\rightleftarrows}} \dots$

are nested principal fiber bundles with a pref. section.

Theorem Any measure $d\mu$ on $\overline{\mathcal{A}}_\Sigma$ is characterized by a a nested sequence of eff. measures

$\{\mathcal{A}_{c_i,A_0};\, d\mu_{c_i}\}$

with $\{c_i\}_i \to \Sigma$ and $d\mu_{c_i} = \pi_* \, d\mu_{c_{i+1}}$

$d\mu_{AL}$ is induced by $\{\mathcal{A}_{c_i,A_0};\, d\mu_{Haar}\}$

$\{d\mu_{c_i}\} \xrightarrow{is}$ sequence of compatible eff. degenerate measures on $\overline{\mathcal{A}}_\Sigma$

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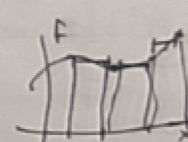
V In progress:

- Renorm. procedure { - renorm. prescription / - study of RG flow
  - Lagrangian (compare with LGT)
  - Hamiltonian
  - General boundary (Local renorm. prescr.)

- Examples
  - 2d Y-M, Topolog~~[illegible]~~
  - 3,4d U(1) Y-M week coupling 70%
    (compare with Madhavan's photons)

- Obs: $Cyl(A_2) \xrightarrow{c} Cyl(c) \to \underset{\text{not continuous!}}{a_{c,A_0}}$

Behind our notion of eff. theory there is an <u>approximation theorem for gauge fields</u> in terms of discontinuous c-constant connections.



We are working on an approx. theo by "polygonals".

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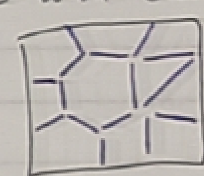

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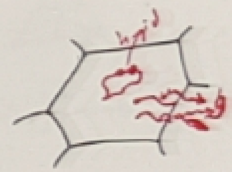

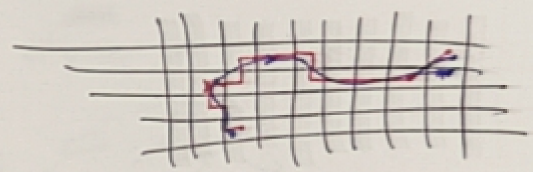

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